2) find the Eigen Value & vector of Rotation Matrix. with $\theta = 0^\circ, 90^\circ, 180^\circ$ & $270^\circ$

A) Rotation Matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(1) $\boxed{\text{when } \theta = 0^\circ}$

then Matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Linear Equation $= (A - \lambda I)X = 0$

calculating $\det(A - \lambda I)$ for finding $\lambda$.

$$\det(A-\lambda I) = \begin{bmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = (1-\lambda)(1-\lambda) = 0$$
$$\Rightarrow \lambda = (1,1)$$

Since the Eigen Values are not distinct we may/may not have all vectors independent of each other.

→ when $\lambda = 1$ then $A - \lambda I = 0$

∴ X can be any values in the plane.

$x_1, x_2$ of Eigen vector can take any combinations in a given plane.

∴ the Eigen space of Identity matrix is entire x-y plane.

(2) $\boxed{\text{when } \theta = 90^\circ}$

then Matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Linear equation $= (A - \lambda I)x = 0$

calculating $\det(A-\lambda I)$ for finding $\lambda$.

$$\det(A-\lambda I) = \left| \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = \left| \begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix} \right|$$
$$= \lambda^2 + 1 = 0 \Rightarrow \lambda = \sqrt{-1} = \pm i \rightarrow \text{complex values}$$

∴ the Values in Eigen Vector is also complex.

(3) when $\theta = 180°$

then Matrix $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Linear equation $= (A - \lambda I)x = 0$

$$\det(A - \lambda I) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix}$$

$\Rightarrow (-1-\lambda)(-1-\lambda) = 0$

$\therefore \lambda = (-1, -1)$

when $\lambda = -1$

then $\left[\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} x$

$\therefore$ the values of vector $x$ can be any combination

$\therefore$ the eigen space for matrix $A$ is entire $x$-$y$ plane.

(4) when $\theta = 270°$

then Matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Linear equation $= (A - \lambda I)x = 0$

$$\det(A - \lambda I) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix} = 0$$

$= \lambda^2 + 1 = 0$

$\therefore \lambda = \sqrt{-1} = \pm i$

$\therefore$ the values of eigen vector is complex value. complex value.

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