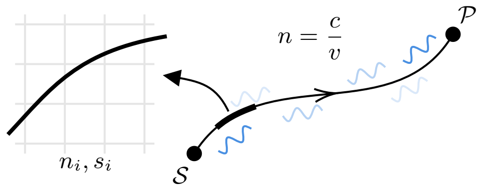
Propagation of Light

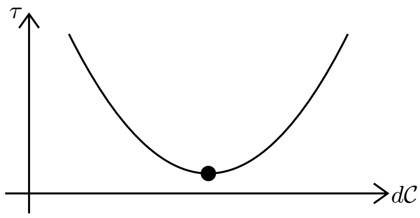
Fermat's Principle:



Optical Path Length: $\tau = \sum_{\text{segment},i} n_i s_i = \int_{\mathcal{C}} n(s)\,\mathrm{d}s$

> Fermat's Principle - *Light travels between two points by paths $\mathcal{C}$ such that the optical path length τ is stationary under infinitesimal changes in the path $\mathcal{C}$*

Straight Path:



Minimum Point: $\frac{\partial \tau}{\partial \mathcal{C}} = 0$

Reflection:

$$\theta_i = \theta_r$$

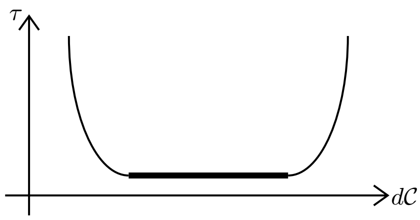
Refraction:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

Imaging Optics

Concepts in Imaging Optics:

> Stigmatic Image - *If an optical system causes a cone of rays diverging from $\mathcal{S}$ to converge at $\mathcal{P}$, then a stigmatic perfect image of $\mathcal{S}$ exists at $\mathcal{P}$*



Lens Minimum Line: $\frac{\partial \tau}{\partial \mathcal{C}} = 0$

Spherical Lenses in the Paraxial Approximation: $\theta \ll 1$

$s_o > 0 : \mathcal{S} \leftarrow \mathcal{V}$ $\qquad$ $s_i > 0 : \mathcal{V} \rightarrow \mathcal{P}$ $\qquad$ $R > 0 : \mathcal{V} \rightarrow \mathcal{C}$

Spherical Surface Reflection:

$$\frac{n_m}{s_o} + \frac{n_\ell}{s_i} = \frac{1}{R}(n_\ell - n_m)$$

Thin Lens Equation:

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_\ell - 1)\left(\frac{1}{R^{(1)}} - \frac{1}{R^{(2)}}\right)$$

Focal Lengths:

$$f_o = \lim_{s_i \to \infty}(s_o) \qquad f_i = \lim_{s_o \to \infty}(s_i)$$

Thin Lens:

$$f = f_o = f_i \qquad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Virtual and Real Images and Objects:

Both Real: $s_0, s_i > f$

Virtual Image: $s_o < f \implies s_i < 0$

Virtual Object: $s_i < f \implies s_o < 0$

Converging and Diverging Lenses:

Converging: $f > 0$ $\qquad$ Diverging: $f < 0$

Combinations of Thin Lenses:

$$\frac{1}{f} = \frac{1}{f^{(A)}} + \frac{1}{f^{(B)}} + \dots$$

Chromatic Aberrations and the Achromatic Doublet:

$$n_\ell \sim \lambda$$

Spherical Mirrors:

$$f = -\frac{R}{2}$$

Finite Imaging, Ray Tracing and Optical Instruments

Newtonian Form of Thin Lens Equation:

$$x_o = s_o - f \qquad x_i = s_i - f$$

$$x_o x_i = f^2$$

Transverse Magnification:

$$M_T = \frac{y_i}{y_o} = -\frac{x_i}{f} = -\frac{f}{x_o} = -\frac{s_i}{s_o}$$

Longitudinal Magnification:

$$M_L = -\frac{f^2}{x_o^2} = -M_T^2$$

Ray Tracing:

> Focal Plane Ray Tracing - *For each ray...*
> *1. Find where it crosses the object focal plane $\mathcal{Q}$,*
> *2. The ray emerges parallel to a line drawn between $\mathcal{Q}$ and the point where the thin lens meets the optical axis*

Optical Instruments:

