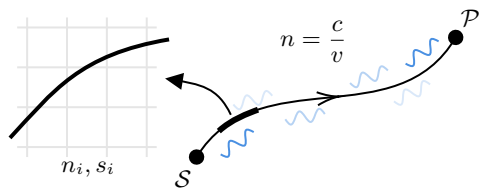


Propagation of Light

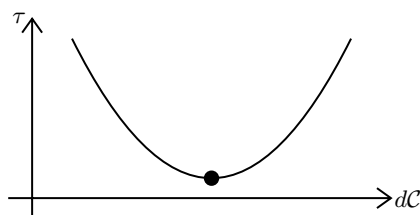
Fermat's Principle:



$$\text{Optical Path Length: } \tau = \sum_{\text{segment}, i} n_i s_i = \int_C n(s) ds$$

FERMAT'S PRINCIPLE - Light travels between two points by paths C such that the optical path length τ is stationary under infinitesimal changes in the path C

Straight Path:



$$\text{Minimum Point: } \frac{\partial \tau}{\partial C} = 0$$

Reflection:

$$\theta_i = \theta_r$$

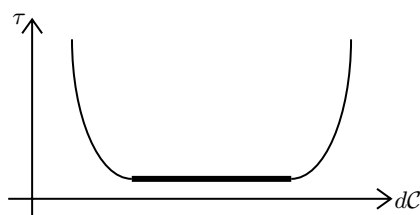
Refraction:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

Imaging Optics

Concepts in Imaging Optics:

STIGMATIC IMAGE - If an optical system causes a cone of rays diverging from S to converge at P , then a stigmatic perfect image of S exists at P



$$\text{Lens Minimum Line: } \frac{\partial \tau}{\partial C} = 0$$

Spherical Lenses in the Paraxial Approximation: $\theta \ll 1$

$$s_o > 0 : S \leftarrow \mathcal{V} \quad s_i > 0 : \mathcal{V} \rightarrow P \quad R > 0 : \mathcal{V} \rightarrow C$$

Spherical Surface Reflection:

$$\frac{n_m}{s_o} + \frac{n_\ell}{s_i} = \frac{1}{R}(n_\ell - n_m)$$

Thin Lens Equation:

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_\ell - 1) \left(\frac{1}{R^{(1)}} - \frac{1}{R^{(2)}} \right)$$

Focal Lengths:

$$f_o = \lim_{s_i \rightarrow \infty} (s_o)$$

$$f_i = \lim_{s_o \rightarrow \infty} (s_i)$$

Thin Lens:

$$f = f_o = f_i \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Virtual and Real Images and Objects:

Both Real: $s_o, s_i > f$

Virtual Image: $s_o < f \Rightarrow s_i < 0$

Virtual Object: $s_i < f \Rightarrow s_o < 0$

Converging and Diverging Lenses:

Converging: $f > 0$

Diverging: $f < 0$

Combinations of Thin Lenses:

$$\frac{1}{f} = \frac{1}{f^{(A)}} + \frac{1}{f^{(B)}} + \dots$$

Chromatic Aberrations and the Achromatic Doublet:

$$n_\ell \sim \lambda$$

Spherical Mirrors:

$$f = -\frac{R}{2}$$

Finite Imaging, Ray Tracing and Optical Instruments

Newtonian Form of Thin Lens Equation:

$$x_o = s_o - f \quad x_i = s_i - f$$

$$x_o x_i = f^2$$

Transverse Magnification:

$$M_T = \frac{y_i}{y_o} = -\frac{x_i}{f} = -\frac{f}{x_o} = -\frac{s_i}{s_o}$$

Longitudinal Magnification:

$$M_L = -\frac{f^2}{x_o^2} = -M_T^2$$

Ray Tracing:

FOCAL PLANE RAY TRACING - For each ray...

