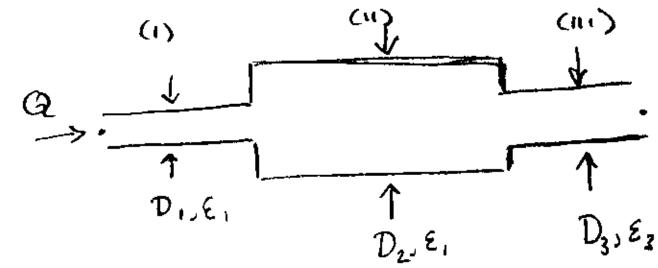
LECTURE 5: LOTS OF PIPES FLOWING TOGETHER

* SORRY IN REAL WORLD APPLICATIONS PIPE SYSTEMS USUALLY HAVE MORE THAN ONE SINGLE PIPE... WHICH IS ALL WE KNOW HOW TO ANALYZE... CRAP!

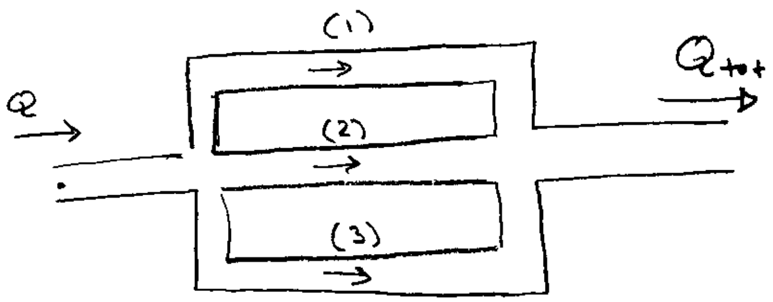
WE HAVE 3 MAIN CONNECTIONS TO CONSTRUCT PIPE SYSTEMS.

1) PIPES IN SERIES



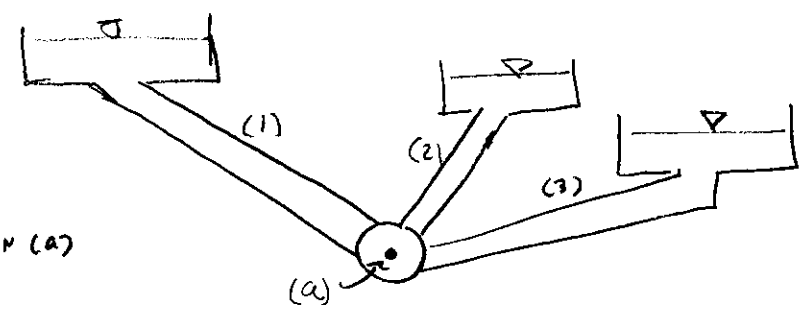
LAW: $Q_{1} = Q_{2} = Q_{III}$

2) PIPE IN PARALLEL



LAW: $Q_{tot} = Q_1 + Q_2 + Q_3$

3) PIPE JUNCTIONS



LAW: $Q_1 + Q_2 + Q_3 = 0$

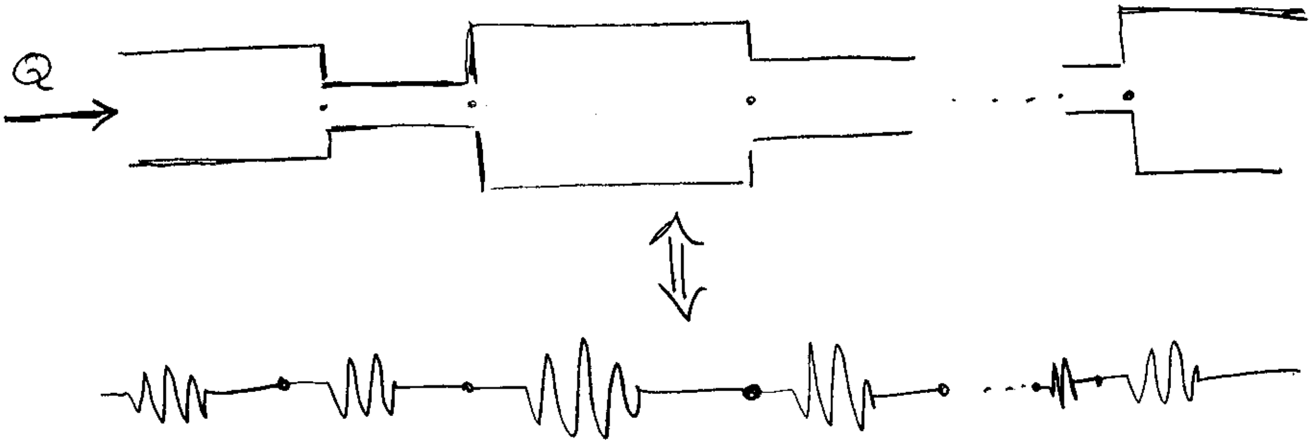
"STATIC PRESSURES AT JUNCTION (a) MUST BE EQUAL"

