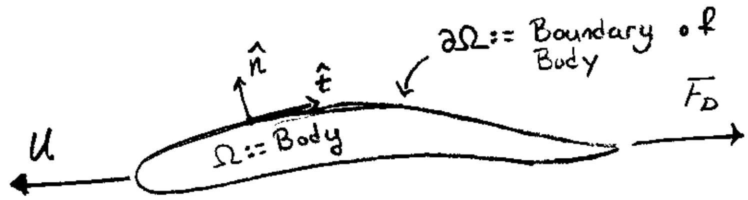
LECTURE 9 : WHAT DO YOU USE THE BOUNDARY LAYER FOR?

* JUST LIKE IN INTERNAL FLOWS WE CONSIDERED...

INTERNAL		EXTERNAL
• Valves	$\Longleftrightarrow$	• Curved Surfaces
• Curves		• Seperation

WE WILL SEE THAT CURVED SURFACE EFFECT THE PRESSURE DISTRIBUTION. BEFORE CONTINUE ON LET'S REVIEW THE RESULTS OF EXTERNAL FLOW ANALYSIS AND THE MOTIVATIONS OF IT SO FAR.

i) What is F_D?
The drag force.



$$\vec{F}_D = \int_{\partial\Omega} p\,\hat{n}\,dA + \int_{\partial\Omega} \tau_w\,\hat{t}\,dA$$

First term:

- This term is called "Form Drag"
- If we consider a flat plate this term is zero.
- This component of drag is due to Geometry of the body Ω

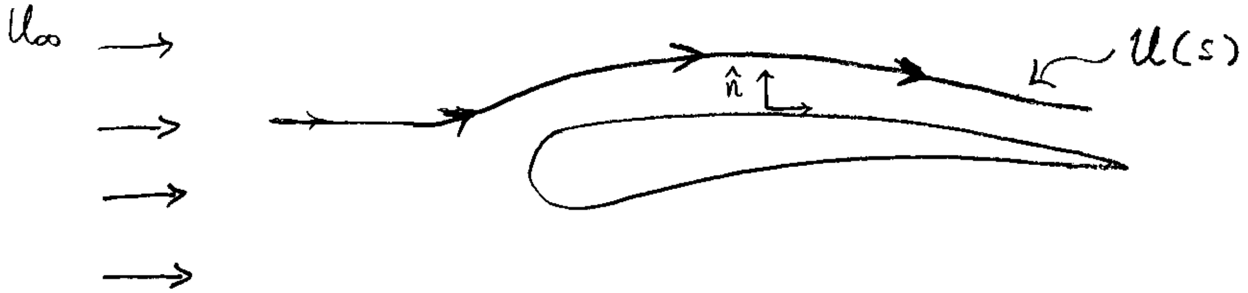
Second term:

- B.L analysis gave us a tool to estimate τ_w
- This component of drag is called "skin friction" and is a result of viscous forces on the surface $\partial\Omega$
- Analysis deliver a way to estimate

C_f

LECTURE 9:

So the last goal is to get a sense on this form drag term to get a total drag force, for any-shape. We look at streamlines!



$U(s)$:= Streamline outside the boundary layer

s := streamline coordinate
coordinate along surface

n := normal to surface coordinate

* ALL of our assumption about the boundary layer said out-side it flow behaved as inviscid. The flow looked parallel, so it was irrotational as well. This means we can . . .

USE BERNOULLIS!

$$\frac{d}{ds}\left(P(s) + \frac{1}{2}\rho U^2(s) + \rho g z(s)\right) = 0$$

$$\boxed{\text{ENERGY IS CONSERVED ALONG A STREAMLINE}}$$

LECTURE 9:

