LECTURE 6: FLUID FLOWING AROUND SOMETHING and NOT THRU IT

NOW WE LOOK AT EXTERNAL FLOW. OUR MAIN GOAL FOR INTERNAL FLOW ANALYSIS WAS ESSENTIALLY SUMMARIZED AS SUCH.

DETERMINE THE FLOW FIELD INSIDE A PIPE

a) LAMINAR CASE
$u(r)$ IS DERIVED FROM PRINCIPLES

b) TURBULENT CASE
$u(r)$ IS EXPERIMENTAL

$\Rightarrow$

USE $u(r)$ TO CALCULATE ENGINEERING QUANTITIES

Q
V_{max}
$\bar{V}$
ΔP
f

$\Rightarrow$

USE QUANTITIES TO PREDICT ENERGY REQUIREMENTS OF A PUMP

Δh or E_{req}

EXTERNAL WILL FOLLOW A SIMILAR PATH WE WILL JUST REPLACE WORDS

DETERMINE THE BOUNDARY LAYER AROUND THE BODY

a) LAMINAR

b) TURBULENT

$\Rightarrow$

USE δ TO CALCULATE ENGINEERING QUANTITIES

τ_w $\quad F_D$
δ^* $\quad F_L$
θ

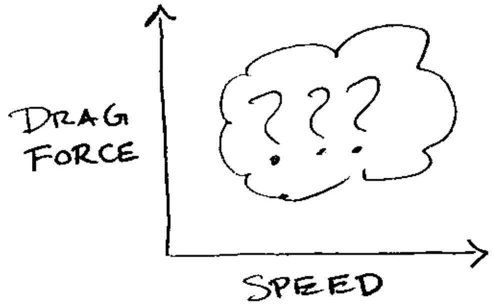
$\Rightarrow$

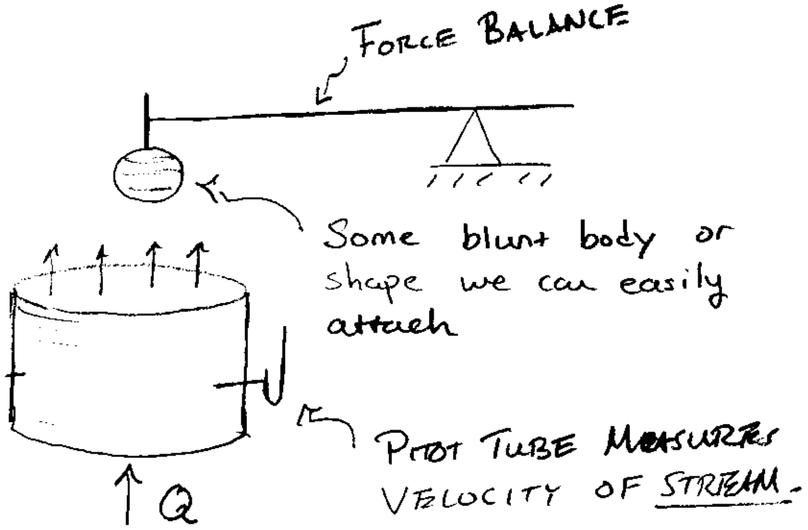
USE QUANTITES TO CALCULATE ENERGY REQUIRMENTS TO MOVE THRU THE FLUID (OVERCOME DRAG)

THAT BEING SAID LETS GO OVER THE PUZZLING EXPERIMENTS THAT MOTIVATED PEOPLE TO GET A THEORETICAL HANDLE ON EXTERNAL FLOW

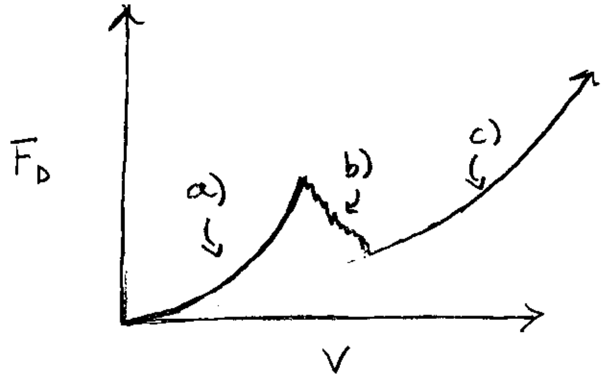
LECTURE 6: FLUID FLOWING AROUND SOMETHING NOT THRU IT

We preform the simple experiment to fill out this graph





RESULTS WERE TROUBLING... LIKE WTF!!?!?