* WE NEED TO INVESTIGATE THESE 3 LAWS AND THEN DEVELOP METHODS TO SOLVE FOR (h_L, D, or Q) IN LARGE PIPE NETWORKS

LECTURE 5 : LOTS OF PIPES FLOWING TOGETHER

DISCLAIMER : UNTIL NOW I HAVE JUST BEEN REWRITING NOTES FROM OTHER PROFESSORS THAT HAVE TAUGHT THE COURSE. FOR THIS ONE LECTURE I AM BULKING UP THIS SECTION AS EVERYONE WANTED "REAL" WORLD PROBLEMS. REAL SYSTEMS HAVE MULTIPLE PIPES SO IF WE WANT TO SOLVE THESE TYPES OF PROBLEMS WE GOTTA UNDERSTAND THIS STUFF.

PIPES ARE RESISTORS!



WE GET A NON-LINEAR OHM'S LAW THOUGH.

$$Q_1 = Q_2 = \cdots \quad Q_N$$

(1) $$V_1 A_1 = V_2 A_2 = \cdots \quad V_N A_N$$ (CONSTANT FLOWRATE)

(2) $$\Delta h = \underbrace{\sum h_L}_{\text{MAJOR LOSSES DUE TO LONG STRAIGHT SECTION}} + \underbrace{\sum h_m}_{\text{MINOR LOSSES DUE TO EXPANION / CONTRACTION / FITTINGS}}$$

LECTURE 5: LOTS OF PIPES FLOWING TOGETHER

WE ALWAYS WANT TO CALCULATE Δh HEAD LOSS BECAUSE IT'S USUALLY WHAT WE DON'T KNOW WHEN IT COMES TO GETTING FLUID FROM POINT A TO B.

NON-LINEARITY

$$\Delta h = \sum_i^N f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_i^N K_i \frac{V_i^2}{2g}$$

OHMS LAW FOR ELECTRICITY SAYS $\Delta V = IR$. FOR PIPES THOUGH IT LOOKS LIKE $\Delta h = V^2 R$, WHICH IS NON-LINEAR. REMEMBER WE USE (1) TO FIGURE OUT VELOCITY VALUES V_i.

$$V_i = \frac{Q_{in}}{A_i}$$

SO

$$\Delta h = \sum_i^N f_i \frac{L_i}{D_i}\left(\frac{Q_{in}}{A_i}\right)^2 \frac{1}{2g} + \sum_i^N K_i \left(\frac{Q_{in}}{A_i}\right)^2 \frac{1}{2g}$$

WE CAN ALSO ALWAYS CALCULATE EFFECTIVE LENGTHS $\mathcal{L}_e$ FOR MINOR LOSSES. THIS JUST MAKES EVERY SUMMATION TERM IDENTICAL

SOME OF THESE ARE EFFECTIVE LENGTHS.

$$\Delta h = \left(\sum_i^{2N} \frac{f_i L_i}{D_i A_i^2 2g}\right) \cdot Q_{in}^2$$

LET'S LOOK AT THE SIMPLEST TYPE OF SERIES PROBLEM.

