



## **Shahjalal University of Science & Technology, Sylhet**

Department of Computer Science and Engineering

Course No: EEE-201D

Assignment No: 01

### **Digital Logic Design**

Submitted To

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(01)

### Problem Statement:

Express each of the function in sum-of-minterms and product of maxterms form.

a)  $(xy + z)(y + xz)$

b)  $\bar{y}z + wx\bar{y} + wx\bar{z} + \bar{w}\bar{x}z$

### Solution:

a) Expression in sum-of-minterms form-

$$\begin{aligned} f(x, y, z) &= (xy + z)(y + xz) \\ &= xy + xyz + yz + xz \\ &= xy(z + \bar{z}) + xyz + (x + \bar{x})yz + x(y + \bar{y})z \\ &= xyz + xy\bar{z} + xyz + xyz + \bar{x}yz + xyz + x\bar{y}z \\ &= xyz + xy\bar{z} + \bar{x}yz + x\bar{y}z \end{aligned}$$



Expression in product of maxterms:

$$f(x, y, z) = (xy + z)(y + xz)$$

$$= (y + z)(x + z)(x + y)(y + z)$$

$$= (x\bar{x} + y + z)(x + y\bar{y} + z)(x + y + z\bar{z})(x\bar{x} + y + z)$$

$$= (x + y + z)(\bar{x} + y + z)(x + y + z)(x + \bar{y} + z)(x + y + z)(x + y + \bar{z})(\bar{x} + y + z)$$

$$= (x + y + z)(\bar{x} + y + z)(x + \bar{y} + z)(x + y + \bar{z})(\bar{x} + y + z)$$



b)

Expression in sum-of-minterms-

$$F(w, x, y, z) = \bar{y}z + wx\bar{y} + w\bar{x}\bar{z} + \bar{w}\bar{x}z$$

$$= (w + \bar{w})(x + \bar{x})\bar{y}z + wx\bar{y}(z + \bar{z}) + wx(y + \bar{y})\bar{z} + \bar{w}\bar{x}(y + \bar{y})z$$

$$= wx\bar{y}z + w\bar{x}\bar{y}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}\bar{y}z + wx\bar{y}\bar{z} + wx\bar{y}z + wx\bar{y}\bar{z} + wx\bar{y}z \\ + w\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$$

$$= wx\bar{y}z + w\bar{x}\bar{y}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}\bar{y}z + wx\bar{y}\bar{z} + wx\bar{y}z \\ + \bar{w}\bar{x}yz$$

Expression in product of maxterms:

$$F(w, x, y, z) = \bar{y}z + wx\bar{y} + w\bar{x}\bar{z} + \bar{w}\bar{x}z$$

$$= m_{13} + m_9 + m_5 + m_1 + m_{12} + m_{14} + m_3$$

$$= \sum (1, 3, 5, 9, 12, 13, 14)$$

$$= \pi (2, 4, 6, 7, 8, 10, 11, 15)$$

$$= (\bar{w} + \bar{x} + \bar{y} + \bar{z})(w + \bar{x} + y + z)(w + \bar{x} + \bar{y} + z)$$

$$(w + \bar{x} + \bar{y} + \bar{z})(\bar{w} + x + y + z)(\bar{w} + x + \bar{y} + z)(\bar{w} + x + \bar{y} + \bar{z})(\bar{w} + \bar{x} + \bar{y} + z)$$



(02)

Problem Statement:

Simplify the boolean expressions using K-maps.

a)  $xyz + wy + wx\bar{y} + \bar{x}y$

b)  $\bar{x}y + y\bar{z} + \bar{y}\bar{z}$

Solution:

a)

$$\begin{aligned} f(w, x, y, z) &= xyz + wy + wx\bar{y} + \bar{x}y \\ &= (w + \bar{w})xyz + w(x + \bar{x})y(z + \bar{z}) + wx\bar{y}(z + \bar{z}) \\ &\quad + (w + \bar{w})\bar{x}y(z + \bar{z}) \end{aligned}$$

