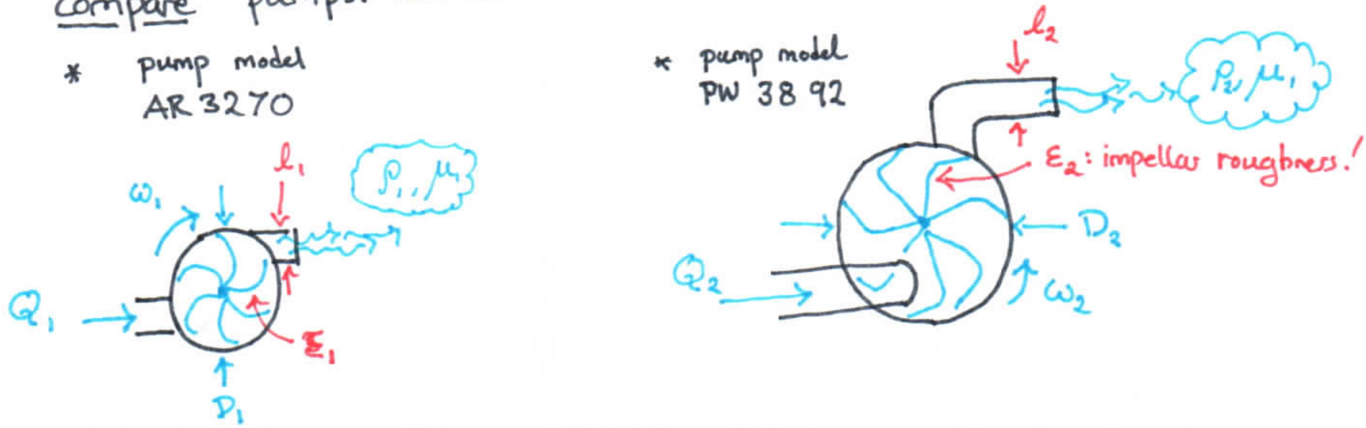
LECTURE 13:

We have methods of choosing pumps based on one we ordered. But we don't have methods to compare pumps. We need non-dimensional expressions!



We can just look at a data sheet of all DIMENSIONAL parameters of each pump and lay em all out...

Extra Credit

Do this process of listing out all dimensional parameters you can think of for a hydro-electric damn. Draw a schematic including a turbine in the wall and go wild with it

FOR A PUMP WE CAN MEASURE, ASSIGN, OR KEEP TRACK OF...

l_i	:= geometric lengths NOT the impellar diameter.	[m]
D	:= impellar diameter	[m]
ε	:= material roughness	[m]
Q	:= volumetic flow	[m³/s]
ω	:= angular velocity	[1/s]
μ	:= working fluid viscosity	[Kg/m·s]
ρ	:= working fluid density	[Kg/m³]

