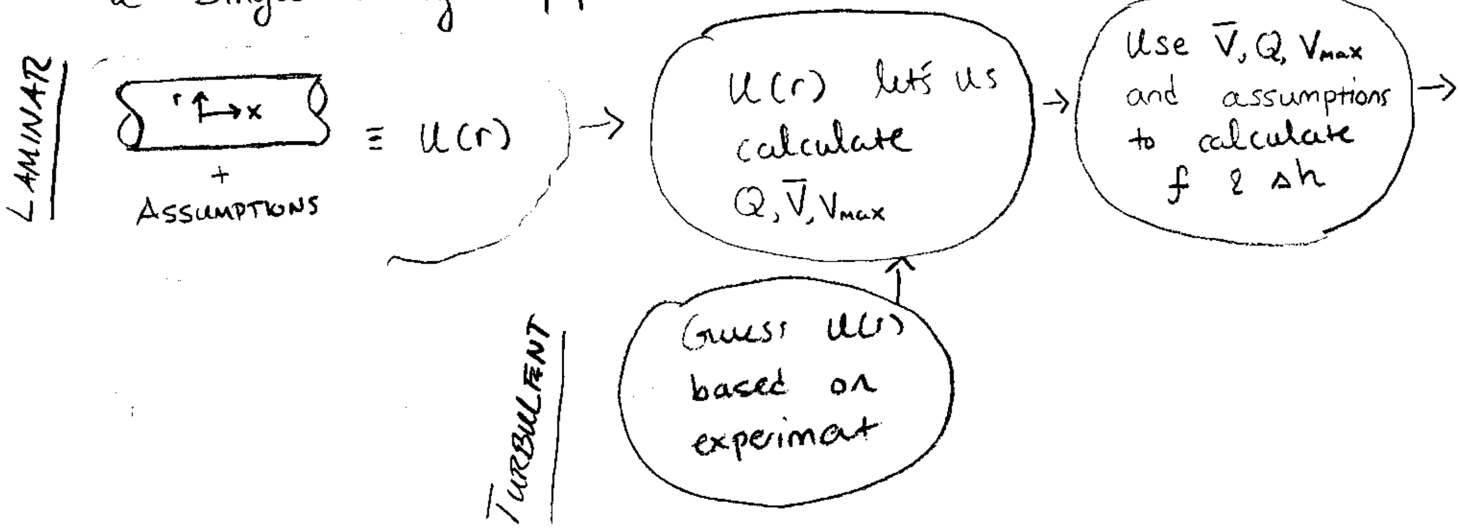
LECTURE 8 : NOTHING'S PERFECT, THE TURBULENT BOUNDARY LAYER.

In a round about way last time we found ways to calculate τ_w and F_D based on a Re value. We have almost mimiced the story of a single-straight pipe too!



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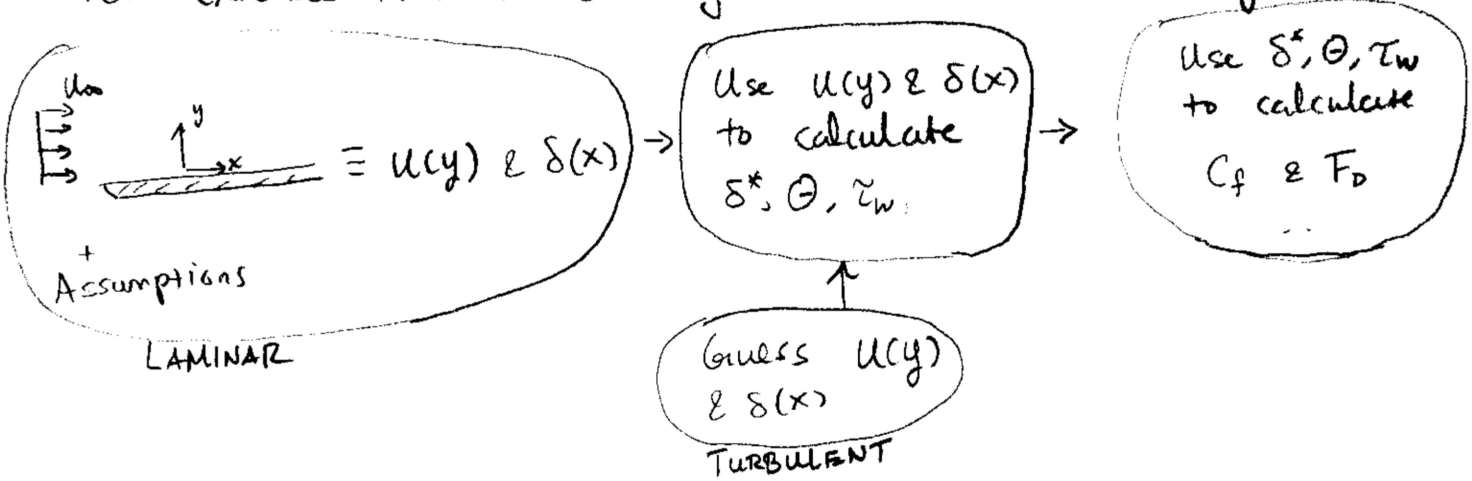
For external flows we're doing the exact same thing



So you can probably guess what's gonna happen... we're going to prescribe a $u(y)$ based on years of experiment and get new C_f values just like we did for f_{turb}.