EXTRA CREDIT
DO THIS EXPERIMENT AT HOME WITH A BLOW DRYER AND SMALL BODIES, TELL ME HOW YOU INCREASED V AIRSTREAM SO MUCH. HAIR DRYER, VACUUM, DECREASE AREA? I DUNNO...

a) AT FIRST DRAG INCREASES QUICKLY. FASTER THAN c)

b) UM... WHAT... BRAIN BROKE... SCIENCE HELP ME!

c) NOTEABLY SLOWER INCREASE THAN IN a). IT'S LIKE COMPARING TWO POLYNOMIALS

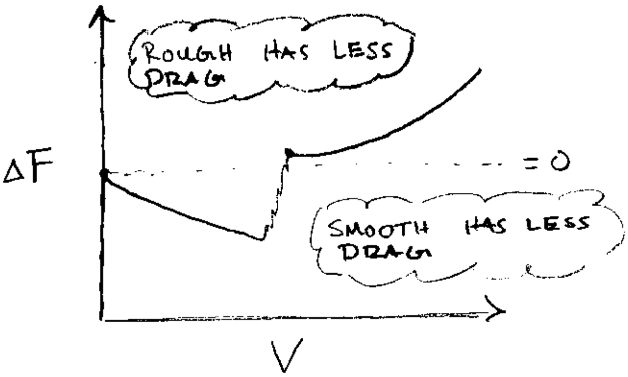
a) $5V^2$ vs c) $0.3V^2$

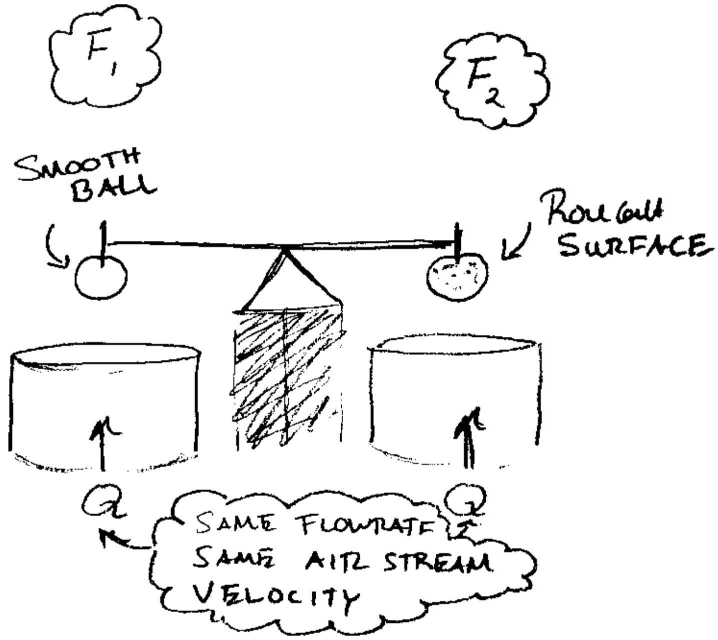
THIS LEADING COEFFICIENT IS REMARKABLY SMALLER!

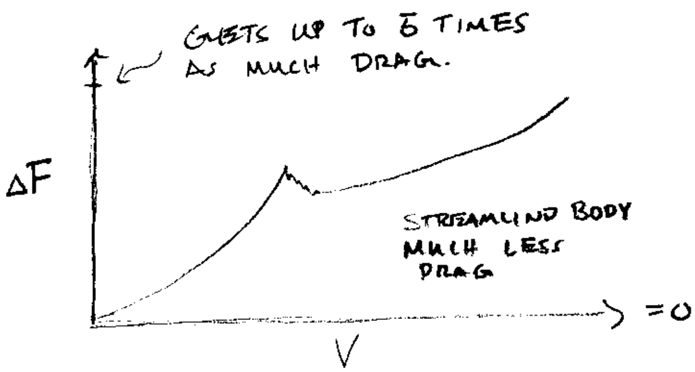
Lecture 6: Fluid Flowing Around Something Not Thru It

It didn't get any better when started comparing <u>bodies</u> and <u>different fluids</u>.

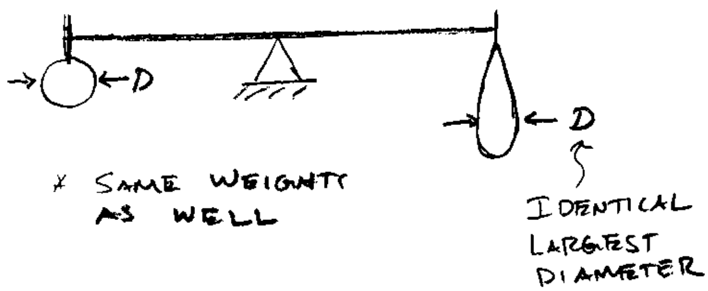
$\Delta F = F_1 - F_2$.

