THE POPULATION PROJECTION AND COMPARISON FOR SWITZERLAND AND DENMARK

An outlook of population projection and comparison for upcoming days

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Executive Summary:

This project paper tries to clarify the basic concepts and differences between population estimation and projection of Switzerland and Denmark. It also explains the tools and techniques for estimation and projection of population. The concepts generate numbers that is intended to indicate the size of the population of a given geographical area at a specific point of time. Most of the developing countries, censuses are taken at every ten years' interval. There are various tools and techniques for estimation and projection of population i.e., mathematical, economic and cohort component. In this project the population is estimated by mathematical component. Before determining the need for a census, it is important that there is an understanding of the role of the census both in terms of what is possible through the selected census enumeration process and how the census itself fits into the overall statistical framework of the country. This study is aimed at analyzing and monitoring the population projection and comparison between Switzerland and Denmark using some selected statistical tools, the data used for the study comprised of from 1861 to 2011 and from 1860 to 2020 respectively. The data was sourced from the Wikipedia and Switzerland and Denmark Statistical office. This study is based on secondary data collection which is obtained from previous studies, relevant journals and census reports etc. In this study, existing literatures related to population estimation and projection is extensively reviewed. The collective data and descriptive information are presented in different ways. Some time-series models were used to estimate the trend and make forecasts of population for 2021 and 2030. At first to analysis the population, the best fitted trend model was found with least percentage error. After obtaining the trend model, the entire data set was fitted to the best model to find the next census population. Then to compare between two countries, the analysis of the percentage growth, population density, birth rate and death rate have been established. From percentage rate, the population of Switzerland and Denmark can be easily compared. Before determining the need for a census, it is important that there is an understanding of the role of the census both in terms of what is possible through the selected census enumeration process and how the census itself fits into the overall statistical framework of the country. In conclusion, the reliable projected data are vital for business and program planning, transparency, prosperity, policy discourse and good governance in different sectors of the nation for the upcoming days.

Introduction:

A census is the procedure of systematically acquiring and recording information about the members of a given population. It is a regularly occurring and official count of a particular population. The term is used mostly in connection with national population and housing census; other common censuses include agriculture, business, traffic. The aim of the census is to provide relevant population data to users in context, but it follows that this data meet users' expectations in terms of quality, appropriately defined. Censuses are blueprints to the building contractor; when they are lacking or containing serious gaps or deficiencies, new strategies must be developed.

The population census can be defined as a statistical operation designed to count the entire population of a country and to collect information on its main demographic, social and economic characteristics. It is a key instrument for assessing the needs of local communities (education, health, transport, etc.) and is even more useful when associated with other data source, as housing or agriculture censuses, or sample surveys. The U.N recommend that countries take a census at least once every ten years.

Many countries have departed from the traditional data collection methods and introduced a new census approach. Individual information is increasingly collected from administrative registers, and from ad hoc or existing sample surveys. For the 2010-2011 census round, almost half of all European countries have opted for an alternative census approach, in most cases based on population registers, either exclusively or in combination with other data sources.

Some Northern European countries, such as Denmark and Finland, use administrative registers for statistical purposes and conduct their census on the basis of information in the registers, rather than through field enumeration. This approach places no burden on individuals and, once registers are established, is less costly to implement. Data are available from registers on a continuous basis, so the census could potentially be taken every year. The population characteristics considered are limited to those available in the registers, so statistical agencies may combine data from different registers by linking data at the individual level in order to expand the range of data. This demands close cooperation between the statistical agency, register authorities and the public administration, and strict legislative oversight. More generally, establishing and maintaining a high-quality register-based statistical system requires significant resources and societal will. In this project we will discuss about the register-based census (Denmark) and Survey based census country (Switzerland) and will compare the difference of their population.

Switzerland, officially the Swiss Confederation, is a landlocked country situated at the confluence of Western, central and Southern Europe. It is a federal republic composed with 26 cantons, with federal authorities based in Bern. Switzerland is bordered by Italy to the south, France to the west, Germany to the north, and Austria and Liechtenstein to the east. It is geographically divided among Swiss Plateau, the Alps, and the Jura, spanning a total area of 41,28 km^2 (15,940 sq mi), and land area of 39,997 km^2 (15,443 sq mi). A0hlthough the Alps occupy the greater part of the territory, the Swiss population of approximately 8.5 million is concentrated mostly on Plateau, where the largest city and economic cents are located, among the Zurich, Geneva, and Basel. These cities are home to several offices of international organizations such as FIFA, the UN's second-largest office, and the main building of Bank for International Settlements. The main international airports of Switzerland are also located in these cities.

The Switzerland confederation was founded in 1921 as a defensive alliance among three cantons. In succeeding years, other localities joined the original three. The Swiss confederation secured its independence from the Holy Roman Empire in 1499. A constitution of 848, subsequently modified in 1874 to allow voters to introduce referenda on proposed laws, replace the confederation with a centralized federal government. Switzerland's sovereignty and neutrality have long been honored by the major European powers.

Extending across the north and south side of the Alps in west-central Europe, Switzerland encompasses a great diversity of landscapes and climates on a limited area of 41,285 square kilometers (15,940 sq mi). The population is about 8 million, resulting in an average population density of around 195 people per square kilometer (500/sq mi). The more mountainous southern half of the country is far more sparsely populated than the northern half. In the largest Canton of Graubünden, lying entirely in the Alps, population density falls to 27 /km2 (70 /sq mi).

The more populous northern part of the country, constituting about 30% of the country's total area, is called the Swiss Plateau. It has greater open and hilly landscapes, partly forested, partly open pastures, usually with grazing herds, or vegetables and fruit fields, but it is still hilly. There are large lakes found here and the biggest Swiss cities are in this area of the country.

Within Switzerland there are two small enclaves: Basinger belongs to Germany, Campione distalia belongs to Italy. Switzerland has no exclaves in other countries.

In other hands, Denmark is a Nordic country in Northern Europe. Denmark proper, which is the southernmost of the Scandinavian countries, consists of a peninsula, Jutland and an archipelago of 443 named islands, with the largest being Zealand, Funen and the North Jutland island. The islands are characterized by flat arable land and sandy coasts, low elevation and a temperate climate. Denmark lies southwest of Sweden and south and south of Norway and is bordered to the south by Germany. The Kingdom of Denmark is constitutionally a unitary state comprising Denmark proper and the two autonomous territories in the North Atlantic Ocean: the Faroe Island and Greenland. Denmark has a total area of 42,943 km^2 (16,580 sq mi) as of 2020, and total area including Greenland and the Faroe Islands is 2,210,579 km^2 (853,509 sq mi). Denmark proper has a population of 5.84 million (as of 2021).

The country occupies a total area of 42,943.9 square kilometres (16,581 sq. mi). The area of inland water is 700 km^2 (270 sq. mi), variously stated as from 500 to 700 km^2 (193–270 sq. mi). Lake Arresø northwest of Copenhagen is the largest lake. The size of the land area cannot be stated exactly since the ocean constantly erodes and adds material to the coastline, and because of human land reclamation projects (to counter erosion). Post-glacial rebound raises the land by a bit less than 1 cm (0.4 in) per year in the north and east, extending the coast. A circle enclosing the same area as Denmark would be 234 kilometres (145 miles) in diameter with a circumference of 736 km (457 mi) (land area only:232.33 km (144.36 mi) and 730 km (454 mi) respectively). It shares a border of 68 kilometres (42 mi) with Germany to the south and is otherwise surrounded by 8,750 km (5,437 mi) of tidal shoreline (including small bays and inlets). No location in Denmark is farther from the coast than 52 km (32 mi). On the south-west coast of Jutland, the tide is between 1 and 2 m (3.28 and 6.56 ft), and the tideline moves outward and inward on a 10 km (6.2 mi) stretch. Denmark's territorial waters total 105,000 square kilometres (40,541 square miles).

The country is flat with little elevation, having an average height above sea level of 31 metres (102 ft). The highest natural point is Møllehøj, at 170.86 metres (560.56 ft). Although this is by far the lowest high point in the Nordic countries and also less than half of the highest point in Southern Sweden, Denmark's general elevation in its interior is generally at a safe level from rising sea levels. A sizeable portion of Denmark's terrain consists of rolling plains whilst the coastline is sandy, with large dunes in northern Jutland. Although once extensively forested, today Denmark largely consists of arable land. It is drained by a dozen or so rivers, and the most significant include the Gudenå, Odense, Skjern, Susa and Vida—a river that flows along its southern border with Germany.

The Kingdom of Denmark includes two overseas territories, both well to the west of Denmark: Greenland, the world's largest island, and the Faroe Islands in the North Atlantic Ocean. These territories are self-governing and form part of the Danish Realm.