LECTURE 5: LOTS OF PIPES FLOWING TOGETHER

GIVEN: ALL PIPE GEOMETRY & Q_{in}

FIND: Δh

SOLUTION: THIS IS JUST BOOK-KEEPING, WE JUST NEED TO DETERMINE.

$$R_{eff} = \left(\sum f_i \cdot \frac{L_i}{\underbrace{D_i A_i^2}_{} 2g} \right)$$

For each component calculate an effective length for this reason

Remember each f requires a Moody Chart or Colebrook equation

For each pipe calculate the cross-sectional area,

! IF IT IS NON-CIRCULAR REMEMBER TO USE THE HYDRAULIC DIAMETER !

* ONCE WE GET ALL THESE TERMS WE CAN CALCULATE THE HEAD LOSS DIRECTLY.

$$\Delta h = R_{eff} \cdot Q_{in}^2$$

WHAT IF WE <u>MEASURED</u> ΔP ACROSS A SERIES OF PIPES, OR HAVE A CONSTRAINT ON ΔP AND WANT TO KNOW Q_{in}.

(THIS REQUIRES ITERATION)

LECTURE 5: LOTS OF PIPES FLOWING TOGETHER.

FIRST LETS RECOGNIZE THAT FOR ALL OF THESE QUESTIONS WE ARE ONLY LOOKING FOR ONE UNKNOWN. THAT IS BECAUSE THE HEAD EQUATION ~~IS~~ JUST ONE EQUATION. IF WE ARE TRYING TO DETERMINE MORE THAN ONE QUANTITY FROM ONE EQUATION WE WILL EITHER HAVE

a) ∞ SOLUTIONS

b) 0 SOLUTIONS $\leftarrow$ USUALLY NOT THE CASE.

GIVEN Δh AND EVERYTHING BUT ONE D_i

FIND D_i

SOLUTION: ONE EQN ONE UNKNOWN $\Rightarrow$ 1 UNIQUE SOLUTION FOR D_i

$$\overset{\checkmark}{\Delta h} = \underbrace{\left(\sum_i^{N-1} \frac{f_i L_i}{D_i A_i^2 2g}\right) Q_{in}^2}_{\text{ALL KNOWN QUANTITES}} + \underbrace{\frac{f_k L_k Q_{in}^2}{D_k A_k^2 2g}}_{\text{• THE ONE UNKNOWN DIAMETER}}$$

$$\Delta h - \sum_i^{N-1} \frac{f_i L_i}{D_i A_i 2g} Q_{in}^2 = \frac{f_k L_k Q_{in}^2}{D_k A_k^2 2g} = \frac{16 f_k L_k Q_{in}}{\pi^2 D_k^5 2g}$$

$$A_k = \frac{\pi D_k^2}{4} \Rightarrow A_k^2 = \frac{\pi^2 D_k^4}{16}$$

$$\therefore \quad D_k = \left[\left(\Delta h - \sum_i^{N-1} \frac{f_i L_i Q_{in}^2 16}{\pi^2 D_i^5 2g}\right) \cdot \frac{\pi^2 2g}{16 f_k L_k Q_{in}}\right]^{1/5}$$

LECTURE 5

* NOW LET'S SAY WE HAVE MULTIPLE UNKNOWN PIPE DIMENSIONS $\{D_1, D_2, D_3, \ldots D_K\}$ SAME SET UP.

$$\Delta h = \left(\sum_i^K \frac{f_i L_i 16}{\pi^2 2g D_i^5}\right) Q_{in}^2$$

$$= \left\{\sum_i^K \left(\frac{f_i L_i 8}{\pi^2 g D_i^5}\right)\right\} Q_{in}^2$$

GOAL WE JUST NEED TO FIND K NON-ZERO NUMBERS $D_1, D_2 \ldots D_K > 0$ SUCH THAT.

$$(*) \qquad 0 \stackrel{!}{=} \left\{\sum_i^K \left(\frac{f_i L_i 8}{\pi^2 g}\right) \frac{1}{D_i^5}\right\} Q_{in}^2 - \Delta h$$

SOLUTION : THE EASIEST WAY IS TO USE A MATLAB FUNCTION fsolve, OR A NUMPY PACKAGE np.fsolve, OR WRITE YOU'RE OWN SOLVER! THERE ARE MANY D_i's THAT SOLVE (*). YOU MAY NEED TO THROW-OUT NEGATIVE OR ZERO DIAMETER SOLS.

