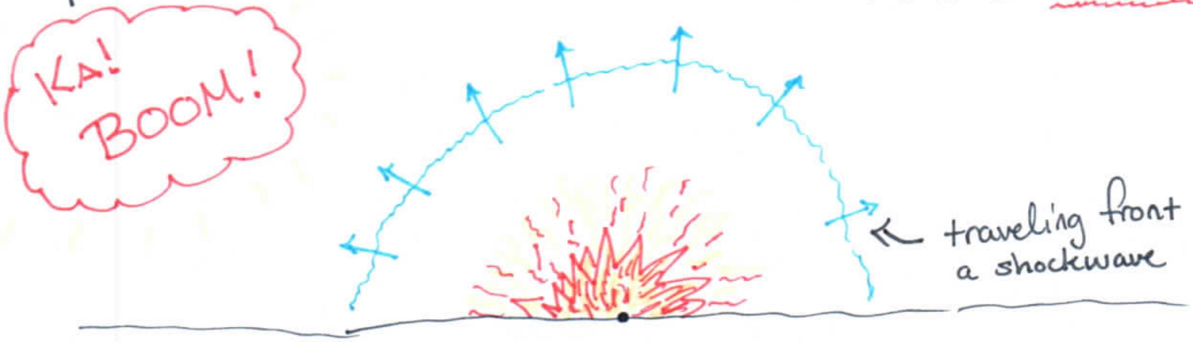
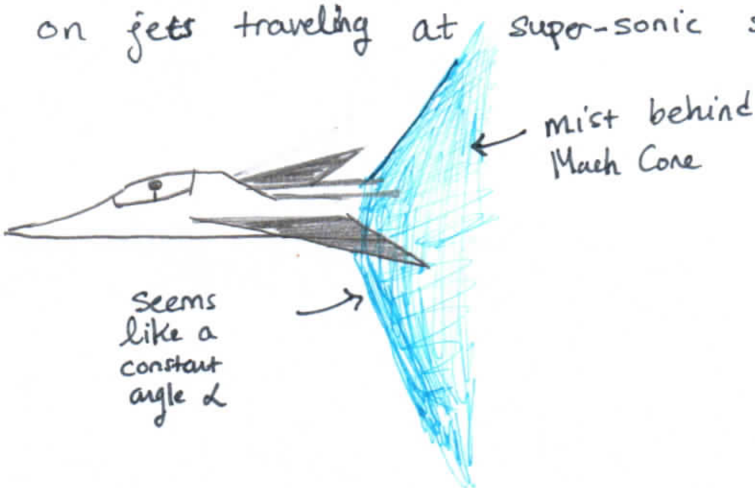
# LECTURE 17 : What the hell is a Shockwave?!

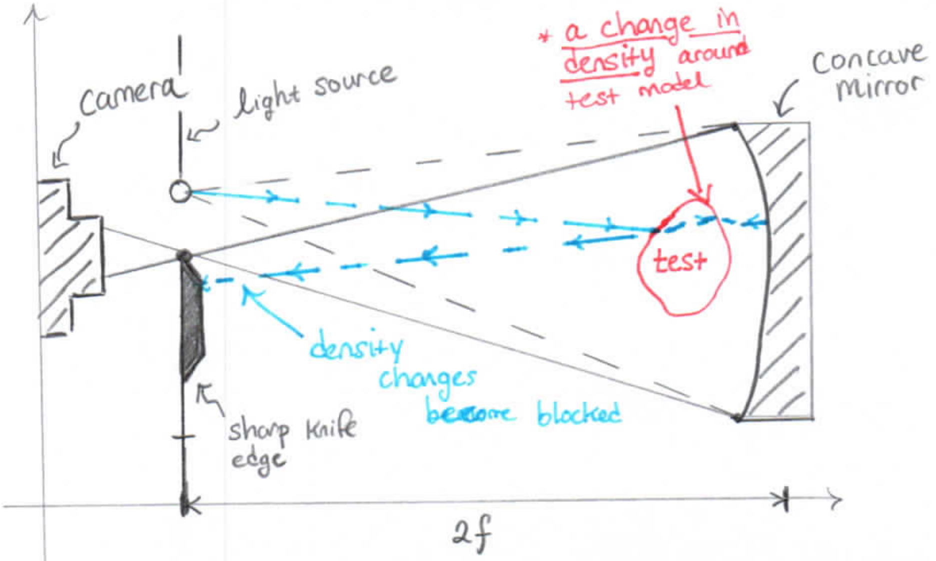
Everyone has at least seen images of shockwaves and can picture them in their head. We see them in EXPLOSIANS!




We see them on jets traveling at super-sonic speeds.




If you're clever enough you can see better images by using a super-sonic wind tunnel, a small model, and schlieren imagery. This really just a clever way to observe index of refraction gradients in an image.




* A change in density causes a change in refraction angle
* Stiffer materials or media slow down the speed of light in that media.
* The image produces shadows where sharp density gradient's appear.

