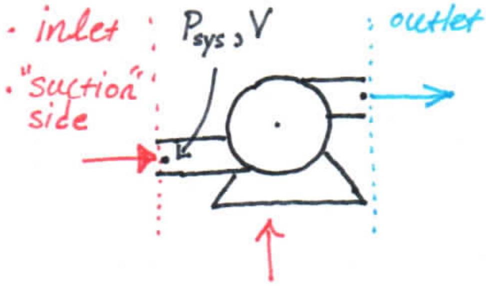
LECTURE 12: Okay, But What Pump Do I Need To Buy...

So we have a simple model of a catrifugal pump that allows us to calculate a h_{pump} for an ideal case. The story isn't done because pumps also must prevent...



WE DONT WANT CAVITATION TO HAPPEN HERE

CAVITATION

WHY SHOULD PREVENT CAVITATION?

i) Pressures become low enough liquid in pump begins to BOIL! BOILING = BUBBLES. Bubbles suck for pump systems

ii) The location of cavitation experiences large acoustic pressure waves ⟺ structural damage.

When does liquid "cavitate" or begin to boil?

$$P_{sys} \overset{!}{=} P_{vapor}$$

So we will define a variable that measures how close we are to this cavitation limit. We call it the NPSH (Net Positive Suction Head)

$$NPSH = \underbrace{\frac{P_1}{\rho g} + \frac{V_1^2}{2g}}_{\text{Head at inlet (suction) side of pump}} - \underbrace{\frac{P_v}{\rho g}}_{\text{Vapor head of liquid, found in table for working liquid}}$$

* If NPSH < 0 Cavitation Occurs! That's why we call it net positive suction head, It should always be positive!

LECTURE 12:

Now we never want NPSH to be close to zero or else small perturbations could cause cavitation, which is hard to stop once it begins.

$$NPSH > \varepsilon > 0$$

↑ How big should this number be?

Once you've made your pump you run experiments to see when cavitation occurs in reality. This value is called called the (Required Net Positive Head) $NPSH_R > 0$.

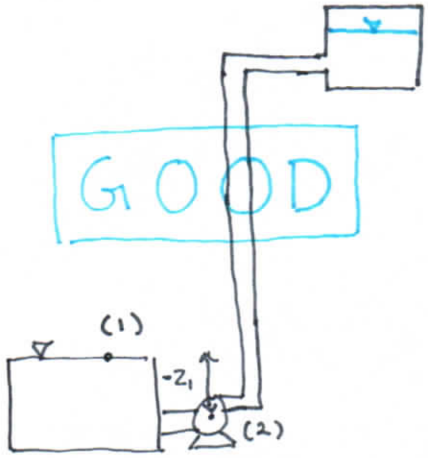
$$\underbrace{NPSH_A}_{\text{What you actually run the pump at}} \geq \underbrace{NPSH_R}_{\text{A lower limit given by the manufacturer or by experiment}}$$

- * This has to be true of all parts in a piping system
- * $NPSH_R$ increases with Q
- * $NPSH_A$ decreases with Q

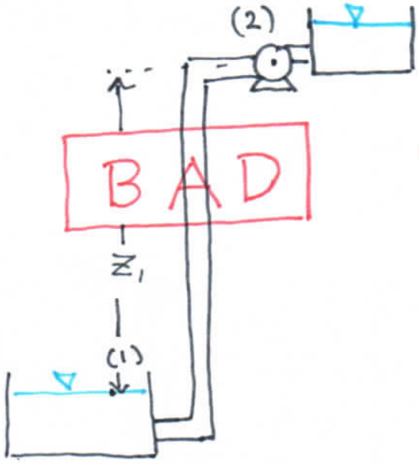
The NPSH condition must be checked at maximum flowrate for a piping system.

$$NPSH_A \geq NPSH_R \quad @ \; Q_{max}$$

Elevation changes greatly effect $NPSH_A$!



VS.