$$\begin{aligned} &= wxyz + \bar{w}xyz + wx\bar{y}z + wx\bar{y}\bar{z} + w\bar{x}yz \\ &\quad + w\bar{x}y\bar{z} + wxy\bar{z} + wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} \\ &\quad + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} \end{aligned}$$

$$\begin{aligned} &= wxyz + \bar{w}xyz + wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} \\ &\quad + wx\bar{y}z + wx\bar{y}\bar{z} + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} \end{aligned}$$

$$\begin{aligned} &= m_{15} + m_7 + m_{14} + m_{11} + m_{10} + m_{13} + m_{12} \\ &\quad + m_3 + m_2 \end{aligned}$$



w \ x \ yz				
	00	01	11	10
00	0	0	1	1
01	0	0	1	0
11	1	1	1	1
10	0	0	1	1

$$f(w, x, y, z) = wx + yz + wy + \bar{w}\bar{x}y$$

b)

$$f(x, y, z) = \bar{x}y + y\bar{z} + \bar{y}\bar{z}$$

$$= \bar{x}y(z + \bar{z}) + (x + \bar{x})y\bar{z} + (x + \bar{x})\bar{y}\bar{z}$$

$$= \bar{x}yz + \bar{x}y\bar{z} + xy\bar{z} + \bar{x}y\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$$

$$= \bar{x}yz + \bar{x}y\bar{z} + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$$

$$= m_3 + m_2 + m_6 + m_4 + m_0$$

x \ yz	00	01	11	10
	1		1	1
1	1			1

$$f(x, y, z) = \bar{z} + \bar{x}y$$



(03)

Problem statement:

Implement the functions from question 1 with a 4x1 multiplexer and external gates.

Here,

$$f(w, x, y, z) = (xy + z)(y + xz)$$

$$= xyz + xy\bar{z} + \bar{x}yz + x\bar{y}z$$

$$= (w + \bar{w})xyz + (w + \bar{w})xy\bar{z} + (w + \bar{w})\bar{x}yz + (w + \bar{w})x\bar{y}z$$

$$= wxyz + \bar{w}xyz + wxy\bar{z} + \bar{w}xy\bar{z} + w\bar{x}yz + \bar{w}\bar{x}yz + wx\bar{y}z + \bar{w}x\bar{y}z$$