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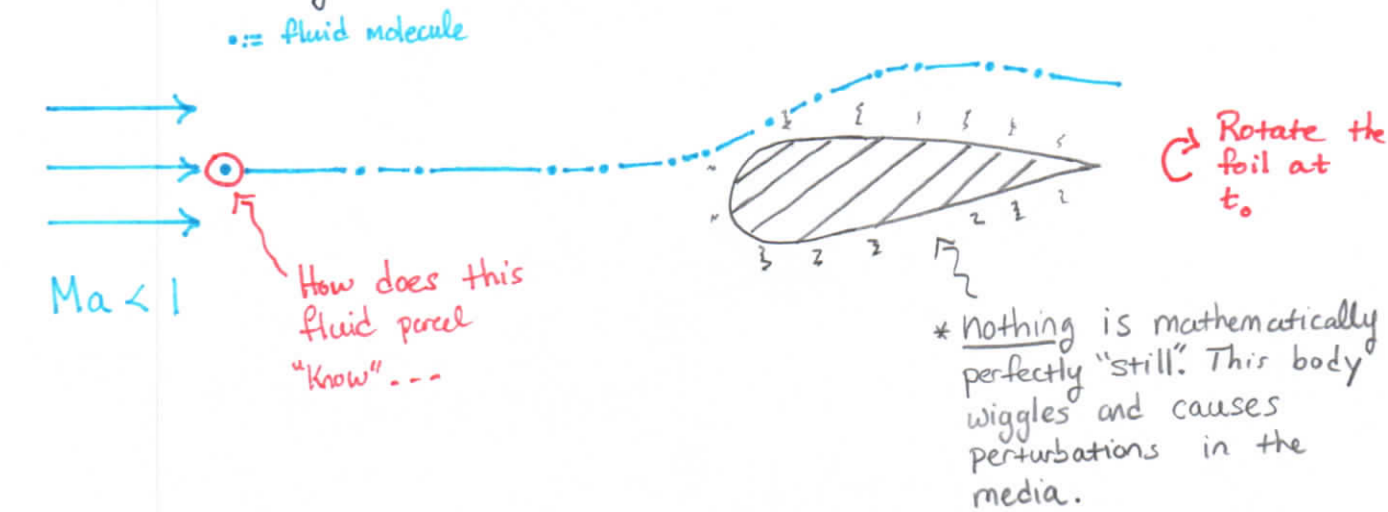
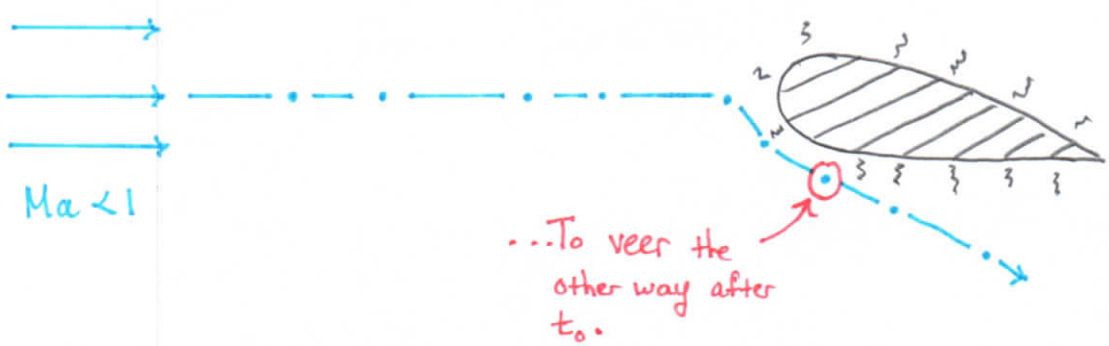
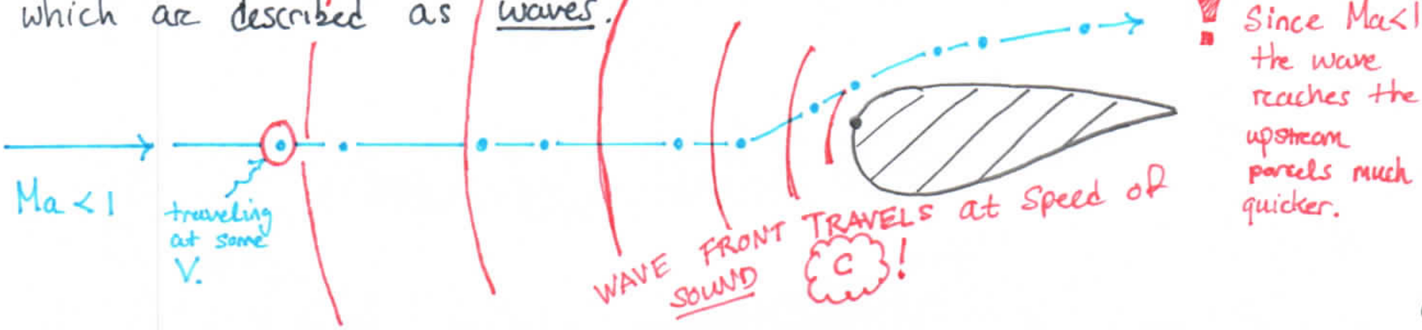
We definitely then can see sharp changes in density around objects moving very fast. But why are they happening? This is the classic rationalization of PRESSURE WAVES eminating from a body. Consider a body sitting in external flow.




