9.13 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.

Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12\times10^{-3}\,m}{\sqrt{1.3\,m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x\sim m, \delta\sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

9.21 A smooth, flat plate of length $\ell = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{\left(1.12\times10^{-6}\,\frac{m^2}{s}\right)x}{0.5\,\frac{m}{s}}} = 7.48\times10^{-3}\sqrt{x}\ m, \text{ where } x\sim m$$

and

$$\tau_w = 0.332\,U^{3/2}\sqrt{\frac{\rho\mu}{x}} = 0.332\left(0.5\,\frac{m}{s}\right)^{3/2}\sqrt{\frac{\left(999\,\frac{kg}{m^3}\right)\left(1.12\times10^{-3}\,\frac{N\cdot s}{m^2}\right)}{x}}$$

$$= \frac{0.124}{\sqrt{x}}\ \frac{N}{m^2}, \text{ where } x\sim m$$

Thus, at $x = 3\,m$

$$\delta = 7.48\times10^{-3}\sqrt{3} = \underline{0.0130\ m}$$

$$\tau_w = \frac{0.124}{\sqrt{3}} = \underline{0.0716\ \frac{N}{m^2}}$$

while at $x = 6\,m$

$$\delta = 7.48\times10^{-3}\sqrt{6} = \underline{0.0183\ m}$$

$$\tau_w = \frac{0.124}{\sqrt{6}} = \underline{0.0506\ \frac{N}{m^2}}$$

9.27 An airplane flies at a speed of 400 mph at an altitude of 10,000 ft. If the boundary layers on the wing surfaces behave as those on a flat plate, estimate the extent of laminar boundary layer flow along the wing. Assume a transitional Reynolds number of $Re_{xcr} = 5 \times 10^5$. If the airplane maintains its 400-mph speed but descends to sea level elevation, will the portion of the wing covered by a laminar boundary layer increase or decrease compared with its value at 10,000 ft? Explain.

At 10,000 ft:

(a) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and from Table C.1, $\nu = \frac{\mu}{\rho} = \frac{3.534 \times 10^{-7}\,\frac{lb\cdot s}{ft^2}}{1.756 \times 10^{-3}\,\frac{slug}{ft^3}}$

$= 2.01 \times 10^{-4}\,\frac{ft^2}{s}$

Hence, with $Re_{x_{cr}} = 5 \times 10^5$,

$$x_{cr} = \frac{\nu\, Re_{x_{cr}}}{U} = \frac{(2.01\times10^{-4}\,\frac{ft^2}{s})(5\times10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.171\ ft}}$$

At sea-level:

(b) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and $\nu = 1.57 \times 10^{-4}\,\frac{ft^2}{s}$

Hence,

$$x_{cr} = \frac{\nu\, Re_{cr}}{U} = \frac{(1.57\times10^{-4}\,\frac{ft^2}{s})(5\times10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.134}}\ ft$$

The laminar boundary layer occupies the first 0.134 ft of the wing at sea level and (from part (a) above) the first 0.171 ft at an altitude of 10,000 ft. This is due mainly to the lower density (larger kinematic viscosity). The dynamic viscosities are approximately the same.

9.38 The drag coefficient for a newly designed hybrid car is predicted to be 0.21. The cross-sectional area of the car is 30 ft². Determine the aerodynamic drag on the car when it is driven through still air at 55 mph.

$$\mathscr{D} = C_D \tfrac{1}{2}\rho V^2 A$$

$$V = 55\,mph \times \frac{88\,ft/s}{60\,mph} = 80.7\,ft/s$$

$$\mathscr{D} = 0.21\left(\tfrac{1}{2}\right)(0.00238\,slugs/ft^3)(80.7\,ft/s)^2(30\,ft^2)$$

$$\underline{\underline{\mathscr{D} = 48.8\ lb}}$$

9.41 The aerodynamic drag on a car depends on the "shape" of the car. For example, the car shown in Fig. P9.41 has a drag coefficient of 0.36 with the windows and roof closed. With the windows and roof open, the drag coefficient increases to 0.45. With the windows and roof open, at what speed is the amount of power needed to overcome aerodynamic drag the same as it is at 65 mph with the windows and roof closed? Assume the frontal area remains the same. Recall that power is force times velocity.