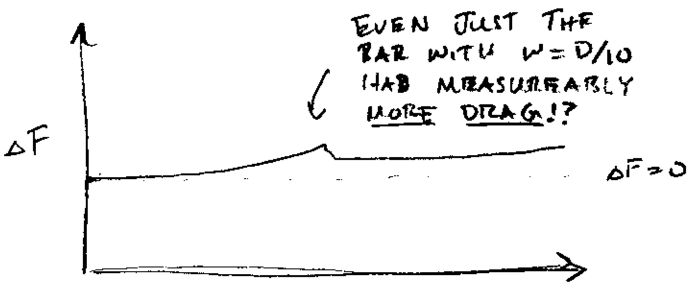


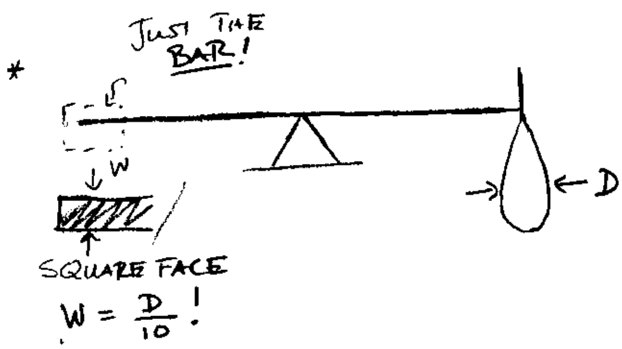




* Same set-up <u>different bodies</u>



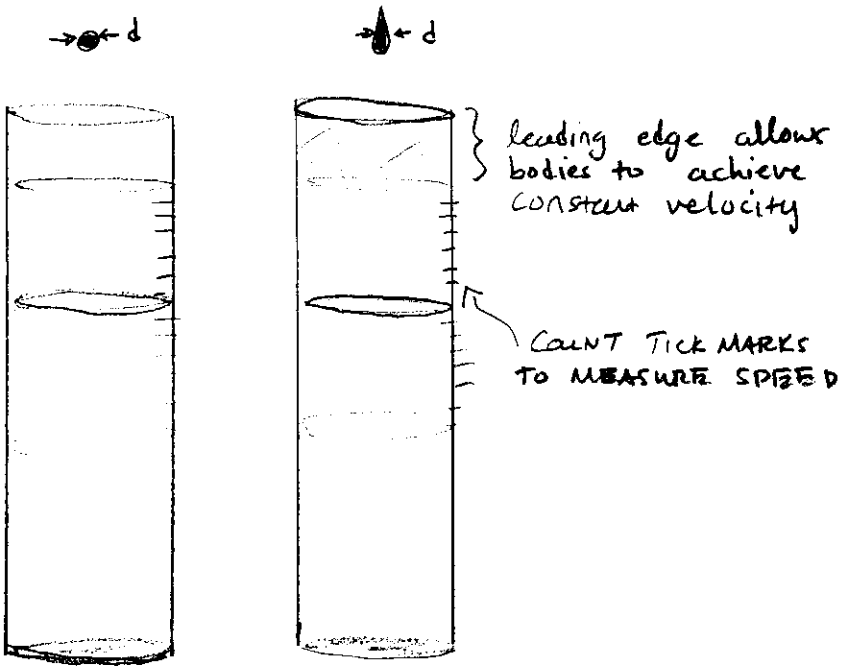




* <u>All</u> plots always produced some shift when the fluid got fast enough.

* It got worse when we <u>changed the fluid</u>

WE THEN TOOK SMALL BODYS AND DROPPED THEM IN TANKS OF GLYCERIN (VERY VISCOUS FLUID)



* MEASURE SPEED

* & HAVE SAME WEIGHT IN GLYCERIN

* AT CONSTANT Velocity

$$\sum F = 0$$

* IF ONE HAS A HIGHER VELOCITY IT OFFERS LESS RESISTENCE

RESULTS

$$V_{ball} > V_{teardrop}$$!!?!

YOOOOO!? THIS IS THE EXACT OPPOSITE IN THE CASE AIR FLOW. SCIENCE CALM DOWN.

