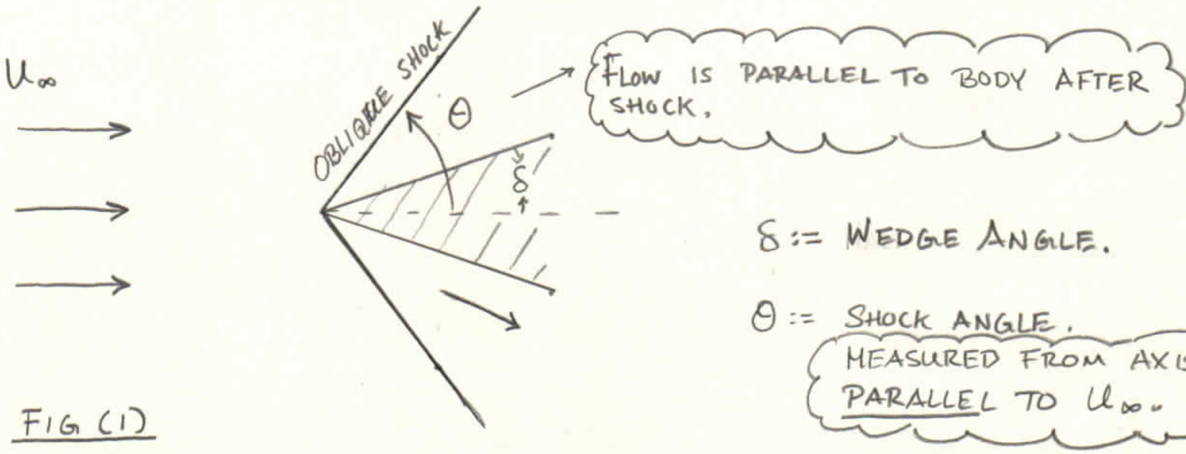
* I RAN OUT OF MY CHEAP PAPER SO USING MY PRECIOUS ROARING SPRING BUFF PAPER FOR THESE NOTES ☹. YOU WILL NOTICE I REASON DIFFERENTLY ON THIS PAPER--- *

PREVIOUSLY WE DEVELOPED INTUITION ON SHOCK-WAVES IN GENERAL BUT ONLY DEVELOPED EQUATIONS FOR SIMPLE 1-D NORMAL SHOCKS. BUT RARELY IS A SHOCK 1-D, USUALLY IT IS AT AN ANGLE WHICH IS 2D!



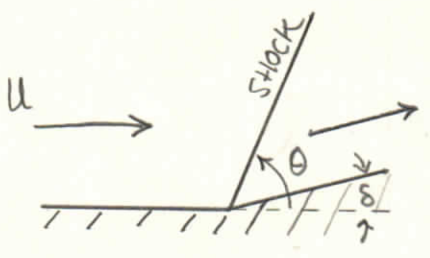
δ := WEDGE ANGLE.

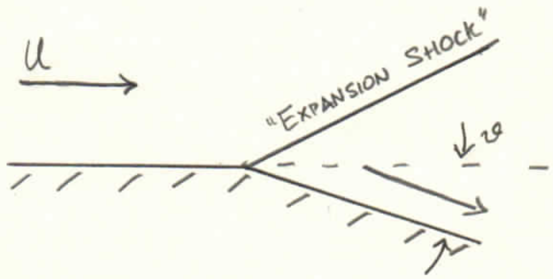
θ := SHOCK ANGLE. MEASURED FROM AXIS PARALLEL TO U_∞.

FIG (1)

NOW WE WISH TO DEVELOP THE EQUATIONS FOR FLOW PROPERTIES IN THIS 2D SHOCK. OBLIQUE ≡ 2D.

WE DONT JUST NEED A WEDGE TO GET OBLIQUE SHOCKS, SHARP ANGLES ON SURFACES DO IT TOO.





* NOTICE THIS IS JUST "HALF" THE PICTURE OF FIG(1)

* WE WILL NOT COVER THIS CASE. IF INTERESTED LOOK UP "PRANDTL-MEYER FLOW"

FIRST LETS SUMMARIZE WHAT WE'VE LEARNED ABOUT SHOCKS SO FAR.

i) ONLY POSSIBLE FOR $Ma > 1$ UPSTREAM.

ii) FLOW PROPERTIES CHANGE DISCONTINOUSLY ACROSS A SHOCK

iii) ENTROPY MUST INCREASE ACROSS SHOCK.

