6.5 Determine the vorticity field for the following velocity vector:

$$\mathbf{V} = (x^2 - y^2)\hat{\mathbf{i}} - 2xy\hat{\mathbf{j}}$$

$$\nabla \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k},$$

where

$u = x^2 - y^2$, $v = -2xy$, and $w = 0$

Thus,

$$\nabla \times \vec{V} = 0\hat{i} + 0\hat{j} + \left[\frac{\partial}{\partial x}(-2xy) - \frac{\partial}{\partial y}(x^2 - y^2)\right]\hat{k}$$

$$= [-2y - (-2y)]\hat{k} = 0\hat{k}$$

Hence,

$$\nabla \times \vec{V} = \underline{\underline{0}}$$

6.16 The stream function for an incompressible, two-dimensional flow field is

$$\psi = ay - by^3$$

where a and b are constants. Is this an irrotational flow? Explain.

For the flow to be irrotational,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and for the stream function given,

$$u = \frac{\partial \psi}{\partial y} = a - 3by^2$$

$$v = -\frac{\partial \psi}{\partial x} = 0$$

Thus,

$$\frac{\partial u}{\partial y} = -6by \qquad \frac{\partial v}{\partial x} = 0$$

so that

$$\omega_z = \frac{1}{2}\left[0 - (-6by)\right] = 3by$$

Since $\omega_z \neq 0$ flow is not irrotational (unless $b = 0$). $\underline{\underline{\text{No}}}$.

6.25 The stream function for an incompressible flow field is given by the equation

$$\psi = 3x^2y - y^3$$

where the stream function has the units of m^2/s with x and y in meters. **(a)** Sketch the streamline(s) passing through the origin. **(b)** Determine the rate of flow across the straight path AB shown in Fig. P6.25.

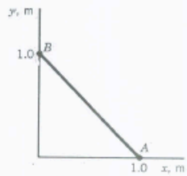
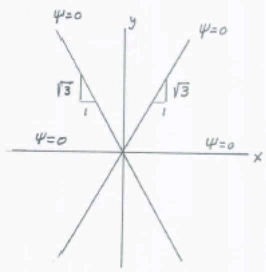


FIGURE P6.25

(a) Lines of constant ψ are streamlines. For $\psi = 3x^2y - y^3$ the streamline passing through the origin $(x=0, y=0)$ has a value $\psi = 0$. Thus, the equation for the streamlines through the origin is



$$0 = 3x^2y - y^3$$

or

$$y = \pm\sqrt{3}\,x$$

A sketch of these streamlines is shown in the figure.

(b) $q = \psi_B - \psi_A$

At B $x=0$, $y = 1\,m$ so that

$$\psi_B = 3(0)^2(1) - (1)^3 = -1\ m^3/s \quad (\text{per unit width})$$

At A $x = 1\,m$, $y = 0$ so that

$$\psi_A = 3(1)^2(0) - (0)^3 = 0$$

Thus,

$$q = \psi_B = \underline{-1\ m^3/s \quad (\text{per unit width})}$$

The negative sign indicates that the flow is from <u>right</u> to <u>left</u> as we look from A to B.

8.4 Blue and yellow streams of paint at 60 °F (each with a density of 1.6 slugs/ft³ and a viscosity 1000 times greater than water) enter a pipe with an average velocity of 4 ft/s as shown in Fig. P8.4. Would you expect the paint to exit the pipe as green paint or separate streams of blue and yellow paint? Explain. Repeat the problem if the paint were "thinned" so that it is only 10 times more viscous than water. Assume the density remains the same.

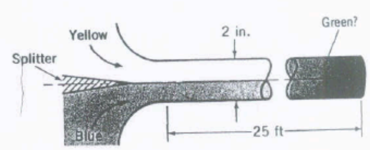


FIGURE P8.4

If the flow is laminar the paint would exit as separate blue and yellow streams.

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V D}{1000\,\mu_{H_2O}} = \frac{1.6\,\frac{slugs}{ft^3}\left(4\,\frac{ft}{s}\right)\left(\frac{2}{12}\,ft\right)}{1000\left(2.34\times10^{-5}\,\frac{lb\cdot s}{ft^2}\right)} = 45.6 < 2100$$

Thus, <u>laminar flow so blue and yellow streams.</u>

If use $\mu = 10\,\mu_{H_2O}$ obtain

$Re = 4560 > 4000$ so have <u>turbulent flow with natural mixing and green paint.</u>

Note: Check to determine if the 25 ft length is greater than the entrance length, ℓ_e.

For laminar flow $\frac{\ell_e}{D} = 0.06\,Re$, or $\ell_e = 0.06(45.6)\left(\frac{2}{12}\,ft\right) = 0.456\,ft < 25\,ft$

For turbulent flow $\frac{\ell_e}{D} = 4.4\,Re^{1/6}$, or $\ell_e = 4.4(4560)^{\frac{1}{6}}\left(\frac{2}{12}\,ft\right) = 2.99\,ft < 25\,ft$

8.5 Air at 200 °F flows at standard atmospheric pressure in a pipe at a rate of 0.08 lb/s. Determine the minimum diameter allowed if the flow is to be laminar.

Maximum $Re = \frac{\rho V D}{\mu}$ for laminar flow is $Re = 2100$.
or with

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}, \quad Re = \frac{\rho\left(\frac{4Q}{\pi D^2}\right)D}{\mu} = \frac{4\rho Q}{\pi \mu D} = 2100$$

Hence,

$$Q = \frac{2100 \pi \mu D}{4\rho} \qquad (1)$$

Given $\gamma Q = 0.08 \frac{lb}{s}$, where $\gamma = g\rho$ and $\rho = \frac{p}{RT}$
Thus,

$$\rho = \frac{(14.7 \times 144 \frac{lb}{ft^2})}{(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(460+200)^\circ R} = 0.00187 \frac{slugs}{ft^3}$$

so that

$$Q = \frac{0.08 \frac{lb}{s}}{(32.2 \frac{ft}{s^2})(0.00187 \frac{slugs}{ft^3})} = 1.33 \frac{ft^3}{s}$$

Hence, with $\mu = 4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2}$ (see Table B.3), Eq. (1) gives

$$D = \frac{4\rho Q}{2100 \pi \mu} = \frac{4(0.00187 \frac{slugs}{ft^3})(1.33 \frac{ft^3}{s})}{2100\pi(4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2})} = \underline{\underline{3.36 \, ft}}$$

8.14 The pressure drop needed to force water through a horizontal 1-in.-diameter pipe is 0.60 psi for every 12-ft length of pipe. Determine the shear stress on the pipe wall. Determine the shear stress at distances 0.3 and 0.5 in. away from the pipe wall.

For a horizontal pipe $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ or $\tau = \frac{r}{2}\frac{\Delta p}{\ell}$
Thus,

$$\tau = r\frac{(0.6 \times 144 \frac{lb}{ft^2})}{2(12\, ft)} = 3.6\, r \frac{lb}{ft^2}, \text{ where } r \sim ft$$

Hence,

$$\tau_w = 3.6\left(\frac{0.5}{12}\right) = \underline{\underline{0.15 \frac{lb}{ft^2}}}$$

and with $r = (0.5 - 0.3)\, in. = 0.2\, in.$,

$$\tau = 3.6\left(\frac{0.2}{12}\right) = \underline{\underline{0.06 \frac{lb}{ft^2}}}$$

Finally, with $r = (0.5 - 0.5)\, in. = 0\, in.$ $\quad \underline{\underline{\tau = 0}}$