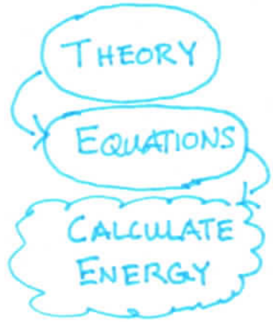
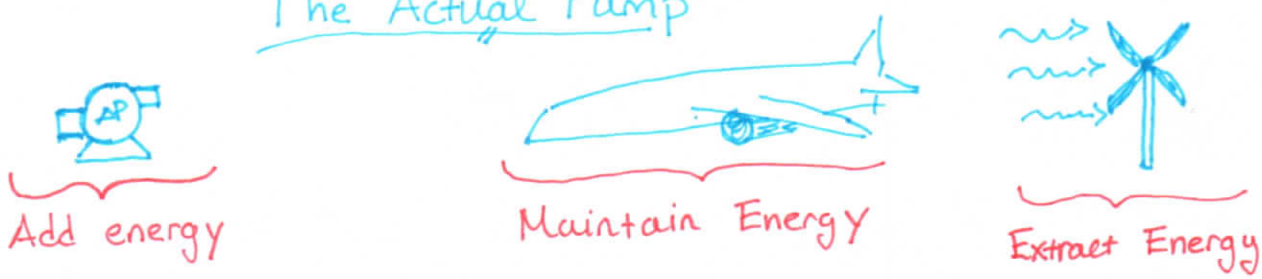
Lecture 11: What Pump Do You Need To Buy?

By this point you should see a pattern of



We have left one part a blackbox all this time.

The Actual "Pump"



There are so many different applications we use a large umbrella term...

Turbomachines

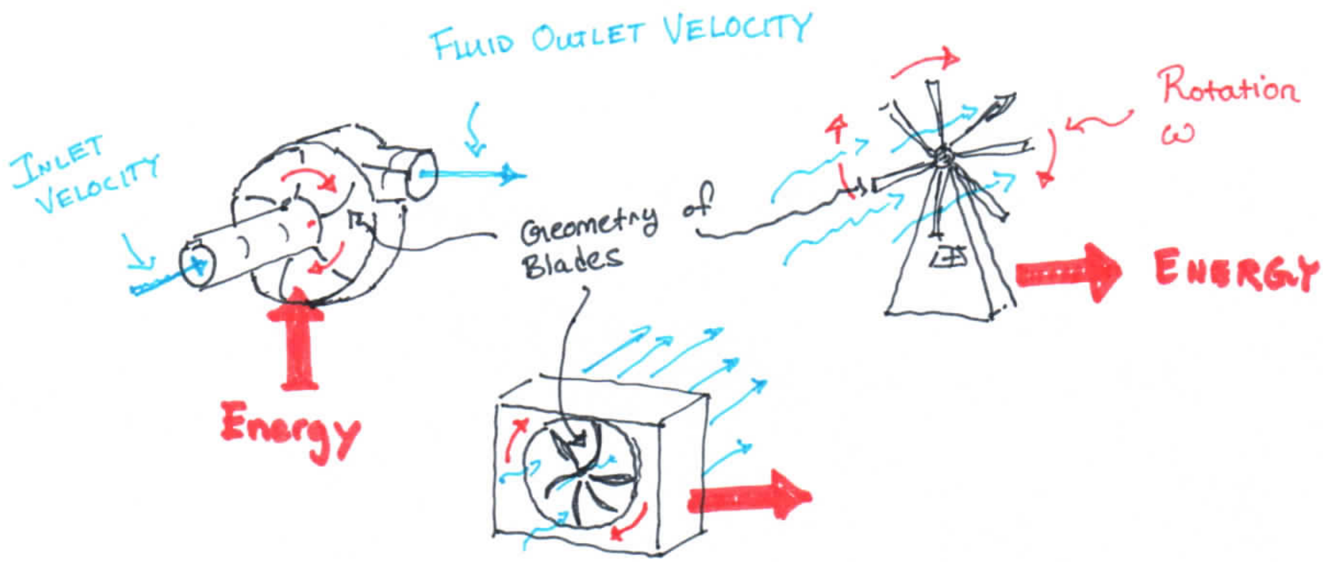
- simple window fans
- propellers
- axial-flow water pumps
- steam turbines
- compressors
- automobile turbo chargers
- etc.

This portion of the class will deal with the details of

How Does The Machine Add/Subtract Energy!?