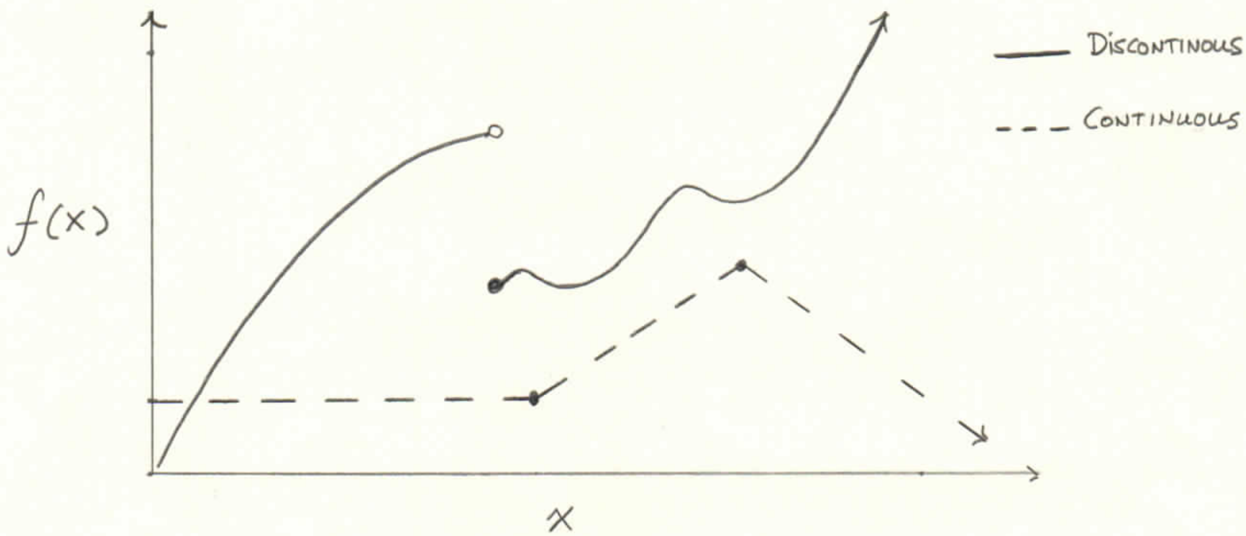
iv) THEY ARISE FROM PRESSURE WAVES PILING UP AT A REGION.

v) THEY DISSAPATE ENERGY BY MEANS OF HEAT.

BUT AN OVERALL FUNCTION TO ALWAYS REMEMBER IS THAT THEY...

<u>SLOW DOWN FLOW DISCONTINOUSLY</u>

A REMINDER OF WHAT DISCONTINOUS "LOOKS" LIKE.

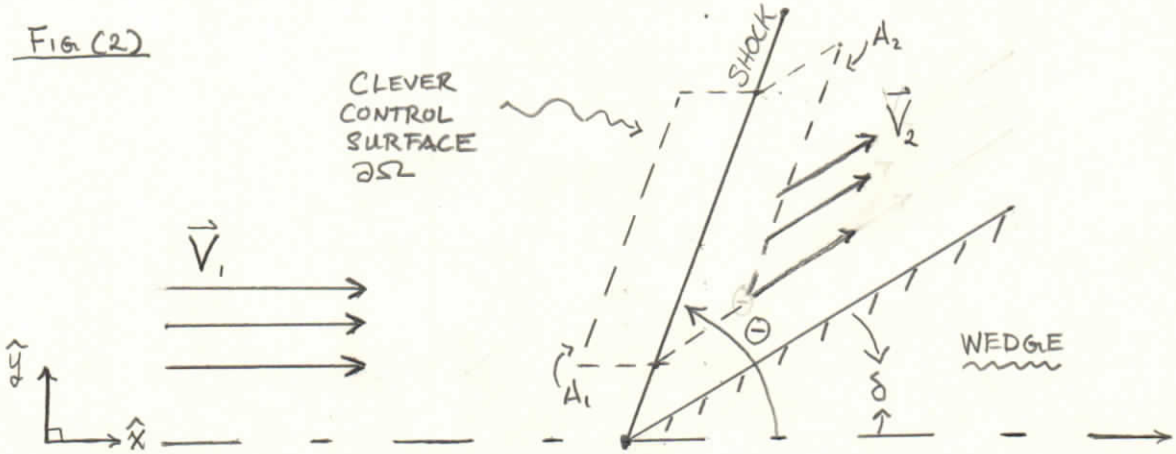


BY FLOW PROPERTIES WE ARE REFERING TO

$$\rho, V, P, T$$

WE WILL SOLVE THE PROBLEM BY <u>ASSUMING THE EXISTENCE</u> OF A SHOCK-LINE AND DRAW A CONTROL SURFACE.