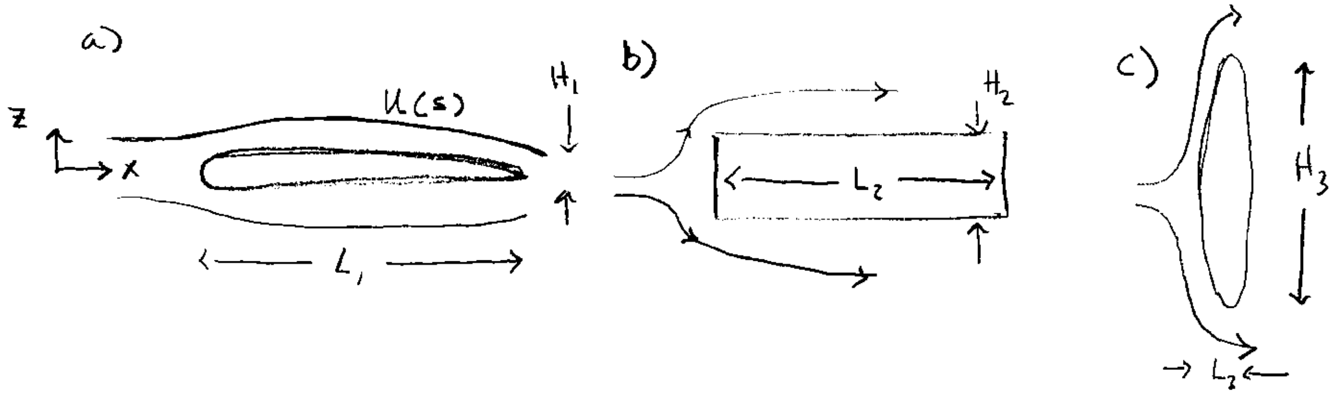
CARRY OUT $\frac{d}{ds}$ TO GET

$$\underbrace{\frac{dP}{ds}}_{\substack{\text{Change in}\\ \text{pressure}\\ \text{along a}\\ \text{streamline}}} + \underbrace{\frac{1}{2}\rho\frac{dU^2}{ds}}_{\substack{\text{Any acceleration}\\ \text{or decelleration}\\ \text{along a}\\ \text{streamline}}} + \underbrace{\rho g\frac{dz}{ds}}_{\substack{\text{Changes in}\\ \text{elevation with}\\ \text{respect to a}\\ \text{streamline coordinate } s.}} \equiv 0$$

WE COMMONLY MAKE THE ASSUMPTION THAT

$$\frac{dz}{ds} \approx 0 \qquad (*)$$

FOR AIRFOILS AND THIN BODIES THIS MAKES SENSE. WITH <u>SCALING</u> YOU CAN EVEN GET A QUANTITATIVE ESTIMATE OF HOW CLOSE TO ZERO IT IS.



EXTRA CREDIT

For a.) look up dimensions of common airplane wings and see if (*) is valid.

For b.) look up dimensions of birdseye views of rectangular skyscrapers and see if (*) is valid

For c.) look up windmill blades to see if (*) is valid.

LECTURE 9:

IF (*) IS TRUE WE GET A FAMOUS RELATIONSHIP.

$$\frac{1}{\rho}\frac{dP}{ds} = -U\frac{dU}{ds} \qquad (1)$$

THE OPPOSITE SIGN IS VERY IMPORTANT. LETS JUST SEE WHY

CASE 1: $\frac{dU}{ds} > 0$ (ACCELERATION)

IMPIES $\Rightarrow$ $\frac{dP}{ds} < 0$ WE CALL THIS A FAVORABLE PRESSURE GRADIENT

MOREOVER, SINCE U INCREASED RE INCREASED! TIE THIS BACK TO δ & τ_w. SO FROM POINT s_0 to s_1 WE SAY.

$$\frac{dU}{ds} > 0 \Rightarrow \frac{dP}{ds} < 0 \quad \& \quad \frac{dRe}{ds} > 0 \Rightarrow \frac{d\delta}{ds} < 0 \Rightarrow \frac{d\tau_w}{ds} > 0$$

{Reynold's Increases} $\Rightarrow$ {Boundary Layer decreases} $\Rightarrow$ {Shear Stress at Wall Increases}