1. Find where it crosses the object focal plane \mathcal{Q} ,
2. The ray emerges parallel to a line drawn between \mathcal{Q} and the point where the thin lens meets the optical axis

Optical Instruments:

Cameras:

$$f/\# \text{ where } \# = \frac{f}{D}$$

Magnifying Glass:

$$M_\theta = \frac{0.25 \text{ [m]}}{f} + 1 \quad D = \frac{1}{f} \text{ [m}^{-1}\text{]} \quad D_{\text{eff}} = D_1 + D_2 + \dots$$

Astronomical Telescopes:

$$M_\theta = \frac{\theta_{\text{out}}}{\theta} \approx \frac{\tan(\theta_{\text{out}})}{\tan(\theta)} = -\frac{f_{\text{obj}}}{f_{\text{eye}}}$$

Principles of Physical Optics

Electromagnetic Radiation:

Maxwell's Equations:

$$\begin{aligned}\nabla \cdot \underline{\mathbf{E}} &= \frac{\rho}{\epsilon_0} & \nabla \times \underline{\mathbf{E}} &= -\frac{\partial \underline{\mathbf{B}}}{\partial t} \\ \nabla \cdot \underline{\mathbf{B}} &= 0 & \frac{1}{\mu_0} \nabla \times \underline{\mathbf{B}} &= \underline{\mathbf{J}} + \epsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t}\end{aligned}$$

EM Wave Equations:

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{1}{\sqrt{\mu \epsilon}} \quad v = \frac{c}{n} \quad n = \sqrt{\mu_r \epsilon_r}$$

Relative Strength of Fields:

The $\underline{\mathbf{E}}$ field dominates except at relativistic speeds....

$$|\underline{\mathbf{B}}| = \frac{|\underline{\mathbf{E}}|n}{c}$$

Scalar Wave Equation and Plane Waves:

Scalar wave equation:

$$\frac{\partial^2 u}{\partial z^2} = \left(\frac{n}{c}\right)^2 \frac{\partial^2 u}{\partial t^2}$$

Plane Wave Solutions:

$$u(\underline{\mathbf{r}}, t) = u_0 e^{i(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t + \phi)}$$

$$\underline{\mathbf{k}} = (k_x, k_y, k_z) \quad |\underline{\mathbf{k}}| = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$$

Plane Waves in a Dielectric Medium:

$$\underline{\mathbf{k}}' = \frac{1}{n} \underline{\mathbf{k}}$$

Spherical Waves:

$$u(r, t) = \frac{u_0}{|\underline{\mathbf{r}}|} e^{i(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t + \phi)}$$

Poynting Vector and Intensity:

Poynting Vector:

$$\underline{\mathbf{S}} = \frac{1}{\mu_0} \underline{\mathbf{E}} \times \underline{\mathbf{B}}$$

$$\underline{\mathbf{E}} = u_0 \cos(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t) \underline{\mathbf{e}}_0 \quad \underline{\mathbf{B}} = \frac{u_0 n}{c} \cos(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t) \underline{\mathbf{e}}_0$$

$$\underline{\mathbf{S}} = \frac{1}{\mu_0} \underline{\mathbf{E}} \times \underline{\mathbf{B}} = \frac{1}{\mu_0} u_0^2 \frac{n}{c} \cos^2(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t)$$

Intensity:

$$\langle \underline{\mathbf{S}} \rangle = \frac{1}{T} \int_0^T \underline{\mathbf{S}} dt \implies I = |\langle \underline{\mathbf{S}} \rangle| = \frac{|u_0|^2 n}{2c\mu_0} = |u_0|^2 \frac{n\epsilon_0 c}{2}$$

$$I(\underline{\mathbf{r}}, t) = |u(\underline{\mathbf{r}}, t)|^2 \frac{n\epsilon_0 c}{2}$$

Superposition and Interference:

PRINCIPLE OF SUPERPOSITION - Linear combinations of solutions to the scalar wave equation are also solutions

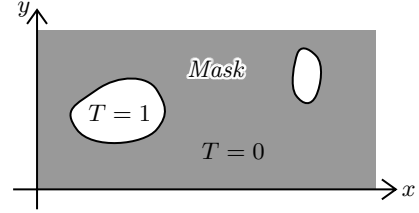
$$I_{\text{tot}} = \frac{n\epsilon_0 c}{2} |u|^2 = \frac{n\epsilon_0 c}{2} |u_1 + u_2|^2$$