Windows and roof closed: $C_D = 0.35$ | Windows open; roof open: $C_D = 0.45$

FIGURE P9.41

$$\text{Power} = \mathcal{P} = F \cdot V$$

The force is the drag force. Let $(\ )_c$ and $(\ )_o$ denote closed and open.

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

We want to find U_o when $\mathcal{P}_o = \mathcal{P}_c$

$$\mathcal{P}_o = U_o \mathcal{D}_o = \tfrac{1}{2}\rho U_o^3 A_o C_{D_o} = \mathcal{P}_c = U_c \mathcal{D}_c = \tfrac{1}{2}\rho U_c^3 A_c C_{D_c}$$

The frontal areas are the same, so $A_o = A_c$

$$U_o^3 C_{D_o} = U_c^3 C_{D_c}$$

$$U_o = U_c \left(\frac{C_{D_c}}{C_{D_o}}\right)^{1/3} = (65\,\text{mph})\left(\frac{0.36}{0.45}\right)^{1/3}$$

$$\underline{\underline{U_o = 60.3\ \text{mph}}}$$

12.19 A centrifugal water pump having an impeller diameter of 0.5 m operates at 900 rpm. The water enters the pump parallel to the pump shaft. If the exit blade angle, β_2 (see Fig. 12.8), is 25°, determine the shaft power required to turn the impeller when the flow through the pump is 0.16 m^3/s. The uniform blade height is 50 mm.

$$\dot{W}_{shaft} = T_{shaft}\,\omega = T_{shaft}\,2\pi N$$

$$T_{shaft} = \rho Q\,(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \qquad (Eq.\ 12.10)$$

With $V_{\theta 1} = 0$

$$T_{shaft} = \rho Q r_2 V_{\theta 2} \qquad (1)$$

From Fig. 12.8c

$$\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2}\cot\beta_2 \qquad (2)$$

For $r_2 = \frac{0.5\,m}{2} = 0.25\,m$ with $\omega = \frac{\left(900\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{60\,\frac{s}{min}} = 94.2\,\frac{rad}{s}$

then

$$U_2 = r_2\omega = (0.25\,m)\left(94.2\,\frac{rad}{s}\right) = 23.6\,\frac{m}{s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.16\,\frac{m^3}{s}\right)}{(2\pi)(0.25\,m)(0.05\,m)} = 2.04\,\frac{m}{s}$$

Thus, from Eq. (2)

$$V_{\theta 2} = (23.6 - 2.04\cot 25^\circ)\,\frac{m}{s} = 19.2\,\frac{m}{s}$$

and from Eq. (1)

$$T_{shaft} = \left(999\,\frac{kg}{m^3}\right)\left(0.16\,\frac{m^3}{s}\right)(0.25\,m)\left(19.2\,\frac{m}{s}\right) = 768\,N\cdot m$$

so,

$$\dot{W}_{shaft} = (768\,N.m)\left(2\pi\,\frac{rad}{rev}\right)\left(900\,\frac{rev}{min}\right)\frac{1}{\left(60\,\frac{s}{min}\right)}\left(\frac{1}{1000\,\frac{N.m}{s.\,kW}}\right) = \underline{\underline{0.08\,kW}}$$

12.25 The performance characteristics of a certain centrifugal pump having a 9-in.-diameter impeller and operating at 1750 rpm are determined using an experimental setup similar to that shown in Fig. 12.10. The following data were obtained during a series of tests in which $z_2 - z_1 = 0$, $V_2 = V_1$, and the fluid was water.

Q (gpm)	20	40	60	80	100	120	140
$p_2 - p_1$ (psi)	40.2	40.1	38.1	36.2	33.5	30.1	25.8
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

Based on these data, show or plot how the actual head rise, h_a, and the pump efficiency, η, vary with the flowrate. What is the design flowrate for this pump?