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CASE 2: $\frac{dU}{ds} < 0$ (Deccelration) $\Rightarrow$ $\frac{dP}{ds} > 0$ Adverse PRESSURE GRADIENT

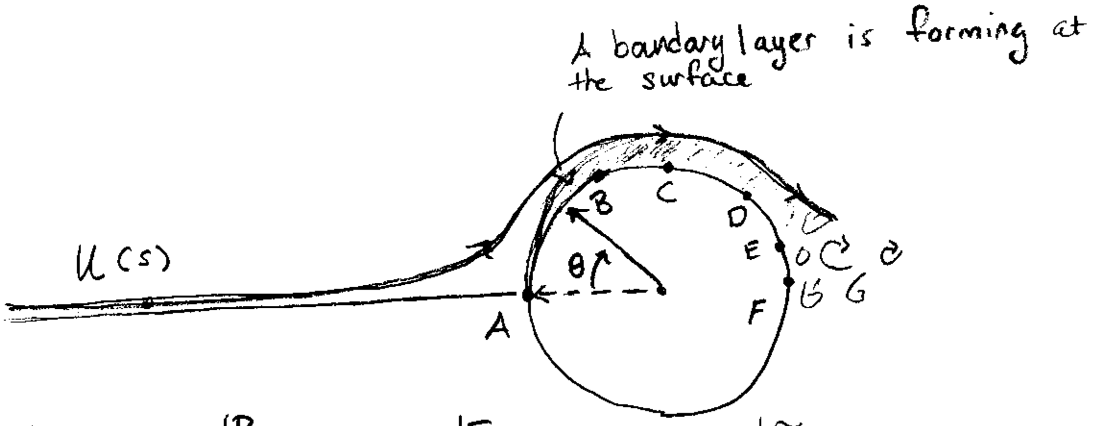
SAME REYNOLDS NUMBER REASONING SAYS.

$$\frac{dU}{ds} < 0 \Rightarrow \frac{dRe}{ds} < 0 \Rightarrow \frac{d\delta}{ds} > 0 \Rightarrow \frac{d\tau_w}{ds} < 0$$

{Slowing Down} $\Rightarrow$ {Increases the Boundary Layer} $\Rightarrow$ {Which decreases shear at the wall}

LECTURE 9:

WE CAN TAKE THIS AN SEE EVERY SITUATION OCCUR WITH FLOW OVER A BALL OR CYLINDER.



$(A \to C)$ $\frac{dU}{ds} > 0 \therefore \frac{dP}{ds} < 0 \Rightarrow \frac{d\delta}{ds} < 0 \Rightarrow \frac{d\tau_w}{ds} > 0$

$(C \to E)$ $\frac{dU}{ds} < 0 \therefore \frac{dP}{ds} > 0 \Rightarrow \frac{d\delta}{ds} > 0$ & $\underbrace{\frac{d\tau_w}{ds} < 0}$

THIS MEANS τ_w IS DECREASING? WHAT HAPPENS IF IT HITS ZERO!?