FIG (2)



TO FORMULATE EQUATIONS FOR FLOW VARIABLES WE CONSERVE QUANTITIES IN AND OUT OF $\partial\Omega$. WE WILL ESTABLISH A COORDINATE SYSTEM THAT IS ALIGNED TO THE SHOCK-LINE. WITH THE TANGENT AXIS PARALLEL TO THE SHOCK-LINE.

$\hat{t} \cdot \hat{x} = \cos\Theta$

$\|V_1\| = (V_{1n}^2 + V_{1t}^2)^{1/2}$

EXTRA CREDIT

PROVE THAT $\|V\|$ IS THE SAME REGARDLESS OF WHICH COORDINATE SYSTEM YOU'RE IN

(1) CONSERVE MASS. LET $A_1 = A_2$

$$\rho_1 V_{1n} = \rho_2 V_{2n} \equiv \frac{\dot{m}}{A}$$

* NOTICE WE HAVE DECOMPOSE OUR VECTORS INTO $\hat{n}$ & $\hat{t}$ PARTS.

(2) CONSERVE $\hat{n}$ MOMENTUM

$$P_1 - P_2 = \rho_2 V_{2n}^2 - \rho_1 V_{1n}^2$$

(3) CONSERVE $\hat{t}$ MOMENTUM

$$0 = \frac{\dot{m}}{A}(V_{1t} - V_{2t})$$

(4) CONSERVE ENERGY (scalar quantity)

$$h_1 + \frac{\|\vec{V}_1\|^2}{2} = h_2 + \frac{\|\vec{V}_2\|^2}{2}$$

(5) EQUATION OF STATE

$$P - \rho R T = 0$$

WE NOW USE THIS SYSTEM OF EQUATIONS (1)–(5) TO DERIVE PROPERTY RELATIONS AND SOMETHING CALLED THE PRANDTL RELATION.

COMBINE (1) ↦ (2)

$$P_2 - P_1 = \rho_1 V_{1n}^2 - \rho_2 V_{2n}^2$$

$$= \rho_1 V_{1n}^2 \left(1 - \frac{\rho_1}{\rho_2} \right)$$

COMBINE (3) ↦ (4)

$$h_1 - h_2 = \frac{V_{1n}^2 + \cancel{V_{1t}^2}}{2} - \frac{(V_{2n}^2 + \cancel{V_{2t}^2})}{2}$$

$$\therefore \quad h_1 - h_2 = \frac{V_{1n}^2 - V_{2n}^2}{2} \qquad (*)$$

EXPRESS $V_{in} = f(P_1, P_2, \rho_1, \rho_2)$.

$$V_{1n} = \left(\frac{P_2 - P_1}{\rho_2 - \rho_1} \frac{\rho_2}{\rho_1} \right)^{1/2} \qquad (a)$$

$$V_{2n} = \left(\frac{P_2 - P_1}{\rho_2 - \rho_1} \frac{\rho_1}{\rho_2} \right)^{1/2} \qquad (b)$$

RECALL $h = \gamma P / (\gamma - 1)\rho$, and sub into (*) ALONG WITH (a), (b).

$$\frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} \frac{\Delta P}{2 \Delta \rho} \frac{\rho_2}{\rho_1} = \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2} \frac{\Delta P}{2 \Delta \rho} \frac{\rho_1}{\rho_2}$$

$$\therefore \quad \boxed{\frac{P_2}{P_1} = \frac{\frac{\gamma+1}{\gamma-1} \frac{\rho_2}{\rho_1} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_2}{\rho_1}}}$$

NOW WE WILL ELLIMINATE $\{\rho_i, P_j\}$ TO GET THE PRANDTL RELATION. RECALL (4).

$$\frac{\|\vec{V}_1\|^2}{2} + h_1 = \frac{\|\vec{V}_2\|^2}{2} + h_2 \equiv h_0 \qquad (a)$$

WE WILL CALCULATE THIS ENERGY AT A STAGNATION VALUE. IDEAL GAS LAW SAYS.

$$h_0 = \frac{\gamma}{\gamma - 1} R T_0$$

WE GET STAG TEMP T_0 FROM CRITICAL $Ma=1$ VALUES.