From Eq. 12.19 with $z_1 = z_2$ and $V_1 = V_2$

$$h_a = \frac{p_2 - p_1}{\gamma}$$

Thus, for the first set of data in the table

$$h_a = \frac{\left(40.2 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.4 \frac{lb}{ft^3}} = 92.8 ft$$

From Eq. 12.23

$$\eta = \frac{\gamma Q h_a / 550}{bhp}$$

and for the first set of data in the table

$$\eta = \frac{\left(62.4 \frac{lb}{ft^3}\right)\left[(20 gpm)/\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\right](92.8 ft)}{(1.58 hp)\left(550 \frac{ft \cdot lb}{s \cdot hp}\right)}$$

$$= 0.297$$

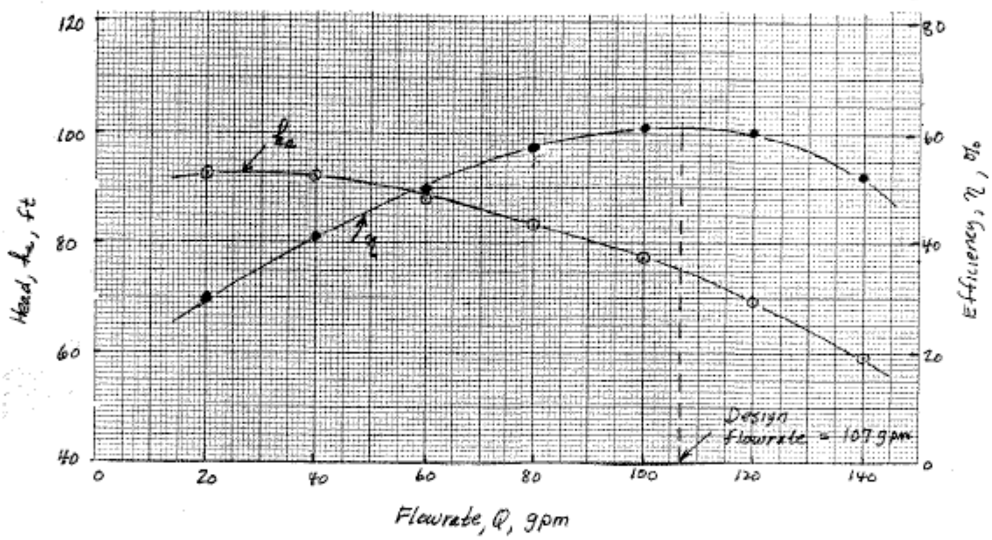
or

$$\eta = 29.7\%$$

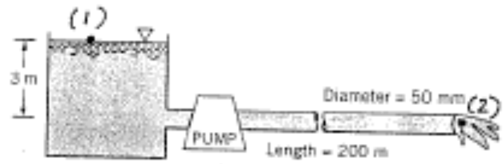
Remaining values for h_a and η can be calculated in a similar manner, and all values are tabulated in the table below.

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6

A plot of the data is shown below. The design flowrate occurs at peak efficiency and is <u>107 gpm.</u>



12.30 Water at 40 °C is pumped from an open tank through 200 m of 50-mm-diameter smooth horizontal pipe as shown in Fig. P12.30 and discharges into the atmosphere with a velocity of 3 m/s. Minor losses are negligible. **(a)** If the efficiency of the pump is 70%, how much power is being supplied to the pump? **(b)** What is the NPSH$_A$ at the pump inlet? Neglect losses in the short section of pipe connecting the pump to the tank. Assume standard atmospheric pressure.