When we rotate the foil we can see that the streamline upstream of the action of rotation adjusts almost simultaneously. SPOOKY ACTION AT A DISTANCE




This paradox is settled by Pressure Waves in the medium. Slight jiggles of the body cause small compressions in the media. These compressions then expand / contract away from the body which are described as waves.



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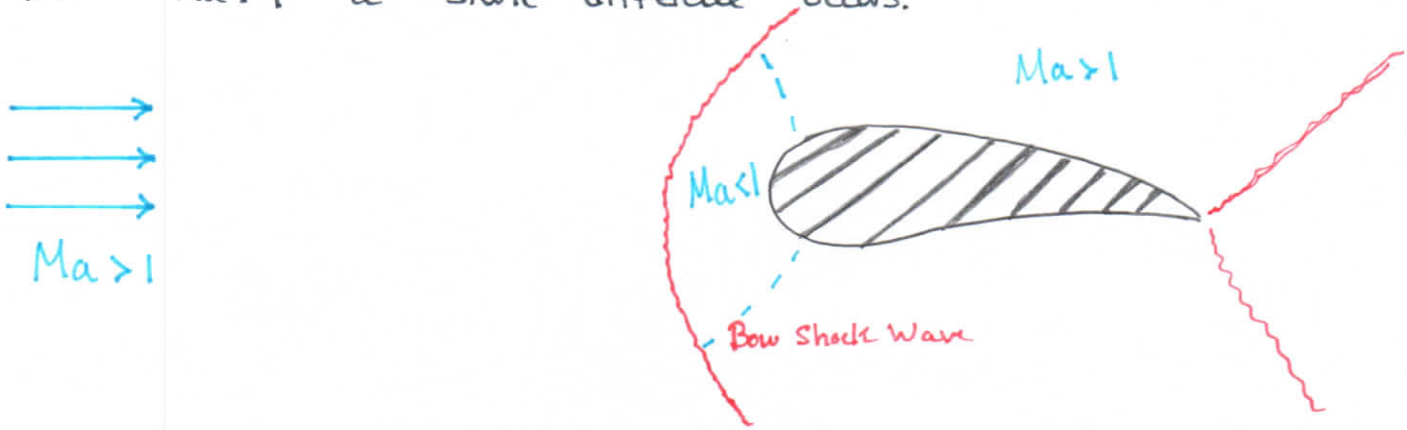
Now we should recall we did tons of work to show $C$ is dependent on thermodynamic properties of the media.

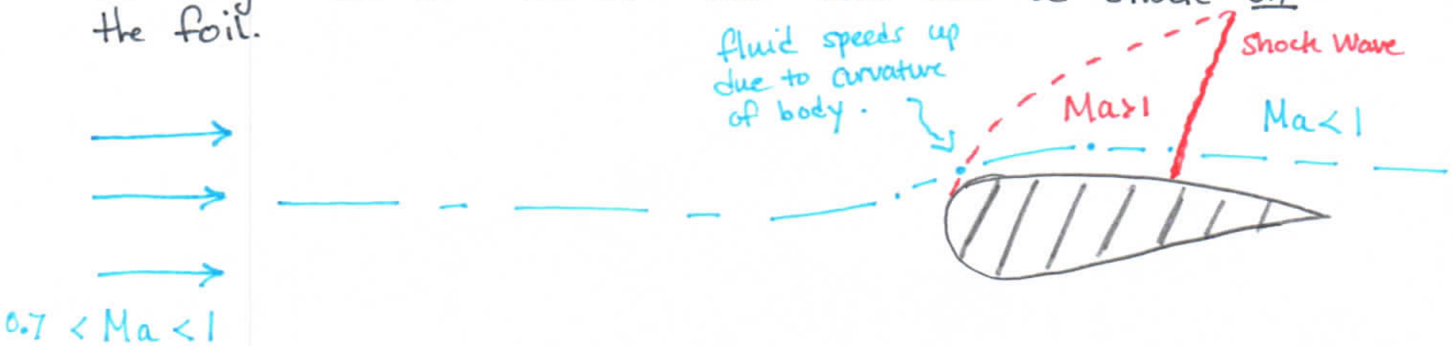
$$C = \sqrt{\left.\frac{\partial P}{\partial \rho}\right|_{s=s_0}}$$

our experiments stilized imaging sharp $d\rho$ so we're on the right track.

So when we preform the same streamline experiment with $Ma > 1$ a stark difference occurs.