* KEEP IN MIND WE STILL NEED TO DETERMINE THE FRICTION FACTOR f_i FOR EACH SECTION! THATS WHY WE PRACTICED IT.

THE OTHER PROBLEM IS TO GET Q_{in}. SAME SET-UP THOUGH.

$$0 = \sum_i^K f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} - \Delta h$$

SAME IDEA ONLY NOW YOU ARE SOLVING FOR f_i VALUES

LECTURE 5: LOTS OF PIPES FLOWING TOGETHER

FLOWRATE PROBLEMS ARE TRICKY JUST BECAUSE RECALL WE NEED REYNOLDS #'S TO SOLVE FOR f's. REYNOLDS NUMBER REQUIRE VELOCITY.

IF WE WRITE EQUATION IN TERMS OF Q_{in} WE SEE THE PROBLEM REDUCES TO FINDING f's.

$$\Delta h = \sum_i^K f_i \frac{L_i}{D_i}\frac{V_i^2}{2g}$$

$$V_i = \frac{Q_{in}}{A_i} = \frac{4\,Q_{in}}{\pi D_i^2}$$

$$V_i^2 = \frac{16\,Q_{in}^2}{\pi^2 D_i^4}$$

$$\therefore \Delta h = \left(\sum_i^K \frac{8 f_i L_i}{\pi^2 D_i^5 g}\right)\cdot Q_{in}^2$$

SO OUR ITERATIONS GO LIKE THIS.

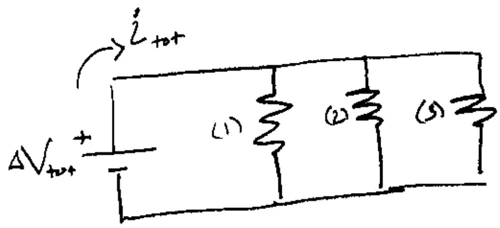
1) GUESS A $Q_{in} = Q_0$
2) Calculate all f_i's based on Q_0
3) $Q_K = \left(\dfrac{\Delta h}{\sum \frac{8 f_i L_i}{\pi^2 D_i^5 g}}\right)^{1/2}$ Where f_i's Come from Q_{K-1}
4) STOP WHEN $Q_K \simeq Q_{K-1}$

* THIS IS SLIGHTLY DIFFERENT THAN THE PROCESS GIVEN IN THE NOTE "process.pdf." BOTH ARE EQUIVALENT.

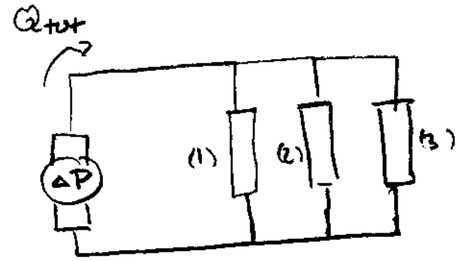
Lecture 5: Lots of Pipes Flowing Together

Parallel Pipes

* Pipes in Parallel are analogous to Resistors in Parallel.



$\approx$



$$\dot{i}_{tot} = \dot{i}_1 + \dot{i}_2 + \dot{i}_3$$

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V$$

Voltage drop across each resistor is equal ΔV_{tot}

$$Q_{tot} = Q_1 + Q_2 + Q_3$$

$$\Delta h_{tot} = \Delta h_1 = \Delta h_2 = \Delta h_3$$

Head loss across each pipe is equal to Δh_{tot}

Head Loss Problem (Complicated for Parallel Pipes)

Given Q_{tot} and pipe dimensions $(L_i, D_i, \varepsilon_i, \mu, \rho)$

Find Δh

Solution: Iterative.