LECTURE II:

AS ALWAYS I LIKE TO ASK WHY WE SHOULD DO THIS MATH AND IT WILL BE GOOD FOR!? WELL IF LOOK BACK AT HISTORY WE SEE PUMPS, TURBOMACHINES LONG BEFORE THIS PHYSICS STUFF. WE ALWAYS KNEW AS HUMANS WE COULD COMBINE SOME PARAMETERS TO MAKE FLUID MOVING DO STUFF.



SO WE WANT TO UNDERSTAND HOW THESE MEASURABLE and DESIGNABLE DIMENSIONS & PARAMETERS "PRODUCE"/TRANSFORM ENERGY SO WE CAN OPTIMIZE FOR A GIVEN DESIGN.

The Plan

1) Pretend all pumps are simple & draw an ideal sketch of how the fluid would move thru the blades, (3D problem → 1D problem)

This is the BEST CASE scenario so designing for this simple model is OVER designing!

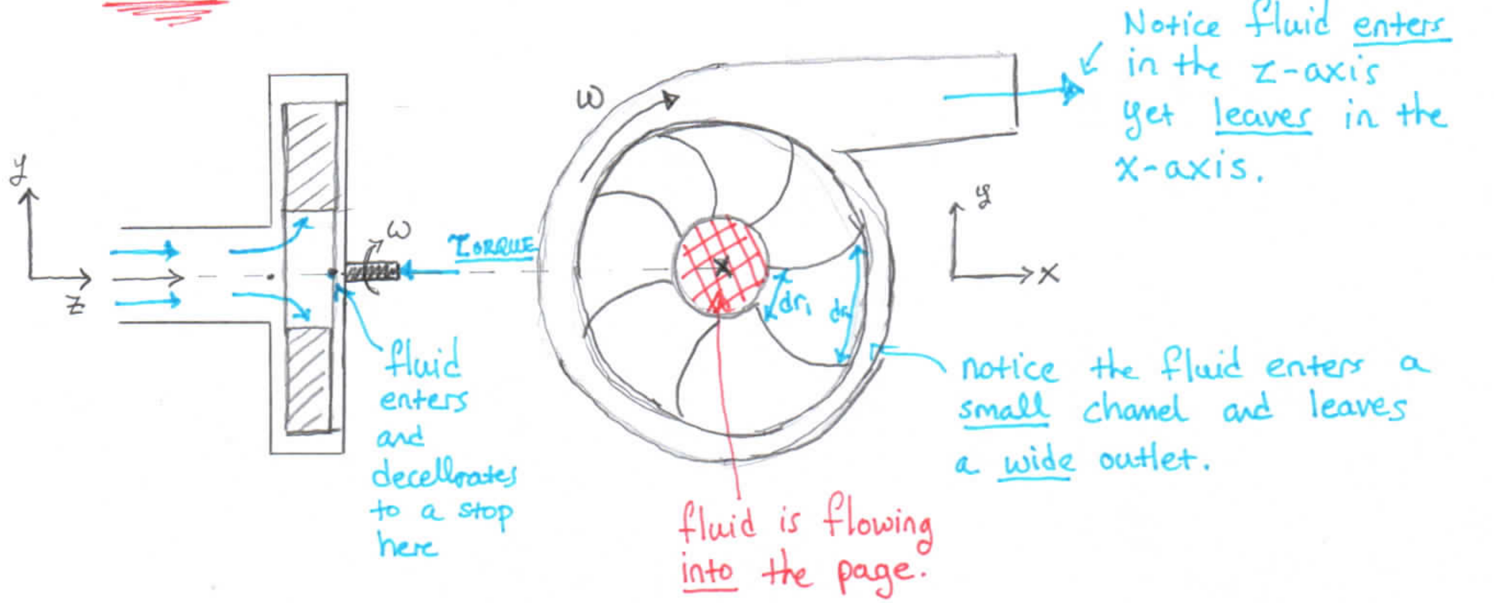
2) Throw math at that picture.
   i) conserve ANGULAR MOMENTUM