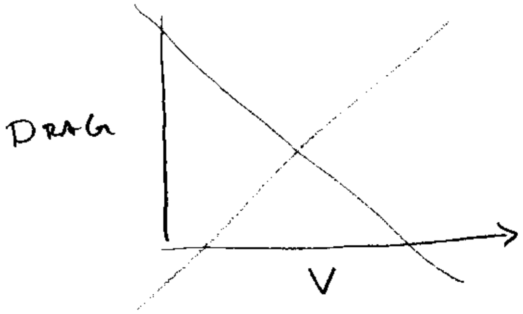
THEY ALSO COMPARED SEVERAL DIFFERENT SIZES

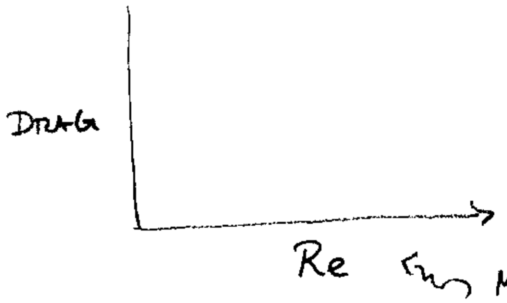
WHEN WE FOUND TWO THAT FELL AT THE SAME VELOCITY WE THEN COMPARED THEIR WEIGHTS IN GLYCERIN AND FOUND EVERY TIME

HAD MORE DRAG!?

LECTURE 6: FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

WE KEEP MEASURING DRAG FORCES AND FOUND THAT THE COMPLEXITIES OF DIFFERENT FLUIDS AND SHAPE AND SPEEDS COULD BE LUMPED TOGETHER IN ... YOU GUESSED IT A REYNOLDS #!





MUCH BETTER INDEPENDENT VAR TO DESCRIBE RESULTS!

THEN AFTER ENOUGH RUNS CLEAR RELATIONSHIPS EMERGED

CASE		LAW
Low Re	EXPERIMENTS!! $\Rightarrow$	DRAG $\sim$ SPEED × VISCOSITY × SIZE
High Re		DRAG $\sim$ SPEED^2 × DENSITY × SIZE^2

LETS UNDERSTAND WHAT THIS $\sim$ MEANS THOUGH.

$$a \sim b$$ "Proportional"

means..

$$\underbrace{a = C \cdot b}$$ "C is the proportionality constant"

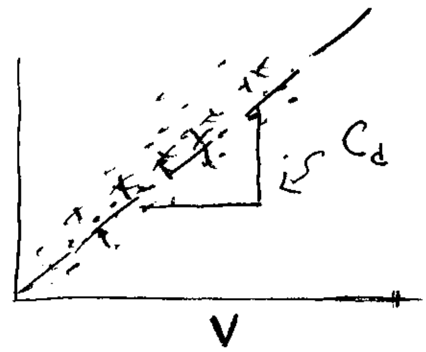
THIS MEANS THE EXPERIMENTS WERE LOOKING AT <u>PLOTS OF LINES</u>!

LECTURE 6: FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

SO RUNS LOOKED LIKE THIS

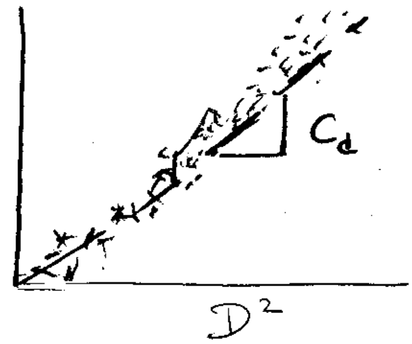
KEEP Re LOW FOR ALL RUNS $\Rightarrow$ CHOOSE 2 QUANTITIES TO KEEP CONSTANT VARY THE THIRD $\Rightarrow$ DRAG

i.e.
μ, D constant
increas V



KEEP Re HIGH FOR ALL RUNS $\Rightarrow$ Choose 2 quantities constant vary third $\Rightarrow$ DRAG

i.e.
V, ρ constant
vary D



THESE PROPORTIONALITY CONSTANTS C_D OR C_L ARE WHAT WE ARE ALWAYS AFTER. THEY ARE ONLY FUNCTIONS OF GEOMETRY OF THE BODY. OR REALLY $C_D(Re)$

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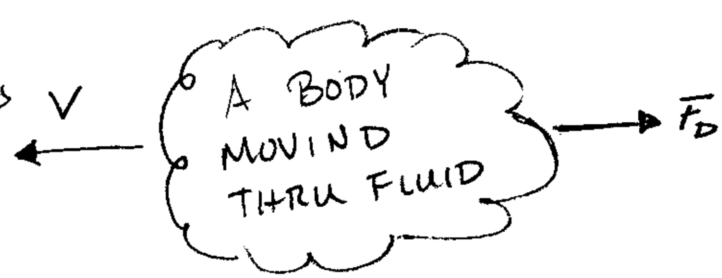
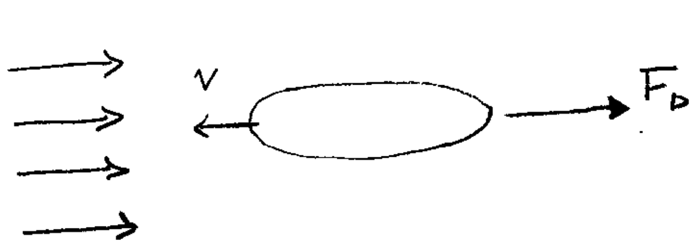
WHY CALCULATE DRAG? ENERGY!!!