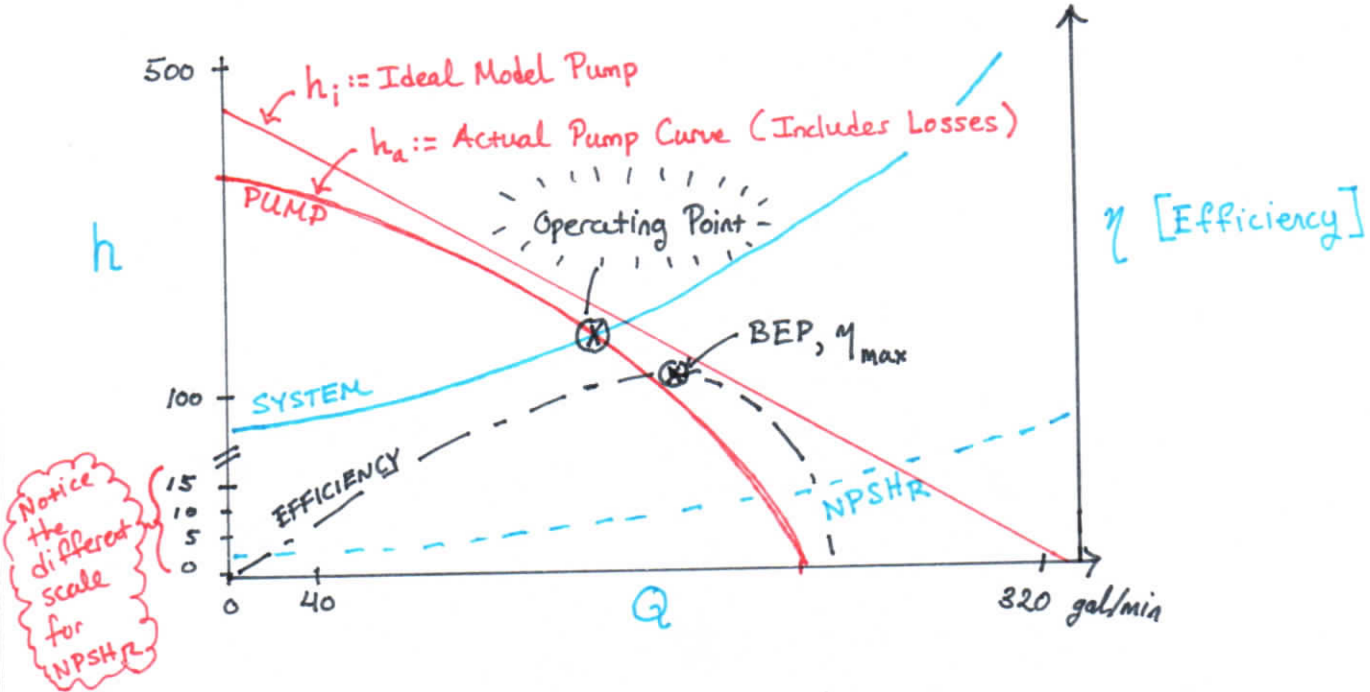
usually tiny ↓

$$NPSH_A = \frac{P_1}{\rho g} - z_1 - \sum h_L - \frac{P_v}{\rho g}$$

↑ can greatly reduce $NPSH_A$

LECTURE 12:

We almost have all the curves now to populate our pump matching plot.



WOW THAT'S A WHOLE LOTTA CURVES! BUT WE GET MOST OF THEM BY NOW

$NPSH_R$ ⇒ Make sure all other curves are FAR above this curve.

PUMP ⇒ Use line at first based on pump parameters, then experiment to get h_a to includ losses

SYSTEM ⇒ This is what you did you're entire project calculating losses in the system. Remember in terms of flowrate you could calculate resistence values and in the end it looks like.

$$h_{sys} = \Delta Z + R_{eff} \cdot Q^2$$

EFFICIENCY ⇒ This is a measure of

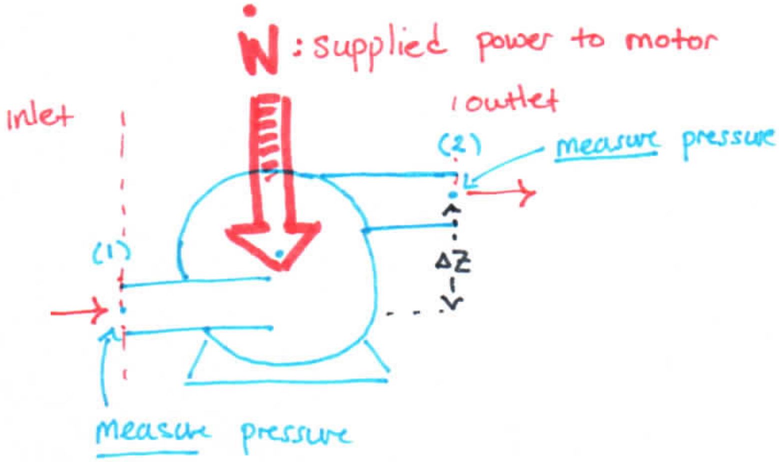
$$\eta = \frac{\text{Power Gained By Fluid}}{\text{Power Supplied to Shaft}}$$

