Three Practice Problems for Compressible Flow

11.37 Determine the static pressure to stagnation pressure ratio associated with the following motion in standard air: **(a)** a runner moving at the rate of 10 mph, **(b)** a cyclist moving at the rate of 40 mph, **(c)** a car moving at the rate of 65 mph, **(d)** an airplane moving at the rate of 500 mph.

With a value of Mach number calculated with

$$Ma = \frac{V}{c} \qquad (1)$$

we can calculate $\frac{p}{p_0}$ with

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]^{\frac{k}{k-1}} \qquad (11.59)$$

For c we use for parts a, b and c

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(519\ {}^\circ R)(1.40)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = 1117\ \frac{ft}{s}$$

or

$$c = \left(1117\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = 761.6\ mph$$

(a) For $V = 10\ mph$

$$Ma = \frac{10\ mph}{761.6\ mph} = 0.0131$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.0131)^2}\right]^{\frac{1.4}{1.4-1}} = \left[\frac{1}{1+(0.2)(0.0131)^2}\right]^{3.5} = 0.99988$$

(b) For $V = 40\ mph$

$$Ma = \frac{40\ mph}{761.6\ mph} = 0.0525$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0525)^2}\right]^{3.5} = 0.998$$

(c) For $V = 65\ mph$

$$Ma = \frac{65\ mph}{761.6\ mph} = 0.0854$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0854)^2}\right]^{3.5} = 0.9949$$

(d) For airplane we assume a nominal altitude of 30,000 ft. From Table C.1 we note a corresponding temperature of $-47.83\ {}^\circ F$. Then

$$c = \sqrt{\frac{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)\left[(-47.83+460)\ {}^\circ R\right](1.4)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}}$$

$$c = 995\ \frac{ft}{s}$$

or

$$c = \left(995\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = 678\ mph$$

Then for

$$Ma = \frac{500\ mph}{678\ mph} = 0.738$$

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.738)^2}\right]^{3.5} = 0.696$$

11.39 The stagnation pressure and temperature of air flowing past a probe are 120 kPa (abs) and 100 °C, respectively. The air pressure is 80 kPa (abs). Determine the air speed and Mach number considering the flow to be **(a)** incompressible; **(b)** compressible.

(a) Assuming incompressible flow we use Bernoulli's equation (Eq. 3.7) to connect the static and stagnation states and get

$$V = \sqrt{\frac{2(P_0 - P)}{\rho_0}} \qquad (1)$$

With the ideal gas equation of state (Eq. 1) we obtain

$$\rho_0 = \frac{P_0}{RT_0} \qquad (2)$$

and combining Eqs. 1 and 2 we obtain

$$V = \sqrt{\frac{2(P_0 - P)RT_0}{P_0}}$$

or

$$V = \sqrt{\frac{2\,[120\ \text{kPa(abs)} - 80\ \text{kPa(abs)}]\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(373\ K)}{[120\ \text{kPa(abs)}]\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = \underline{\underline{267\ \frac{m}{s}}}$$

For Mach number we need

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (3)$$

To determine T we use the equation of motion (Eq. 11.54) to obtain

$$T = T_0 - \frac{V^2(k-1)}{2kR} = 373K - \frac{\left(267\ \frac{m}{s}\right)^2(1.4-1)}{2(1.4)\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}$$

or $T = 337.5\ K$

With Eq. 3 we obtain

$$Ma = \frac{267 \frac{m}{s}}{\sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \frac{(337.5 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = \underline{\underline{0.725}}$$

(b) For compressible flow

$$\frac{p}{p_0} = \frac{80 \text{ kPa(abs)}}{120 \text{ kPa (abs)}} = 0.67$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.78}}$$

Also from Fig. D.1 we read

$$\frac{T}{T_0} = 0.89$$

and thus

$$T = (0.89)(373 K) = 332 \ K$$

Thus,

$$V = Ma\sqrt{RTk} = (0.78)\sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \frac{(332 \ K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

and

$$V = \underline{\underline{285}} \ \frac{m}{s}$$

11.45 An ideal gas is to flow isentropically from a large tank where the air is maintained at a temperature and pressure of 59 °F and 80 psia to standard atmospheric discharge conditions. Describe in general terms the kind of duct involved and determine the duct exit Mach number and velocity in ft/s if the gas is air.

To determine the duct exit Mach number, Ma_{exit}, we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{P_{exit}}{P_0}\right)^{\frac{k-1}{k}}} - 1\right]\left(\frac{2}{k-1}\right)} \qquad (1)$$

or for air

$$Ma_{exit} = \text{Fig. D.1 value as a function of } \frac{P_{exit}}{P_0} \qquad (2)$$

To determine exit velocity, V_{exit}, we use

$$V_{exit} = (Ma_{exit})\, c_{exit} = Ma_{exit}\sqrt{R\,T_{exit}\,k} \qquad (3)$$

where

$$T_{exit} = \frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma_{exit}^2} \qquad (4)$$

or for air

$$T_{exit} = T_0\left(\frac{T_{exit}}{T_0} \text{ value from Fig. D.1 for } Ma_{exit}\right) \qquad (5)$$

$$\frac{P_{exit}}{P_0} = \frac{14.7\text{ psia}}{80\text{ psia}} = 0.1838$$

and thus from Fig. D.1, the corresponding values are

$$Ma_{exit} = \underline{\underline{1.8}}$$

and

$$\frac{T_{exit}}{T_0} = 0.62$$

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Then with Eq. 5 we obtain

$$T_{exit} = (519\,°R)(0.62\ ) = 322\,°R$$

and with Eq. 3 we conclude that

$$V_{exit} = (1.8)\sqrt{\frac{\left(1716\ \frac{ft\cdot lb}{slug\cdot °R}\right)(322\ °R)(1.4)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{1580\ \frac{ft}{s}}}$$

A converging-diverging nozzle is required because the exit flow is supersonic.