Denmark and Switzerland are small and successful countries with exceptionally content populations. However, they have very different political institutions and economic models. They have followed the general tendency in the West toward economic convergence, but both countries have managed to stay on top. They both have a strong liberal tradition, but otherwise their economic strategies are a welfare state model for Denmark and a safe haven model for Switzerland. The Danish welfare state is tax-based, while the expenditures for social welfare are insurance-based in Switzerland. The political institutions are a multiparty unicameral system in Denmark, and a permanent coalition system with many referenda and strong local government in Switzerland. Both approaches have managed to ensure smoothly working political power-sharing and economic systems that allocate resources in a fairly efficient way. To date, they have also managed to adapt the economies to changes in the external environment with a combination of stability and flexibility.

Objective:

The main objective of this project is to explain the tools and techniques for estimation and projection of population and comparison between two approximately same sized countries in regional and national levels. The specific objectives of this study are to introduce basic concepts related to population estimation, projection for the upcoming days. Population projection is empirically based on calculations of past or future, population numbers under specified assumptions about changes in population growth or its components. Basically, there are three techniques for population projection: Mathematical Method, Economic Method and Cohort Component Method. Here the mathematical least square method is used for projection. The fitted trend is termed as the best in the sense that the sum of squares of deviations of observations, from it, is minimized. So, the best fitted model is obtained by this method and finally the next census year is established from the best fitted model.

Data:

Population data is essential for planning purposes for any country,

So, the population census data for Switzerland is shown in the below:

Table, A: Census Population Data of Switzerland:

YEAR	POPULATION	YEAR	POPULATION
1861	2515396	1941	4268964
1871	2673468	1951	4717200
1881	2840501	1961	5360153
1891	2972041	1971	6193064
1901	3318985	1981	6335243
1911	3756842	1991	6757188
1921	3883360	2001	7197638
1931	4070042	2011	7870134

Source: *Federal Statistical Office of Switzerland.*

The population census data for Denmark is shown in the below:

Table, B: Census Population Data of Denmark:

YEAR	POPULATION	YEAR	POPULATION
1860	1608362	1950	4281275
1870	1784741	1960	4585256
1880	1969039	1970	4937579
1890	2172380	1980	5122065
1900	2432000	1990	5135409
1910	2737000	2000	5330020
1920	3079000	2010	5554800
1930	3550656	2020	5822863
1940	3844312		

Source: *Statistics Denmark.*

Methodology:

Population projections are calculations of future birth rate, death rate and migration of population based on their past and present conditions. They are neither predictions, nor forecasts, nor estimates. Rather they are in between predictions and forecasts.

According to a UN Study, "Population projections are calculations which show the future course of fertility, mortality and migration. They are in general purely formal calculations, developing the implications of the assumptions that are made."

Methods of Population Projections:

There are three methods of population projection – Mathematical Method, Growth Component Method, and Economic Method.

Here the mathematical method is used for population projection.

Mathematical Method:

The mathematical method is the earliest one to be used for population projection.

The resistance or the sum of the obstacles opposed to the unlimited growth of population increases in proportion to the square of the velocity with which the population tends to increase. It means that the growth of population declines in proportion to the increase in density of population. Here, in order to obtain the best fitted model, the four mathematical method have used, and these are: Linear Model, Exponential model, Quadratic model, and S-Curve model.

Fitting of a straight line by the method of least squares (Linear Model):

Let t Y be the value of the time series at time t. Thus, Y_t is the independent variable depending on t.

Assume a straight-line trend to be of the form $Y_t = a+bt$………… (1)

Where Y_{tc} is used to designate the trend values to distinguish from the actual t Y values, a is the Y-intercept and b is the slope of the trend line.

Now the values of a and b to be estimated from the given time series data by the method of least squares.

In this method we have to find out a and b values such that the sum of the squares of the deviations of the actual values Y_t and the computed values Y_{tc} is least.

i.e., $S = \sum (Y_t - Y_{tc})^2$ should be least

i.e., $S = \sum (Y_t - a - b)^2$ ………… (2) Should be least

Now differentiating partially (2) w.r.to a and equating to 0 we get,

$$\frac{\partial S}{\partial a} = 2\sum(Y_t - a - bt)(-1) = 0$$

$$\Rightarrow \sum(Y_t - a - bt) = 0$$

$$\Rightarrow \sum Y_t = \sum a + b\sum t$$

$$\Rightarrow Y_t = na + b\sum t \quad \text{…………………} (3)$$

Now differentiating partially (2) w.r.to b and equating to zero we get

$$\frac{\partial S}{\partial b} = 2\sum(Y_t - a - bt)\,(-t) = 0$$

$$\Rightarrow \sum t(Y_t - a - bt) = 0$$

$$\Rightarrow \sum Y_t = a\sum t + b\sum t^2 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (4)$$

The equations (3) and (4) are called „normal equations"

Solving these two equations we get the values of a and b say a ˆ and b ˆ

Now putting these two values in the equation (1) we get

$$Y_{tc} = \hat{a} + \hat{b}t$$

which is the required straight line trend equation.

Note: The method for assessing the appropriateness of the straight-line modal is the method of first differences. If the differences between successive observations of a series are constant (nearly constant) the straight line should be taken to be an appropriate representation of the trend component

Fitting of a Quadratic trend by the method of least squares:

The mathematical form of a parabolic trend is given by:

$$Y_t = a + bt + ct^2$$

Here a, b and c are constants to be determined from the given data.

Using the method of least squares, the normal equations for the simultaneous solution of a, b and c are:

$$\sum Y = na + b\sum t + c\sum t^2$$

$$\sum tY = a\sum t + b\sum t^2 + c\sum t^3$$

$$\sum t^2Y = a\sum t^2 + b\sum t^3 + c\sum t^4$$

By selecting a suitable year of origin, i.e., define X = t – origin such that $\sum X = 0$, the computation work can be considerably simplified. Also note that if $\sum X = 0$, then $\sum X^3$ will also be equal to zero. Thus, the above equations can be rewritten as:

$$\sum Y = na + c\sum X^2 \quad \ldots\ldots\ldots\; (1)$$

$$\sum XY = b\sum X^2 \quad \ldots\ldots\ldots\; (2)$$

$$\sum X^2Y = a\sum X^2 + c\sum X^4 \ldots\ldots\ldots\; (3)$$

From equation (2), we get,

$$b = \sum XY / \sum X^2$$

From equation (1), we get,

$$a = \sum Y - c\sum X^2 /n$$

From equation (3), we get,

$$c = \sum X^2Y - a\sum X^2 /\sum X^4$$

This are the three equations to find the value of constants a, b and c.

Note: The method for assessing the appropriateness of the second-degree equation is the method of second differences. If the differences are taken of the first differences and the results are constant (nearly constant) the second-degree equation be taken to be an appropriate representation of the trend component.

Fitting of an Exponential trend by the method of least squares:

The general form of an exponential trend is:

$$Y_t = a.b^t$$

Taking logarithms of both sides, we gave,

$$\log Y = \log a + \log b$$

This is linear equation in log Y and t can be fitted in a similar way as done in case of linear trend. Let A=log a and B =log b, then the above equation can be written as:

$$\log Y = A + Bt$$

The normal equations based on the principle of least squares are:

$$\sum \log Y = n A + B \sum t$$

$$\text{And } \sum t \log Y = n \sum t + B \sum t2$$

By selecting a suitable origin, i.e., defining $X = t$ – origin such that $\sum X = 0$, the computation work can be simplified. The values of A and B are given by:

$$A = \sum \log Y / n$$

$$B = \sum X \log Y / \sum X^2$$

Thus, the fitted trend equation can be written as:

$\log Y = A + BX$

or $Y = \text{Antilog } [A + BX]$

$= \text{Antilog } [\log a + X \log b]$

$= \text{Antilog } [\log a.b^x]$

$= a.\ bx$

Note: The first property implies that the position of fitted trend equation is such that the sum of deviations of observations above and below this equal to zero. The second property implies that the sums of squares of deviations of observations, about the trend equations, are minimum.

