Calculation according to normal stress theory

→ Equilibrium in x-direction

$$\sum F_x = 0$$

$$F_x - \mu(F_1 + F_2 + F_3) + R = 0 \quad ----①$$

→ Equilibrium in z-direction

$$\sum F_z = 0$$

$$F_1 \sin\lambda + F_2 \cos\gamma - F_3 - F_2 - W = 0 \quad --②$$

→ Equilibrium in y-direction

$$\sum F_y = 0$$

$$F_1 \sin\lambda - F_2 \sin\gamma + F_y = 0 \quad -----③$$

→ Moment about x-direction

$$\frac{F_1 \cos\lambda\, b}{4} + \frac{F_2 \cos\gamma\, b}{2} - \frac{F_3 b}{2} - F_2 \frac{d}{2} - F_y h + \mu(F_1 + F_2)\frac{b}{2}$$

$$- \frac{\mu F_3 b}{2} + F_x h + Rb = 0 \quad ----④$$

→ Moment about y-direction

$$-F_1 \cos\lambda \frac{b}{2} - F_2 \cos\lambda \frac{b}{2} + F_3 \frac{b}{2} + F_2 \frac{d}{2} + F_y h + RZ$$

$$- \mu(F_1 + F_2)\frac{b}{2} - \mu F_3 \frac{b}{2} = 0 \quad ---⑤$$

Moment about z-axis

$$\frac{Rb}{2} + f_y h - f_x \frac{d}{2} + \mu(f_1 + f_2)\frac{b}{2} - \mu f_3 \frac{b}{2} = 0 \quad \text{--- (6)}$$

from equation (4)

$$f_3 = f_1\left(\frac{\cos\lambda}{2} + \mu\right) + f_2(\cos\gamma + \mu) - f_2\frac{d}{2} - \frac{2f_y h}{b}$$

$$+ \frac{2f_x h}{b} + 2R \quad \text{---- (7)}$$

Putting f_3 in equation (2) we get

$$f_1\cos\lambda + f_2\cos\gamma = f_z - f_1\left(\frac{\cos\lambda}{2} + \mu\right) - f_2(\cos\gamma + \mu)$$

$$+ \frac{f_2 d}{b} + \frac{2f_y h}{b} - \frac{2f_x h}{b} - 2R + W$$

$$\text{--- (8)}$$

Assumption

$$\lambda + \gamma = 90^\circ$$

$$\gamma = 90 - \lambda \quad \text{---- (9)}$$

Solving equation (9) & equation (3) we get

$$f_1 = \frac{f_z - f_y - \frac{f_y}{\sin\lambda}(\cos\gamma + \mu) - \frac{f_2 d}{b} + \frac{2f_y h}{b} - \frac{2f_x h}{b} - 2R + W}{\frac{5\cos\lambda}{2} + \sin\lambda + 2\mu}$$

$$F_2 = \frac{F_2 - F_y - \frac{F_y}{\sin\gamma}(\cos\gamma + \mu) + \frac{F_2 d}{b} + \frac{2F_y h}{b} - \frac{2F_x h}{b} + 2R + W}{\frac{5\cos\lambda}{2} + \sin\lambda + 2\mu} + \frac{F_y}{\sin\gamma}$$

$$F_3 = F_1\left(\frac{\cos\lambda}{2} + \mu\right) + F_2\left(\cos\gamma + \mu\right) - \frac{F_2 d}{b} - \frac{2F_y h}{b} - \frac{2F_x h}{b} + 2R$$

* According to maximum Normal stress theory

$$\sigma_{allowed} = \frac{\sigma_{ut}}{FOS}$$

→ Assumption :

FOS = 3

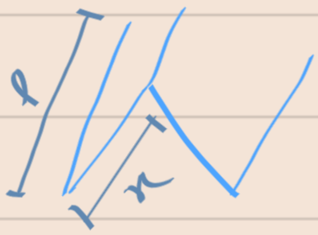
$$\Rightarrow \frac{F}{A} = \frac{\sigma_{ut}}{3}$$