3) Use expressions to calculate a theoretical head provided by the pump. h_p

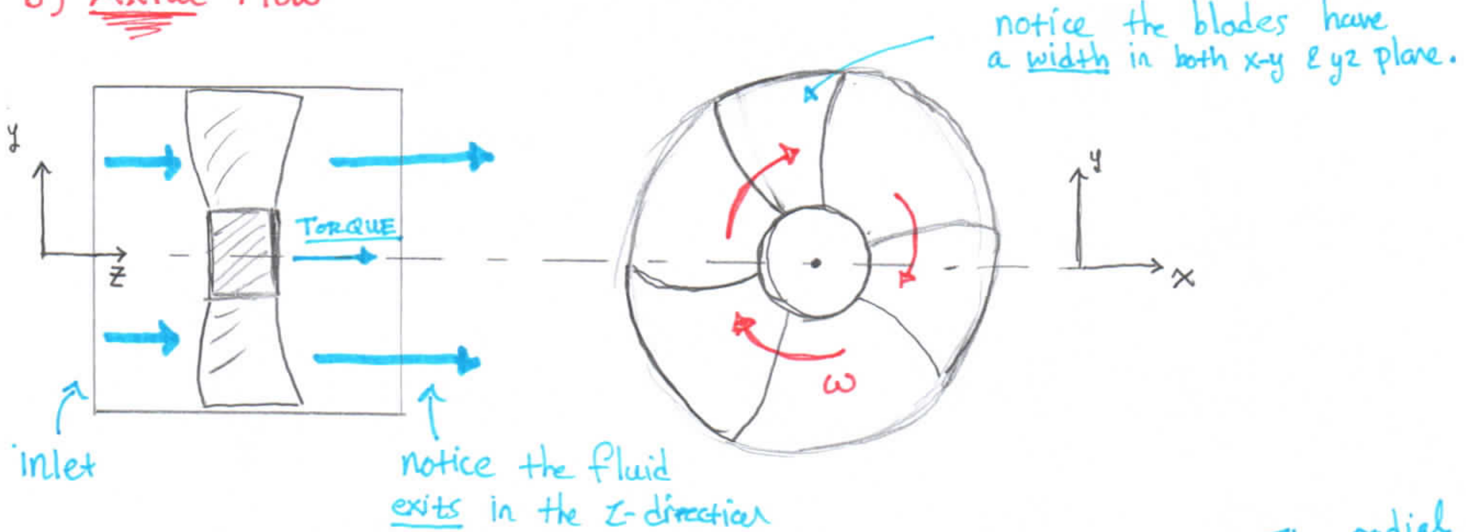
4) Turn expressions into graphs $h(Q)$ and math pumps to systems (MOST IMPORTANT PART!) Then use the graphs to quickly reason parameter effects on pump.

The Picture(s)