Fitting of a S-Curve trend by the method of least squares:

A logistic curve describing the population growth was proposed and named by P. F. Verhulst (Verhulst 1838, 1845) to describe the growth in the size of a population or organ. In figure, the curve shows an early growth JK at an increasing rate i.e., geometric growth or log growth, $\frac{dP}{dt} \propto P$ the transitional middle curve KM follows arithmetic increase i.e., $\frac{dP}{dt}$ = constant and later growth MN the rate of change of population is proportional to difference between saturation population and existing population, i.e., $\frac{dP}{dt} = (P_s-P)$. Verhaulst has put forward a mathematical solution for this logistic curve JN which can be represented by an autocatalytic first order equation, given by

$$Log_e\left(\frac{Ps-P}{P}\right) - Log_e\left(\frac{Ps-P0}{P0}\right) = -K.P_s.t$$

where, P = Population at any time t from the origin J

P_s = Saturation population

P_0 = Population of the city at the start point J

K = Constant

t = Years

From the above equation we get,

$$Loge\left(\frac{Ps-P}{P}\right)\left(\frac{P0}{Ps-P0}\right) = -K.P_s.t$$

After solving we get,

$$P = \frac{Ps}{1+\left(\frac{Ps-P0}{P0}\right)\left(\frac{1}{Loge}\right)(-K.Ps.t)}$$

Substituting $\frac{Ps-P0}{P0}$ = m (a constant)

And -K. P_s = n (another constant)

We get $P = \frac{Ps}{1+m\left(\frac{1}{Loge}\right)(n.t)}$

This is the required equation of the logistic curve, which will be used for predicting population. McLean further suggested that if only three pairs of characteristic values P_0, P_1, P_2 at times $t = t_0 = 0$, t_1 and $t_2 = 2t_1$ extending over the past record are chosen, the saturation population Ps and constant m and n can be estimated by the following equation, as follows:

$$Ps = \frac{2P0P1P2-P1.P1(P0+P2)}{P0P2-P1.P1}$$

$$m = \frac{P_S-P0}{P0}$$

$$n = \frac{2.3}{t_1} log_{10}\left(\frac{P0(Ps-P1)}{P1(Ps-P0)}\right)$$

Result and Analysis:

The analysis of this comparison between Switzerland and Denmark are divided in few steps. First Collection of census data of these country. Second, Find a best fitted model for the data and Third, calculation and lastly, comparison. Now let's discuss about the steps. At first the Switzerland (1861-2011) and Denmark (1860-2010) population data from is collected from different online sources. In order to find the best model for the data set, only 14 data set will be considered and put last two population census years. Then the data set were fitted in different models e.g., Linear, Exponential (Growth curve), Quadratic and Logistic of both countries differently and project the last two population census year (previously mentioned).

A. ***Switzerland population projection for 2001 and 2011:***

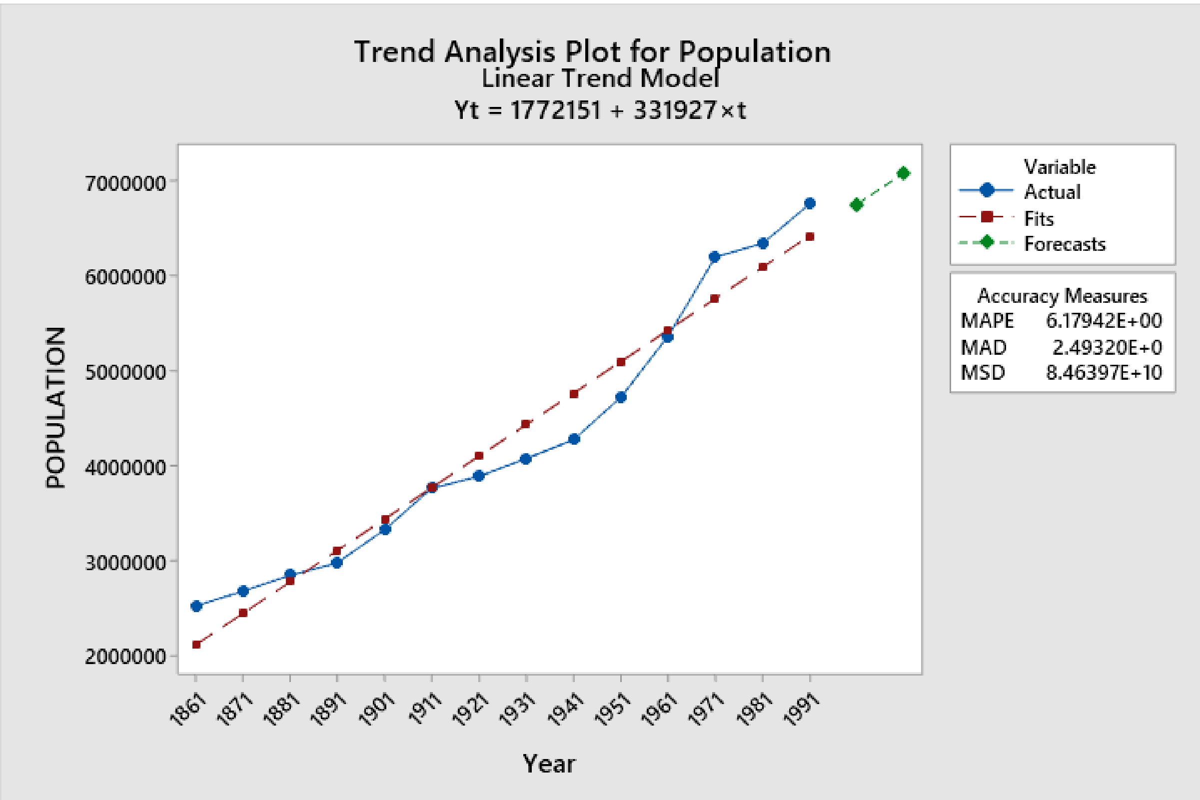
The graphs for the models (linear, quadratic, exponential and s-curve) for 2001 and 2011 are shown below:

1.1 Linear trend model:

$$Y_t = 1772151 + 331927*t$$

(Here Y_t is the Switzerland census population at time t)

Fig: A.1.1, Trend Analysis Plot for Switzerland Population (Linear Trend Model)



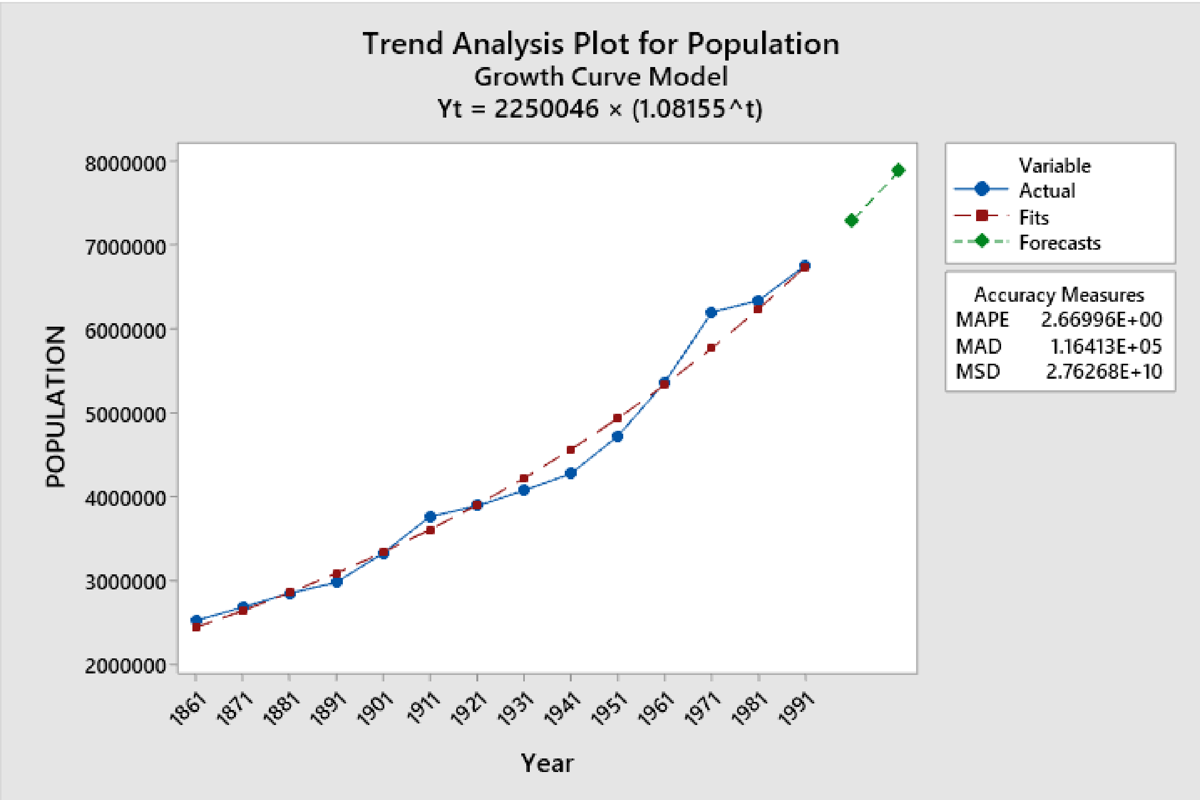
In the above model fig A.1.1, The actual trend line (blue line), is showing a constant rate in the growth rate of population which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2001 and 2011 which are 6751056 and 7082983 respectively.