LECTURE 12 :

The efficiency curve describes how much money you're wasting. It costs you to turn the shaft. It's up to mother nature to see how much energy she actually takes from that torque.

$$h_a = \frac{\Delta P}{\rho g} + \cancel{\Delta z} + \cancel{\frac{\Delta V^2}{2g}}$$

≈0 in usual pump designs



We define efficiency as naturally.

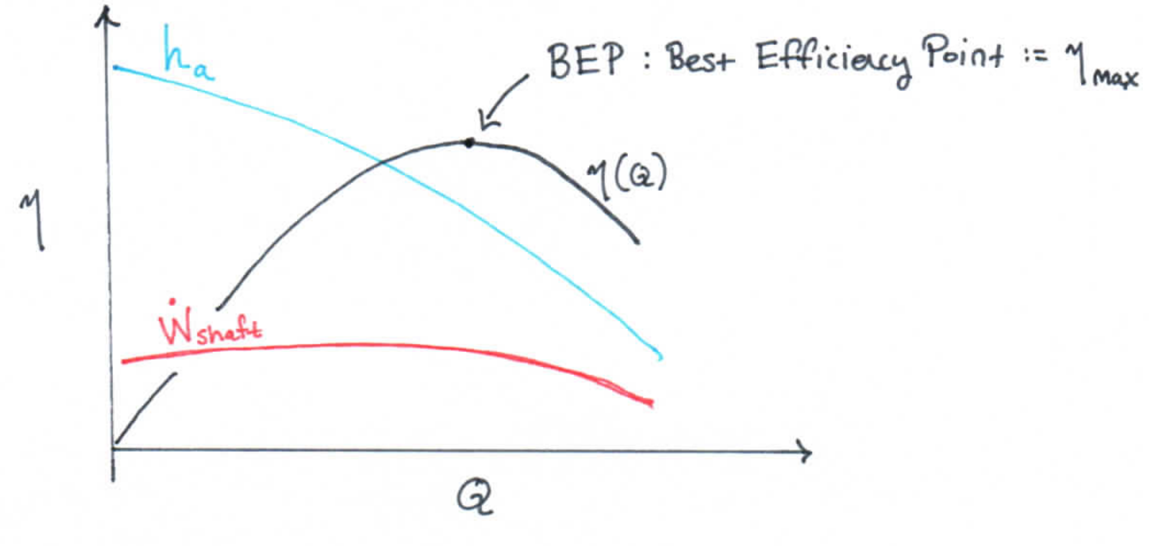
$$\eta = \frac{P_f}{\dot{W}_{shaft}} := \frac{\text{"What you get"}}{\text{"What you pay for"}}$$

Where,

[S I] $\quad P_f = \rho g Q h_a \qquad h_a :=$ actual head rise provided by pump

[Barley Corn Units] $\quad P_f = \dfrac{\rho g Q h_a}{550} = [hp] \qquad h_a = [ft]$, $\rho g := [lb/ft^3]$, $Q = [ft^3/s]$

This produces an efficiency plot which produces an $\eta(Q)$ curve.