$$= m_{15} + m_7 + m_{14} + m_6 + m_{11} + m_3 + m_{13} + m_5$$



$s_0 \ s_1$   
 $w \ x$

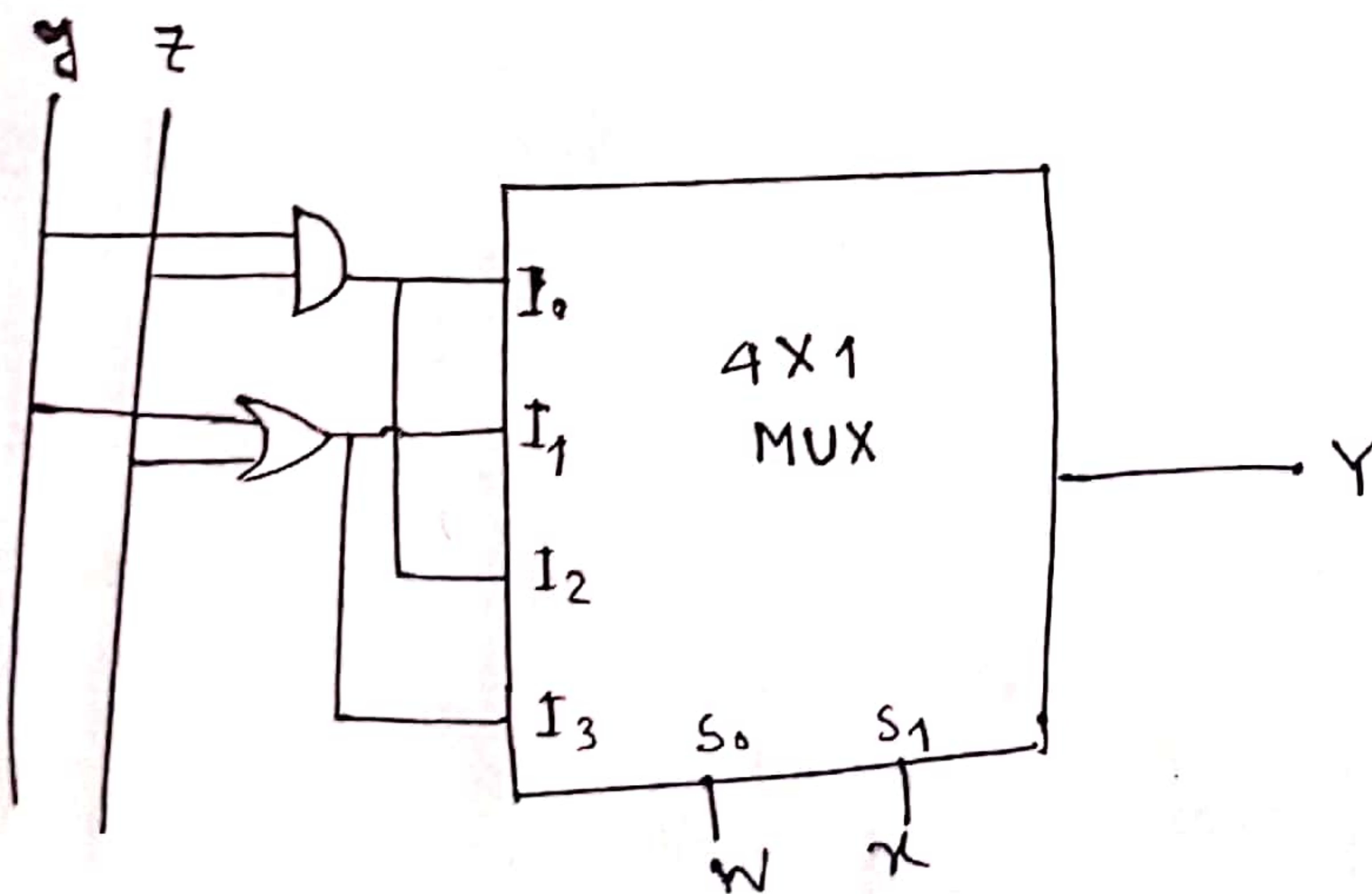
$y \ z$	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	0	1	1	1
10	0	0	1	0

$$I_0 = yz$$

$$I_1 = z + y$$

$$I_2 = yz$$

$$I_3 = z + y$$





b)

$$f(w, x, y, z) = \bar{y}z + wx\bar{y} + wx\bar{z} + \bar{w}\bar{x}z$$

$$= wx\bar{y}z + wx\bar{y}\bar{z} + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$$