→ Given dimension

$l = 0.15\,m$

$$\Rightarrow A = \frac{3F}{\sigma_{ut}} \Rightarrow l \times x = \frac{3F}{\sigma_{ut}}$$

$$x = \frac{3F}{l \cdot \sigma_{ut}}$$



$$x = \frac{3F}{l\,\sigma_{ut}}$$

x → length of 'V' edge (u, v, w)

l → length of carriage

F → Normal Forces (F_1, F_2, F_3)

σ_{ut} → Ultimate tensile strength

$$u = \frac{3f_1}{l\sigma_{ut}} \qquad v = \frac{3f_2}{l\sigma_{ut}} \qquad w = \frac{3f_3}{l\sigma_{ut}}$$

$$u = \frac{3 \times 2606}{0.15 \times 6 \times 10^6} = 0.029\ m = 2.9\ cm$$

$$v = \frac{3 \times 3546}{0.15 \times 6 \times 10^6} = 0.039\ m = 3.9\ cm$$

$$w = \frac{3 \times 1613}{0.15 \times 6 \times 10^6} = 0.017\ m = 1.7\ cm$$

* **Calculation for notch safety**

for this case,

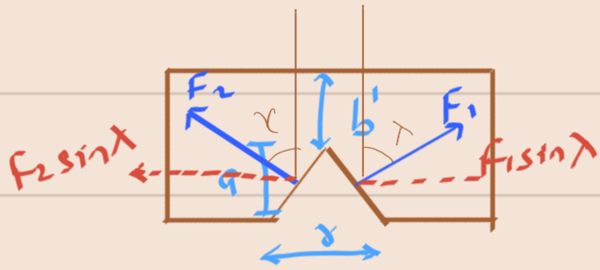
theoritical stress concentration factor (K_t)

$$K_t = 1 + \frac{2a}{r} = \frac{\sigma_{max}}{\sigma_{nominal}}$$ under static loading

$$K_t = 1 + \frac{2(0.08)}{0.04} = 5$$

* Assumption: only static loading no cycling load is acting, because cutting is happening continiously, so cutting forces will be continious not cyclic load.

$$\sigma_{\substack{nominal \\ theoritical}} = \left(1+\frac{2a}{r}\right)\sigma_0$$



$$\sigma_{\substack{nominal \\ theoritical}} = 5 \times 6 \times 10^6 = 3 \times 10^7 \ N/m^2$$

$$\sigma_{\substack{nominal \\ Practical}} = \frac{F_1 \sin\lambda + F_2 \sin\gamma}{l \times b'}$$

$$\sigma_{\substack{nominal \\ Practical}} = \frac{2606 \sin 45^\circ + 3546 \sin 45^\circ}{0.15 \times b'}$$

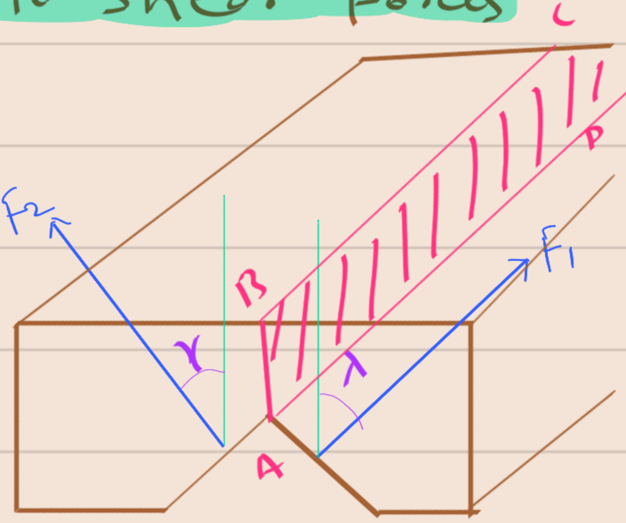
for safe design $\sigma_{\substack{nominal \\ Practical}} \leq \sigma_{\substack{nominal \\ theoritical}}$