USUALLY WE THINK OF THE FLUID MOVING PAST



So a gust of wind matters



LECTURE 6 : FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

SO IF WE CAN A PRIORI "fancy word for predicting the future" GUESS WHAT THE DRAG FORCE WILL BE THEN.

$$\underbrace{\text{POWER}}_{\text{WHAT KIND OF ENGINE DO I NEED?}} := \mathcal{P} = F_D \cdot V$$

$$= \{\text{DRAG}\} \cdot \{\text{DESIGN VELOCITY}\}$$

YOUR BOSS TELLS YOU THIS **NUMBER**

NOW THE BEAUTY OF DRAG COEFFICIENTS IS THIS

$$C_D := \frac{F_D}{\frac{1}{2}\rho U_\infty^2 A_{frontal}} \quad (\text{HIGH } Re)$$

So if I KNOW THIS NUMBER FOR A GIVEN BODY

$C_D = f(Re)$ and all that I can change in design is the D in Re

$$\underbrace{F_D = C_D \frac{1}{2}\rho U_\infty^2 A_{frontal}}_{\text{EASY CALCULATION IF } C_D \text{ IS KNOWN!}} \qquad (1)$$

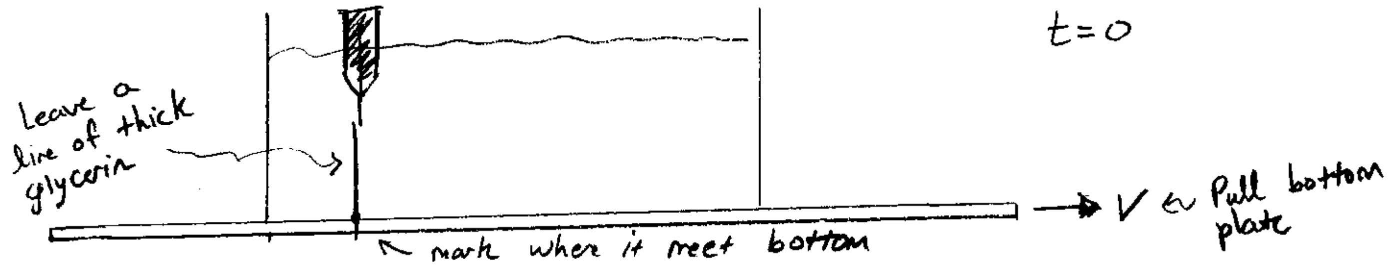
EXTRA CREDIT

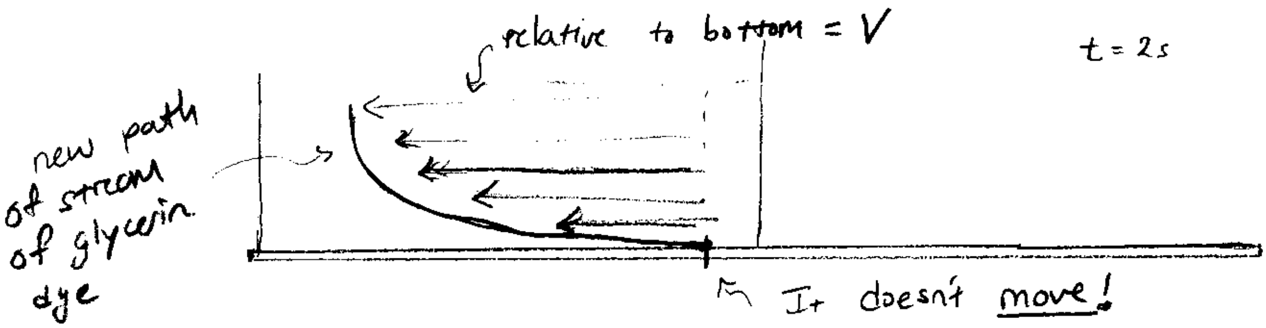
$$\rho\left\{\frac{\partial u}{\partial t} + (u\cdot\nabla)u\right\} = -\nabla P + \mu\nabla^2 u$$

Force is $P \cdot A = F$, Show AT HIGH REYNOLDS NUMBER NAVIER STOKES PREDICTS (1) should be the case. Show the same law with LOW REYNOLDS. What Assumptions do you have to make?

