Homework Sheet

classmate
Date
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Day-37/180 → # Vector in C++

1. Prove that the average time complexity of push_back() operation in Vector is O(1) time

```
vector<int> v;

for( int i=0; i<=n; i++)
    v.push_back(i);
```

As we can observe in table as size of v tends to capacity of v, the average is given as

Average T.C. $= \frac{2^{n+1}-1}{2^n} = \frac{2^n \cdot 2^1}{2^n} - \frac{1}{2^n}$

$= 2 - \frac{1}{2^n} \approx 1.---$

Average T.C. $= O\left(2-\frac{1}{2^n}\right) = O(1.---) = O(1)$ ⟸ Hence, proved.

i	n	Capacity of v	Size of v	Operations on v	Total Operations	Average ($\frac{\text{Total}}{n}$)	O(1)
		0	0				
i =	1=2^0	1	1	1	1=2^1-1	1	O(1)
→	2=2^1	2	2	2	3=2^2-1	1.5	O(1)
→	3	4	3	3	6=	2	
→	4=2^2	4	4	1	7=2^3-1	1.75	O(1)
→	5	8	5	5	12	2.4	
will go till i<=n →	6	8	6	1	13	2.16	
→	7	8	7	1	14	2	
→	8=2^3	8	8	1	15=2^4-1	1.875	O(1)
→	9	816	9	9	24	2.6	
→	10	16	10	1	25	2.5	
→	11	16	11	1	26	2.36	
→	12	16	12	1	27	2.25	
→	13	16	13	1	28	2.15384	
→	⋮	⋮	⋮	⋮	⋮	⋮	
→	16=2^4	16	16	1	31=2^5-1	1.93	O(1)
→	⋮	⋮	⋮	⋮	⋮	⋮	
→	32=2^5	32	32	1	63=2^6-1	1.96	O(1)
→	2^n	2^n	2^n	1	2^n-1	1.---	O(1)