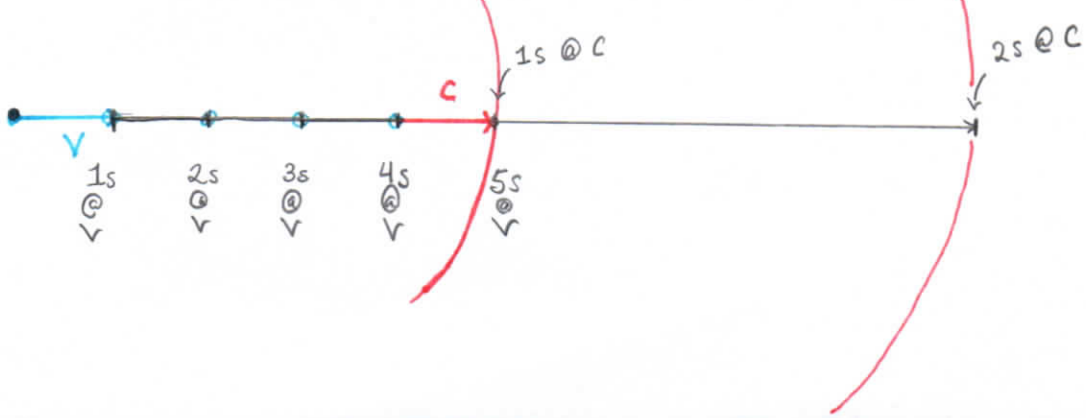



Actually even at $Ma < 1$ we can see a shock on the foil.




Let's now just consider a projectile moving at a speed $V$ eminating spherical pressure waves due to jiggles. We draw a coordinate system that has tick marks every $V \cdot 1s = [m]$ we also consider the speed of pressure waves traveling at $c$. For $Ma < 1$ notice.

$$Ma = \frac{1s @ C}{1s @ V} = 0.2$$



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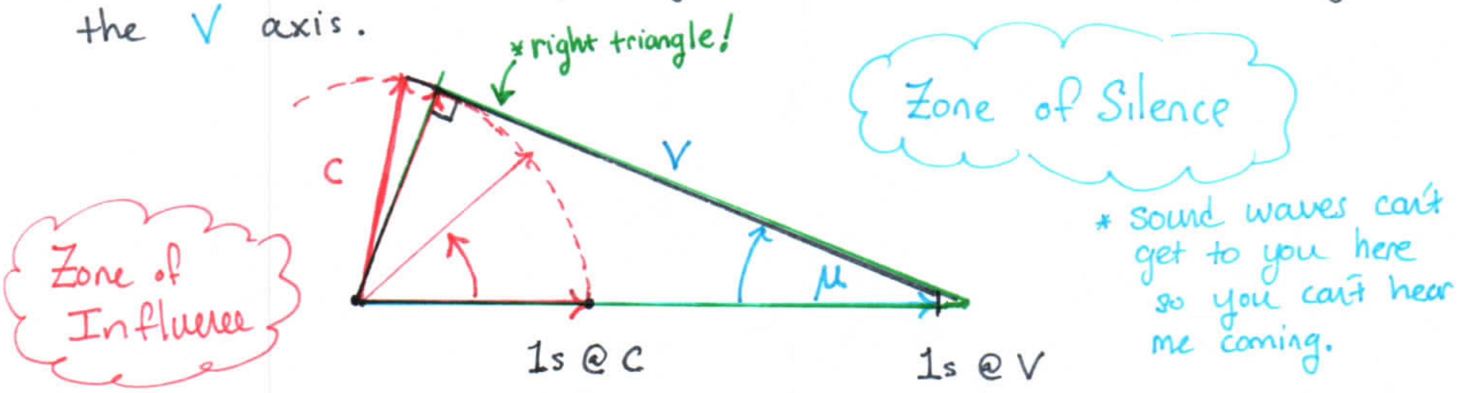
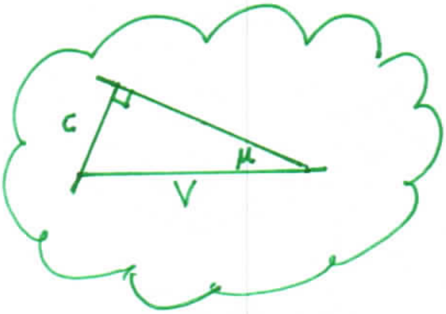
Now as V increases what happens as we surpass the speed of sound. Conceptually this means we're stretching the V axis.