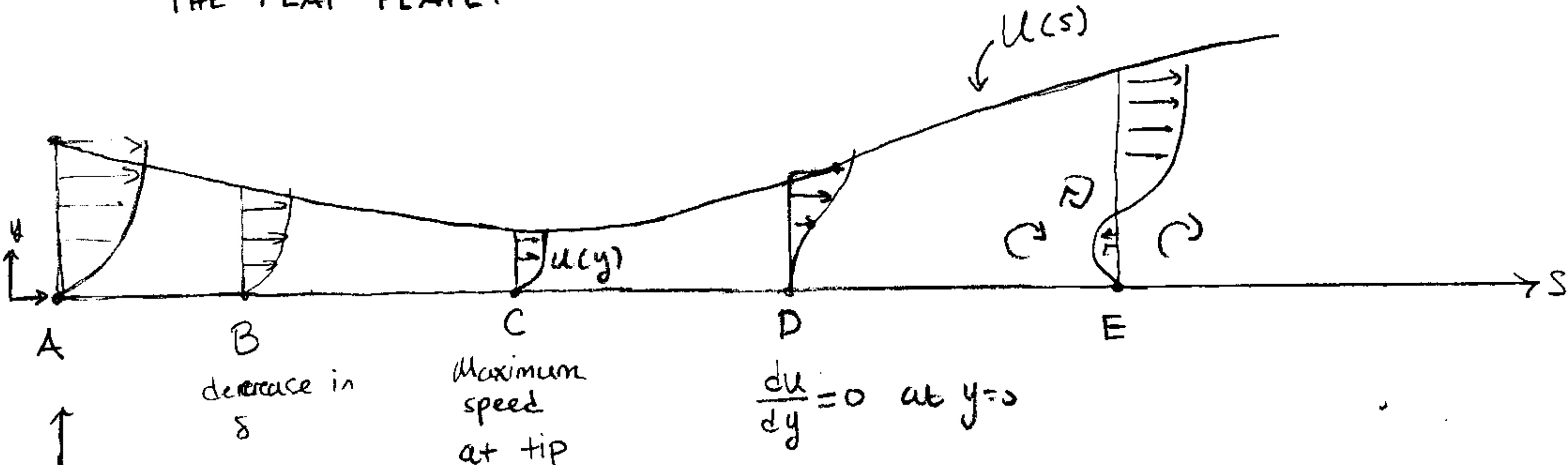
THE BOUNDARY LAYER SEPERATES WHEN $\tau_w = 0$ OR IN TERMS OF VELOCITY, SINCE $\tau = \mu \frac{\partial u}{\partial y}$.

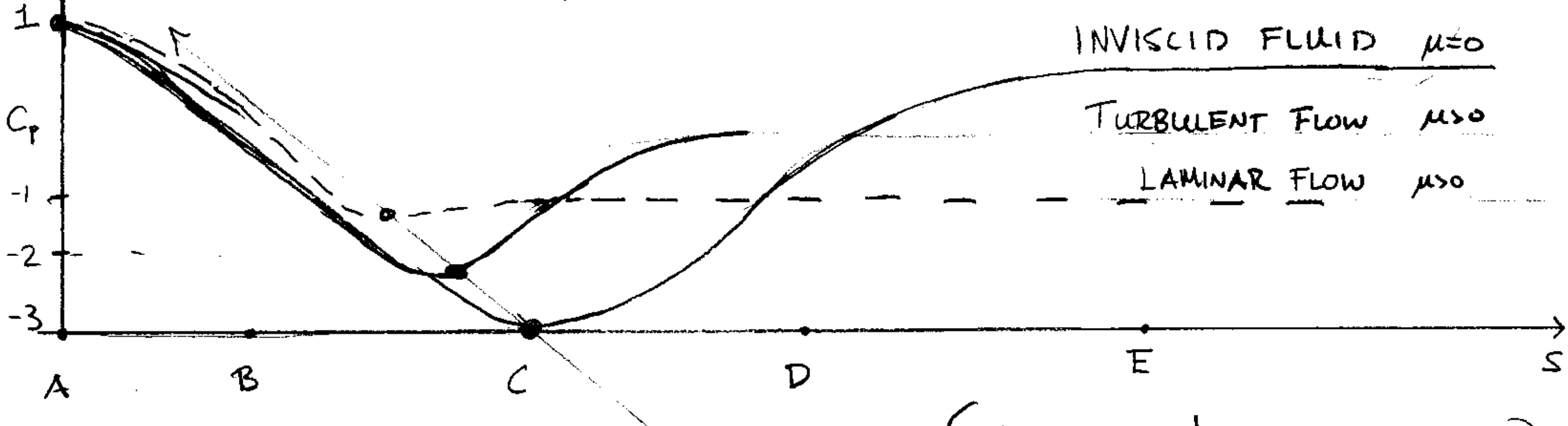
SEPERATION OCCURS $\quad \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$

IF WE "UNWRAP" THE CYLINDER A DRAW $u(y)$ ALONG THE ABSTRACT FLAT PLATE WE SEE WHATS HAPPENING TO $u(y)$.