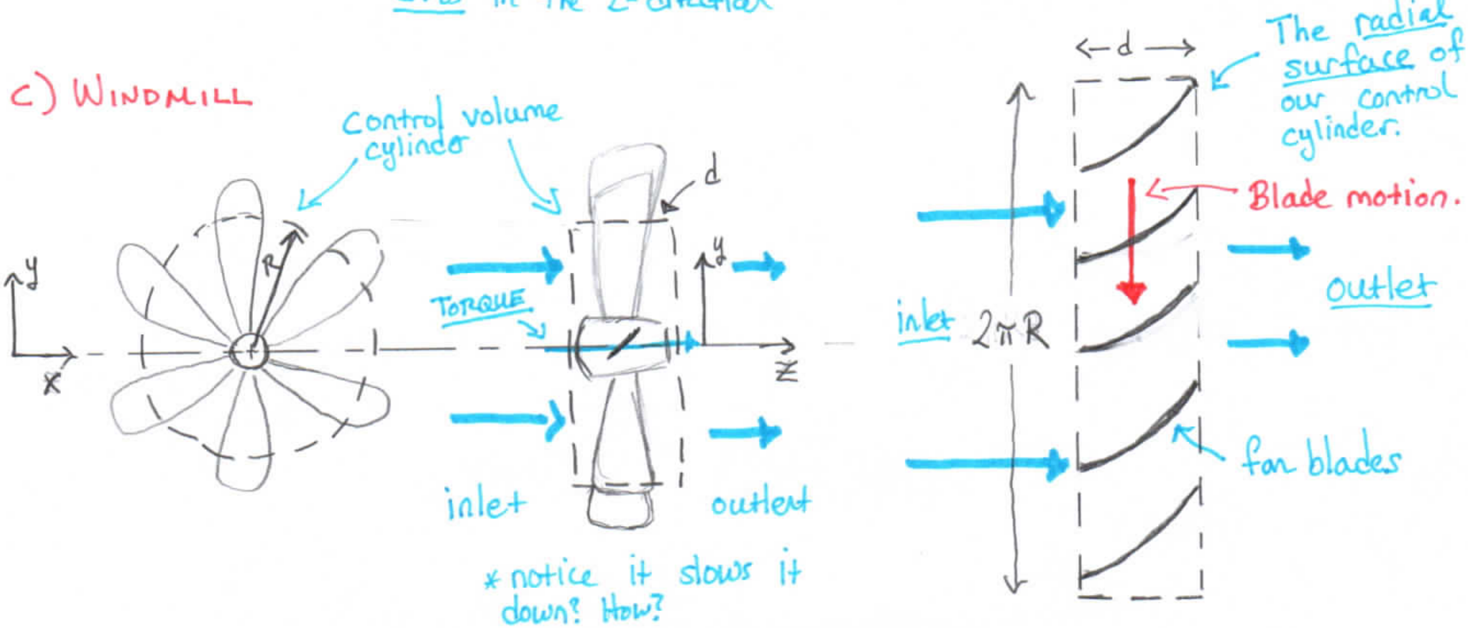
a) Radial Flow



b) Axial Flow

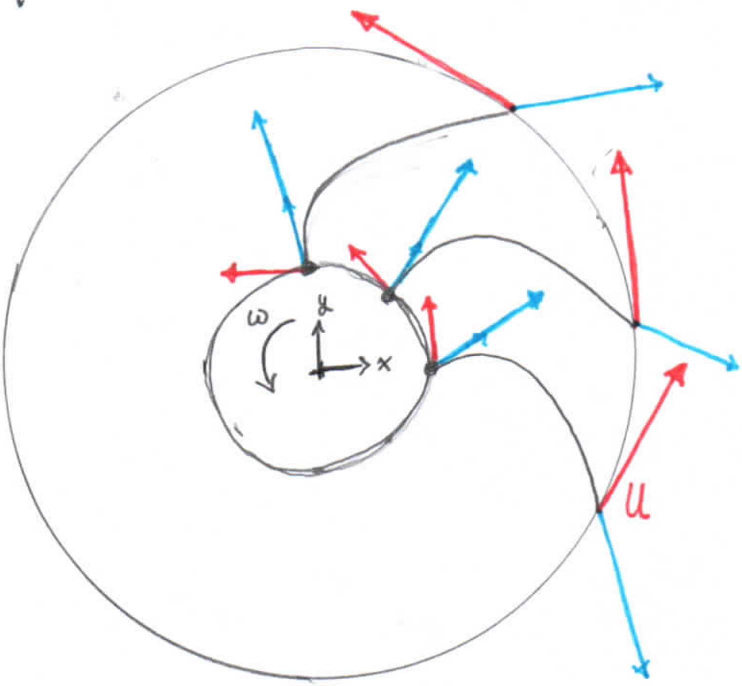


c) Windmill



LECTURE II:

From drawing enough picturer we can see all we want to describe is this inlet/outlet fluid velocity. This all happes at the impellar section of the pump. Let's focus on the Radial Flow xy impellar. We've worked so much with pumps. We ever calculated energy requirements of pumps so let's dig deeper to see how could design a pump to meet that requirement.



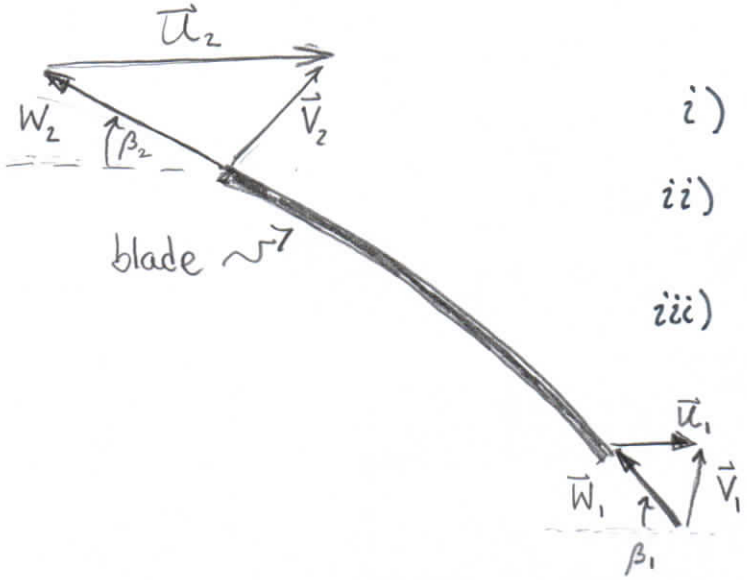
$\vec{U} := r\omega$ Radial Blade Speed

$\vec{W} :=$ fluid velocity relative to blade

$\vec{V} :=$ Absolute Velocity of fluid.

$$\vec{V} = \vec{W} + \vec{U}$$

* Because the blade is moving and the fluid we have to consider this relative velocity crap. IT IS CHANGES IN $\vec{V}$ WE CARE ABOUT.