Cameras:

$$f/\#\ \text{where}\ \# = \frac{f}{D}$$

Magnifying Glass:

$$M_\theta = \frac{0.25\ [\mathrm{m}]}{f} + 1 \qquad \mathcal{D} = \frac{1}{f}\ [\mathrm{m}^{-1}] \qquad \mathcal{D}_{\text{eff}} = \mathcal{D}_1 + \mathcal{D}_2 + \dots$$

Astronomical Telescopes:

$$M_\theta = \frac{\theta_{\text{out}}}{\theta} \approx \frac{\tan(\theta_{\text{out}})}{\tan(\theta)} = -\frac{f_{\text{obj}}}{f_{\text{eye}}}$$

Principles of Physical Optics

Electromagnetic Radiation:

Maxwell's Equations:

$$\nabla \cdot \underline{\mathbf{E}} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \underline{\mathbf{E}} = -\frac{\partial \underline{\mathbf{B}}}{\partial t}$$

$$\nabla \cdot \underline{\mathbf{B}} = 0 \qquad \frac{1}{\mu_0}\nabla \times \underline{\mathbf{B}} = \underline{\mathbf{J}} + \varepsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t}$$

EM Wave Equations:

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad v = \frac{c}{n} \qquad n = \sqrt{\mu_r \varepsilon_r}$$

Relative Strength of Fields:

The $\underline{\mathbf{E}}$ *field dominates except at relativistic speeds....*

$$|\underline{\mathbf{B}}| = \frac{|\underline{\mathbf{E}}| n}{c}$$

Scalar Wave Equation and Plane Waves:

Scalar wave equation:

$$\frac{\partial^2 u}{\partial z^2} = \left(\frac{n}{c}\right)^2 \frac{\partial^2 u}{\partial t^2}$$

Plane Wave Solutions:

$$u(\underline{\mathbf{r}}, t) = u_0\; e^{i(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t + \phi)}$$

$$\underline{\mathbf{k}} = (k_x, k_y, k_z) \qquad |\underline{\mathbf{k}}| = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Plane Waves in a Dielectric Medium:

$$\underline{\mathbf{k}}' = \frac{1}{n}\underline{\mathbf{k}}$$

Spherical Waves:

$$u(r, t) = \frac{u_0}{|\underline{\mathbf{r}}|}\; e^{i(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t + \phi)}$$

Poynting Vector and Intensity:

Poynting Vector:

$$\underline{\mathbf{S}} = \frac{1}{\mu_0}\underline{\mathbf{E}} \times \underline{\mathbf{B}}$$

$$\underline{\mathbf{E}} = u_0 \cos(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t) E_0 \qquad \underline{\mathbf{B}} = \frac{u_0 n}{c} \cos(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t) E_0$$

$$\underline{\mathbf{S}} = \frac{1}{\mu_0}\underline{\mathbf{E}} \times \underline{\mathbf{B}} = \frac{1}{\mu_0} u_0^2 \frac{n}{c} \cos^2(\underline{\mathbf{k}} \cdot \underline{\mathbf{r}} - \omega t)$$

Intensity:

$$\langle \underline{\mathbf{S}} \rangle = \frac{1}{T}\int_0^T \underline{\mathbf{S}}\, \mathrm{d}t \implies I = |\langle \vec{\mathbf{S}} \rangle| = \frac{|u_0|^2 n}{2c\mu_0} = |u_0|^2 \frac{n\varepsilon_0 c}{2}$$

$$I(\underline{\mathbf{r}}, t) = |u(\underline{\mathbf{r}}, t)|^2 \frac{n\varepsilon_0 c}{2}$$

Superposition and Interference:

PRINCIPLE OF SUPERPOSITION - *Linear combinations of solutions to the scalar wave equation are also solutions*

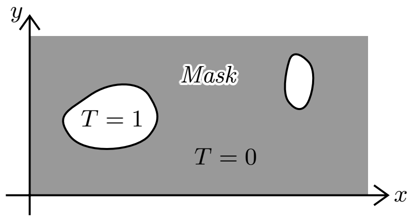
$$I_{\text{tot}} = \frac{n\varepsilon_0 c}{2}|u|^2 = \frac{n\varepsilon_0 c}{2}|u_1 + u_2|^2$$

Scalar Diffraction Theory

Huygens–Fresnel Principle:

HUYGENS–FRESNEL PRINCIPLE - *Each element of a wavefront acts as a source of secondary wavelets, diverging spherically from it. The secondary wavelets interfere and the resultant field is a superposition of all these wavelets*

Kirchoff–Fresnel Formula:



$$\text{Transmission Function: } T(x, y) = \begin{cases} 0, & \textit{Opaque} \\ \updownarrow & \\ 1, & \textit{Transparent} \end{cases}$$

$$u_p(x_s, y_s) = \frac{Cu_0}{\lambda R_0} \iint_{-\infty}^{\infty} T(x, y) \exp\left[ik\left(\frac{x^2 + y^2}{2R_0} - \frac{xx_s + yy_s}{R_0}\right)\right] \mathrm{d}x\, \mathrm{d}y$$

The Rayleigh Distance: $(x, y) \sim a$

$$R_R = \frac{a^2}{\lambda}$$

$R_0 \lesssim R_R \implies$ *Fresnel* $\qquad R_0 \gg R_R \implies$ *Fraunhofer*

Fraunhofer Diffraction:

$$u_p(x_s, y_s) = \frac{Cu_0}{\lambda R_0} \iint_{-\infty}^{\infty} T(x, y) \exp\left[-ik\left(\frac{xx_s + yy_s}{R_0}\right)\right] \mathrm{d}x\, \mathrm{d}y$$

Fraunhofer Diffraction and Fourier Optics

Fraunhofer Diffraction and Fourier Transforms:

$$u = \frac{x_s}{\lambda R_0} \approx \frac{x_s}{\lambda L} \approx \frac{\theta_x}{\lambda} \qquad v = \frac{y_s}{\lambda R_0} \approx \frac{y_s}{\lambda L} \approx \frac{\theta_y}{\lambda}$$

$$u_p(u, v) = \frac{Cu_0}{\lambda L} \iint_{-\infty}^{\infty} T(x, y) \exp\left[-2\pi i(xu + yv)\right] \mathrm{d}x\, \mathrm{d}y$$

$$u_p(u, v) = \frac{Cu_0}{\lambda L} \mathcal{F}\left[T(x, y)\right] = \frac{Cu_0}{\lambda L} \overline{T}(x, y)$$

Important Functions and their Fourier Transforms:

Normalised Sinc:

$$\operatorname{sinc}(\omega) = \frac{\sin(\pi\omega)}{\pi\omega}$$

The Rectangle:

$$\operatorname{rect}(x/a) = \begin{cases} 1, & |x| < a/2 \\ 1/2, & |x| = a/2 \\ 0, & \textit{elsewhere.} \end{cases}$$

$$\cdots \implies \mathcal{F}\left[\operatorname{rect}(x/a)\right] = a \operatorname{sinc}(au)$$

The Dirac Delta:

$$\delta(x) = \lim_{a \to 0} \frac{1}{a} \operatorname{rect}(a/x)$$

$$\cdots \implies \mathcal{F}\left[\delta(x - b)\right] = e^{-i2\pi bu}$$

The Dirac Comb:

$$\operatorname{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x - n)$$

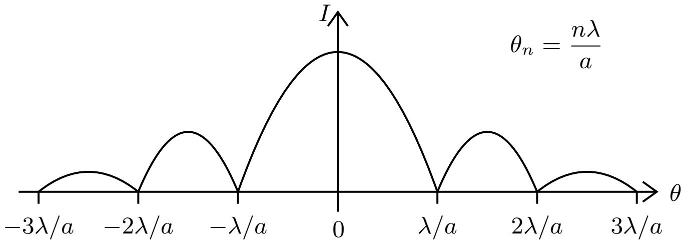
$$\cdots \implies \mathcal{F}\left[\operatorname{comb}(x/d)\right] = d \operatorname{comb}(du)$$

The Convolution Theorem:

$$\mathcal{F}[f(x) * g(x)] = \mathcal{F}[f(x)] \cdot \mathcal{F}[g(x)]$$

Single Slit of Width a:

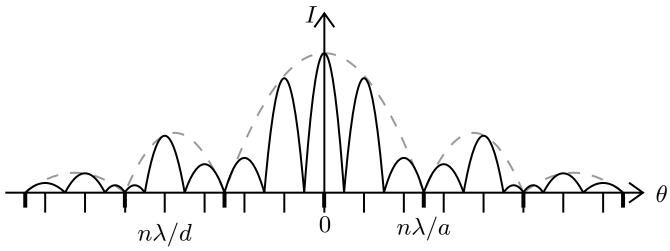
$$T(x) = \text{rect}\left(\frac{x}{a}\right) \implies I(\theta) = \frac{I_0|C|^2}{\lambda^2 L^2} a^2 \,\text{sinc}^2\left(\frac{a\theta}{\lambda}\right)$$



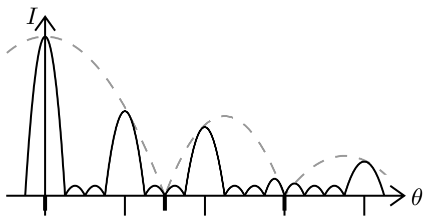
Young's Double Slit Diffraction:

$$T(x) = \text{rect}\left(\frac{x}{a}\right) * \delta(x + d/2) + \text{rect}\left(\frac{x}{a}\right) * \delta(x - d/2)$$

$$\implies I(\theta) = \frac{4|C|^2 I_0}{\lambda^2 L^2} a^2 \,\text{sinc}^2\left(\frac{a\theta}{\lambda}\right) \cos^2\left(\frac{\pi d\theta}{\lambda}\right)$$



N-Slit Diffraction:



"Primary maxima every λ/d with $(N-2)$ subsidiary maxima"

Resolving Power and Rayleigh Criterion:

$$\frac{\Delta\lambda}{\lambda} \gg \frac{1}{mN} \equiv \frac{1}{R_G} \qquad \alpha > \frac{1.22\lambda}{D}$$

Polarization

Polarization of EM Plane Waves:

$$\underline{\mathbf{E}} = E_0^x \; x\cos(kz - \omega t) + E_0^y \; y\cos(kz - \omega t + \varepsilon)$$

$$\text{Linear: } \underline{\mathbf{E}} = (E_0^x \; x + E_0^y \; y) \; \cos(kz - \omega t)$$

$$\text{Circular: } \underline{\mathbf{E}} = E_0\big[x\cos(kz - \omega t) \pm y\sin(kz - \omega t)\big]$$

Polarizers and Malus's law:

Linear In - Linear Out:

$$I(\theta) = I_0 \cos^2(\theta)$$

Unpolarized In - Linear Out:

$$I_0 = \int_0^{2\pi} \langle I \rangle \, \mathrm{d}\theta = \int_0^{2\pi} \frac{I_0}{2\pi} \, \mathrm{d}\theta \implies I_{\text{out}} = \int_0^{2\pi} \frac{I_0\cos^2(\theta)}{2\pi} \, \mathrm{d}\theta = \frac{1}{2} I_0$$

Birefringence and Optical Activity:

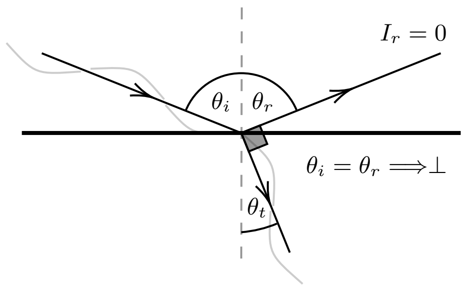
"Optical properties of a medium depends on the direction of propagation and polarization of light"

Rotation Constant:

$$\beta = \frac{\pi}{\lambda}(n_L - n_R)$$

Left $: \beta < 0 : n_R > n_L$ *Right* $: \beta > 0 : n_L > n_R$

Brewster's Angle:



Optical Cavities and Lasers

The Fabry-Perot interferometer:

$$\delta = \frac{4\pi n_m d}{\lambda} \qquad I_r = \mathcal{R} I_i$$

Finesse and Free Spectral Range:

$$\mathbb{F} = \frac{\pi}{2}\sqrt{\frac{4\mathcal{R}}{(1-\mathcal{R})^2}}$$

$$\Delta\Lambda_{\text{FSR}} = \frac{\lambda^2}{2n_m d} \qquad \Delta f_{\text{FSR}} = \frac{c}{2n_m d}$$

Longitudinal Laser Modes:

Laser Action: $I_t = g I_i$