$$\frac{T_0}{T_*} = \frac{\gamma+1}{2}$$

$$\therefore T_0 = T_*\left(\frac{\gamma+1}{2}\right)$$

AT CRITICAL PROPERTIES WE HAVE.

$$C_*^2 = \gamma R T_*$$

$$\therefore T_0 = \frac{C_*^2}{\gamma R} \cdot \frac{\gamma+1}{2}$$

PLUG IT ALL BACK INTO (a) NOW.

$$\frac{\|\vec{V}_1\|^2}{2} + \frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1} = \frac{\|\vec{V}_2\|^2}{2} + \frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2} = \frac{\gamma+1}{2(\gamma-1)} C_*^2$$

SOLVE FOR P_i/ρ_i.

$$\frac{P_1}{\rho_1} = \frac{\gamma+1}{2\gamma} C_*^2 - \frac{\gamma-1}{\gamma}\frac{\|\vec{V}_2\|^2}{2} \qquad (**)$$

$$\frac{P_2}{\rho_2} = \frac{\gamma+1}{2\gamma} C_*^2 - \frac{\gamma-1}{\gamma}\frac{\|\vec{V}_1\|^2}{2} \qquad (***)$$

NOW TAKE (**), (***) AND SUB THEM INTO THE $\hat{n}$ MOMENTUM, REARRANGED AS

$$\frac{1}{V_{1n}}\frac{P_1}{\rho_1} - \frac{1}{V_{2n}}\frac{P_2}{\rho_2} = V_{2n} - V_{1n}$$

$$\frac{1}{V_{1n}}\left\{\frac{\gamma+1}{2\gamma}C_*^2 - \frac{\gamma-1}{\gamma}\frac{\|\vec{V}_2\|^2}{2}\right\} - \frac{1}{V_{2n}}\left\{\frac{\gamma+1}{2\gamma}C_*^2 - \frac{\gamma-1}{\gamma}\frac{\|V_1\|^2}{2}\right\} =$$

EXPAND $\|V_i\|^2 = V_{in}^2 + V_{it}^2$. AND RECALL $V_{1t} = V_{2t} =: V_t$

$$\left(\frac{\gamma+1}{2\gamma}C_*^2\,\frac{V_{2n}-V_{1n}}{V_{1n}V_{2n}}\right) + \frac{\gamma-1}{2\gamma}\left(\frac{V_{2n}^2+V_t^2}{V_{2n}} - \frac{V_{1n}^2+V_t^2}{V_{1n}}\right) = V_{2n} - V_{1n}$$

COLLECT TERMS OF $(V_{2n}-V_{1n})$... DO IT YOUR-SELF!

$$\left(\frac{\gamma+1}{2\gamma}\frac{C_*^2}{V_{1n}V_{2n}}\right)(V_{2n}-V_{1n}) + \frac{\gamma-1}{2\gamma}(V_{2n}-V_{1n}) + \left(\frac{\gamma-1}{2\gamma}V_t^2\right)\underbrace{\left(\frac{1}{V_{2n}} - \frac{1}{V_{1n}}\right)}_{=\frac{V_{2n}-V_{1n}}{V_{2n}V_{1n}}} = V_{2n} - V_{1n}$$

WE CAN CANCEL NOW ALL $V_{2n}-V_{1n}$ TERMS TO GET THE PRANDTL RELATION.

$$V_{2n}V_{1n} = C_*^2 - \frac{\gamma-1}{\gamma+1}V_t^2$$

OR INTERMS OF MACH Ma DIVIDE THROUGH BY C_*.

$$\boxed{Ma_2 Ma_1\Big|_{\hat{n}} = 1 - \frac{\gamma-1}{\gamma+1}Ma_t^2}$$

NOTICE NOW THAT FOR NORMAL SHOCKS $Ma_t \equiv 0$. WHICH MEANS IF $Ma_1 > 1$, THEN $Ma_2 < 1$! * $Ma_1 < 1$ VIOLATES 2^{nd} LAW * SO NEED NOT BE CONSIDERED *

BUT NOW FOR OBLIQUE SHOCKS $V_t \neq 0$ WHICH OPENS UP CASES. THIS RELATION HELPS ESTIMATE Ma_2 IF V_t IS KNOWN OR VICE VERSA.

$$Ma_1 Ma_2 \Big|_{\hat{n}} = 1 - \frac{\gamma-1}{\gamma+1} V_t^2 < 1$$

$Ma_1 > 1$ OR ELSE 2^{nd} VIOLATION

THE LARGER V_t IS THE MORE $Ma_{2n} < 1$ MUST BE

NOW WE WANT THE PROPERTY RELATIONS. THAT IS. FIND...

$$\frac{P_2}{P_1} = f_1(Ma_1, \gamma)$$

$$\frac{\rho_2}{\rho_1} = f_2(Ma_1, \gamma)$$

$$\frac{T_2}{T_1} = f_3(Ma_1, \gamma)$$

$$Ma_2 = f_4(Ma_1, \gamma)$$

* THIS IS THE MOST IMPORTANT ONE!