1.2 Growth Curve Trend Model:

$$Y_t = 2250046 + (1.08155)^{t}$$

(Here Y_t is the Switzerland census population at time t)

Fig: A.1.2, Trend Analysis Plot for Switzerland Population (Growth Trend Model)



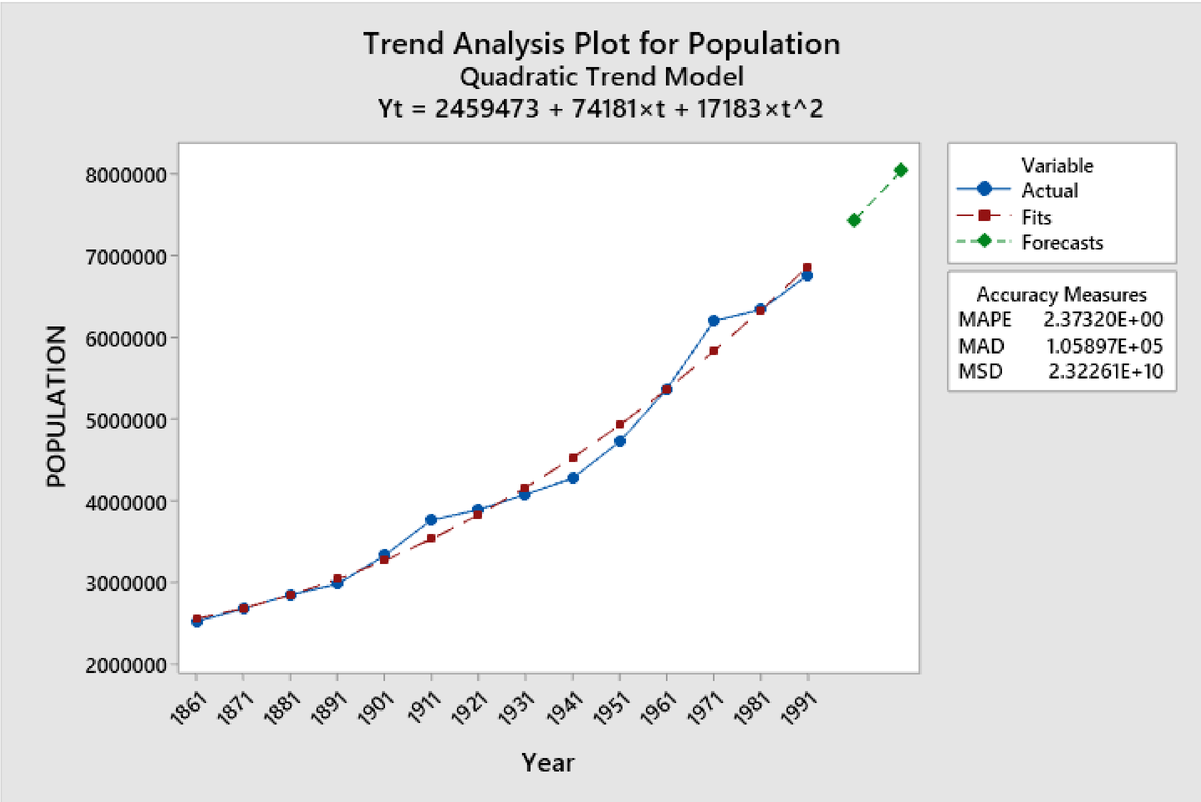
In the above model fig A.1.2, The actual trend line (blue line), is showing a constant rate in the growth rate of population from 1861 to 1991 which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2001 and 2011 which are 7292282 and 7886935 respectively.

1.3 Quadratic Trend Model:

$$Y_t = 2459473 + 74181*t + (17183*t^2)$$

(Here Y_t is the Switzerland census population at time t)

Fig: A.1.3, Trend Analysis Plot for Switzerland Population (Quadratic Trend Model)



In the above model fig A.1.3, The actual trend line (blue line), is showing a constant rate in the growth rate of population from 1861 to 1991 which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2001 and 2011 which are 7438378 and 8045234 respectively.

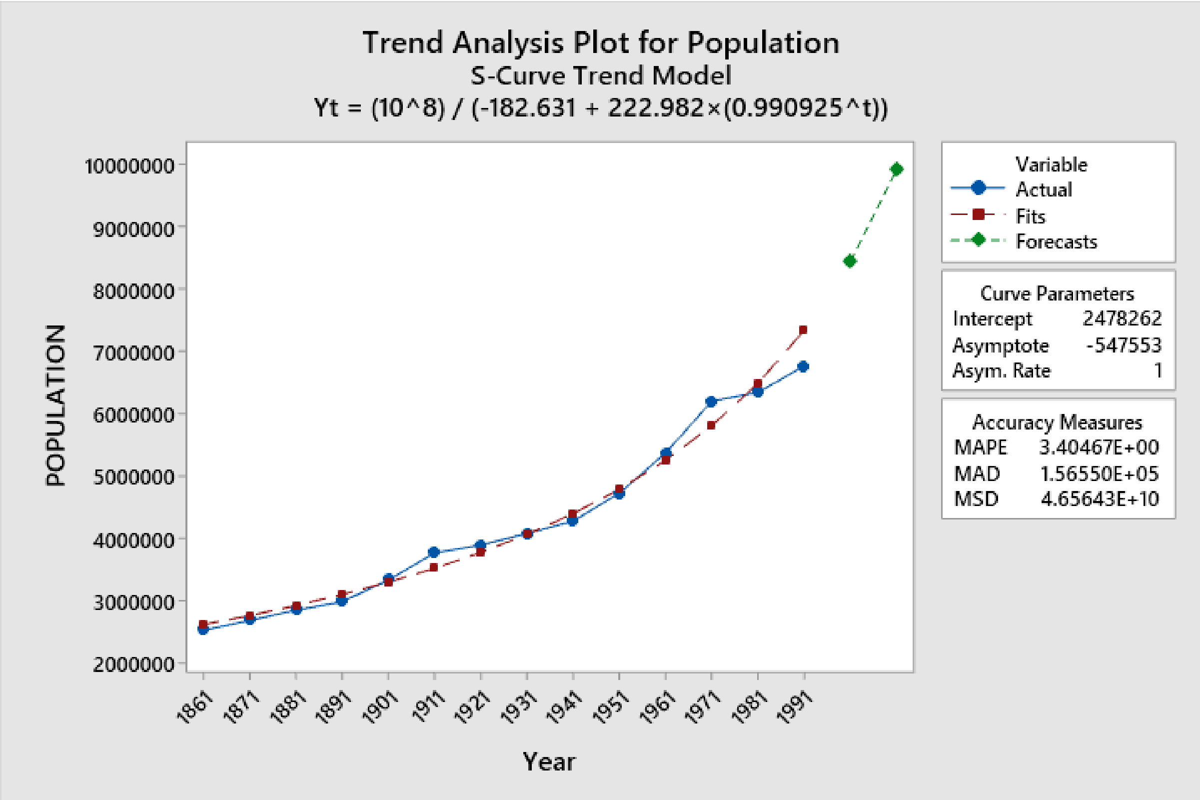
1.4 S-Curve Trend Model:

$$Y_t = 10^8 / (-182.631 + 222.982 * (0.990925^t))$$

(Here Y_t is the Switzerland census population at time t)

In the model fig A.1.4, The actual trend line (blue line), is showing a constant rate in the growth rate of population from 1861 to 1991 which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2001 and 2011 which are 8436490 and 9912355 respectively.

Fig A.1.4, Trend Analysis Plot for Switzerland Population (S-curve Trend Model)



After obtaining the population census for 2001 and 2011, the comparison of the census projections is made with the observed population of Switzerland.

The formula of the percentage error is:

$$\% \text{ ERROR} = \frac{(\text{Observed population} - \text{estimated population})}{\textit{observed population}} * 100$$

The percentage error comparison table are shown in below:

Table: A.2.1, Percentage Error for Linear Model:

year	observed population	estimated population	Error
2001	7197638	6751056	6.204563219
2011	7870134	7082983	10.00174838

Table: A.2.2, Percentage Error for Growth Curve Model:

year	observed population	estimated population	Error
2001	7197638	7292282	-1.314931371
2011	7870134	7886935	-0.213477941

Table: A.2.3, Percentage Error for Quadratic Model:

year	observed population	estimated population	Error
2001	7197638	7438378	-3.344708361
2011	7870134	8045234	-2.224866819

Table: A.2.4, Percentage Error for S-Curve Model:

year	observed population	estimated population	Error
2001	7197638	8436490	-17.21192425
2011	7870134	9912355	-25.94899909

So, from the above tables we can see that growth curve model has the least percentage error among all the other models.

Now another method to find the best fitted model for the data is to find the least measure for the Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAD), Mean Square Deviation (MSD). The Tables for this comparison are shown below:

Fig: A.3, Comparison of the MAPE, MAD, MSD for four models:

MODEL	MAPE	MAD	MSD
Linear Model	6.17942E+00	2.49320E+0	8.46397E+10
Growth curve Model	**2.66996E+00**	**1.16413E+05**	**2.76268E+10**
Quadratic Model	2.37320E+00	1.05897E+05	2.32261E+10
S- Curve Model	3.40467E+00	1.56550E+05	4.65643E+10

From the above table the Growth Curve model has the minimum MAPE, MAD, MSD. So, the best fitted trend model among four models for Switzerland is Growth Curve Model.