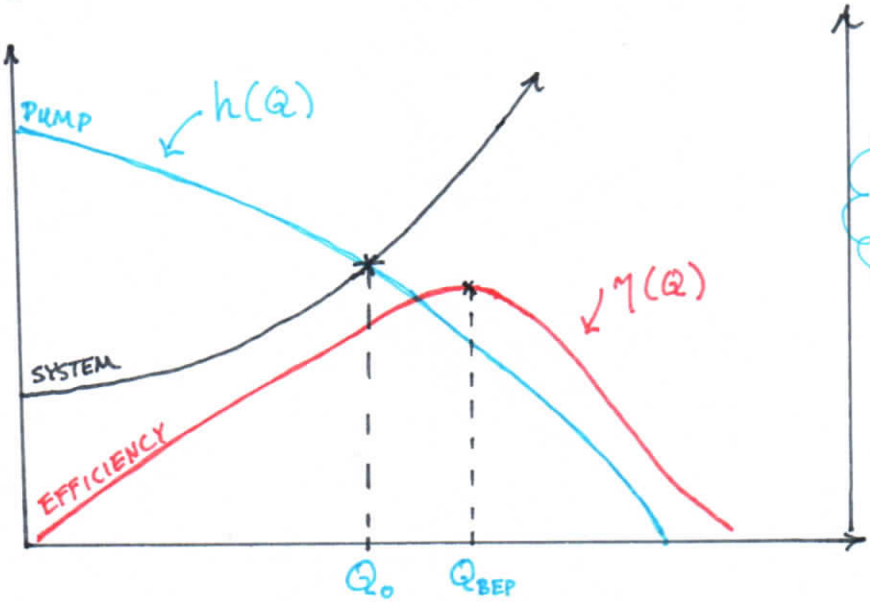
LECTURE 13

So we've listed all the independent variables of ANY pump we could ever pick. Now motivated by our DIMENSIONAL analysis of pumps we've developed lets take stock in what we ended up caring about.

Look at the all mighty pump performance graph if you're lost, what y-axis stuff are we plotting always?

[m] h

I'm always looking for the head rise of the pump



$$\eta \equiv \frac{P_f}{\dot{W}_{shaft}}$$

Looks like I always want to plot an efficiency curve as well. Remember the goal of pump selection.

$$\min_{\{variables\}} (Q_o - Q_{BEP})$$

The information I WANT from a pump then is...

$h :=$ actual head rise of fluid. Sometimes denoted "h_a". I don't know why? head is head no subscript needed... $[m]$

$\dot{W}_{shaft} :=$ Shaft work supplied to the pump. What you're paying for! $[W] = [kg \cdot m^2/s^3]$

$P_f :=$ Power supplied to the fluid. What you get! $[W] \equiv [kg \cdot m^2/s^3]$

don't forget $P_f = \rho g Q h$

Mathematically then all we've stated then are the existence of functions such that

$$h = f_1(l_i, D, \varepsilon, Q, \omega, \mu, \rho)$$

$$\dot{W}_{shaft} = f_2(l_i, D, \varepsilon, Q, \omega, \mu, \rho)$$

$$P_f = f_3(l_i, D, \varepsilon, Q, \omega, \mu, \rho)$$

Where f_1, f_2, and f_3 are dimensional functions. If we want NON-DIMENSIONAL equations ϕ_1, ϕ_2, and ϕ_3 we must have all inputs and out-puts be non-dimensional as well. WARNING TANGENT ... Personally I hate and dispise Buckingham Pi garbage. It is some of the most unmotivated horse cud I've ever seen in physical math. People that practice it have no clue or intuition of the variables it produces yet feel they've accomplished something by doing something a computer can do.

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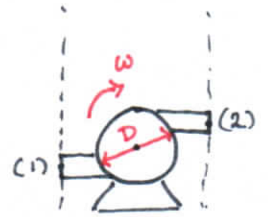
1) What should we compare h_{pump} to? We should compare it to the losses in the pump cause that would reference dimensions of the pump.



$$\left(\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g}\right) = \left(\frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}\right) + \sum h_L - h_{pump}$$

these terms do NOT reference the pump

I should compare h_{pump} to PUMP losses. Divide then the whole equation by $\sum h_L$

$$\sum h_L \sim K\frac{V^2}{2g}$$

$$\sim \frac{K}{8}\frac{(\omega D)^2}{g}$$

$$\boxed{\therefore\ h^* = \frac{h_{pump}}{\sum h_L} \sim \frac{g h_a}{\omega^2 D^2}}$$

What does $\dot{W}_{shaft}$ scale like?

Recall from the simple centrifugle model we showed.

$$\dot{W}_{shaft} = \rho Q \omega (r_2 V_{\theta 2} - \cancel{r_1 V_{\theta 1}})$$ assume no swirle.

In the sense of scales though $r_i \sim D$, $V_{\theta i} \sim V \sim (\omega D)$.

$$\dot{W}_{shaft} \sim \rho Q \omega r_2 V_{\theta 2}$$ ← notice this references PUMP dimensions.

$$\sim \rho A V \omega D V$$

$$\sim \rho D^2 (\omega D) \omega D (\omega D)$$

$$\therefore \boxed{\dot{W}_{shaft} \sim \rho \omega^3 D^5} \Rightarrow \dot{W}^*_{shaft} = \frac{\dot{W}_{shaft}}{\rho \omega^3 D^5}$$

We already know as well that $\eta = \rho g Q h / \dot{W}_{shaft}$ so that completes dependent variables. Now let's scale all independent variables.