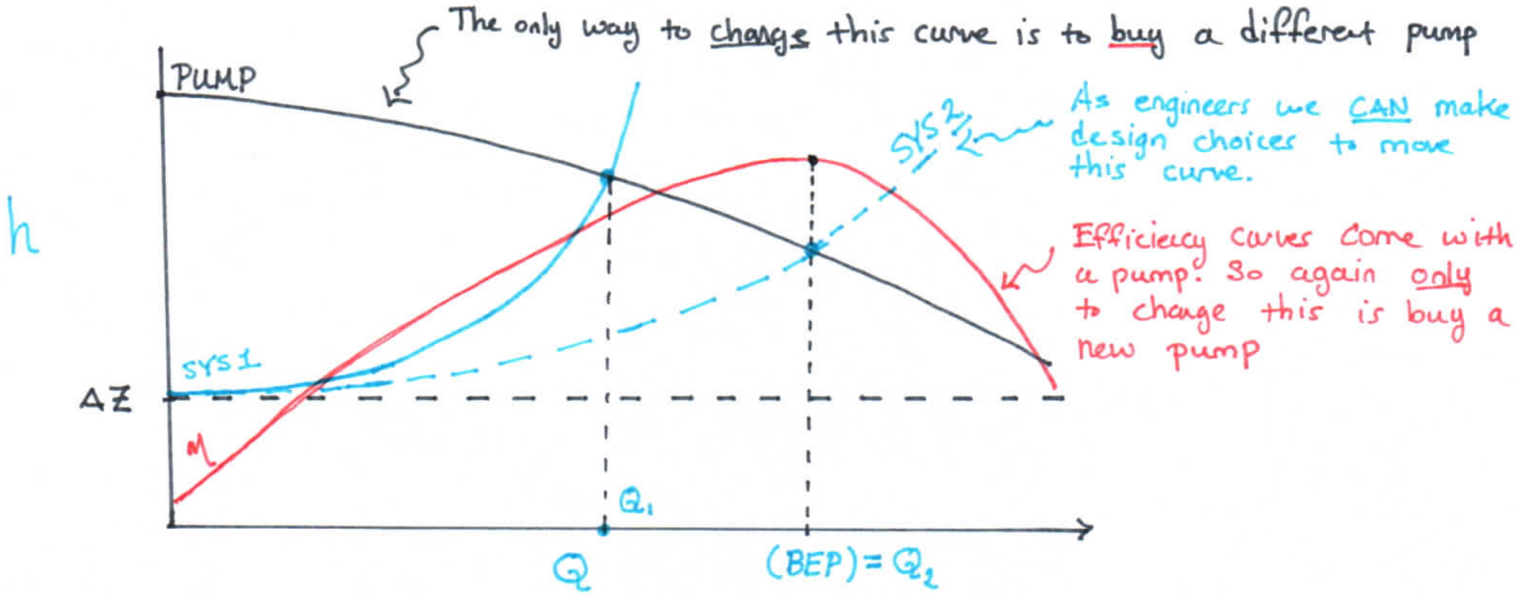


LECTURE 12:

WE NOW CAN DEFINE THE <u>GOAL</u> OF PUMP SELECTION.

* MAKE OPERATING FLOW RATE = BEP

Graphically this is fun.



The only way to <u>change</u> this curve is to <u>buy</u> a different pump

As engineers we <u>CAN</u> make design choices to move this curve.

Efficiency curves come with a pump. So again <u>only</u> to change this is buy a new pump

Let's see just how this system curve changes mathematically. As <u>always</u> we are moving fluid from point A to point B. Energy equation says.

$$\left(\cancel{\frac{P}{\rho g}} + z + \cancel{\frac{V^2}{2g}}\right)_A = \left(\cancel{\frac{P}{\rho g}} + z + \cancel{\frac{V^2}{2g}}\right)_B + \underbrace{\sum h_L}_{\text{Major + Minor}} - h_p$$

* open tank to open tank usually
$P_A = P_{atm} = P_B$ and $V_A = V_B = 0$

$$h_{\substack{sys \\ \cancel{pump}}} = \Delta Z + \sum h_L$$

$$= \Delta Z + \sum h_{major} + \sum h_{minor}$$

$$\boxed{h_{sys} = \Delta Z + K_{tot} Q^2}$$, K_{tot} := total system flow resistence

You did an <u>entire</u> project calculating system resistance values. You should get a sense now of <u>HOW MANY VARIABLES GO INTO IT</u>

LECTURE 12

Recall just how many ways there are to change K_{tot}.

$$K_{TOT} = \sum_{i}^{N} \frac{f_i L_i 8}{\pi^2 g D_i^5}$$

some of these are L_{eff} coming from Minor losses

Extra Credit ~

Calculate system sesitivity vector

a) $\nabla_L K_{TOT}$ $\quad \nabla_L := \{\sum \frac{\partial}{\partial L_i}$

b) $\nabla_D K_{TOT}$ $\quad \nabla_D := \langle \frac{\partial}{\partial D_1}, \frac{\partial}{\partial D_2} \cdots \rangle$

c) $\nabla_f K_{TOT}$ $\quad \nabla_f := \langle \frac{\partial}{\partial f_1}, \frac{\partial}{\partial f_2}, \cdots \rangle$