Switzerland 2021 Population Projection:

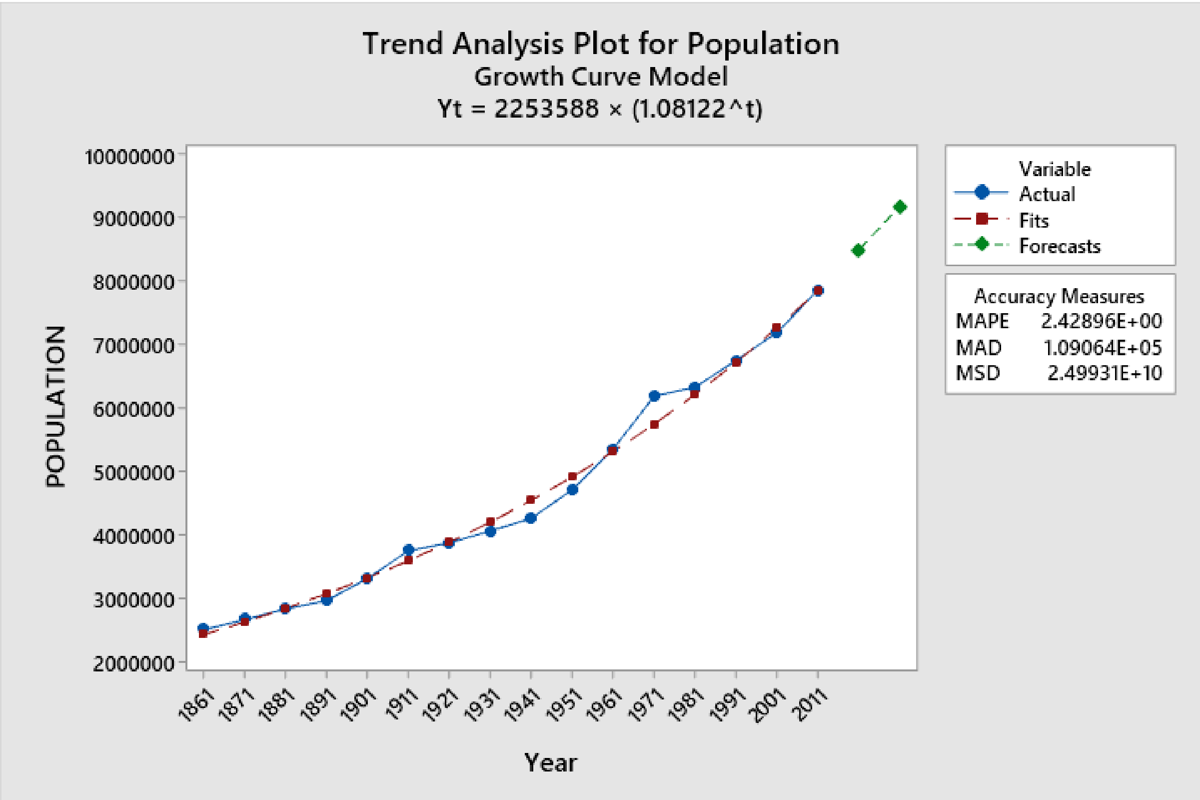
The Growth curve model has been found as the best fitted model for Switzerland population, now the entire data set including 2001 and 2011 is projected on S- curve model for 2021 population for Switzerland. The 2021 Switzerland population is projected 8500510. The fitted graph is shown in bellow:

The Exponential (Growth Curve) Trend model is:

$$Y_t = 225388 * (1.08122)^t$$

Where Y_t= Switzerland census population at time t

Fig: A.4, Trend Analysis Plot for 2021 Switzerland Population (Growth Curve Trend Model):



In the above model, the actual trend line (blue line), is showing a constant rate in the growth rate of population from 1961 to 2011 which is gradually increasing. The fitted line (red line) and the forecast line (green line) is showing the actual population and forecast population 2021 respectively, is increasing and the population of 2021 is 8500510.

Postcensal Estimation of Switzerland:

Postcensal estimates are produced using data from the most recent census and estimates the components of demographic growth since that data.

For Switzerland, let t =0, t=1 and t=2 which are the time points when the last two censuses were done. Here t=0 denotes the population of 2000, and t=1 denotes the population of 2011.

$$P_0\,(2001) = 7197638 \; (t=0)$$

$$P_1\,(2011) = 7870134 \; (t=1)$$

Since the best fitted model is Exponential model, so the equation is:

$$P_t = ab^t;$$

where P_t = Population at the time point and t= time point or year.

Now putting t= 0,1, we get the equations:

$$P_0 = a \qquad \ldots\ldots \text{(i)}$$

$$P_1 = ab \qquad \ldots\ldots \text{(ii)}$$

By solving (i), (ii), we get,

$$a = P_0$$

$$b = P_1/P_0$$

Now from the data we get,

a =7197638

b =1.093432

Hence the model is:

$$P_t = [7197638^* \, (1.093432)^t]$$

By putting t=1.1, the population of 2012 is:

$$P_{2012} = [7197638^* \, (1.093432)^{1.1}]$$

$$= 7940746$$

Similarly, the Postcensal estimation from 2013 to 2020 are:

$$P_{2013} = 8011985$$

$$P_{2014} = 8083869$$

$$P_{2015} = 8156399$$

$$P_{2016} = 8229579$$

$$P_{2017} = 8303416$$

P_{2018}= 8377916

P_{2019}= 8453084

P_{2020}= 8528926

Fig: A.5, Comparison of Projection and Postcensal Estimation:

Methods	Value of a, b
By Exponential Curve graph projection	a= 2253588, b=1.08122
By Postcensal estimation	a =7197638, b = 1.093432

B. Denmark Population projection for 2010 and 2020:

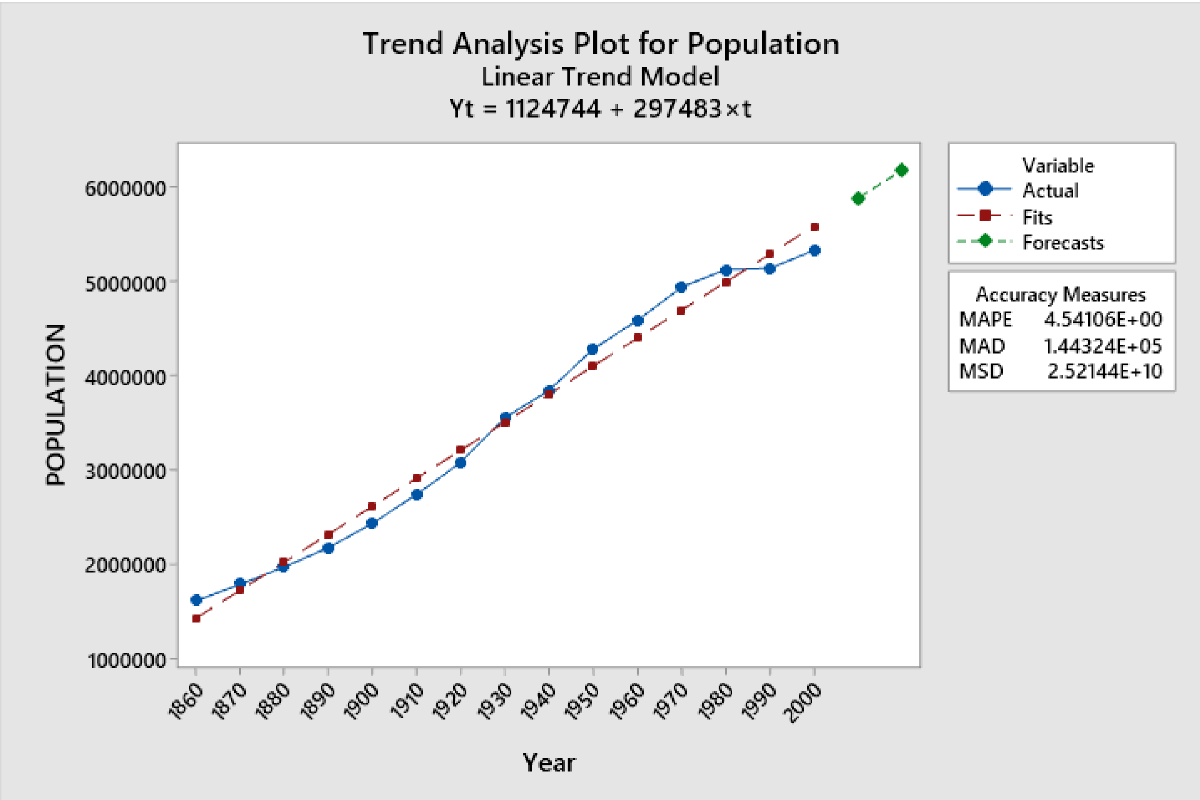
The graphs of the models (linear, quadratic, exponential and s-curve) for 2010 and 2020 are shown below:

B.1.1. Linear trend model:

$$Y_t = 1124744 + 297483*t$$

(Here Y_t is the Denmark census population at time t)

Fig: B.1.1, Trend Analysis Plot for Denmark Population (Linear Trend Model):



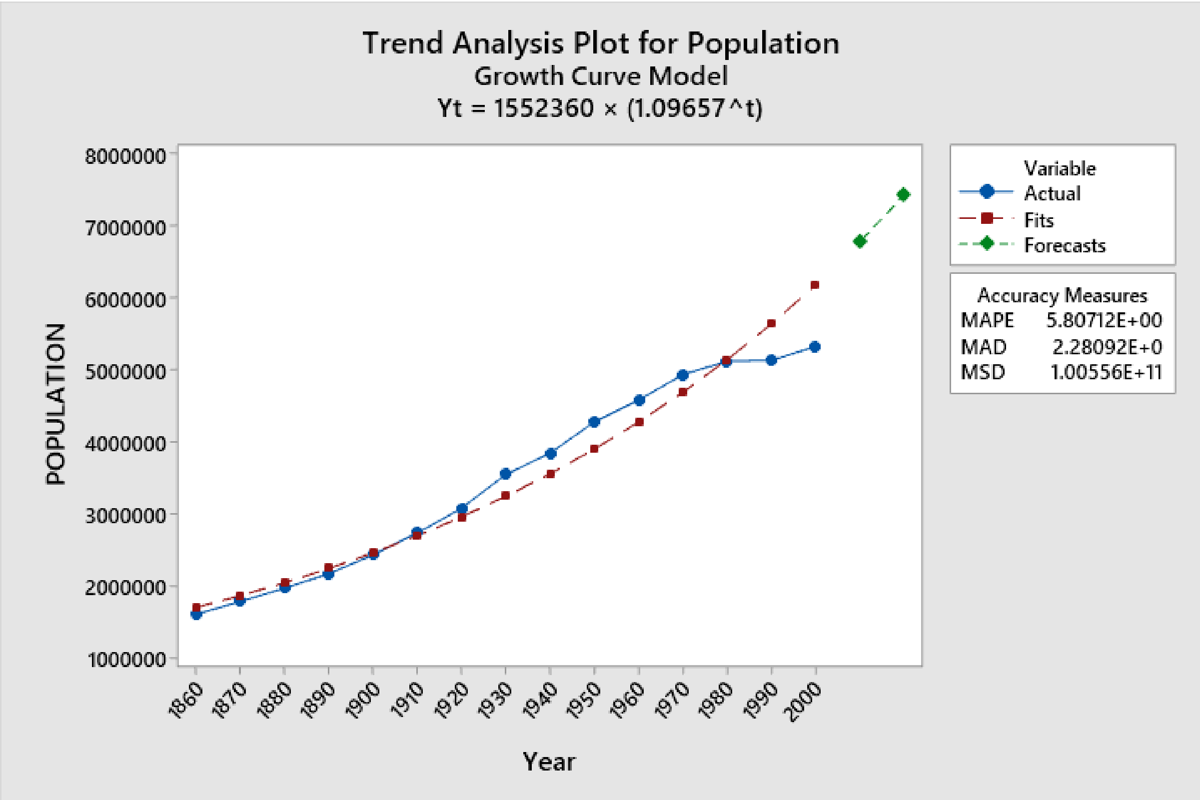
In the above model fig B.1.1, The actual trend line (blue line), is showing a constant rate in the growth rate of population which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2010 and 2020 which are 5884468 and 6181951 respectively.

B.1.2. Growth Curve Trend Model:

$$Y_t = 1552360 + (1.09657)^{t}$$

(Here Y_t is the Denmark census population at time t)

Fig, B.1.2: Trend Analysis Plot for Denmark Growth Population (Growth Trend Model):



In the above model fig B.1.2, The actual trend line (blue line), is showing a constant rate in the growth rate of population from 1861 to 2000 which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2000 and 2020 which are 6785118 and 7440339 respectively.

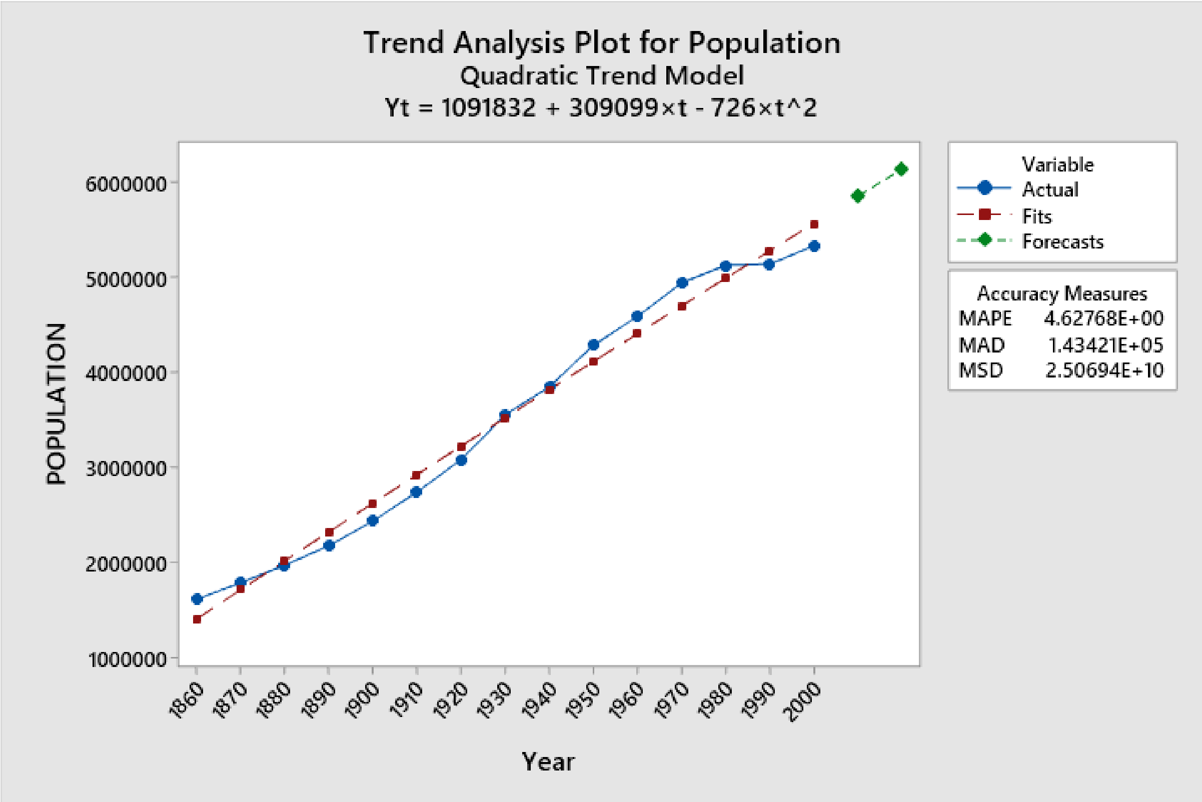
B.1.3. Quadratic Trend Model:

$$Y_t = 1091832 + 309099 * t - (726 * t^2)$$

(Here Y_t is the Denmark census population at time t

In the model fig B.1.3, The actual trend line (blue line), is showing a constant rate in the growth rate of population from 1860 to 2000 which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2010 and 2020 which are 5851556 and 6136697 respectively.