For a pump...

i) $|V_2| > |V_1|$

ii) Kinetic Energy is increased due to WORK DONE by blades.

iii) Housing collects & decelerates fluid, causing a pressure rise or head rise.

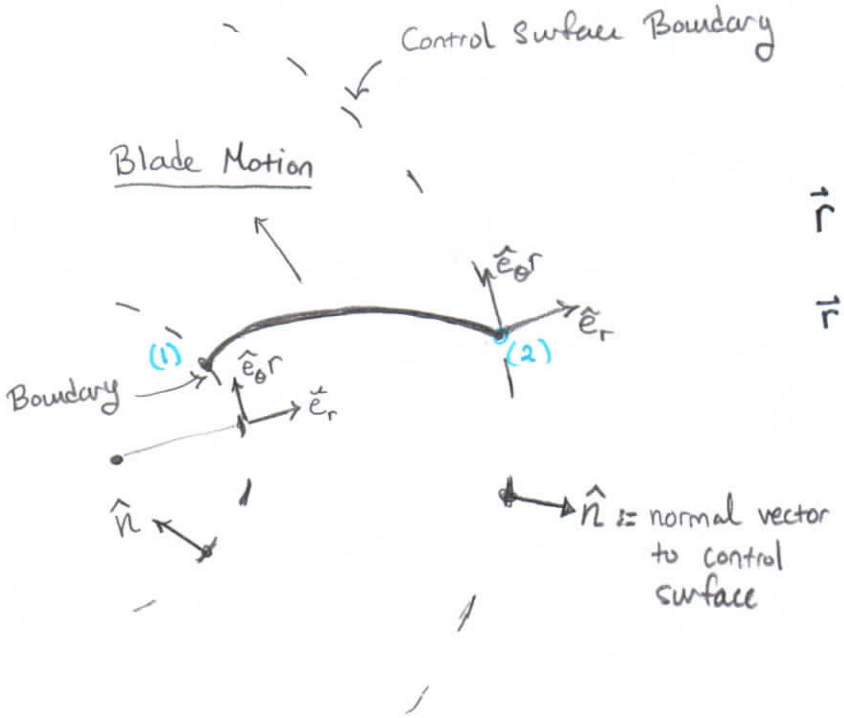
LECTURE 11:

We are concerned with changes in $\vec{V}$. We will conserve angular momentum to get an equation.

$$\underbrace{\sum (\vec r \times \vec F)}_{\text{Sum of external torques on control volume}} = \underbrace{\int_{\partial\Omega} (\vec r \times \vec V)\, \rho \vec V \cdot \hat n \, dA}_{\text{Flux of angular momentum across the control surface.}}$$

notice we're using $\vec V$, not $\vec U$ or $\vec W$.

* notice since there is a $(\vec V \cdot \hat n)$ we will have to break up the velocity into components to calculate this.



$$\vec V = V_r \hat e_r + r V_\theta \hat e_\theta$$

$$\vec r \times \vec F = T_{shaft} \hat e_z$$

$$\vec r \times \vec V = r V_\theta \hat e_z$$

Conservation of Mass
$\dot M_1 = \dot M_2$

$$T_{shaft} = r_1 V_{\theta_1} \rho (-V_1 A_1) + r_2 V_{\theta_2} \rho V_2 A_2$$

$$= \underbrace{-\rho V_1 A_1}_{\dot m_1} r_1 V_{\theta_1} + \underbrace{\rho V_2 A_2}_{\dot m_2} r_2 V_{\theta_2}$$

$$\boxed{T_{shaft} = \dot m \left[r_2 V_{\theta_2} - r_1 V_{\theta_1} \right]}$$

LECTURE 11:

Usually knowing the torque is meaningless we care about Shaft power $\dot{W}_s$

$$\dot{W}_s = T_{shaft} \cdot \omega$$

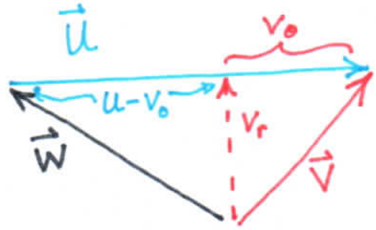
Power Requirement of Pump

$$\dot{W}_s = \dot{m}\left[\omega r_2 V_{\theta_2} - \omega r_1 V_{\theta_1}\right]$$

$$= \dot{m}\left[U_2 V_{\theta_2} - U_1 V_{\theta_1}\right]$$

$$\frac{\dot{W}_s}{\dot{m}} := \boxed{w_s = \left[U_2 V_{\theta_2} - U_1 V_{\theta_1}\right]}$$