Functionally $f(Re, \varepsilon/D)$ so these derivatives are actually complicated. Look at just <u>one</u>.

$$\frac{\partial}{\partial D}\left\{\frac{f(Re, \frac{\varepsilon}{D}) \cdot L \cdot 8}{\pi^2 g D^5}\right\}$$

$$\frac{\frac{\partial f}{\partial D} \cdot L \cdot 8}{\pi^2 g D^5} + \frac{f L 8}{\pi^2 g} \frac{\partial}{\partial D}\left(\frac{1}{D^5}\right)$$

* Product Rule

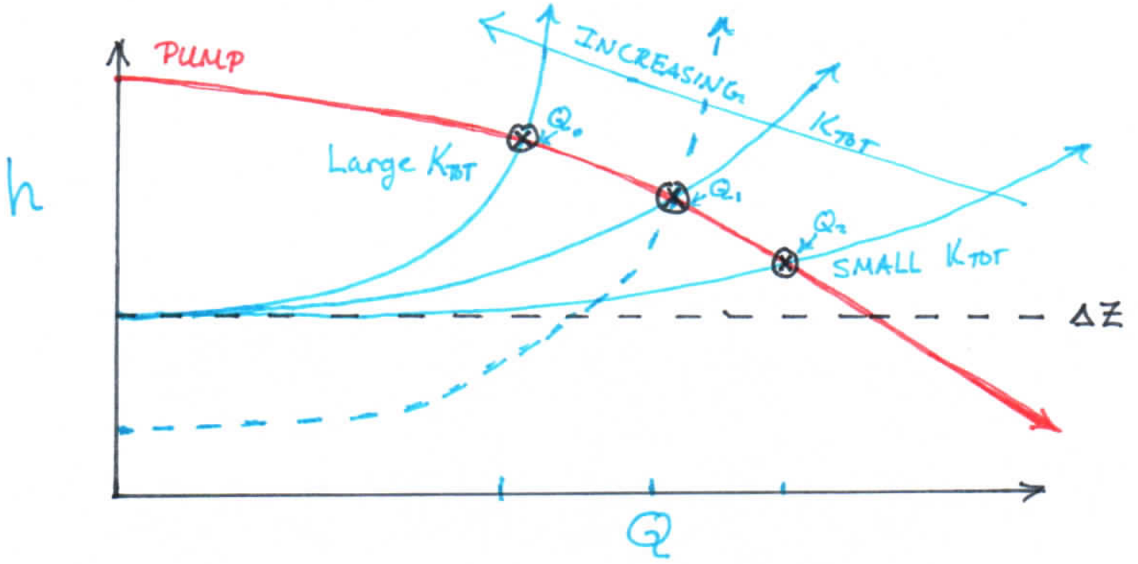
$$\frac{8\left(\frac{\partial Re}{\partial D}\frac{\partial f}{\partial Re} + \frac{\partial \varepsilon/D}{\partial D}\frac{\partial f}{\partial \varepsilon/D}\right)L}{\pi^2 g D^5} - \frac{40 f L}{\pi^2 g D^6}$$

* Linearization for multivariable functions.

WITHOUT CALCULATING DERIVATIVES YOU SHOULD STILL HAVE A SENSE OF HOW TO CHANGE K_{TOT} BY DESIGN CHOICES.

a) Change Diameter
b) new material
c) Shorten pipe length
d) change Components K_i
e) friction factors

JUST TO BEAT A DEAD HORSE YOU SHOULD UNDERSTAND WHAT K_{TOT} DOES TO SYSTEM CURVE, ALSO WHAT ΔZ DOES.