Scalar Diffraction Theory

Huygens-Fresnel Principle:

HUYGENS-FRESNEL PRINCIPLE - Each element of a wavefront acts as a source of secondary wavelets, diverging spherically from it. The secondary wavelets interfere and the resultant field is a superposition of all these wavelets

Kirchoff-Fresnel Formula:



$$\text{Transmission Function: } T(x, y) = \begin{cases} 0, & \text{Opaque} \\ \updownarrow \\ 1, & \text{Transparent} \end{cases}$$

$$u_p(x_s, y_s) = \frac{Cu_0}{\lambda R_0} \iint_{-\infty}^{\infty} T(x, y) \exp \left[ik \left(\frac{x^2 + y^2}{2R_0} - \frac{xx_s + yy_s}{R_0} \right) \right] dx dy$$

The Rayleigh Distance:

$$(x, y) \sim a$$

$$R_R = \frac{a^2}{\lambda}$$

$$R_0 \lesssim R_R \implies \text{Fresnel}$$

$$R_0 \gg R_R \implies \text{Fraunhofer}$$

Fraunhofer Diffraction:

$$u_p(x_s, y_s) = \frac{Cu_0}{\lambda R_0} \iint_{-\infty}^{\infty} T(x, y) \exp \left[-ik \left(\frac{xx_s + yy_s}{R_0} \right) \right] dx dy$$

Fraunhofer Diffraction and Fourier Optics

Fraunhofer Diffraction and Fourier Transforms:

$$u = \frac{x_s}{\lambda R_0} \approx \frac{x_s}{\lambda L} \approx \frac{\theta_x}{\lambda} \quad v = \frac{y_s}{\lambda R_0} \approx \frac{y_s}{\lambda L} \approx \frac{\theta_y}{\lambda}$$

$$u_p(u, v) = \frac{Cu_0}{\lambda L} \iint_{-\infty}^{\infty} T(x, y) \exp [-2\pi i(xu + yv)] dx dy$$

$$u_p(u, v) = \frac{Cu_0}{\lambda L} \mathcal{F}[T(x, y)] = \frac{Cu_0}{\lambda L} \overline{T}(x, y)$$

Important Functions and their Fourier Transforms:

Normalised Sinc:

$$\text{sinc}(\omega) = \frac{\sin(\pi\omega)}{\pi\omega}$$

The Rectangle:

$$\text{rect}(x/a) = \begin{cases} 1, & |x| < a/2 \\ 1/2, & |x| = a/2 \\ 0, & \text{elsewhere.} \end{cases}$$

$$\dots \implies \mathcal{F}[\text{rect}(x/a)] = a \text{sinc}(au)$$

The Dirac Delta:

$$\delta(x) = \lim_{a \rightarrow 0} \frac{1}{a} \text{rect}(x/a)$$

$$\dots \implies \mathcal{F}[\delta(x-b)] = e^{-i2\pi bu}$$

The Dirac Comb:

$$\text{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$$

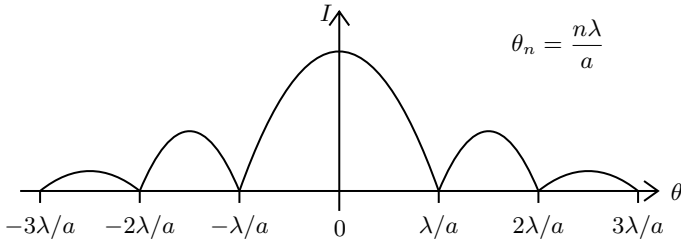
$$\dots \implies \mathcal{F}[\text{comb}(x/d)] = d \text{comb}(du)$$

The Convolution Theorem:

$$\mathcal{F}[f(x) * g(x)] = \mathcal{F}[f(x)] \cdot \mathcal{F}[g(x)]$$

Single Slit of Width a :