FOR THE INTEREST STUDENT (AND SINCE I'VE DONE THIS 3 TIMES) TRY IT YOURSELF MANIPULATING EQUATIONS AROUND TO GET THESE f_i EXPRESSIONS. I'LL JUST STATE THEM.

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) Ma_1^2 \sin^2\theta}{2 + (\gamma-1) Ma_1^2 \sin^2\theta} \quad ; \quad \frac{P_2}{P_1} = \frac{2\gamma Ma_1^2 \sin^2\theta - (\gamma-1)}{\gamma+1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{\rho_1}{\rho_2}\right) \quad \text{and} \quad \Delta S = c_p \ln \left\{ \frac{(T_2/T_1)}{(P_2/P_1)^{\gamma-1/\gamma}} \right\}$$

MOST IMPORTANTLY THOUGH! PLOT THIS FUNCTION

$$Ma_2 = \csc(\theta - \delta)\left(\frac{2 + (\gamma - 1)Ma_1^2 \sin^2\theta}{2\gamma Ma_1^2 \sin^2\theta - (\gamma - 1)}\right)^{1/2}$$

* THE MAIN TAKE AWAY OF THIS IS THAT Ma_2 CAN BE $Ma_2 > 1$ <u>OR</u> $Ma_2 < 1$. SO FOR OBLIQUE SHOCKS DOWNSTREAM CAN <u>STILL</u> BE SUPERSONIC WITHOUT VIOLATING 2nd LAW

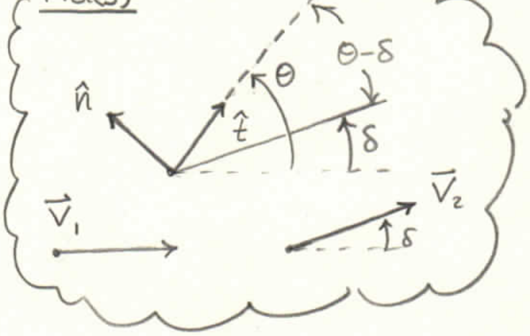
WE HAVE ASSUMED WE'VE KNOWN δ UNTIL NOW. LETS TRY TO <u>SOLVE</u> FOR δ GIVEN A θ AND Ma_1.

$$\left.\frac{Ma_2}{Ma_1}\right|_{\hat{n}} = \frac{2 + (\gamma - 1)Ma_1^2 \sin^2\theta}{(\gamma + 1)Ma_1^2 \sin^2\theta}$$

BUT BY VECTOR DECOMPOSITION LAWS, AND $V_{2t} = V_{1t}$

$$V_{2n} = \|V_2\|\sin(\theta - \delta) \qquad \& \qquad V_{2t} = \|V_2\|\cos(\theta - \delta)$$
$$V_{1n} = \|V_1\|\sin(\theta) \qquad\qquad V_{1t} = \|V_1\|\cos(\theta)$$
$$\frac{\|V_2\|}{\|V_1\|} = \frac{\cos\theta}{\cos(\theta - \delta)}$$

FIG(3)



WE CAN REWRITE THEN

$$\frac{V_{2n}}{V_{1n}} = \left(\frac{\|V_2\|}{\|V_1\|}\right)\frac{\sin(\theta - \delta)}{\sin(\theta)}$$

FINALLY,

$$\frac{\tan(\theta - \delta)}{\tan(\theta)} = \frac{2 + (\gamma - 1)Ma_1^2 \sin^2\theta}{(\gamma + 1)Ma_1^2 \sin^2\theta}$$

WE NEED TO BREAK OUT THEM TRIG IDENTITIES NOW!