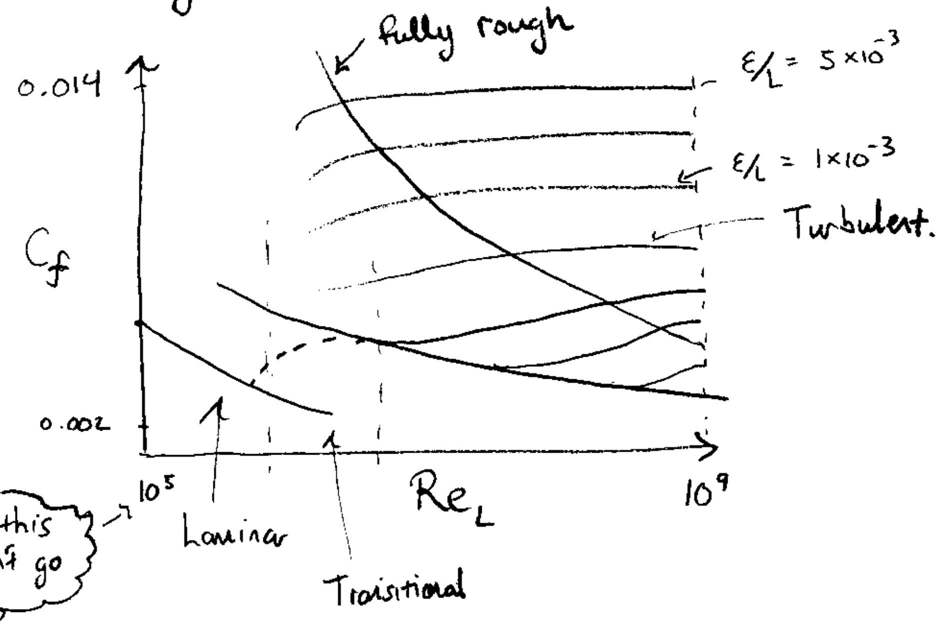
LECTURE 8:

So first based on experiment we know we have a different critical reynold's #.

$$Re < 5 \times 10^5 \qquad \text{(Laminar)}$$

$$Re > 5 \times 10^5 \qquad \text{(Turbulent)}$$

To cut to the chase we eventually ever develop an "External" Moody Chart.



Notice this scale doesn't go to zero!

Extra Credit: make this plot using the equations developed

This C_D is the drag-coefficient for external flows. To reiterate we use it like this.

$$F_D = C_f \; \frac{1}{2} \rho W L U_\infty^2$$

$$h_L = f \; \frac{L}{D} \frac{V^2}{2g}$$

Stuff you have to calculate, some thinking required

stuff you have to book-keep and keep track off

LECTURE 8

We define the drag-coefficient as a ratio of viscous skin friction to the inertial force.

$$C_f = \frac{F_D}{\frac{1}{2}\rho W L U_\infty^2}$$

so if we can derive a $\tau_w(x)$ we win.

$$= \frac{W \int_0^L \tau_w \, dx}{\frac{1}{2}\rho U_\infty^2 W L}$$

Now, how do we figure out $\tau_w(x)$? We have two ways...

1.) (Simple-Way) $\quad \tau_w(x) = \rho U_\infty^2 \frac{d\Theta}{dx}$

2.) (Exact Way) $\quad \tau_w(x) = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$

This requires you to solve those governing equations using a stream function approach and another method called similarity-solutions

Why is 1.) Easy? Well because remember we have a definition for Θ.

$$\Theta(x) = \int_0^{\delta(x)} \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

So all you need to provide is $u(y)$ and $\delta(x)$!

LECTURE 8:

So you'll notice what's worse for external flow is we need to estimate $\delta(x)$ and $u(y)$. More often we are trying to estimate.

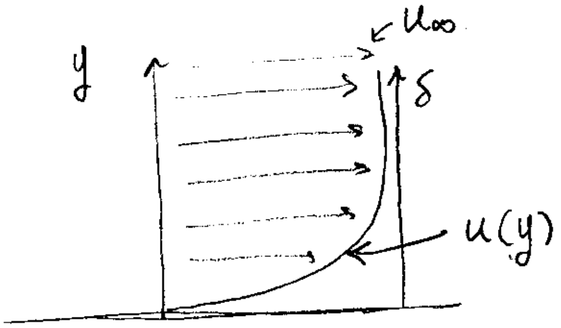
$$\boxed{\delta(x) = \frac{C_o \cdot x}{Re_x^a}}$$

So experimentally we need to determine (C_o & a) from lots of measurements.