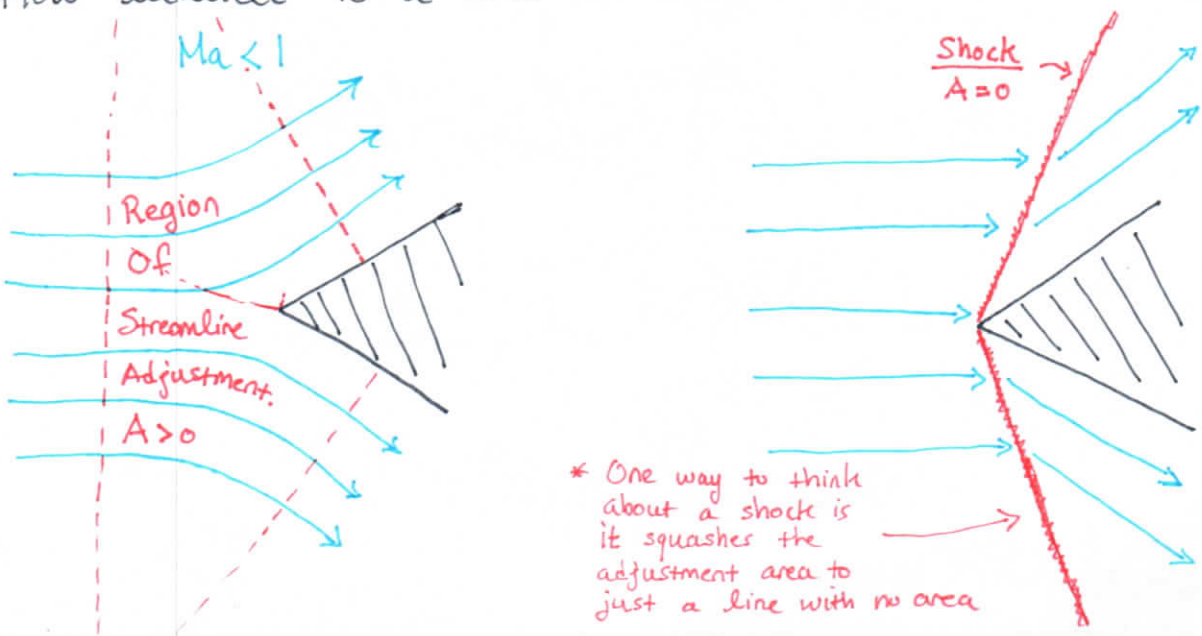




$$\sin \mu = \frac{c}{V} = Ma^{-1} < 1 \quad \text{(for supersonic)}$$

$$\therefore \quad \mu = \sin^{-1}(Ma^{-1} \leq 1)$$

This angle only makes sense as well if $Ma > 1$. This is a way you can determine speed from looking at an image of a supersonic jet. Just like relativity refers to a "light-cone" compressible flow has it's own "sound-cone".

A shock wave seems to be a discontinuous adjustment to flow localized to a line of action.



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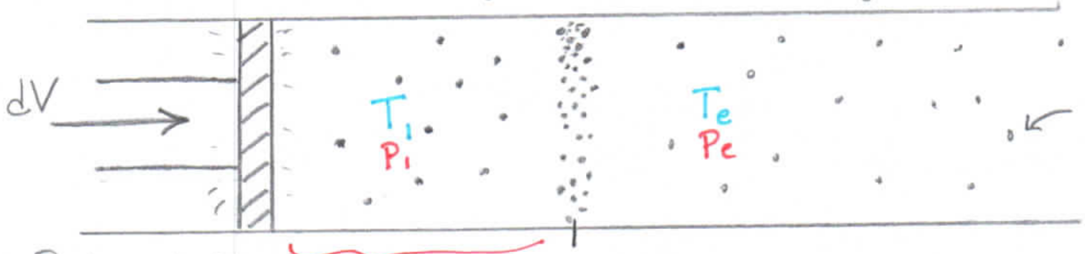
Now we take a simple piston and try to investigate the origin of a shock, first we study "Normal" shocks which are in the same direction as flow. Consider a piston.

** Piston compresses gas infront of it causing a wave to travel forward



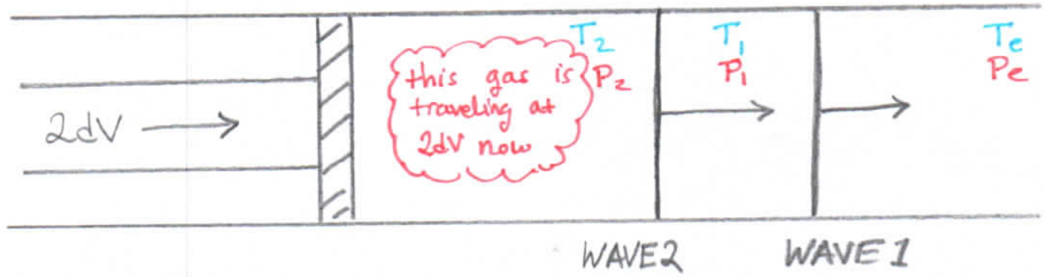

* Piston starts at rest and is accelerated to a dV

this gas is traveling at dV now.

* $T_1 > T_e$ because of a speed increase.