LENGTHS ~ DIAMETER (Largest Reference length...)

* $D \gg l_i > \varepsilon$ [It's important that $D \gg l_i$ actually].

$\left\{ \frac{l_i}{D}, \frac{\varepsilon}{D} \right\}$ non dimensional length vars!

FLOWRATE ~ VOLUME OF PUMP / SEC

$$Q \sim \frac{\omega D^3}{\cancel{2\pi}} \Rightarrow Q^* = Q/\omega D^3$$

REYNOLDS ALWAYS OR.. INERTIAL FORCES ~ VISCOUS FORCES

$$Re = \frac{\rho V D}{\mu} = \frac{\rho \omega D^2}{\mu}$$

Putting everything together we get our theoretical non-dimensional relationships.

This is just non dimensional head rise $\rightarrow$ $C_H := \left\{ \begin{matrix} \text{Head Rise} \\ \text{Coefficient} \end{matrix} \right\} = \phi_1 \left(\frac{l_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, Re \right)$

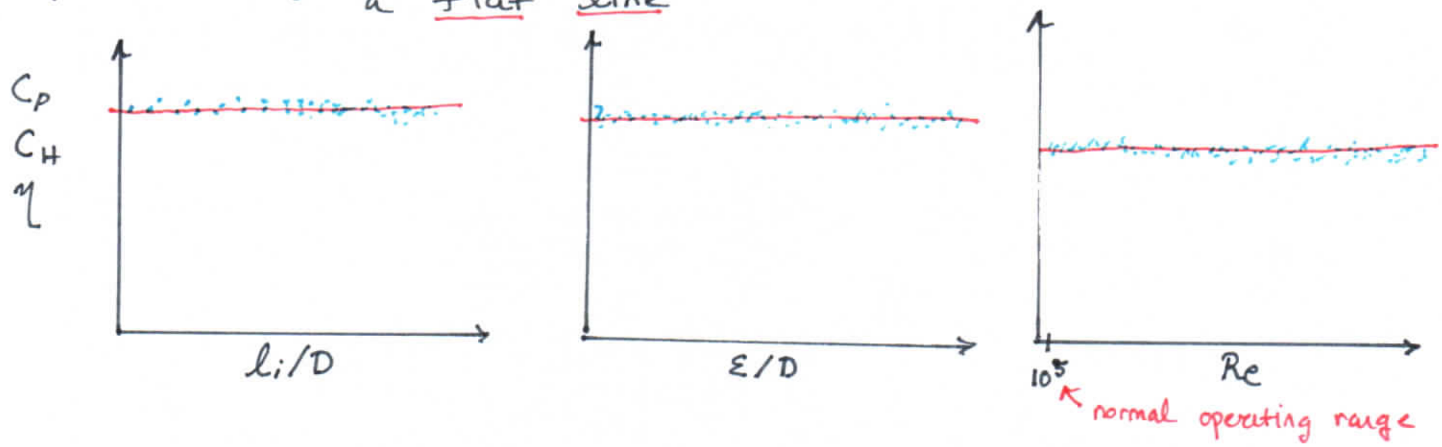
This is just non-dimensional power output $\rightarrow$ $C_P := \left\{ \begin{matrix} \text{Power} \\ \text{Coefficient} \end{matrix} \right\} = \phi_2 \left(\frac{l_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, Re \right)$

Efficiency! $\rightarrow$ $\eta := \left\{ \frac{\text{What you } \cancel{\text{pay for}} \text{ get for}}{\text{what you paid for}} \right\} = \phi_3 \left(\frac{l_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, Re \right)$

Now you really never see laminar pumps in industry. Usually the case that the Reynolds INSIDE The pump

$$Re \rightarrow \infty$$

It took experiments to really show however that l_i/D and ε/D had little effect on (C_H, C_P, η). This means that mathematically $\frac{\partial \phi_i}{\partial (l_i/D)} \approx 0$. So the figure the experiment produced was a flat line



Conclusion from this is that the only pertinent variable is!

$$\{C_H, C_P, \eta\} = \phi_i \left(\frac{Q}{\omega D^3} \right) = \phi_i (C_Q)$$

We call this the flow coefficient!