Let's rewrite each term in terms of $\vec{V}, \vec{W}, \vec{U}$.



$$|\vec{V}|^2 = V_r^2 + V_\theta^2$$

$$|W|^2 = V_r^2 + (U - V_\theta)^2$$

Solve now for UV_θ.

$$V^2 = \left[W^2 - (U - V_\theta)^2\right] + V_\theta^2$$

$$= W^2 - (U^2 - 2UV_\theta + \cancel{V_\theta^2}) + \cancel{V_\theta^2}$$

$$= W^2 - U^2 + 2UV_\theta$$

$$\therefore UV_\theta = \frac{1}{2}\left(V^2 + U^2 - W^2\right)$$

Now we can look at w_s in terms of all the velocities contribution.

$$w_s = \frac{1}{2}\left[\underbrace{(V_2^2 - V_1^2)}_{\text{Increase in Kinetic Energy}} + \underbrace{(U_2^2 - U_1^2)}_{\text{Difference of Kinetic Energy for a particle stuck on a blade tip}} - \underbrace{(W_2^2 - W_1^2)}_{\text{Decrease in Kinetic energy as measured from relative velocity.}}\right]$$

All this equation is good for is reasoning contributions to power based on pump design properties. I don't know why we study it though...

$$w_s = \left[U_2 V_{\theta_2} - U_1 V_{\theta_1}\right] \text{ (much better equation)}$$

Using w_s we want to get an expression for head rise due to a pump. What variables do we want it in though?

$$\underbrace{\dot{W}_{shaft}}_{\text{Shaft power ACTUALLY transfered to the fluid.}} = \rho g Q h_i = \dot{m} g h_p \Rightarrow h_p = \frac{w_s}{g}$$

* This is less than power supplied to motor

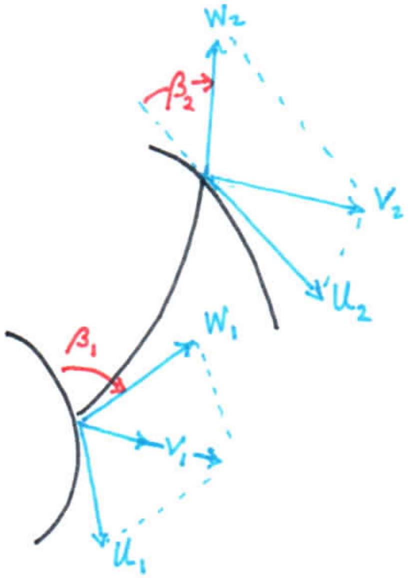
$$\therefore\ \boxed{h_p = \frac{1}{g}\left(U_2 V_{\theta 2} - U_1 V_{\theta 1}\right)}$$

Our expression for $h_p(U_2, U_1, V_{\theta 1}, V_{\theta 2})$ is useless though because its not interms of variables I can design.

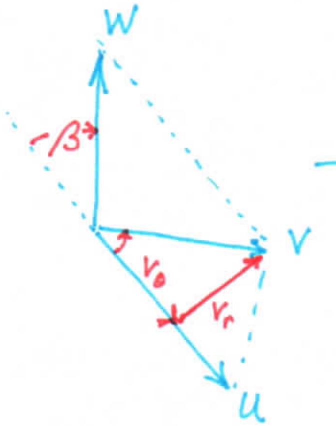
$$U = r \cdot \omega$$

r: geometry of impellars I can control this

ω: a radial velocity is known based on the output of my motor



V_θ = "swirl velocity"



Extra Credit show this

$$V_r \cot\beta = U - V_\theta$$

We make the assumption for turbomachinery that

$$U_1 V_{\theta 1} \approx 0$$

This makes sense only because we know at first

$$\left.\begin{array}{l} V_{\theta 1}^2 < U_1^2 \\ r_1 < r_2 \end{array}\right\} \text{for a pump only.}$$

So only one term matters for the head.

$$h_p = \frac{U_2 V_{\theta 2}}{g} - \cancel{\frac{U_1 V_{\theta 1}}{g}}$$

negligable in comparison.