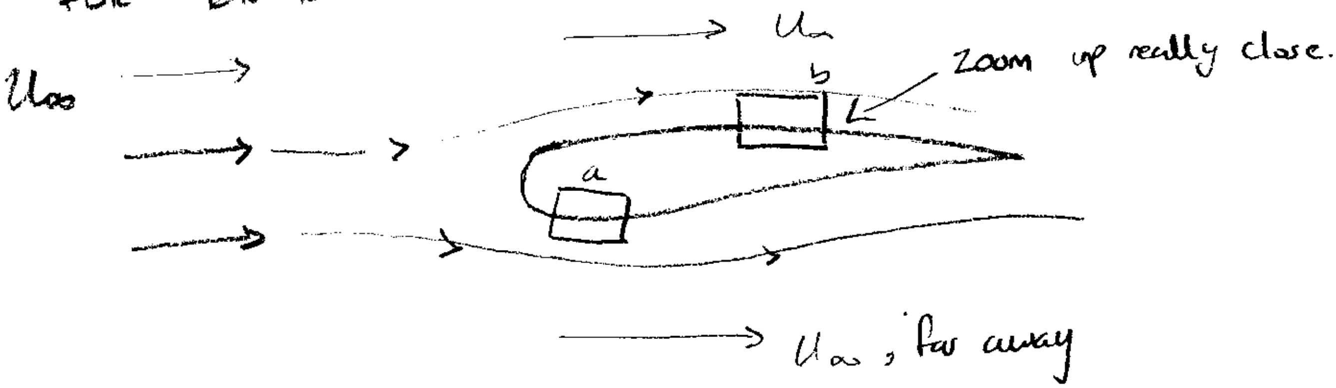
LECTURE 6: FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

LETS NOW DO A LITTLE PHYSICS AND JUST TRY TO THINK ABOUT HOW DRAG COMES ABOUT. FIRST PRINCIPLE WE NEED IS <u>NO-SLIP</u>. THIS CAN BE <u>SEEN</u> EXPERIMENTALLY IN A SIMPLE SET UP.





SO OUR FIRST PRINCIPLE TO HOLD ON TO FOR EXTERNAL FLOW IS JUST THAT!

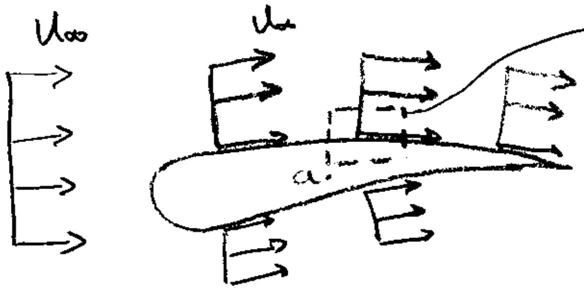


IF <u>ZOOMED</u> INTO a. OR b. I KNOW AT THE WALL $u=0$! FAR AWAY THOUGH I ALSO KNOW $u = U_\infty$

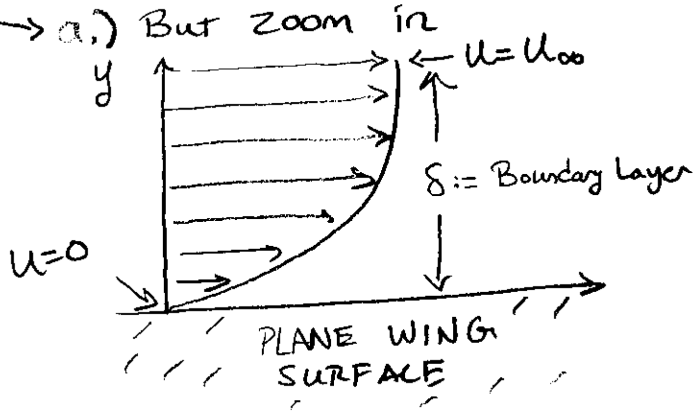
Extra Credit
Give a <u>molecular</u> rationale for no-slip. Consider attractive forces of fluid molecules and solid molecules. Do these change as we get close to the wall

LECTURE 6 : FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

So no matter how large the velocity is or high Re is this no slip condition holds. So there must exist some distance δ for the velocity profile to get to $u(0)=0$ to $u(\delta)=U_\infty$. This is called the Boundary Layer δ.