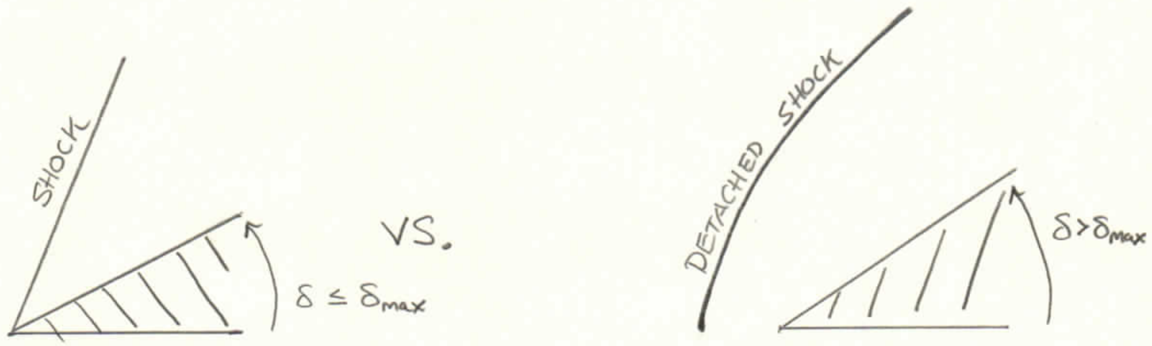
(I SERIOUSLY HAVE A SHEET OF THESE ON MY WALL...)

$$\tan(\theta-\delta) = \frac{\tan(\theta)-\tan(\delta)}{1+\tan(\theta)\tan(\delta)}$$

NOW WE CAN SOLVE FOR $\tan(\delta)$!

$$\delta(Ma_1,\theta,\gamma) = \arctan\left\{\frac{(Ma_1^2\sin^2\theta - 1)\cot\theta}{(\gamma+1)Ma_1^2\sin^2\theta}\right\} \quad (!!!)$$

THIS EQUATION IS SUPER INTERESTING AND DESCRIBES WHEN SHOCKS ARE NOT CONNECTED TO THE NOSE!



THIS IS SEEN IN (!!!) BECAUSE FOR A GIVEN Ma_1, θ THERE EXISTS A δ_{max} PAST WHICH NO SOLUTION EXISTS.

FOR AIR $\gamma = 1.4$ THIS ANGLE CAN BE APPROXIMATED

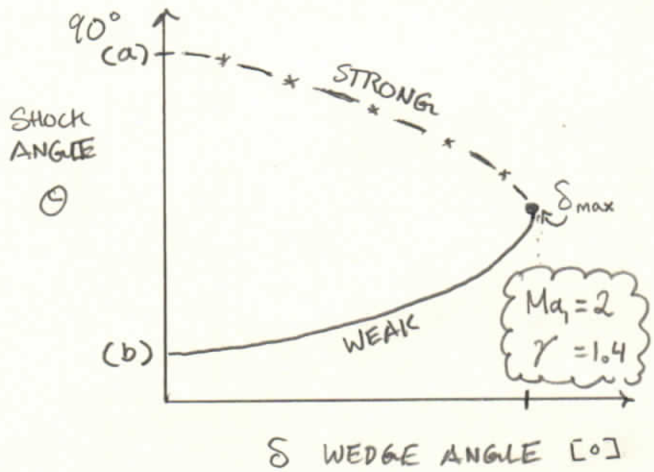
$$\delta_{max} \approx 0.9\,(Ma_1 - 1)^{3/2}$$

THE SIMPLEST TAKE AWAY FROM OBLIQUE SHOCK ANALYSIS IS JUST

i) OBLIQUE PROBLEMS REDUCE TO NORMAL SHOCK PROBLEMS FOR THE $\hat{n}$ COMPONENT OF VELOCITY

ONE LAST PROPERTY OF OBLIQUE SHOCKS IS A CONCEPT OF <u>WEAK SHOCKS</u> AND <u>STRONG SHOCKS</u>.

WHEN WE PLOT $\delta(\theta)$ OR EQ (!!!) WE GET A COMMON PLOT.



* BECAUSE THIS GRAPH IS DOUBLED VALUED THE SOLUTIONS ARE SEPERATED INTO 2 CASES CALLED

STRONG SHOCKS — * —
WEAK SHOCKS ———

* THE 3rd CASE IS <u>NO SOLUTION</u> WHICH IS A DETATCHED SHOCK.