Fig B.1.3: Trend Analysis Plot for Denmark Population (Quadratic Trend Model)



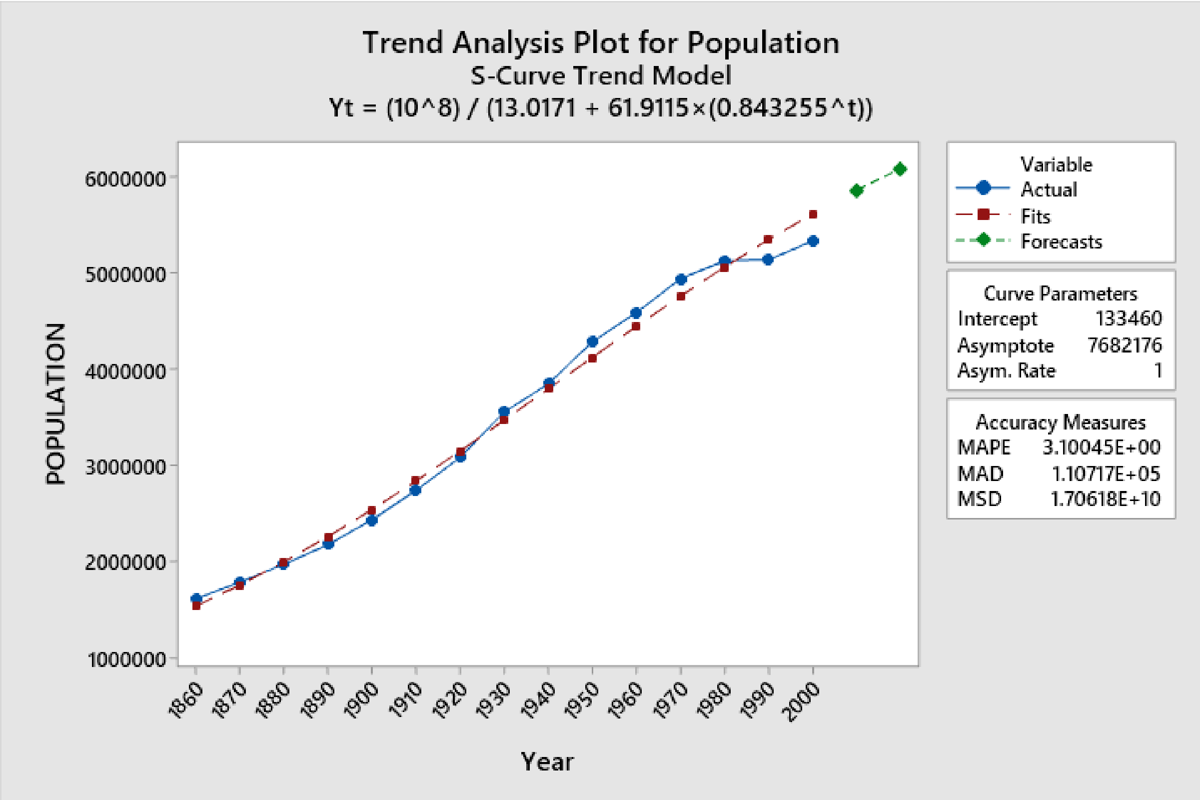
B.1.4. S-Curve Trend Model:

$$Y_t = 10^8 / (13.0171 + 61.9115 * (0.843255^t))$$

(Here Y_t is the Denmark census population at time t)

In the model fig B.1.4, The actual trend line (blue line), is showing a constant rate in the growth rate of population from 1860 to 2000 which is gradually increasing. The fitted line (red line) is also increasing for the entire time. And the forecast line (green line) is showing the forecast population of 2010 and 2020 which are 5860299 and 6086554 respectively.

Fig, B.1.4: Trend Analysis Plot for Denmark Population (S-curve Trend Model):



After obtaining the population census for 2010 and 2020, the comparison of the census projections is made with the observed population of Denmark.

The formula of the percentage error is:

$$\%\text{ERROR} = \frac{(\text{Observed population} - \text{estimated population})}{\textit{observed population}} * 100$$

The percentage error comparison table are shown in below:

Table: B.2.1, Percentage Error for Linear Model:

year	**observed population**	**estimated population**	**Error**
2010	5554800	5884468	-5.934831137
2020	5822863	6181951	-6.166863277

Table: B.2.2, Percentage Error for Growth Curve Model:

year	**observed population**	**estimated population**	**Error**
2010	5554800	6785118	-22.14873623
2020	5822863	7440339	-27.77801916

Table: B.2.3, Percentage Error for Quadratic Model:

year	observed population	estimated population	Error
2010	5554800	5851556	-5.342334557
2020	5822863	6136697	-5.389685452

Table: B.2.4, Percentage Error for S- Curve Model:

year	observed population	estimated population	Error
2010	5554800	5860299	-5.499729963
2020	5822863	6086554	-4.528545494

So, from the above tables we can see that S-Curve model has the least percentage error among all the other models.

Now another method to find the best fitted model for the data is to find the least measure for the Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAD), Mean Square Deviation (MSD). The Tables for this comparison are shown below:

Fig: B.3, Comparison of the MAPE, MAD, MSD for four models:

MODEL	MAPE	MAD	MSD
Linear Model	4.54106E+00	1.44324E+00	2.52144E+00
Growth curve Model	5.80712E+00	2.28092E+00	1.00556E+00
Quadratic Model	4.62768E+00	1.43421E+00	2.50694E+00
S- Curve Model	**3.10045E+00**	**1.10717E+00**	**1.70618E+00**

From the above table the S-Curve model has the minimum MAPE, MAD, MSD. So, the best fitted trend model among four models for Denmark Is S-Curve Model.

Denmark Population Projection for 2030:

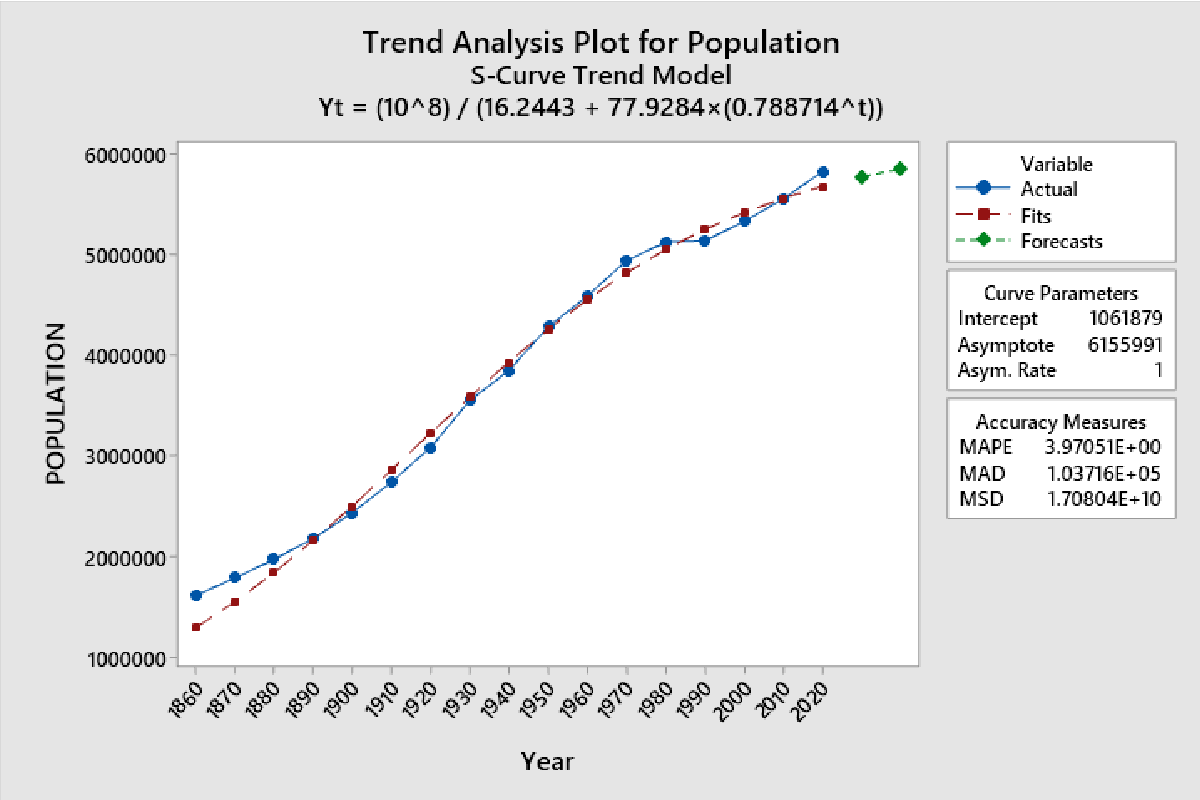
The S-curve model has been found as the best fitted model for Denmark population, now the entire data set including 2010 and 2020 is projected on S- curve model for 2030 population for Denmark. The 2030 Denmark population is projected 5769877. The fitted graph is shown in bellow:

The S-Curve Trend model is:

$$Y_t = \frac{10^8}{16.2443+77.9284\times(0.788714^t)}$$

Where Y_t= Denmark census population at time

Fig: B.4, Trend Analysis Plot for 2030 Denmark Population (S-curve Trend Model):



In the above model, the actual trend line (blue line), is showing a constant rate in the growth rate of population from 1960 to 2020 which is gradually increasing. But the fitted line (red line) and the forecast line (green line) is the actual population and forecast population 2030 respectively, which is decreasing and the population of 2030 is 5769877.

Postcensal Estimation of Denmark:

Postcensal estimates are produced using data from the most recent census and estimates the components of demographic growth since that data.

