Shahjalal University of Science &Technology, Sylhet

Department of Computer Science and Engineering

Course No: CSE-239

Assignment No: 01

Numerical Analysis

Submitted To

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Problem statement:
Define accuracy, precision, round-off error, true error and approximate error.

Solution:

Accuracy:
Accuracy refers to how closely a computed or measured agrees with the true value.

Precision:
Precision refers to how closely individuals computed or measured values agree with each other.

Round-off Error:
The error which occurs when numbers having limited significant figures are used to represent exact number.

Approximate Error:

The approximate error in some data is the discrepancy between an exact value and some approximation to it.

True error:

The error which we get after subtracting approximation from true value is called true error.

(02)

Problem statement:

Mention the purpose of tolerance value?

Solution:

In numerical analysis many types of errors can be occured. So, the calculated value will be different slightly from the exact value. If the percentage of error is less than a fixed value we will accept the calculated value which is close to real value. The fixed value here is tolerance value.

To compare the calculated value at each step of a numerical method and whether it is close to the desire output value we use tolerance value.

If the solution of that numerical method is convergence, then the errors at each step will decreases and approach near the tolerance value.

Problem Statement:

Initiate a 3X3 matrix A. Here all the cell values should be randomly selected by you. Now calculate the inverse matrix A^{-1} using LU Decomposition method. Show the full calculation.

Solution:

Consider the following system of linear equation,

$$x_1 - 2x_2 + 3x_3 = 7 \quad \text{---} (1)$$
$$2x_1 + x_2 + x_3 = 4 \quad \text{---} (2)$$
$$-3x_1 + 2x_2 - 2x_3 = -10 \quad \text{---} (3)$$

From, (2) - (1)×2 we get,

$$(2x_1 + x_2 + x_3) - (2x_1 - 4x_2 + 6x_3) = 4 - 14$$
$$\Rightarrow 5x_2 - 5x_3 = -10 \quad \text{---} (4)$$

From, (3) - (-3)×(1), we get,

$$(-3x_1 + 2x_2 - 2x_3) + 3(x_1 - 2x_2 + 3x_3) = -10 + 21$$
$$\Rightarrow -4x_2 + 7x_3 = 11 \quad \text{---} (5)$$

From,

⑤ $- ④ \times \left(\frac{-4}{5}\right)$ we get,

$$-4x_2 + 7x_3 + \frac{4}{5}(5x_2 - 5x_3) = 11 - 8$$

$$\Rightarrow 3x_3 = 3$$

Here, $f_{21} = 2$

$f_{31} = -3$

$f_{32} = \frac{-4}{5}$

So,

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 2 & -2 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -5 \\ 0 & 0 & 3 \end{bmatrix}$$

and, $$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{-4}{5} & 1 \end{bmatrix}$$

We know,

$$AA^{-1} = I$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 2 & -2 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ⑥$$

According to LU decomposition,

$$[A]\{X\} = \{B\}$$

$$\Rightarrow [A]\{X\} - \{B\} = 0$$

After Forward Elimination,

$$[U]\{X\} = \{D\} \quad \text{——— ⑦}$$

$$\Rightarrow [U]\{X\} - \{D\} = 0$$

$$\Rightarrow [L]\left([U]\{X\} - \{D\}\right) = [A]\{X\} - \{B\}$$

So,

$$[L][U] = [A]$$

$$[L]\{D\} = \{B\} \quad \text{——— ⑧}$$

From equation ⑥ we get,

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 2 & -2 \end{bmatrix} \begin{Bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

From eq. 8

$$[L]\{D\} = \{B\}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{-4}{5} & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

So, $d_1 = 1$

$2d_1 + d_2 = 0$

$\Rightarrow d_2 = -2$

and,

$-3d_1 - \frac{4}{5} d_2 + d_3 = 0$

$\Rightarrow -3 - \frac{4}{5}(-2) + d_3 = 0$

$\Rightarrow d_3 = \frac{7}{5}$

From eq. 7

$$[U]\{X\} = \{D\}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -5 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -2 \\ \frac{7}{5} \end{Bmatrix}$$