$$\left(1+\frac{2a}{r}\right)\sigma_0 = \frac{F_1 \sin\lambda + F_2 \sin\gamma}{l \times b'}$$

$$b' = \frac{F_1 \sin\lambda + F_2 \sin\gamma}{\left(1+\frac{2a}{r}\right)\sigma_0 \, l}$$

$$b' = 0.04 \ m = 4 \ cm$$

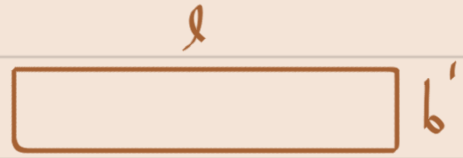
* Calculation at Notch due to shear forces



→ maximum shear will act on plane ABCD

→ Assuming

$$\tau_{max} = \frac{\sigma_{ut}}{2} = \frac{6 \times 10^6}{2}$$



→ for safe design

$$\tau_{allow} \leq \tau_{max}$$

→ $$\tau_{allow} = \frac{\text{Total forces along plane}}{\text{Area}}$$

$$= \frac{F_1 \cos\lambda + F_2 \cos\gamma}{l \times b'}$$

$$\tau_{allow} = \frac{2606 \cos 45^\circ + 3546 \cos 45^\circ}{0.15 \times b'}$$

Now for safe design $\tau_{allow} \leq \tau_{max}$

$$\frac{F_1 \cos\lambda + F_2 \cos\gamma}{l \times b'} = \tau_{max}$$

$$\Rightarrow \quad b' = \frac{F_1 \cos\lambda + F_2 \cos\gamma}{l \times \tau_{max}}$$

$$\Rightarrow \quad b' = 0.11 \text{ m} = 11 \text{ cm}$$

* Design of lead screw (Buckling)

→ Torque required to generate axial force 'F'.

$$T = \frac{FL}{2\pi\eta} \quad \text{--(I)}$$

$T \rightarrow$ Torque

$F \rightarrow$ axial force required to move carriage forward

$\eta \rightarrow$ efficiency

$$T = \frac{(F_x + R)L}{2\pi\eta} = \frac{(500 + 50) \times (0.15)}{2 \times \pi \times 0.3} = 43.76$$

→ Torsion Stress due to torque

$$\tau = \frac{Tc}{J} \quad \text{--(II)}$$

$c = \frac{d}{2}$

$J = \frac{\pi d^4}{32}$

$$\tau = \frac{T \cdot \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3}$$

$$\tau = \frac{16 \times 43.76}{\pi d^3}$$

Now, for safety from failure

$$\tau \leq \tau_{max}$$

$$\frac{16T}{\pi d^3} = \tau_{max} \Rightarrow d^3 = \frac{16T}{\pi \tau_{max}}$$

$$d = \sqrt[3]{\frac{16T}{\pi \tau_{max}}} = \sqrt{\frac{16 \times 43.76}{\pi \times 1.03 \times 10^7}}$$

$$d = 0.0077 \text{ m} = 7.7 \text{ mm}$$

→ Stability for Buckling (axially)

$$F_{cr} = \frac{\pi^2 E I}{(L_{eff})^2}$$

$E \rightarrow$ Modulus of elasticity

$I = \frac{\pi d^4}{64}$

L_{eff} = effective length

$$F_{cr} = \frac{\pi^2 E \left(\frac{\pi d^4}{64}\right)}{(L_{eff})^2}$$

$$d^4 = \frac{64 F_{cr} L_{eff}^2}{\pi^3 E}$$

$$d = \sqrt[4]{\frac{64 F_{cr} L_{eff}^2}{\pi^3 E}}$$

$$d = \sqrt{\frac{64 \times (F_n + R) \times (0.5 \times 0.15)^2}{\pi^3 \times 2.1 \times 10^{10}}} = 0.015 \text{ m} = 1.5 \text{ cm}$$