LECTURE 9:

* FOR CURVED SURFACES $U(s)$ IS NOT CONSTANT, UNLIKE THE FLAT PLATE.





$$C_p := \frac{(P-P_0)}{\frac{1}{2}\rho U^2}$$

{ This is the FORM DRAG COEFFICIENT SIMILAR TO OUR PREVIOUS SKIN DRAG COEFF }

* INVISCID THEORY FULLY RECOVERS PRESSURE

* INCREASING SPEED MAKES MINIMUM OCCUR SOONER

* LAMINAR FLOW HAS A LOWER PRESSURE RECOVERY.

* TURBULENT FLOW HAS A DELAYED SEPERATION POINT

* NO MATTER HOW SMALL μ IS SEPERATION WILL OCCUR AND ADD TO THE FORM DRAG COMPONENT OF $\vec{F}_D$.

LECTURE 9:

FROM THIS SIMPLE EXAMPLE WE GAINED SOME INSIGHTS THAT.

CURVED SURFACES $\Rightarrow$ MAKE $U_\infty(s)$ AND **NOT** CONSTANT.

THIS MEANS VARIATIONS IN STREAMLINES TELL US ABOUT SHEAR. RECALL.

$$\tau_w = \rho U_\infty^2 \Theta$$

only if $U_\infty =$ constant

IF $U_\infty(s)$ WE GET AN INTEGRAL MOMENTUM EQUATION, RECALL DEFS OF Θ AND δ^*.

$$\tau_w(s) = \rho \frac{d}{ds}\left(U_\infty^2 \cdot \Theta\right) + \rho \delta^* U_\infty(s) \frac{d U_\infty(s)}{ds}$$

$$= \rho \Theta \frac{d U_\infty^2}{ds} + \rho U_\infty^2 \frac{d\Theta}{ds} + \frac{1}{2}\rho \delta^* \frac{d U_\infty^2}{ds}$$

OR ANOTHER FORM.

$$\tau_w = \rho U_\infty^2 \frac{d\Theta}{ds} + \rho\left(\Theta + \frac{\delta^*}{2}\right)\frac{d U_\infty^2}{ds} \qquad (**)$$

THIS IS TRUE FOR TURBULENT OR LAMINAR FLOWS! Looking at (**) WE CAN DETERMINE A SEPERATION POINT

EXTRA CREDIT
Using (**) what equation would govern the seperation point.., this is pretty simple...

LECTURE 9:

So what do you USE Boundary layer theory for?

Well... basically.

ESTIMATE WHAT C_f & C_p WILL LOOK LIKE FOR A GIVEN BODY IN A FLOW FIELD

$$C_f = \frac{F_{SKIN}}{\frac{1}{2}\rho U^2 A} \quad \& \quad C_p = \frac{F_{FORM}}{\frac{1}{2}\rho U^2 A}$$

$$C_L = \frac{F_{LIFT}}{\frac{1}{2}\rho U^2 A}$$

FOR THE SITUATION...