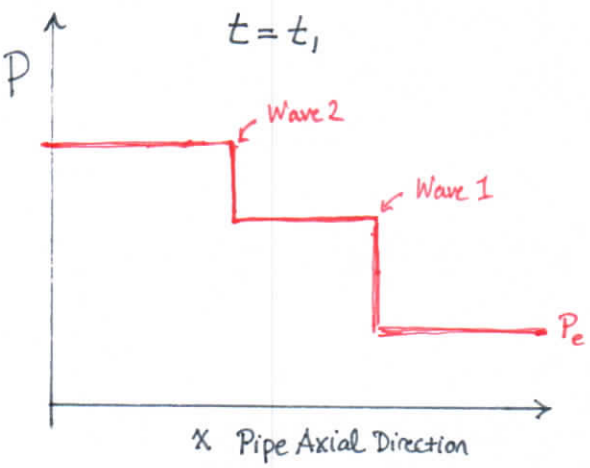
* $P_1 > P_e$ cause it's compressed.

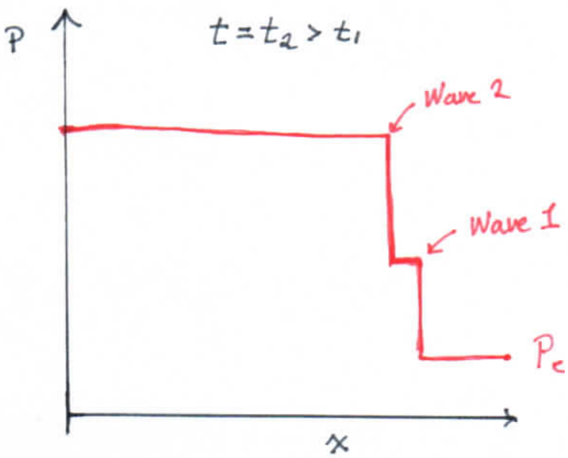
Now imagine we keep incrementing the piston by dV exhaustively and consider the traveling waves.



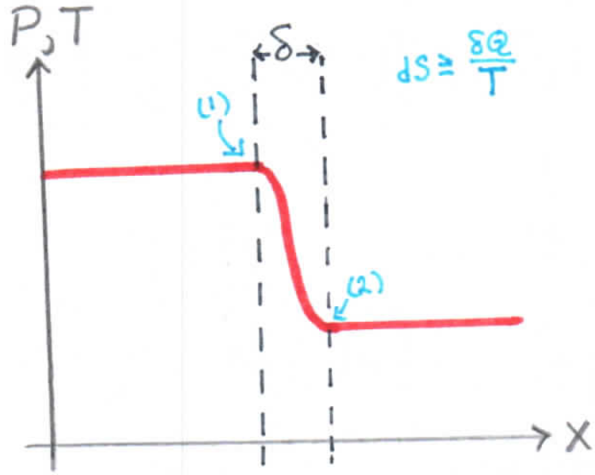

* Since $T_1 > T_e$ and $c = \sqrt{\gamma RT}$ the second wave is traveling FASTER than the first

We could keep doing this making dV ever so small creating a continuous acceleration. Now since for every increment the wave speed is increasing due to slight temperature elevations realize the waves will at some point all meet at some $x_0$ for some $t_*$. A time snap shot series of pressure would look like such.



In a limiting case we build-up a normal shock. Across this front properties abruptly change. In reality it's continuous though.




$\delta \approx 10^{-5}$ in "Shockthickness"

* What occurs "inside" δ is
  - Viscous Dissipation
  - Heat Conduction
  - Acoustic Losses

**2nd Law Time**

Since there are dissipative effects across δ entropy must increase across a shock wave.

Now since we are comfortable with the existence of a shock front we will use control volumes to develope equations for flow properties behind and infront of the shock.

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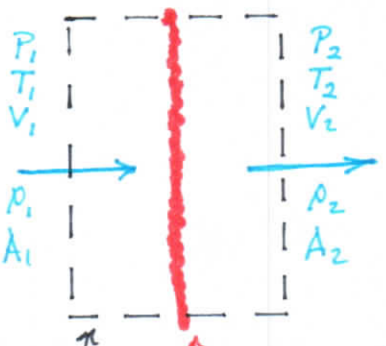


$A_1 = A_2$

This should be second nature now as to what to balance in/out.

$$\rho_1 V_1 \cancel{A_1} - \rho_2 V_2 \cancel{A_2} = 0 \qquad \text{Mass} \qquad (1)$$

$$P_1 \cancel{A_1} - P_2 \cancel{A_2} - \iint_{C.S.} V_x \left(\rho \vec{V} \cdot \cancel{dA}\right) = 0 \qquad \text{Momentum} \qquad (2)$$

$$h_1 + \frac{V_1^2}{2} - h_2 - \frac{V_2^2}{2} = 0 \qquad \text{Energy} \qquad (3)$$

$$P - \rho R T = 0 \qquad \text{Equation of State} \qquad (4)$$

$$Ma - \frac{V}{\sqrt{\gamma R T}} = 0 \qquad \text{Mach Def.} \qquad (5)$$

**Goal**: Use (1-5) to express flow properties in terms of $Ma, \gamma$.

$\frac{T_2}{T_1} = f_1(Ma, \gamma)$, $\frac{P_2}{P_1} = f_2(Ma, \gamma)$, $\frac{\rho_2}{\rho_1} = f_3(Ma, \gamma)$, $\frac{V_2}{V_1} = f_4(Ma, \gamma)$

Upstream Mach Value actually

So let's face our fears head on and just DO IT!