* From a distance or to the naked eye it looks like a constant velocity profile around the wing



* For a plane wing $\delta \approx 1\text{mm}$ (size of credit card)

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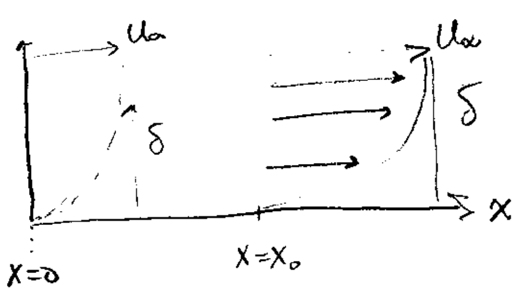
Just like entrance lengths people figured out $\delta(Re)$. So calculate a Re and you get δ for free.



$$Re_x = \frac{\rho U_\infty x_0}{\mu}$$

$$\boxed{\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}}$$

EXACT FORMULA

LECTURE 6: FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

JUST AS WE KNOW FROM EXPERIMENT THIS EQUATION FOR δ TELLS US A CORRECT INTUITION

LOW Re $\rightarrow$ $\delta \sim \dfrac{1}{\sqrt{Re}}$ $\rightarrow$ LARGE δ

SO Re<<1

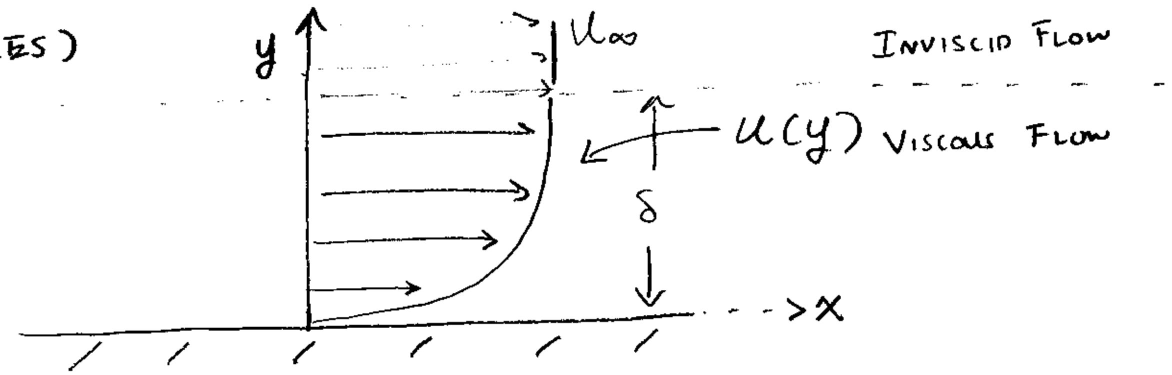
BIG NUMBER

HIGH Re $\rightarrow$ $\delta \sim \dfrac{1}{\sqrt{Re}}$ $\rightarrow$ SMALL δ

Re>>1

* LIKE FOR AN AIRPLANE WING.

WHAT DOES δ HAVE TO DO WITH F_D THOUGH? WELL LETS DRAW THE BOUNDARY LAYER PICTURE AGAIN... (IN GRAD-SCHOOL YOU DRAW THIS IMAGE TO MANY TIMES)



WHAT DOES IT TAKE TO CHANGE VELOCITY? KINETIC ENERGY THM (PHYSICS I!)

$$\Delta KE = W$$

$$\begin{Bmatrix}\text{Change in} \\ \text{Kinetic} \\ \text{Energy}\end{Bmatrix} = \begin{Bmatrix}\text{Work Done} \\ \text{on} \\ \text{System}\end{Bmatrix}$$

$$\frac{1}{2}\rho V\left(U_\infty^2 - \cancel{0^2}\right) = F_{visc} \cdot x$$

no-slip

$$= \{\text{Viscous Force}\} \cdot \{\text{A distance}\}$$

LECTURE 6 : FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

RECALL FOR A NEWTONIAN LIQUID . W = width

$$\tau = \mu \frac{\partial u}{\partial y} = \text{PRESSURE}$$

$$\frac{1}{2}\rho(\delta \times \cancel{w})U_\infty^2 = \mu \frac{\partial u}{\partial y} \times (x\cancel{w})$$ ~ multiply by area to get force

$$\cancel{\tfrac{1}{2}}\rho\,\delta\cancel{\rho} D\, U_\infty^{\cancel{2}} \sim \mu \frac{U_\infty}{\delta} \cancel{x}$$ (magical scale arguement)