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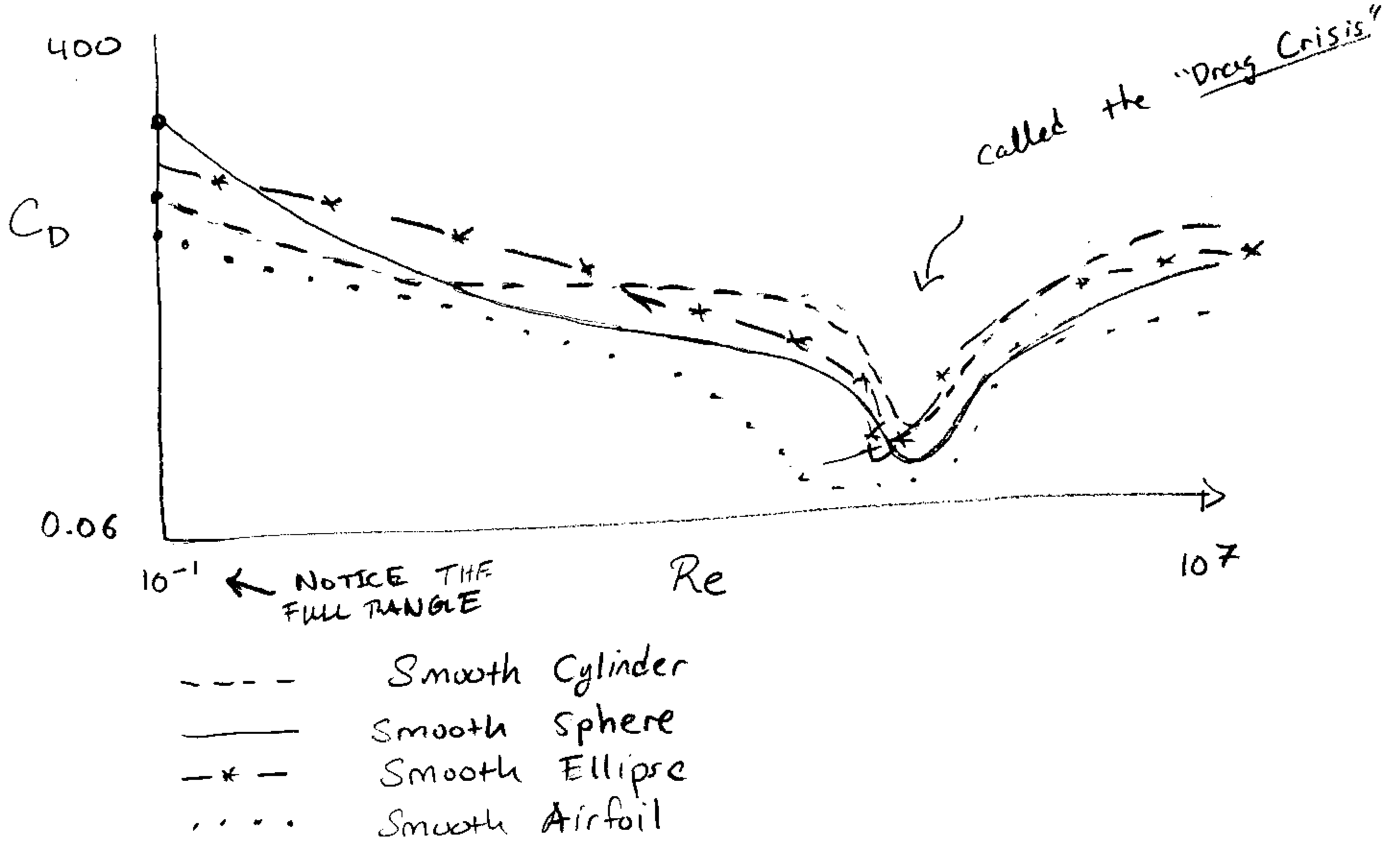
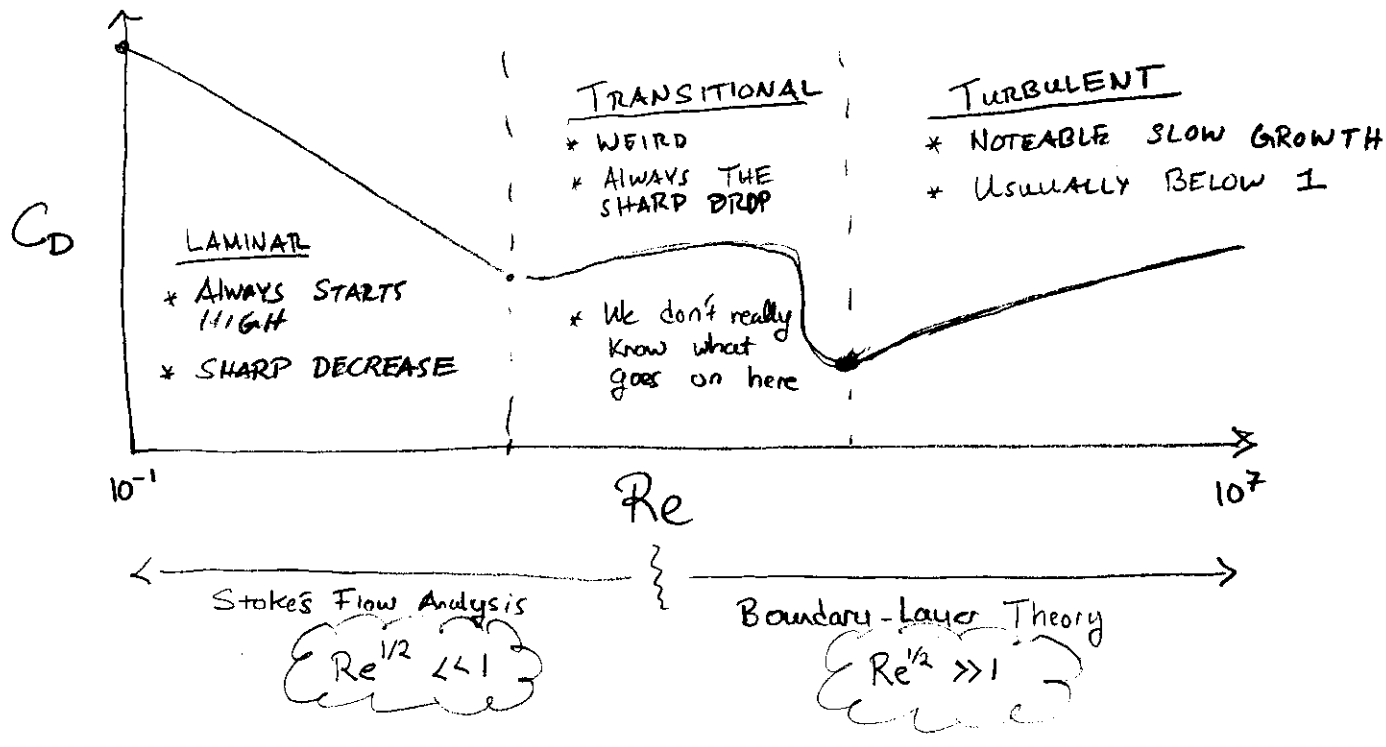
NOW FOR INTERNAL FLOW I THINK USING A MOODY CHART IS OUTDATED, BUT AS YOU CAN SEE EXTERNAL FLOW IS SO MUCH COMPLEX THAT I BELIEVE CHARTS RULE.