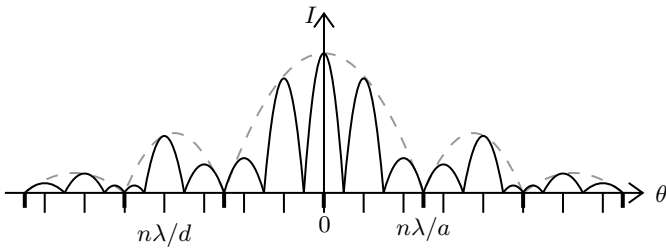
$$T(x) = \text{rect}\left(\frac{x}{a}\right) \implies I(\theta) = \frac{I_0 |C|^2}{\lambda^2 L^2} a^2 \text{sinc}^2\left(\frac{a\theta}{\lambda}\right)$$



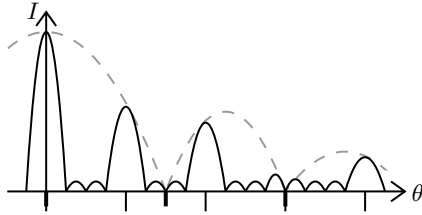
Young's Double Slit Diffraction:

$$T(x) = \text{rect}\left(\frac{x}{a}\right) * \delta(x + d/2) + \text{rect}\left(\frac{x}{a}\right) * \delta(x - d/2)$$

$$\implies I(\theta) = \frac{4|C|^2 I_0}{\lambda^2 L^2} a^2 \text{sinc}^2\left(\frac{a\theta}{\lambda}\right) \cos^2\left(\frac{\pi d\theta}{\lambda}\right)$$



N -Slit Diffraction:



"Primary maxima every λ/d with $(N - 2)$ subsidiary maxima"

Resolving Power and Rayleigh Criterion:

$$\frac{\Delta\lambda}{\lambda} \gg \frac{1}{mN} \equiv \frac{1}{R_G} \quad \alpha > \frac{1.22\lambda}{D}$$

Polarization

Polarization of EM Plane Waves:

$$\underline{\mathbf{E}} = E_0^x x \cos(kz - \omega t) + E_0^y y \cos(kz - \omega t + \epsilon)$$

$$\text{Linear: } \underline{\mathbf{E}} = (E_0^x x + E_0^y y) \cos(kz - \omega t)$$

$$\text{Circular: } \underline{\mathbf{E}} = E_0 [x \cos(kz - \omega t) \pm y \sin(kz - \omega t)]$$

Polarizers and Malus's law:

Linear In - Linear Out:

$$I(\theta) = I_0 \cos^2(\theta)$$

Unpolarized In - Linear Out:

$$I_0 = \int_0^{2\pi} \langle I \rangle d\theta = \int_0^{2\pi} \frac{I_0}{2\pi} d\theta \implies I_{\text{out}} = \int_0^{2\pi} \frac{I_0 \cos^2(\theta)}{2\pi} d\theta = \frac{1}{2} I_0$$

Birefringence and Optical Activity:

"Optical properties of a medium depends on the direction of propagation and polarization of light"

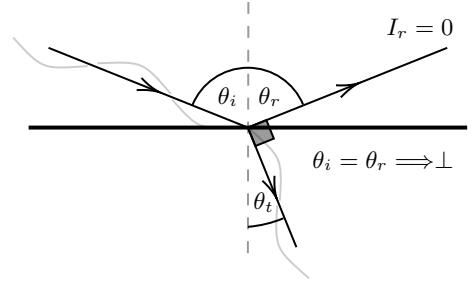
Rotation Constant:

$$\beta = \frac{\pi}{\lambda} (n_L - n_R)$$

Left: $\beta < 0 : n_R > n_L$

Right: $\beta > 0 : n_L > n_R$

Brewster's Angle:



Optical Cavities and Lasers

The Fabry-Perot interferometer:

$$\delta = \frac{4\pi n_m d}{\lambda} \quad I_r = \mathcal{R} I_i$$

Finesse and Free Spectral Range:

$$\mathbb{F} = \frac{\pi}{2} \sqrt{\frac{4\mathcal{R}}{(1 - \mathcal{R})^2}}$$

$$\Delta\Lambda_{\text{FSR}} = \frac{\lambda^2}{2n_m d} \quad \Delta f_{\text{FSR}} = \frac{c}{2n_m d}$$

Longitudinal Laser Modes:

$$\text{Laser Action: } I_t = g I_i$$