1) Guess Q_i such that
   $Q_{tot} = \sum Q_i$

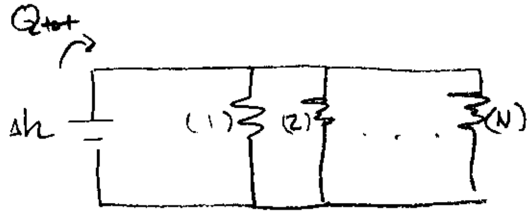
2) Compute h_i for each pipe

3) Are all h_i equal

   Yes? $\Rightarrow$ Stop
   No? $\Rightarrow$ Update new Q_i, back to 2)

LECTURE 5 : LOTS OF PIPES FLOWING TOGETHER

PARALLEL Q PROBLEMS



GIVEN : $h, L_i, D_i, \varepsilon_i, \rho, \mu$

FIND : Q_{tot}

SOLUTION : SINCE Δh IS THE SAME FOR EACH BRANCH THIS PROBLEM REDUCES TO N-SINGLE PIPE PROBLEMS, REFER TO "process.pdf" FOR BASIC FLOW RATE PROBLEM

FOR $i = 1 : N$ (SOLVE N PROBLEMS)

1. GUESS f_i VALUE $f_i = f_0$
2. $$V_i = \sqrt{\frac{2 g h D_i}{f_i L}}$$
3. COMPUTE Re
4. FIND f_{new} WITH Re FROM STEP 3
5. IF $f_{new} \approx f_{old}$ STOP ; ELSE
   $f \Leftarrow f_{new}$
   RETURN TO STEP 2

ONE MORE MULTI-PIPE PROBLEM!

LECTURE 5 : LOTS OF PIPES FLOWING TOGETHER

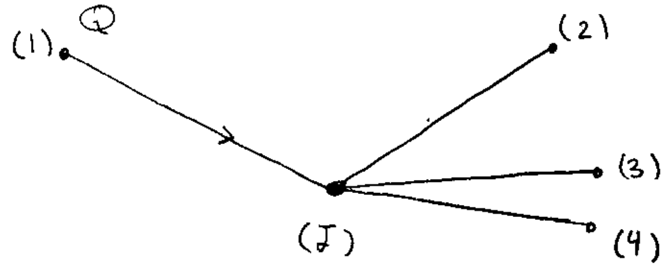
JUNCTIONS

$$Q_{1\to J} = Q_{J\to 2} + Q_{J\to 3} + Q_{J\to 4}$$

or

$$\underbrace{0 = \sum Q_{i\to J}}_{} = \sum V_i A_i$$

SUM OF ALL FLOWRATES AT A JUNCTION MUST BE ZERO!



THE OTHER EQUATION WE GET IS ENERGY (BERNOULLI) FROM (1) $\to$ (K).

(1) $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum h_{major} + \sum h_{minor}$$

$\vdots$

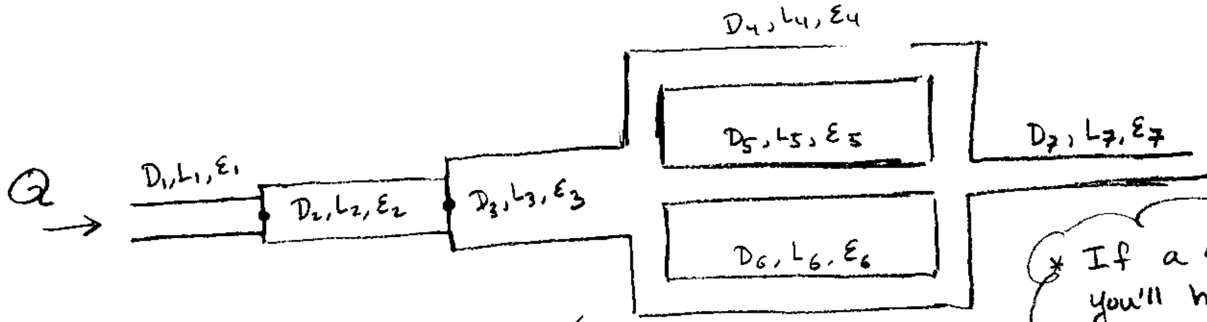
(3) $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_4}{\rho g} + \frac{V_4^2}{2g} + z_4 + \sum h_{major} + \sum h_{minor}$$