LECTURE 13

This last conclusion has far reaching consequences. That is this.

If:

$$\left(C_Q \right)_{PUMP\ 1} = \left(C_Q \right)_{PUMP\ 2} \qquad (1)$$

$$\left(\frac{Q}{\omega D^3} \right)_{PUMP\ 1} = \left(\frac{Q}{\omega D^3} \right)_{PUMP\ 2}$$

THEN $\Rightarrow$

$$\left(C_H \right)_{PUMP\ 1} = \left(C_H \right)_{PUMP\ 2} \qquad (2)$$

$$\left(C_P \right)_{PUMP\ 1} = \left(C_P \right)_{PUMP\ 2} \qquad (3)$$

$$\left(\eta \right)_{PUMP\ 1} = \left(\eta \right)_{PUMP\ 2} \qquad (4)$$

Extra Credit

{ EXPRESS $\eta(C_Q, C_H, C_P)$ write η in terms of C_Q, C_H, C_P }

PROBABLY ON EXAM...

Now we can start USING these relationships. What if I now compare a pump with itself but at DIFFERENT speeds?

If... $D_1 = D_2$ * It's the same pump so diameter is the same.

$$\therefore \frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad \text{* From satisfying (1)}$$

$$\therefore \frac{h_1}{h_2} = \left(\frac{\omega_1}{\omega_2} \right)^2 \qquad \text{* From (2)}$$

$$\therefore \frac{\dot{W}_1}{\dot{W}_2} = \left(\frac{\omega_1}{\omega_2} \right)^3 \qquad \text{* From (3)}$$

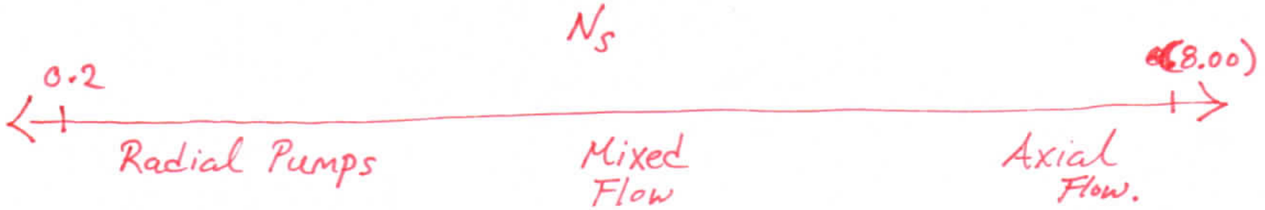
Extra Credit
do the algebra to show this

Extra Credit

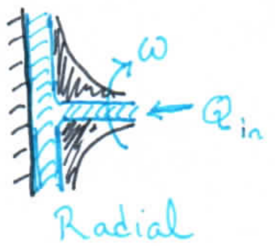
If $D_1 \neq D_2$ but instead $\omega_1 = \omega_2$. Change in impellar designs. Derive the next 3 equations (1), (2), (3) results

SPECIFIC SPEED N_s

We are usually conserned with pumps operating at optimum efficiency for a design. We also would like a parameter independent of impellar diameter. This would allow us to put all pumps on an ordered scale.



This means if you could calculate this N_s parameter you could drop your finger on this chart and see what TYPE of pump RUNNING AT OPTIMUM SPEED you need! We want our parameter to compare then.



vs.

* LAPTOP FANS

* BOX FANS

And it should compare these fans based on their intrinsic physics and not refer to dimensional length.