Q_0 : Large K_{TOT} values drive the operating point up the pump curve

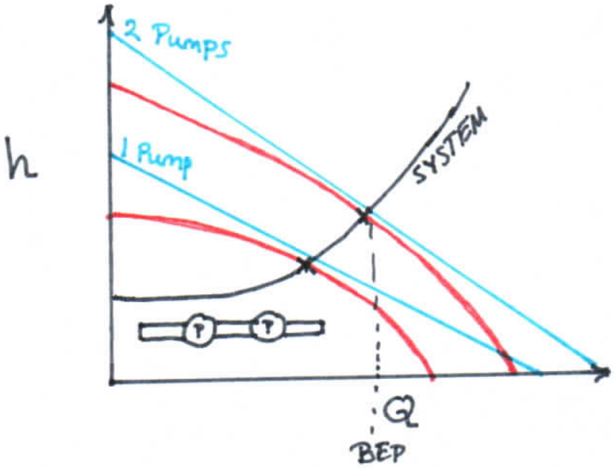
Q_1 : NOTICE THAT WE COULD ACHIEVE THIS OPERATING POINT BY LOWERING K OR Redesigning so ΔZ is lower for the large K system. You usually can't do this though because of Constraints OR $NPSH_R$ IS violated.

Q_2 : lowering K_{TOT} drives the operating point down the pump curve.

We can also achieve Q_0 = BEP by using multiple pumps in series or parallel. This requires more budget but sometimes pumps are cheaper.

LECTURE 12

Series pumps raise the entire pump curve.

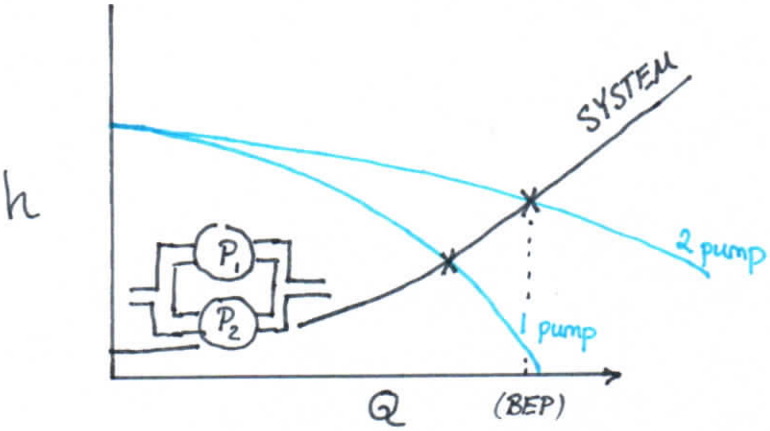


$$\left.\begin{aligned} h_1 &= \frac{U_{exit}^2}{g} - \frac{U_{exit}\cot\beta_2}{2\pi r_2 b_2 g} Q_1 \\ h_2 &= \frac{\tilde{U}_{exit}^2}{g} - \frac{\tilde{U}_{exit}\cot\beta_2}{2\pi r_2 b_2 g} Q_2 \end{aligned}\right\} \text{Ideal Model}$$

$Q = Q_1 = Q_2$ * Constant flow rate in series Pipes

$$\therefore\ h_1 + h_2 = h_p = \underbrace{\frac{U^2 + \tilde{U}^2}{g}}_{\text{* y intercept increases}} - \underbrace{\frac{(U+\tilde{U})\cot\beta_2}{2\pi r_2 b_2 g}}_{\text{*slope becomes steeper}} Q$$

Parallel pumps flatten the curve.