$$\delta^2 \sim \frac{\mu}{\rho \times U_\infty}$$

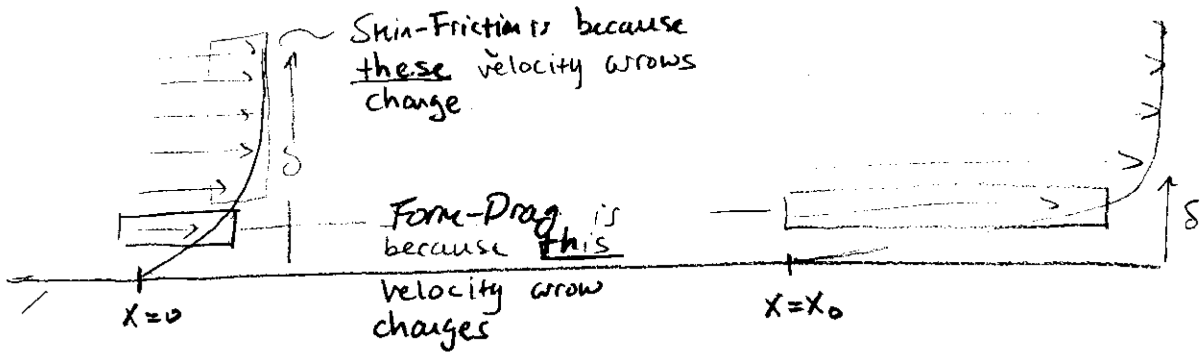
$$\therefore\ \delta \sim \frac{1}{\sqrt{Re_x}}$$ Ta-da!

NOW DRAG CAN BE THOUGHT OF AS AN INTEGRAL. AND TWO PARTS.

(*) $$F_D = \int \tau_w \, dA := \text{"Skin Friction"}$$

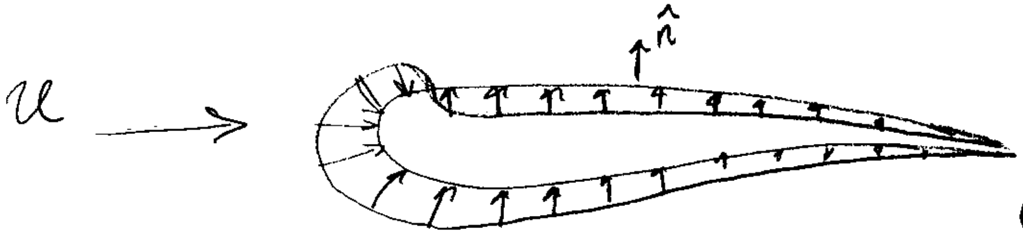
(**) $$F_L = \int p \cdot \hat{n} \, dA := \text{"Form Drag"}$$

We are after expressions for Skin friction with Boundary layer analysis, Form Drag comes from Pressure Distribution. Some picture will prove to you.



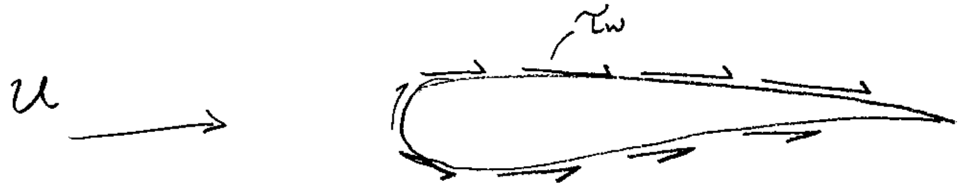
LECTURE 6 : FLUID FLOWING AROUND SOMETHING AND NOT THRU IT

THESE 2 DIRECTIONS OF CHANGING CREATE ORTHOGONAL AXES FOR A BODY CLOSE TO THE SURFACE



NORMAL PRESSURE AT SURFACE

FORM-DRAG



TANGENTIAL SHEAR AT SURFACE

SKIN-FRICTION

$$\vec{D} = \underbrace{\int_{\text{surface}} P\,\hat{n}\,dA}_{} + \int_{\text{surface}} \tau_w(\delta)\,\hat{t}\,dA$$

We can use Bernoulli's to estimate total lift, but we don't cover the "real" way to get this term.

Shear Stress at the wall is a function of boundary layer thickness.

JUST LIKE WE CALCULATED f_{lam} & f_{turb} TO ESTIMATE ENERGY REQUIREMENTS WE WILL DO THE SAME FOR EXTERNAL FLOWS BUT NOW WE LABEL THEM $C_{D,lam}$ & $C_{D,turb}$!

{ NEXT CLASS IS SIMILAR TO L3 FOR INTERNAL FLOW... LOTS OF MATH }