(1) Combine Equation of State + Energy.

$$P = \rho R T \Rightarrow dh = c_p dT$$ (We've shown this many a times)

$$T_1 + \frac{V_1^2}{2} = T_2 + \frac{V_2^2}{2}$$

Convert $V_i \to M_i$ using Mach def.

$$T_1\left(1 + \frac{\gamma-1}{2}Ma_1^2\right) = T_2\left(1 + \frac{\gamma-1}{2}Ma_2^2\right)$$

(2) We convert Momentum as well

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$$P_1\left(1 + \frac{\rho_1 V_1^2}{P_1}\right) = P_2\left(1 + \frac{\rho V_2^2}{P_2}\right)$$

but, $\frac{\rho}{P} = \frac{1}{RT}$ and $C^2 = \gamma R T$ and $Ma = \frac{V}{c}$

$$\therefore \quad P_1(1+\gamma Ma_1^2) = P_2(1+\gamma Ma_2^2)$$

(3) Now we combine everything into continuity.

$$\rho_1 V_1 = \rho_2 V_2 \Rightarrow \frac{P_1}{RT_1} Ma_1\sqrt{\gamma R T_1} = \frac{P_2}{RT_2} Ma_2 \sqrt{\gamma R T_2}$$

$$\Rightarrow \frac{P_1}{\cancel{R}T_1} Ma_1 \sqrt{T_1}\,\cancel{\sqrt{\gamma R}} = \frac{P_2}{\cancel{R}T_2} Ma_2 \sqrt{T_2}\,\cancel{\sqrt{\gamma R}}$$

$$\therefore \quad \left(\frac{Ma_1}{Ma_2}\right) = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right)^{1/2}$$

$$= \frac{(1+\gamma Ma_1^2)}{(1+\gamma Ma_2^2)}\left\{\frac{\left(1 + \frac{\gamma-1}{2}Ma_2^2\right)}{\left(1 + \frac{\gamma-1}{2}Ma_1^2\right)}\right\}^{1/2}$$

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Now we must assume we <u>Know</u> $Ma_1$ (upstream condition) and use the last equation to determine $Ma_2$ (after shock).

This turns out to be a quadratic equation for the variable $Ma_2^2$.

Square the last equation ... assume $Ma_1$ is known.

$$\frac{Ma_1^2}{Ma_2^2} = \frac{(1+\gamma Ma_1^2)^2}{(1+\gamma Ma_2^2)^2} \cdot \frac{\left(1+\frac{\gamma-1}{2}Ma_2^2\right)}{\left(1+\frac{\gamma-1}{2}Ma_1^2\right)}$$

Extra Credit
set-up quadratic for $Ma_2^2$. step by step ...

We get the quadratic equation.

$$(*) \quad Ma_2^4\left(\frac{\gamma-1}{2} - \gamma^2 F(Ma_1)\right) + Ma_2^2\left(1-2\gamma F(Ma_1)\right) - F(Ma_1) = 0$$

Where $F(Ma_1)$ is an expression <u>only</u> in terms of $Ma_1$ and can be considered a constant.

$$F(Ma_1) = \frac{M_1^2\left(1+\frac{\gamma-1}{2}Ma_1^2\right)}{(1+\gamma Ma_1^2)^2}$$

Solve (*) using quadratic formula, for roots of $Ma_2^2$.

$$Ma_2^2 = \frac{Ma_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}Ma_1^2 - 1} := \text{Real root of } (*)$$

Now using L'Hopital we can evaluate $Ma_1 \to \infty$. Or use "clever algebra".

$$\lim_{Ma_1\to\infty} Ma_2 = \sqrt{\frac{\gamma\cancel{k}-1}{2\cancel{k}\gamma}}$$

Sorry I meant $K=\gamma$!

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We could also just look at this $F(Ma_1)$ expression which smoothly transforms the quadratic (*). We now have all our expressions, this is because we know $Ma_2(Ma_1)$!

$$\frac{T_2}{T_1} = \frac{[2\gamma Ma_1^2 - (\gamma-1)][2 + (\gamma-1)Ma_1^2]}{(\gamma+1)^2 Ma_1^2} \qquad (1)$$

Extra Credit sub in $Ma_2$ and show these!