WE CAN LOOK AT (!!!) TO IDENTIFY THE PHYSICS

(a) IF $\cot\theta = 0$, THE SHOCK IS NORMAL SINCE $\theta = 90°$

(b) IF $Ma_1^2 \sin^2\theta - 1 = 0$, $\sin\theta = \pm 1/Ma_1$ THIS MEANS THAT θ IS SMALL SO WE ARE ON THE BOTTOM CURVE.

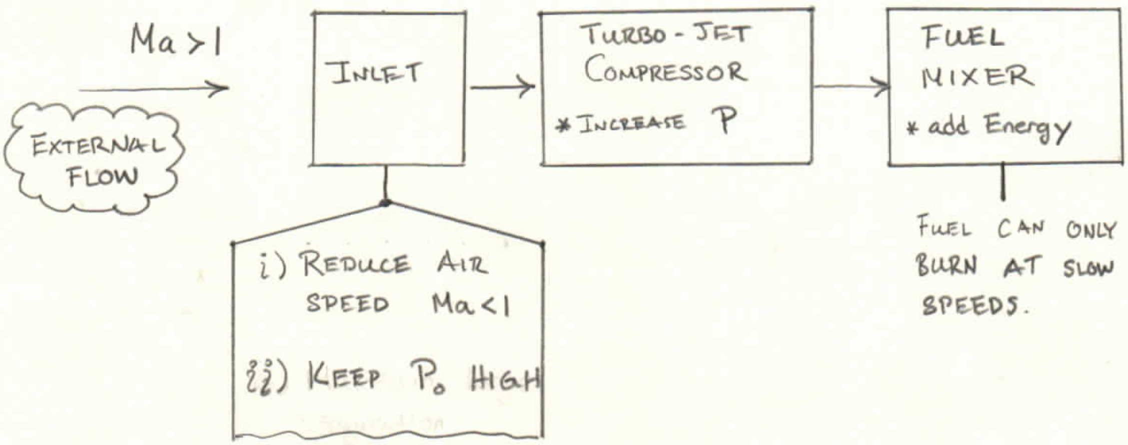
WE CAN USE THE <u>PRANDTL RELATION</u> TO SHOW

I. WEAK SHOCKS	II. STRONG SHOCKS
i) $1 - \varepsilon < Ma_2$	$Ma_2 < 1 - \varepsilon$
* FOR $\varepsilon \ll 1$	* FOR $\varepsilon \ll 1$
ii) $P_2/P_1 \sim 1$	$P_2/P_1 \ll 1$ (LARGE BACK PRESSURE)
iii) $\Delta S \sim 0$ (ISENTROPIC)	$\Delta S \gg 0$

THE APPLICATION OF THIS THEORY IS SUMMARIZED WITH A SUPERSONIC INLET. THE PURPOSE OF AN INLET IS TO SLOW FLOW DOWN.

WHY SLOW FLOW DOWN?

FIG (4) : SUPER SONIC JET ENGINE FUNCTIONAL DIAGRAM.

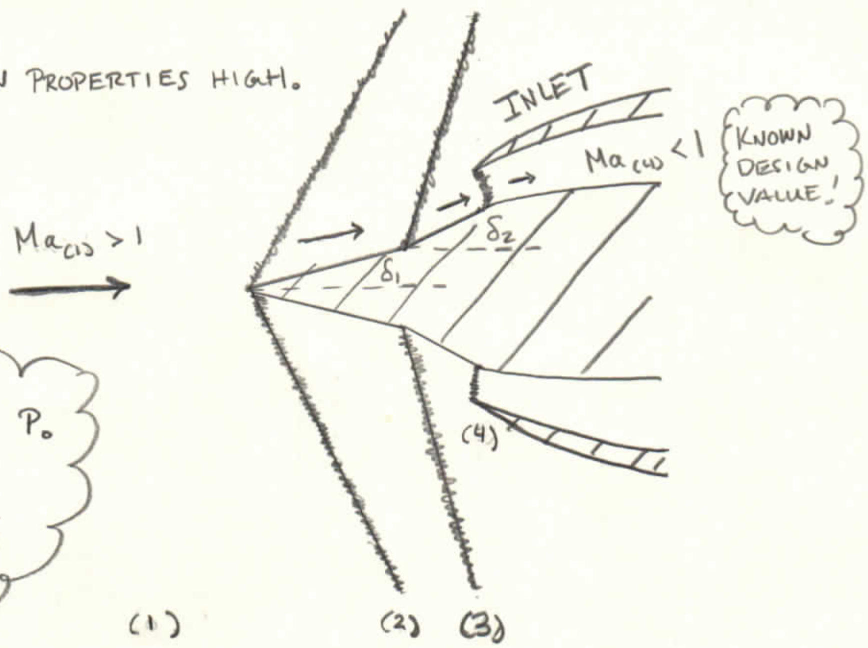


A JET ENGINE WORKS ON A BASIC PRINCIPLE. TAKE AIR AND THROW IT OUT THE BACK REALLY FAST.