So, $3x_{31} = \frac{7}{5} \Rightarrow x_{31} = \frac{7}{15}$

and,

$5x_{21} - 5x_{31} = -2$

$\Rightarrow 5x_{21} - 5 \cdot \frac{7}{15} = -2$

$$\Rightarrow 5x_{21} + \frac{7}{3} = -2$$

$$\Rightarrow x_{21} = \frac{1}{15}$$

and,

$$x_{11} - 2x_{21} + 3x_{31} = 1$$

$$\Rightarrow x_{11} - 2 \cdot \frac{1}{15} + 3 \cdot \frac{7}{15} = 1$$

$$\Rightarrow 15x_{11} - 2 + 21 = 15$$

$$\Rightarrow x_{11} = \frac{-4}{15}$$

Again, From eq. 6

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 2 & -2 \end{bmatrix} \begin{Bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

From eq. 8,

$$[L]\{D\} = \{B\}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{-4}{5} & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

So, $d_1 = 0$

and, $2d_1 + d_2 = 1$

$$\Rightarrow d_2 = 1$$

and,

$$-3d_1 - \frac{4}{5}d_2 + d_3 = 0$$

$$\Rightarrow \frac{-4}{5} + d_3 = 0$$

$$\Rightarrow d_3 = \frac{4}{5}$$

From eq. 7

$$[U]\{X\} = \{D\}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -5 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ \frac{4}{5} \end{Bmatrix}$$

So, $3x_{32} = \frac{4}{5}$

$$\Rightarrow x_{32} = \frac{4}{15}$$

and, $5x_{22} - 5x_{32} = 1$

$$\Rightarrow 5x_{22} - 5 \cdot \frac{4}{5} = 1$$

$$\Rightarrow 5x_{22} - \frac{4}{3} = 1$$

$$\Rightarrow x_{22} = \frac{7}{15}$$

and, $x_{12} - 2x_{22} + 3x_{32} = 0$

$$\Rightarrow x_{12} = 2x_{22} - 3x_{32}$$

$$= 2 \cdot \frac{7}{15} - 3 \cdot \frac{4}{15}$$

$$= \frac{2}{15}$$

Again, From eq. 6

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 2 & -2 \end{bmatrix} \begin{Bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

From eq. 8

$$[L]\{D\} = \{B\}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{-4}{5} & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

So,

$$d_1 = 0$$

and, $2d_1 + d_2 = 0$

$$\Rightarrow d_2 = 0$$

and,

$$-3d_1 - \frac{4}{5} d_2 + d_3 = 1$$

$$\Rightarrow d_3 = 1$$

From eq. 7

$$[U]\{X\} = \{D\}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -5 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

So, $3x_{33} = 1$

$$\Rightarrow x_{33} = \frac{1}{3}$$

and, $5x_{23} - 5x_{33} = 0$

$$\Rightarrow x_{23} = x_{33} = \frac{1}{3}$$

and, $x_{13} - 2x_{23} + 3x_{33} = 0$

$$\Rightarrow x_{13} - 2\cdot\frac{1}{3} + 3\cdot\frac{1}{3} = 0$$

$$\Rightarrow x_{13} = \frac{-1}{3}$$

So, the required inverse matrix is,

$$A^{-1} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-4}{15} & \frac{2}{15} & \frac{-1}{3} \\ \frac{1}{15} & \frac{7}{15} & \frac{1}{3} \\ \frac{7}{15} & \frac{4}{15} & \frac{+1}{3} \end{bmatrix} = \frac{1}{15}\begin{bmatrix} -4 & 2 & -5 \\ 1 & 7 & 5 \\ 7 & 4 & 5 \end{bmatrix}$$