(4) $$0 = \sum V_i A_i$$

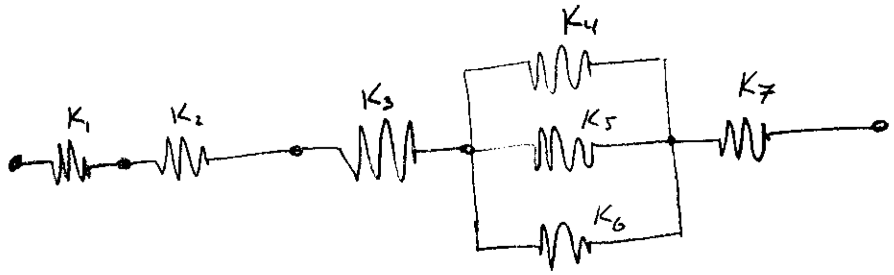
* I CAN'T STRESS THIS ENOUGH THAT IS IMPORTANT TO APPRECIATE THAT WE ALL LOOKING FOR 4 VALUES $\{V_1, V_2, V_3, V_4\}$ AND WE'VE WRITTEN 4 EQUATION! SO WE KNOW THAT A SOLUTION EXISTS.

LECTURE 5 : LOTS OF PIPES FLOWING TOGETHER

FOR ANY PIPE PROBLEM MOST OF THE WORK IS IN BOOK-KEEPING PIPE DIMENSIONS TO CALCULATE FLOW RESISTENCES AND ACCOUNTING FOR MINOR LOSSES (LOT'S OF TABLES...)



* If a pipe is non-circ you'll have to calculate hydraulic diameters on top of this book-keeping



FLOW-RESISTANCE IS A FUNCTION OF PIPE GEOMETRY!

$$\Delta h = \underbrace{\frac{8fL}{\pi^2 D^5 g}} Q^2$$

$$\Delta h = k \cdot Q^2$$

THIS TERM IS CALLED THE RESISTANCE.

SO A GREAT FIRST-STEP IS TO CALCULATE ALL THESE NUMBERS K_i FOR EACH PIPE. IF YOU FIND OUT THAT YOU CAN'T CALCULATE A K BECAUSE YOU'RE MISSING A DIMENSION D_i THEN YOU KNOW YOU HAVE A PIPE SIZING PROBLEM.

LECTURE 5 : LOTS OF PIPES FLOWING TOGETHER

* SO PROJECT 1 WILL BE A FULL ANALYSIS FOR A MULTI-PIPE SYSTEM.

I NEED TO GET WATER FROM THE RESERVOIR UP TO THE HOSPITAL, ONCE IT'S THERE I NEED TO ESTIMATE THE PUMP I NEED TO DISTRIBUTE WATER TO 3 MAIN LEVELS.

OUR INTERN WENT OUT AND TOOK A FULL LEDGER OF THE PIPE SYSTEM DIMENSION'S } EXCEL DATA SHEET

THE HOSPITAL HAS A BUDGET FOR SOME REMODELING AS WELL. THEY WANT TO KNOW IF THE COST OF A REMODEL WILL BE WORTH IT. PUMPS COST ENERGY = \$.

REMODELING WOULD CHANGE ALL THE VALUES OF ε ROUGHNESS AND SOME DIAMETERS D (WE CAN'T ALTER PIPE LENGTHS)

THEY WANT AN ANSWER IN 1 MONTH TO SO WE DON'T HAVE TIME TO RUN A FULL BUILDING SIMULATION WITH CFD.

MORE DETAILS TO COME! BUT THIS IS WHERE WE'RE HEADED AND THE TYPE OF PROBLEM THIS JUNIOR YEAR ENGINEERING COURSE CAN HELP YOU SOLVE