For Denmark, let t =0, t=1 and t=2 which are the time points when the last two censuses were done. Here t=0 denotes the population of 2000, t=1 denotes the population of 2010 and t=2 denotes the population of 2020.

$$P_0\,(2000) = 5330020\ (t=0)$$

$$P_1\,(2010) = 5554800\ (t=1)$$

$$P_2\,(2020) = 5822863\ (t=2)$$

Since the best fitted model is S-Curve model, so the equation is:

$$P_t = 10^8 / (a+bc^t)$$

where P_t = Population at the time point and t= time point or year.

Now putting t= 0,1,2, we get the equations:

$P_0 = 10^8/(a+b)$; ……. (1)

$P_1 = 10^8/(a+bc)$; ……. (2)

$P_2 = 10^8/(a+bc^2)$ ……. (3)

By solving (1), (2), (3) we get,

$a = 18.7616 - b$

$b = 0.759/(1-c)$

$c = 1.092$

Now from the data we get,

$a = 18.8536$

$b = -0.092$

$c = 1.092$

Hence the model is:

$$P_t = 10^8/\{18.8536+(-0.092)(1.092)^t\}$$

So, the population of 2021 is:

$$P_{2021} = 10^8/\{18.8536+(-0.092)(1.092)^t\}$$

$= 5335346$

Similarly, the Postcensal estimation from 2022 to 2030 are:

$P_{2022} = 5335625$

$P_{2023} = 5335906$

$P_{2024} = 5336190$

$P_{2025} = 5336476$

$P_{2026} = 5336764$

$P_{2027} = 5337055$

$P_{2028} = 5337349$

$P_{2029} = 5337646$

Fig: B.5, Comparison of Projection and Postcensal Estimation:

Methods	Value of a, b, c
By S-Curve graph projection	a= 16.2443, b=77.9284, c=0.788714
By Postcensal estimation	a =18.8536, b = -0.092, c=1.092

Comparison between Switzerland and Denmark:

To compare between two different countries, percentage growth rate, population density, birth and death rate are best methods. It shows the picture of different kind of variables as in Industrial growth, agriculture growth, economic growth, educational growth and many others of an individual country.

1. Percentage Growth Rate of Switzerland and Denmark:

The equation of population percentage growth rate is:

$$\% \text{ Growth Rate} = \frac{(Population\ of\ time2 - Population\ of\ time1)}{Population\ of\ time1} * 100$$

Where, Population of time 2 is next year population and Population of time 1 is present year population.

Table: A.6, Population Percentage Growth Rate of Switzerland:

YEAR	POPULATION	PERCENTAGE GROWTH RATE (%)
1861	2515396	-
1871	2673468	6.284179509
1881	2840501	6.24780248
1891	2972041	4.630873216
1901	3318985	11.673594
1911	3756842	13.1924971
1921	3883360	3.367668909
1931	4070042	4.807228792
1941	4268964	4.887467992
1951	4717200	10.49987772
1961	5360153	13.62997117
1971	6193064	15.53894077
1981	6335243	2.295777986
1991	6757188	6.660281224
2001	7197638	6.51824398
2011	7870134	9.34328734

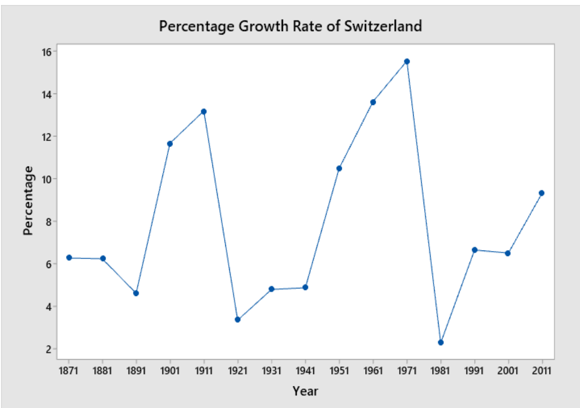
In the above table, the percentage rate of Switzerland Population census from 1871 to 2011 is increasing, then decreasing then again increasing.

Table: B.6, Population Percentage Growth Rate of Denmark:

YEAR	POPULATION	PERCENTAGE GROWTH RATE (%)
1860	1608362	-
1870	1784741	10.96637449
1880	1969039	10.32631626
1890	2172380	10.32691582
1900	2432000	11.95094781
1910	2737000	12.54111842
1920	3079000	12.49543296
1930	3550656	15.31848003
1940	3844312	8.27047171
1950	4281275	11.36648118
1960	4585256	7.10024467
1970	4937579	7.683823978
1980	5122065	3.736365535
1990	5135409	0.260519927
2000	5330020	3.789591053
2010	5554800	4.217244963
2020	5822863	4.825790307

In the above table, the percentage rate of Switzerland Population census from 1870 to 2020 is increasing, then decreasing.

The graphs of percentage growth rate for these two countries are shown in the below:



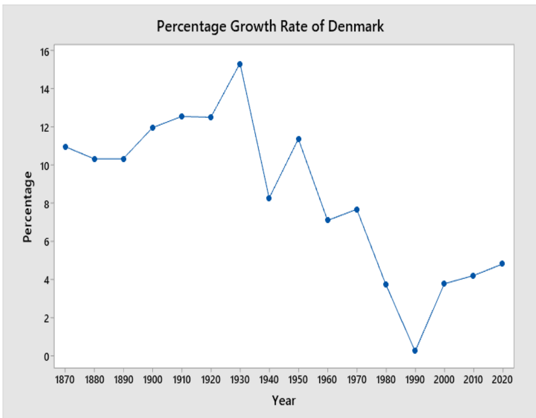


Fig: A.7, Growth Rate of Switzerland

Fig: B.7, Growth Rate of Denmark

2. Population Density of Switzerland and Denmark:

One of the important indices of population concentration is the density of population. It is defined as the number of persons per sq. km.

Fig A.8: Population Density for Switzerland

Year	*Population Density*
1950	113.05
1960	127.89
1970	148.97
1980	152.19
1990	161.12
2000	173.01
2010	189.12
2020	209.6

Fig B.8: Population Density for Denmark

Year	*Population Density*
1950	99.45
1960	106.74
1970	114.89
1980	119.38
1990	119.78
2000	124.45
2010	129.42
2020	134.95

3. Birth Rate of Switzerland And Denmark:

Birth rate is a term used to describe the number of babies born per 1000 people annually in a population.

Fig A.9: Birth Rate for Switzerland

Year	*Birth Rate*
1950	17.04
1960	18.03
1970	16.307
1980	11.67
1990	11.945
2000	10.785
2010	10.179
2020	10.225

Fig B.9: Birth Rate for Denmark

Year	*Birth Rate*
1950	18.343
1960	17.054
1970	15.458
1980	11.523
1990	11.892
2000	12.374
2010	11.215
2020	10.865

4. Death Rate of Switzerland and Denmark:

Death Rate is the name used to describe the number of deaths per 1000 individuals in a po pulation annually.

Fig A.10: Death Rate for Switzerland

Year	*Death Rate*
1950	10.28
1960	9.812
1970	9.328
1980	9.22
1990	9.24
2000	8.748
2010	8.061
2020	8.091

Fig B.10: Death Rate for Denmark

Year	*Death Rate*
1950	8.923
1960	9.42
1970	10.038
1980	10.73
1990	11.629
2000	11.096
2010	9.843
2020	9.848

Conclusion:

The purpose of this project is to analysis the population census for Switzerland (2021) and Denmark (2030) and compare them.

First, for Switzerland population projection 2021, the best fitted model is exponential trend line.

The Exponential (Growth Curve) trend model is:

$$Y_t = 225388 * (1.08122)^t$$

Where Y_t= Switzerland census population at time t.

The total Population projection 2021 of Switzerland is: 8500510.

Second, For Denmark population projection 2030, the best fitted model is S-Curve trend line.

The S-Curve trend model is:

The S-Curve Trend model is:

$$Y_t = \frac{10^8}{16.2443+77.9284\times(0.788714^t)}$$

Where Y_t= Denmark census population at time t.

The total Population projection 2030 of Denmark is: 5769877.

Despite having approximately same geometrical are there is a huge difference between these two countries.

For Switzerland the percentage growth rate is increasing but for Denmark, the growth rate is decreasing.

The total population of these countries are lower than other European countries which have almost same geographical area, like Netherland. Netherland geographical area is 41,543 km^2 and the population is 1.73crores (2019).

Largely as a result of Switzerland's and Denmark's low birth rate (about half the world average), the countries' population grew only slightly during the last decades of the 20th century. The continuing fall in net immigration and increase in deaths while the birth rate dropped. At this rate the percentage will decrease more in upcoming decades.

Acknowledgement:

It is great pleasure for me to undertake this project. I feel highly grateful doing the project entitled- ***The Population Projection and Comparison for Switzerland and Denmark.***

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This study work has indeed helped me to explore more knowledgeable avenues related to my topic and it will sure help me a lot in my future too.

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