■ **FIGURE F12.30**

(a)
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\,\frac{\ell}{D}\,\frac{V_2^2}{2g} \qquad (1)$$

where $p_1 = p_2 = 0$, $V_1 = 0$, $V_2 = 3\ m/s$, $z_1 = 3m$, and $z_2 = 0$. Thus, Eq. (1) becomes

$$z_1 + h_p = \frac{V_2^2}{2g}\left(1 + f\,\frac{\ell}{D}\right) \qquad (2)$$

Also,
$$Re = \frac{VD}{\nu} = \frac{(3\frac{m}{s})(0.05m)}{(6.580\times10^{-7}\frac{m^2}{s})} = 2.28\times10^5$$

and from Fig. 8.23 for smooth pipe $f = 0.0152$. Thus, from Eq. (2)

$$h_p = \frac{(3\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\left[1 + 0.0152\left(\frac{200\ m}{0.05m}\right)\right] - 3m = 25.3\ m$$

Hence,

$$\text{Power gained by fluid} = \gamma Q h_p$$
$$= (9.731\times10^3\frac{N}{m^3})(\frac{\pi}{4})(0.05m)^2(3\frac{m}{s})(25.3m)$$
$$= 1.45\times10^3\ \frac{N\cdot m}{s} = 1.45\ kW$$

and

$$\text{Power supplied to pump} = \frac{\text{Power gained by fluid}}{\text{Efficiency}}$$
$$= \frac{1.45\ kW}{0.7} = \underline{\underline{2.07\ kW}}$$

(b) From Eq. 12.24
$$NPSH = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma} \qquad (3)$$

where p_s and V_s refer to the pressure and velocity at the pump inlet, respectively. Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + h_L$$

so that with $p_1 = p_{atm}$, $V_1 = 0$, $z_s = 0$, and $h_L = 0$

(con't)

$$\frac{p_{atm}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g}$$

and therefore from Eq.(3) the available NPSH is

$$NPSH_A = \frac{p_{atm}}{\gamma} + z_1 - \frac{p_v}{\gamma} \qquad (4)$$

Note that this result corresponds to Eq. 12.25 with z_1 positive (since pump is below reservoir) and $\Sigma h_L = 0$.

From Table B.2 the water vapor pressure at 40°C is $7.376 \times 10^3\ N/m^2$ (abs) and $\gamma = 9.731 \times 10^3\ N/m^3$. Thus, from Eq.(4) with $p_{atm} = 101\ kPa$

$$NPSH_A = \frac{(101 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})} + 3m - \frac{(7.376 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})}$$

$$= \underline{\underline{12.6\ m}}$$

12.37 A centrifugal pump having the characteristics shown in Example 12.4 is used to pump water between two large open tanks through 100 ft of 8-in.-diameter pipe. The pipeline contains 4 regular flanged 90° elbows, a check valve, and a fully open globe valve. Other minor losses are negligible. Assume the friction factor $f = 0.02$ for the 100-ft section of pipe. If the static head (difference in height of fluid surfaces in the two tanks) is 30 ft, what is the expected flowrate? Do you think this pump is a good choice? Explain.

Application of the energy equation between the two free surfaces, points (1) and (2), gives

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 30\text{ ft}$, Eq. (1) becomes

$$h_p = 30\text{ ft} + \Sigma h_L \qquad (2)$$

The head loss term can be expressed as

$$\Sigma h_L = \left[4(0.3) + 10 + 2 + 0.02\frac{(100\text{ ft})}{(\frac{8}{12}\text{ ft})}\right]\frac{V^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

with the minor loss coefficients obtained from Table 8.3. Also,

$$V = \frac{Q}{A} = \frac{Q(\text{ft}^3/\text{s})}{(\frac{\pi}{4})(\frac{8}{12}\text{ ft})^2}$$

and Eq. (2) becomes

$$h_p = 30 + 2.06\left[Q(\text{ft}^3/\text{s})\right]^2$$

or the system equation can be written as

$$h_p = 30 + 1.02 \times 10^{-5}\left[Q(\tfrac{\text{gal}}{\text{min}})\right]^2 \qquad (3)$$

The intersection of the system curve (Eq. 3) with the pump curve, as shown on the figure, indicates that

$$\underline{Q = 1740\ \frac{\text{gal}}{\text{min}}}$$

Since the efficiency at this flowrate is near peak efficiency, as shown on the figure, <u>this pump would be satisfactory</u>.