NOW FOR SUPER SONIC JETS WE USE SHOCKS TO DECELERATE INCOMING FLOW TO Ma < 1

* WE WANT STAGNATION PROPERTIES HIGH.

$$\left.\frac{P_{o,4}}{P_{o,1}}\right| \text{LARGE!}$$



RECALL!

STAGNATION PRESSURE P_o IS A MEASURE OF AVAILABLE ENERGY FOR A GIVEN STATE OF FLOW

A STAGNATION PROPERTY IS DEFINED AS A PROPERTY ATTAINED IF FLOW AT STATE A IS BROUGHT TO REST ISENTROPICALLY!

USING MULTIPLE OBLIQUE SHOCKS DECELERATE FLOW WHILE MAINTAINING HIGH STAGNATION PRESSURE.

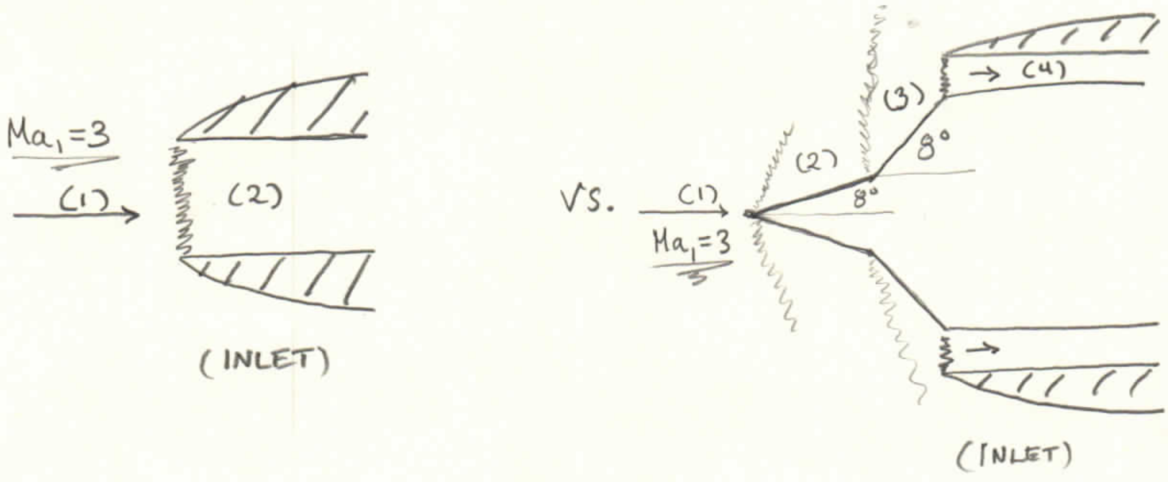
$$\frac{P_{o,4}}{P_{o,1}} = \frac{P_{o,4}}{P_{o,3}} \frac{P_{o,3}}{P_{o,2}} \frac{P_{o,2}}{P_{o,1}}$$

$$= \frac{P_{o,4}}{P_{o,3}} \left(\frac{P_{o,3}}{P_3} \frac{P_3}{P_2} \frac{P_2}{P_{o,2}} \right) \left(\frac{P_{o,2}}{P_2} \frac{P_2}{P_1} \frac{P_1}{P_{o,1}} \right)$$

YOU SERIOUSILY JUST USE TABLES TO DETERMINE

$$Ma_{(2)},\ Ma_{(3)},\ Ma_{(4)}$$

WHICH IDENTIFY ROWS IN TABLES IN BACKS OF BOOKS... THEN YOU JUST LOOK UP ALL THE PRESSURE RATIOS AND MULTIPLY A BUNCH OF NUMBERS. THE MAIN CALCULATION COMPARES A NORMAL SHOCK DIFFUSER AN THE WEDGE DESIGN.



$$\frac{P_{o,2}}{P_{o,1}} = 0.328 \quad < \quad \frac{P_{o,4}}{P_{o,1}} = 0.560$$

* OBLIQUE SHOCK DIFFUSER SLOW DOWN FLOW WHILE MAINTAINING LARGE STAGNATION INLET CONDITIONS!