$$\frac{P_2}{P_1} = \frac{2\gamma Ma_1^2 - (\gamma-1)}{\gamma+1} \qquad (2)$$

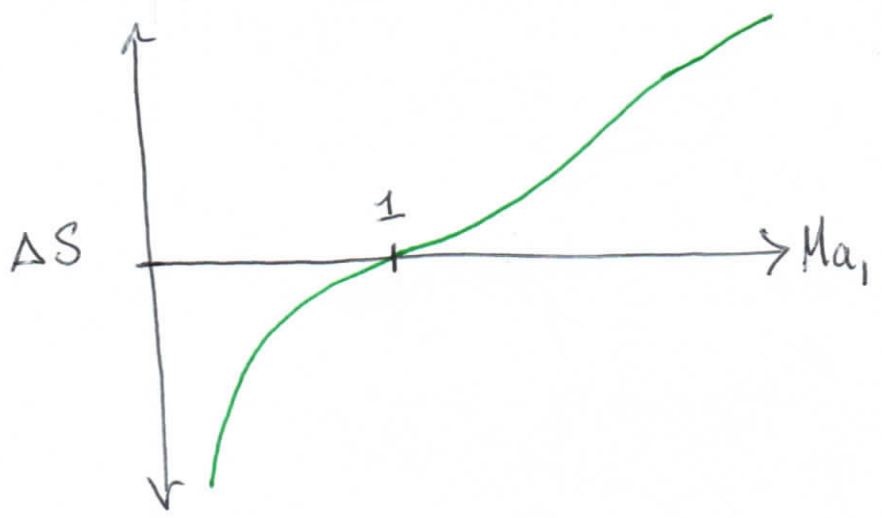
$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)Ma_1^2}{2+(\gamma-1)Ma_1^2} \qquad (3)$$

We can actually calculate entropy change (for an ideal gas) across the shock wave.

$$\Delta S = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

graph using (1),(2)

The plot looks like...




* Notice this means Shocks are <u>only</u> possible if $Ma_1 \geq 1$
* This is due to the 2nd Law...

# LECTURE 17

The last properties we want are STAGNATION VALUES.
This is basically a dirty algebra trick....
We denote $P_{i0}$ so $P_{1,0}$ = Stagnation upstream property.

$$\frac{P_{2,0}}{P_{1,0}} = \frac{(P_{2,0}/P_2)}{(P_{1,0}/P_1)} \cdot \left(\frac{P_2}{P_1}\right)$$

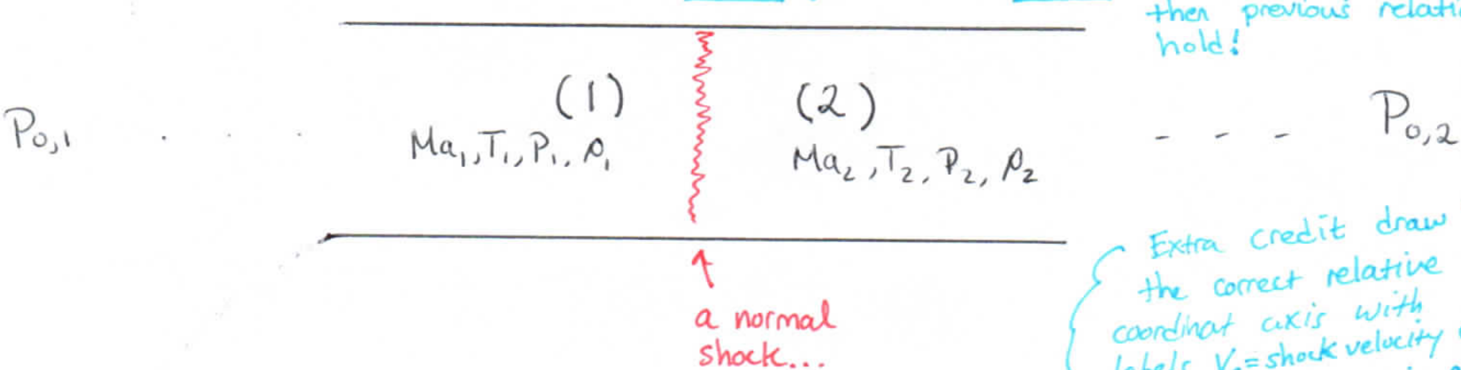
$$= \frac{Ma_1}{Ma_2} \left\{ \frac{\left(1+\frac{\gamma-1}{2} Ma_2^2\right)}{\left(1+\frac{\gamma-1}{2} Ma_1^2\right)} \right\}^{\frac{\gamma+1}{2(\gamma-1)}}$$

Extra Credit, simplify by subbing in $Ma_2$ and reducing

Normal Shock analysis is heavily dependent on table use as well. They are read as such---

| $Ma_1$ | $Ma_2$ | $P_2/P_1$ | $\rho_2/\rho_1$ | $T_2/T_1$ | $P_{0,2}/P_{0,1}$ | $P_1/P_{0,2}$ |
|---|---|---|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

*If the shock is moving just use a relative coordinate system. then previous relations hold!




Extra credit draw the correct relative coordinat axis with labels $V_s$ = shock velocity and $V_g$ = gas velocity, and (1), (2)

Tricky problems involve combining isentropic relations and normal shock relations/tables