$$+ wx\bar{y}\bar{z} + wx\bar{y}z + \bar{w}\bar{x}yz$$

$$= m_{13} + m_9 + m_5 + m_1 + m_{12} + m_{14} + m_3$$

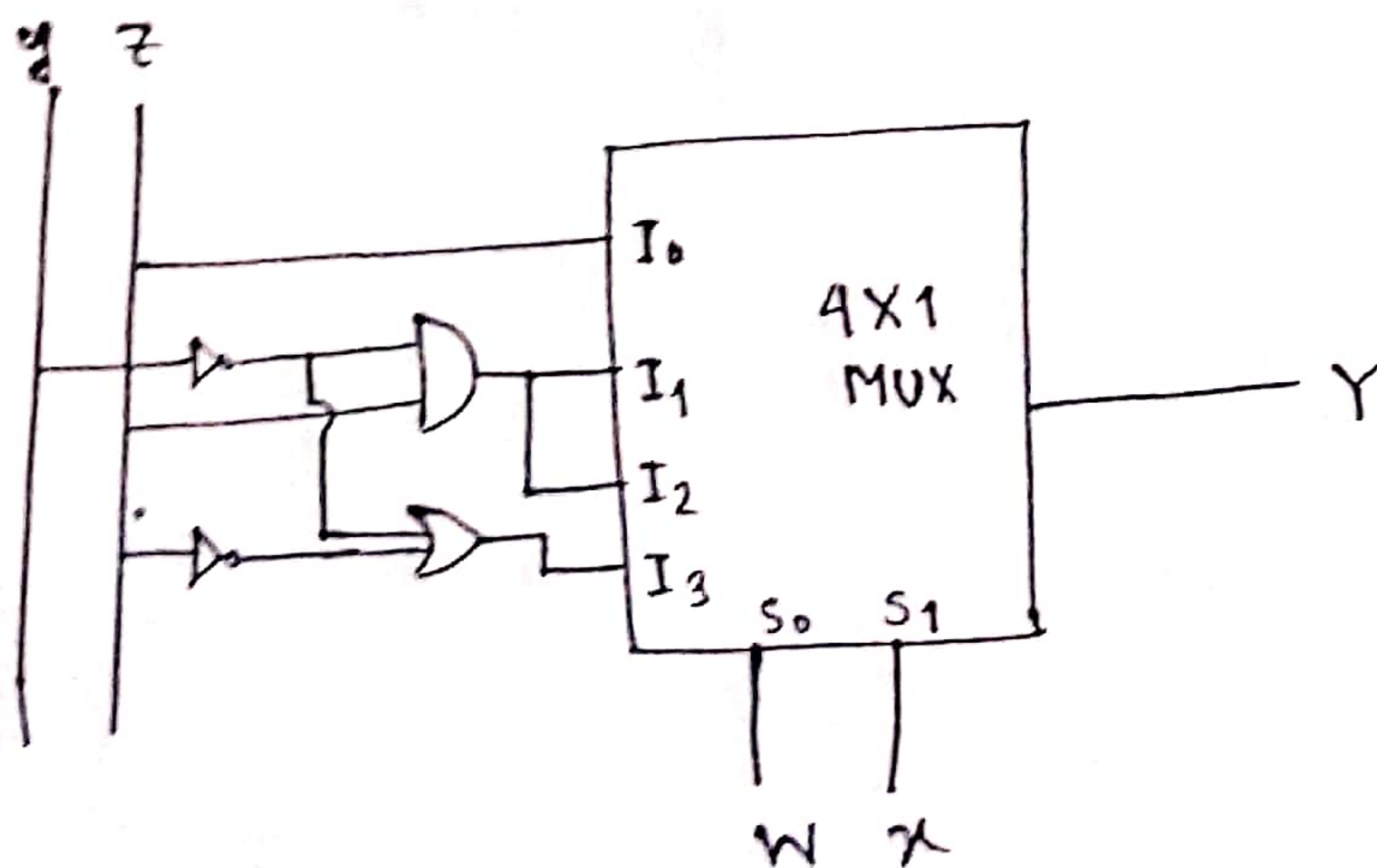
$wx \backslash yz$	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	1	1	0	1
10	0	1	0	0

$$I_0 = z$$

$$I_1 = \bar{y}z$$

$$I_2 = \bar{y}\bar{z}$$

$$I_3 = \bar{y} + \bar{z}$$





(04)

Problem statement:

Design a four-bit combinational circuit 2's complementer.

(The output generates the 2's complement of the input binary number).

Solution:

Input				Output			
w	x	y	z	F <sub>3</sub>	F <sub>2</sub>	F <sub>1</sub>	F <sub>0</sub>
0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	0	0
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1



For  $F_3$ :

$wx \backslash yz$	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	0

$$F_3 = \bar{w}z + \bar{w}y + \bar{w}x\bar{y} + w\bar{x}\bar{y}\bar{z}$$

For  $F_2$ :

$wx \backslash yz$	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	1	0	0	0
10	0	1	1	1

$$F_2 = x\bar{y}\bar{z} + \bar{x}z + \bar{x}y$$



For  $F_1$ :

$w_x \backslash yz$	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	1	0	1
10	0	1	0	1

$$F_1 = \bar{y}z + y\bar{z}$$

For  $F_0$ :

$w_x \backslash yz$	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$$F_0 = z$$



Hence,

$$F_3 = \bar{w}z + \bar{w}y + \bar{w}x\bar{y} + w\bar{x}\bar{y}\bar{z}$$

$$F_2 = x\bar{y}\bar{z} + \bar{x}z + \bar{x}y$$

$$F_1 = \bar{y}z + y\bar{z}$$

$$F_0 = z$$