Extra Credit

Devise an experiment to experimentally find constants a & C_o.

We can actually provide a guess of $\delta(x)$ and $u(y)$ if we provide a non-dimensional function.



$$u^* = \frac{u(y^*)}{U_\infty}$$

and,

$$y^* = \frac{y}{\delta}$$

Now the integral just needs $u^*(y^*)$.

$$\Theta(x) = \delta(x) \cdot \int_0^1 u^*(1 - u^*)\, dy^*$$

Extra Credit: show why the upper limit is 1 now not δ

LECTURE 8:

Prandtl found a good $u^*(y^*)$ for turbulent boundary layers was

$$u^*(y^*) = y^{*\,1/7} = y^{*\,a}$$

Plug this in and we find.

$$\Theta(x) = \delta(x)\,\frac{7}{72} = C_1 \cdot \delta(x)$$

Now the last step is to assign $\delta(x)$, take a derivative.

$$\tau_w(x) = \rho U_\infty^2 \frac{d\delta}{dx} \cdot C_1$$

$$\delta(x) = \frac{C_0 \cdot x}{[Re(x)]^a}$$

$$Re(x) = \frac{\rho U_\infty x}{\mu}$$

Notice with the chain rule and calculations we get

$$\tau_w(x) = C_0 C_1 (1-a)\, \rho U_\infty^2\, Re_x^{-a}$$

extra credit
do this calculation

* Notice that the power-matches the power of $\delta(x)$.

LECTURE 8

Now we're in a position to get C_D based on our integral definition.

$$C_f = \frac{\cancel{w} \int_0^L \tau_w(x)\, dx}{\frac{1}{2}\rho U_\infty^2 \cancel{w} L}$$

Only true for $\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$ and $\delta(x) = \frac{C_0 x}{[Re(x)]^a}$ →

$$= \frac{\cancel{\rho U_\infty^2} C_0 C_1 (1-a) \int_0^L Re(x)^{-a}\, dx}{\cancel{\frac{1}{2}\rho U_\infty} L}$$

$$= (C_0 C_1) \cdot Re_L^{-a}$$

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The last toy example was to show the framework for arriving at a C_f much like getting those f_{turb} formulas. We started by assuming a $u^*(y^*)$ function. Just like internal flows we actually the same log-function which comes from experiment.

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln\left(\frac{y u_*}{\nu}\right) + B \; ; \; \kappa = 0.41 \;\&\; B = 5$$

$$u_* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$$

So in much the same steps we can arrive at

$$C_D = \frac{0.455}{\log_{10}(Re_L)^{2.58}}$$

Turbulent Smooth Pipe

When we consider rough plates.

$$C_f = \left[1.89 - 1.62 \log_{10}\left(\frac{\varepsilon}{L}\right)\right]^{-2.5}$$ Completely Turbulent

Or

$$C_f = \frac{0.455}{\left[\log_{10}(Re_L)\right]^{2.58}} - \frac{1700}{Re_L}$$ Transitional

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It's pretty amazing what you can get from simply calculating a Reynold's Number right! That completes only half of what goes into drag force. Recall

$$F_{D,TOT} = \underbrace{\int p \cdot \hat{n}\, dA}_{\text{Form Drag}} + \underbrace{\int \tau_w \cdot \hat{t}\, dA}_{\text{Skin friction}}$$

Form Drag:

- This is up next and we will see a boundary layer seperates, why dimples make golf balls fly farther, and why an airplain generates lift,

Skin friction:

- tangential shear forces due to viscous forces of liquid
- Analysis of the Boundary Layer determined methods to calculate friction coefficients C_f
- Analysis of integrals provided methods to calculate δ, δ^*, Θ
- Everything is a function of $(Re, \frac{\varepsilon}{L})$ just like f.

LECTURE 8

* A NOTE ON HOMEWORK PROBLEMS. BASED ON WHAT ASSUMED VELOCITY PROFILE YOU HAVE $u^*(y^*)$ YOU GET DIFFERENT CONSTANTS OF PROPORTIONALITY FOR $Re^{-1/2}$.

Profile	$\delta Re^{1/2}/x$	$\hat{C}_f Re_x^{1/2}$ (pointwise coefficient)	$C_f Re_L^{1/2}$ (total skin friction coefficient)
* Blasius (Exact)	5.00	0.664	1.328
Linear $\frac{u}{U_\infty} = \frac{y}{\delta}$	3.46	0.578	1.156
Parabolic $\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$	5.48	0.730	1.460
Cubic $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^3\frac{1}{2}$	4.64	0.646	1.292
Sine $\frac{u}{U} = \sin\left(\frac{\pi}{2}\left(\frac{y}{\delta}\right)\right)$	4.79	0.655	1.310

Make sure you use the Blasius constants for calculations. I don't know why we give the others also... remember for shear.

$$\tau_w = 0.332\, U_\infty^{3/2}\sqrt{\frac{\rho\mu}{x}}$$ ← This comes from the def of $\hat{C}_f$.