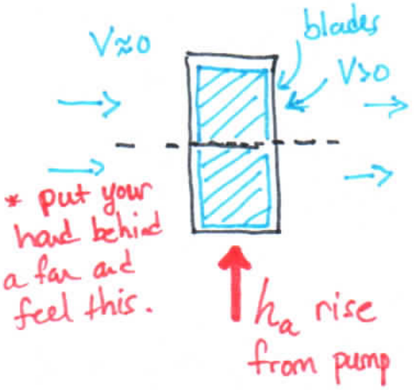
LECTURE 13:

Let's then compare characteristic <u>shear</u> for the designs.

$$N_s := \frac{\tau_{w,radial}}{\tau_{w,axial}} = \frac{C_0 U_R^{3/2}\sqrt{\rho\mu/D}}{C_1 U_A^{3/2}\sqrt{\rho\mu/D}} \sim \frac{U_R^{3/2}}{U_A^{3/2}}$$

So we need to find characteristic velocities intrinsic to the pump design itself and how it fundamentally interacts with the fluid.

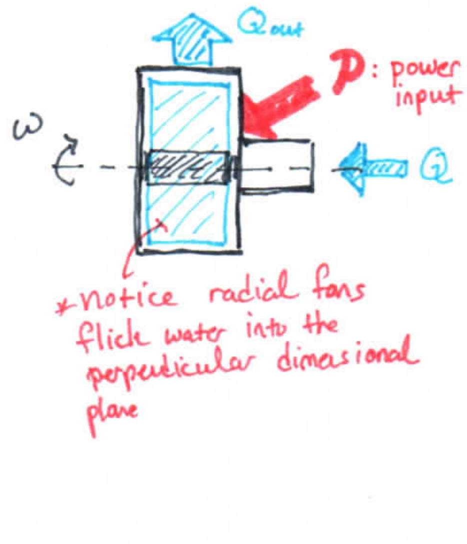
Axial



Bernoulli or The head equation says that

$$\Rightarrow \quad \frac{U_A^2}{g} \sim h_a \quad \Rightarrow \therefore U_A^{3/2} = (g h_a)^{3/4}$$

Radial



Power in terms of Drag from external flow...

Keep a ωD^3 as a flowrate expression

$$\Rightarrow \quad \mathcal{P} \sim \rho A C_D U_R^3$$

$$\therefore U_R^3 \sim \frac{\mathcal{P}_f}{\rho A C_D}$$

$$\sim \frac{\cancel{\rho}\omega^3 D^{\cancel{5}}}{\cancel{\rho}\cancel{D^2}}$$

If you don't keep a ωD^3 term you keep <u>no</u> information about in coming flow

$$\left\{ \begin{array}{l} \sim \omega^2(\omega D^3) \\ \sim \omega^2 Q \end{array} \right.$$

$$\Rightarrow \therefore U_R^{3/2} = \omega\sqrt{Q}$$

There ya have it! We have a non-dimensional parameter that now establishes a ranked scale of pumps.

$$\begin{Bmatrix}\text{Specific}\\ \text{Speed}\end{Bmatrix} := N_s = \frac{\omega\sqrt{Q}}{(g\,h_a)^{3/4}} \qquad [\mathrm{SI}]$$

If you are some sort of sadist that enjoys barly corn units, first of all... dude... why man. But if you must...

$$N_{sd} = \frac{\omega\,[\mathrm{rpm}]\sqrt{Q\,[\mathrm{gpm}]}}{(h_a[\mathrm{ft}])^{3/4}} \qquad [\text{stoopid units}]$$

TAKE A SECOND TO APPRECIATE HOW MAGICALLY AMAZING THIS THING IS!!!

Given a required list of $\{\omega, h, Q\}$ you can calculate a N_s. Then you can immediately determine what TYPE of pump you need!?!?

$N_s \lesssim 1$ (USE RADIAL ~~FAN~~ FLOW)

$N_s \approx 1$ (USE MIXED ~~FAN~~ FLOW)

$N_s \gtrsim 1$ (USE AXIAL FLOW)

IT STILL BLOWS MY MIND THIS WORKS AT ALL...