LECTURE 9:

WE ALREADY DETERMINED THE C_f CHART... IT'S LOOKS IDENTICAL TO THE MOODY CHART. BUT C_D HAS A DIFFERENT SHAPE.

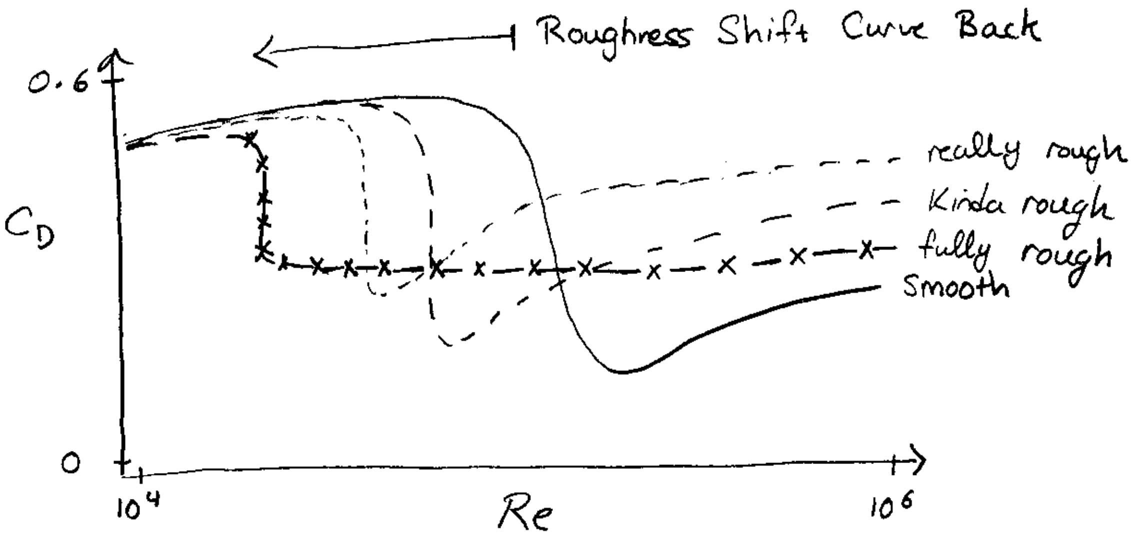


All shapes basically have the same shape.



LECTURE 9

For the same body with a roughsurface ε we see that the "Drag Crisis" is shifted back and recovers more pressure.



* Notice fully rough (a golf ball) transitions at the lowest Re <u>and</u> has the lowest C_D of rough-surfaces after the crisis.

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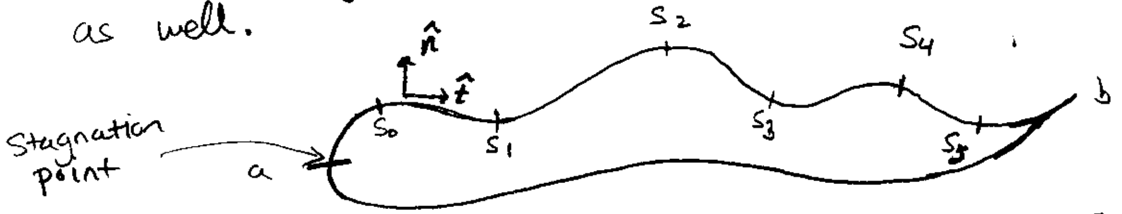
So that's really it. I may well could have just given you charts for C_f and C_D and set you loose calculating Drag Forces, but you wouldn't appreciate

1) Where that Drag comes from
2) A Boundary Layer <u>EXISTS</u>
3) Why the curves look so weird

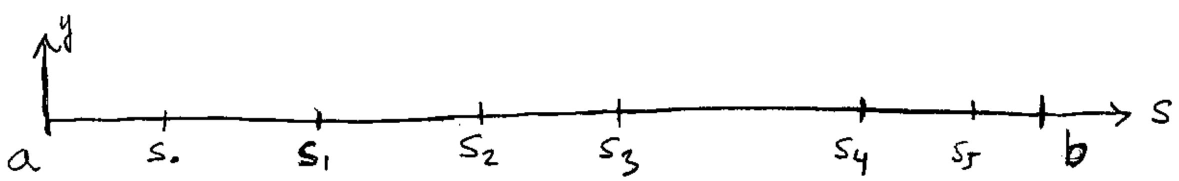
VERY HARD EXTRA CREDIT... DEF WILL BE ON EXAM...

We just said that δ must always exist because of no-slip. So what's up with this B.L. seperation? After seperation what is no-slip violated? The answer is **NO**. No-Slip or $u(0)=0$ is never violated. Explain then what B.L. seperation IS !?!?

It's a great conceptual trick to "unwrap" surfaces and highlight sections of curvature and draw boundary layers. There always exists a stagnation point as well.



* The worst wing ever designed



Extra Credit

fill in velocity profiles and boundary layers similar to the cylinder example. Where do you think it will seperate?

That's it for External Flows! We don't cover a whole other topic of "Circulation" in this class but it is a fascinating way to describe the lift force!

I hope you appreciate though how

1) To calculate power we needed to calculate a drag force

2) To understand the drag-force we had to <u>zoom</u> into a surface <u>using</u> the Navier Stokes.

3) Using our equations we could calculate friction factors, $C_f(Re)$

4) Using the concepts of streamlines we could understand pressure distributions

$$C_D(Re)$$