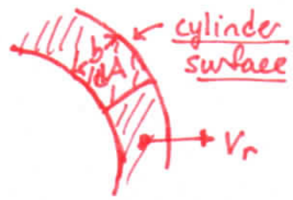
ALL OF THIS TO GET THE IDEAL CENTRIFUGAL PUMP EQUATION.

$$h = \frac{u_2^2}{g} - \frac{u_2 V_{r2} \cot\beta_2}{g}$$

$Q_2 = AV$

$= \underbrace{2\pi r_2 b_2}\, V_{r2}$

b_2 is blade width

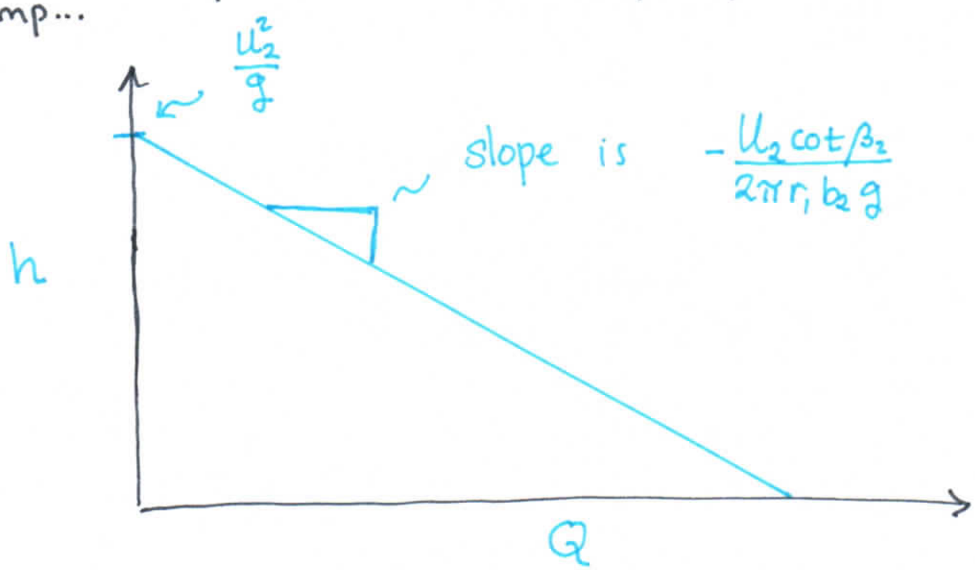


$$= \frac{u_2^2}{g} - \frac{u_2 \cot\beta_2}{2\pi r_2 b_2 g} Q$$

And essentially saying ...

$$h_p := f(\text{designable parameters} + \text{requirements})$$

Now we plot this ideal pump. This fake, never gonna happen pump...



Extra Credit

* If $Q = 0$ we have $h = \frac{u_2^2}{g}$. Why? It seems like if $Q = 0$ we should have no outlet velocity u_2? Explain why $h > 0$ if $Q = 0$.

LECTURE 11

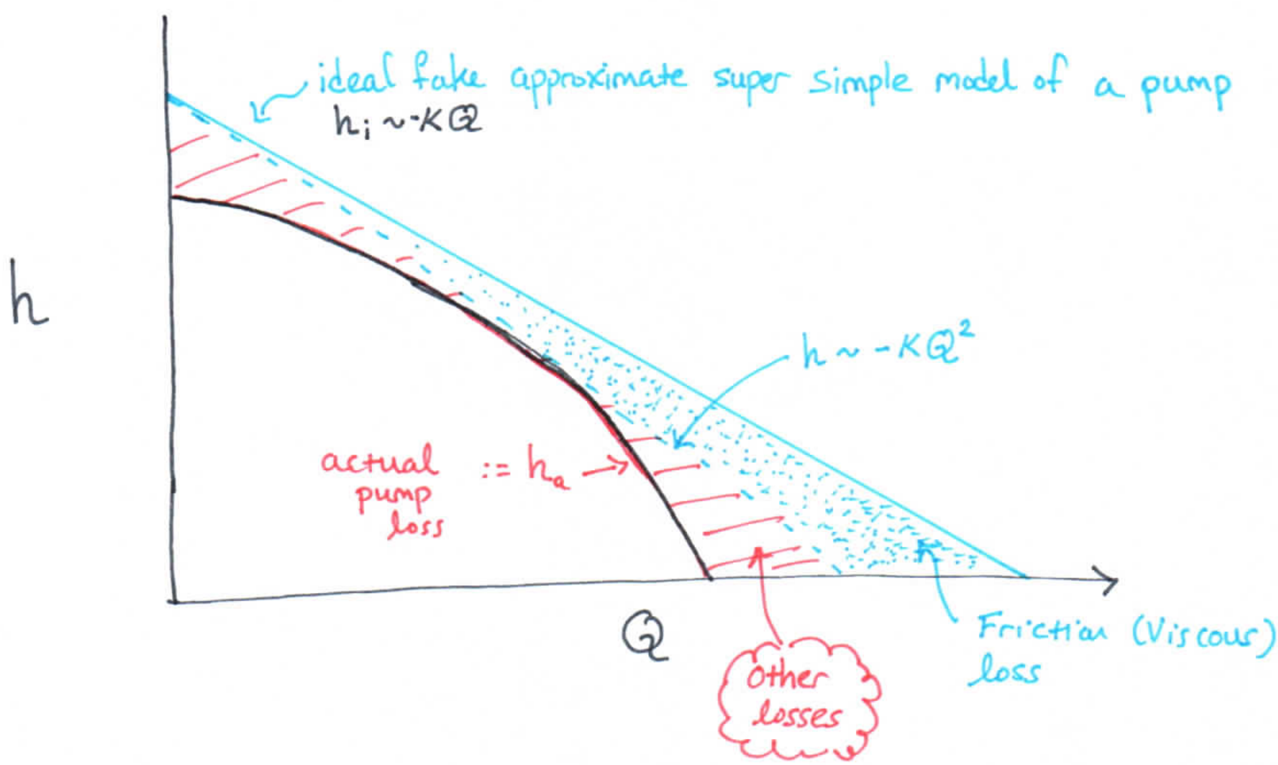
The simplest model for a pump gave us a line to describe $h(Q)$. In reality this curve is <u>less</u> due to losses such as.