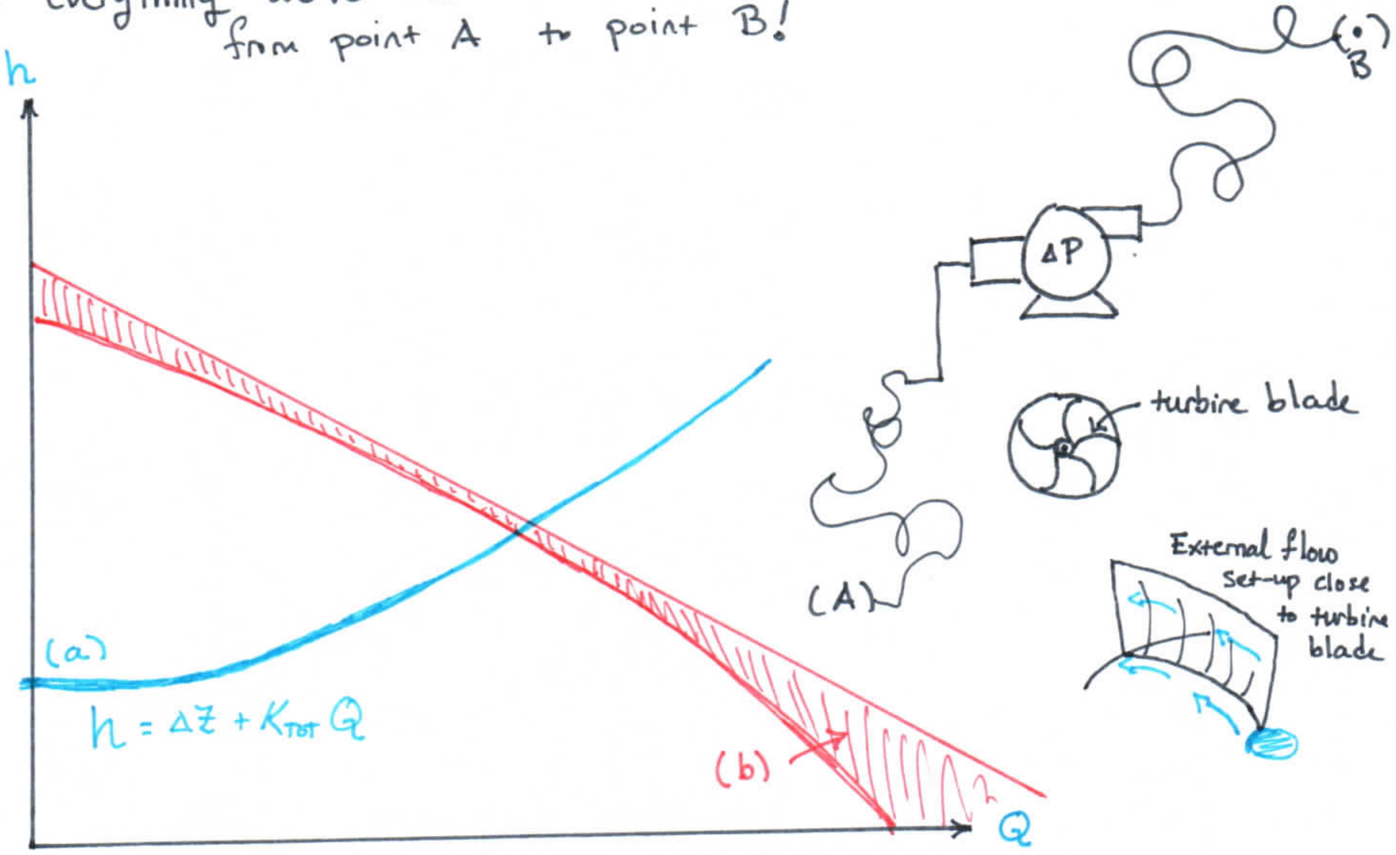


extra credit derive the ideal summation of the pumps. Recall parallel pipe laws.

So you can see we as engineers have multiple methods to make sure our pump is operating at Maximum Efficiency. This amounts to moving the intersection point of system + pump curves closer to the (BEP) η_{max} of our pump.

LECTURE 12:

Now I really like to always draw everything back to a why and how. We can see the applications of everything we've studied so far. We want to get fluid from point A to point B!



(a) We had to study internal pipe flow to develop methods for estimating K_{TOT} before building anything

$$K_{TOT} = \frac{8 f(Re, \varepsilon/D) L}{\pi^2 g D^5}$$

really all our math helped us develop equations for this thing.

(b) External flow actually helps us describe why the pump curve is not a line. Seperation, skin friction, and 3D vorticies.

* You can always take measurements of your actual pump impellar and use the ideal equation to approximate.

LECTURE 12:

Pump Selection Summary

1) Estimate K_{TOT} some way

2) Plot a system curve, look for Pump Curves...

3) Make sure $NPSH_A \geq NPSH_R$

4) Look up efficiency curve for the pump you're considering. Determine (BEP)

5) Engineer the design to get as close as you can to the BEP.

$$\min_{\{\text{all variables}\}} (Q - Q_{BEP})^2$$

Options to achieve this are...

1) Alter pipe dimensions to change K_{TOT}
2) Pumps in parallel?
3) Pumps in series
4) Use a different pump? (Repeat from 3)
5) Buy a pump that with a closer Q_{BEP}?
6) Use your imagination...