Problem Set 3

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Optimization methods

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Exercise 1

Check, if set is a cone:

a) $A = \{x \in \mathbb{R}^n : x_i \geq 0\}$

The set is a cone.

Proof. We have to check, if $\forall_{x \in A} \forall_{\alpha \in \mathbb{R}, \alpha \geq 0} \alpha x \in A$
Let $x \in A, x = (x_1, x_2, \ldots, x_n), \alpha \in \mathbb{R}, \alpha \geq 0$
Then, $\alpha x = (\overbrace{\alpha x_1}^{R_+}, \overbrace{\alpha x_2}^{R_+}, \ldots, \overbrace{\alpha x_n}^{R_+}) \in A$ ∎

c) $C = \{x \in \mathbb{R}^n :< x, a > \leq 0\}, a \in \mathbb{R}^n$

The set is a cone.

Proof. Let $x \in C, \alpha \in \mathbb{R}, \alpha \geq 0$
Then $< \alpha x, a >= \alpha < x, a > \leq 0 \implies \alpha x \in C$ ∎

d) $D = (a) := \{x \in \mathbb{R}^n : x = \alpha a, \alpha \geq 0\}, a \in \mathbb{R}^n$

The set is a cone.

Proof. Let $x \in D, \alpha \in \mathbb{R}, \alpha \geq 0$
Then $\alpha x \in D$ (definition of D) ∎

e) $E = \{x \in \mathbb{R}^n : x = \alpha a, \alpha > 0\}, a \in \mathbb{R}^n$

The set is a cone for $a = 0$.
If $\alpha = 0 \wedge a \neq 0 \implies \alpha x \notin E \implies E$ - not a cone.

f) $F = \{x \in \mathbb{R}^2 : x_1 = 2\}$

The set is a cone.
If $\alpha = 2 \wedge x = (2, 4) \implies \alpha x = (4, 8) \notin F \implies F$ - not a cone.