1) Skin friction (external flow) increases with Q.

2) Flow Seperation (external flow)

3) 3D Effects, vorticies can be generated which take energy away from the ideal scenario

4) Leakage, clearance between fluid and casing provide path for fluid to move back behind impellars.

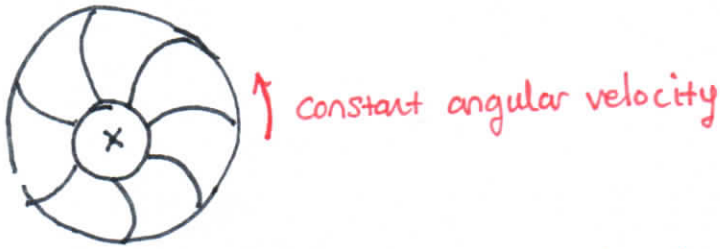


In general, $h_a \sim -KQ^{\alpha}$. The actual pump curve can be experimentally investigated to obtain a best fit α.

LECTURE 12:

Radial stuff is always difficult, But let's recover what we did to get an idea of whats important.

If we pretend that a pump looks like...



Neglect any viscous effects and neglect the inlet "swirl."

$u_1 = r_1 \omega$

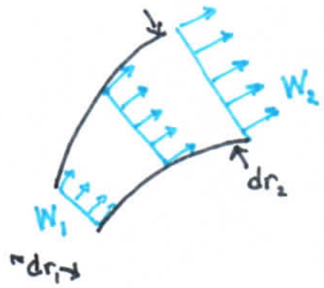
$u_2 = r_2 \omega$

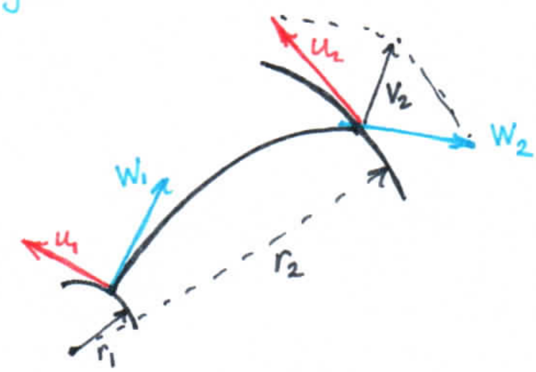
$W_1 = Q/A_1$

$W_2 = Q/A_2$

$V_1 = ?$

$V_2 = ?$





We get an expression for power based on conservation of angular momentum.

$$h = \frac{1}{g} u_2 V_{\theta 2}$$

We then can use the geometry of the blade to express this in terms of tuneable paremeters.

$$h = \frac{u_2^2}{g} - \frac{u_2 \cot \beta_2}{2\pi r_2 b_2 g} \cdot Q$$

We also wrote this some other way for some reason...

$$h = \frac{1}{2g}\left[(V_2^2 - V_1^2) + (u_2^2 - u_1^2) - (W_2^2 - W_1^2)\right]$$

And now we have a way to estimate a pump's head rise...