# 4

# FREQUENCY RESPONSE

## OUTLINE



## OBJECTIVES

1. Predict the features of a circuit near resonance.
2. Estimate a frequency response using Bode straight-line approximations.
3. Define the decibel, and discuss its uses.
4. Apply the Bode SLA technique to systems with mild resonances.
5. Use the MATLAB frequency response tools.

## INTRODUCTION

By retaining the $s$ variable in our transfer functions, we are poised to handle questions regarding any of the possible $e^{st}$ signals. In point of fact, however, exponentially increasing forcing functions would eventually destroy real systems, and exponentially decreasing forcing functions might easily be obscured by even transient natural responses. The signal of greatest interest, by far, is the pure sinusoid, and the frequency response of a system is understood to mean evaluating the transfer or driving point functions along the path $s = +j\omega$.

Techniques have been developed over the years for estimating the frequency response of a system while making a minimum number of calculations. These techniques often provide a point of view that can also be helpful in the design process.

Today, computers are routinely used to plot the exact frequency response of driving point or transfer functions. The frequency response of a system is measurable and is likely to be a criterion in its selection among competing products. The abilities to estimate frequency response and to use the tools that graph or measure frequency response are essential skills in engineering and technology.

## 4.1 RESONANT SYSTEMS: *L-C* CIRCUITS

Systems with poles or zeros very close to the $j\omega$ axis are resonant systems. They have very vigorous responses over small frequency ranges, and their behavior at frequencies far from resonance is generally considered unimportant. A resonant circuit is dominated by inductive and capacitive circuit elements. After completing this section you will be able to:

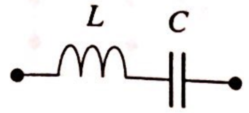
- Predict the features of a circuit near resonance.
- Estimate the resonant frequency and bandwidth of the circuit.
- Estimate the maximum or minimum value of the function.
- Use a pole-zero diagram to graphically calculate function values.

When evaluated for $s = j\omega$, the impedances of inductors and capacitors are purely imaginary and of opposite sign. As a result, although each separately may help limit current, a combination of the two may cause one to cancel the effect of the other, leading to a striking circuit behavior known as *resonance*.

$$s = j\omega$$

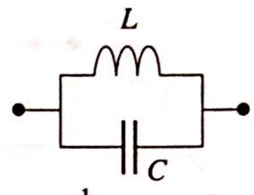
$$Z_L = sL = j\omega L \qquad Z_C = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{-j}{\omega C} \tag{4.1}$$

An *R-L-C* circuit is said to be resonant when its driving point impedance is purely real (resistive). If an inductor and capacitor are in series, the circuit *resonates* when the magnitudes of the inductive and capacitive impedances are equal, and the resulting circuit impedance looks like a short circuit. Parallel resonance, sometimes called




$$Z(s) = Ls + \frac{1}{Cs} = \frac{L(s^2 + 1/LC)}{s}$$

$$Z(j\omega_o) = 0 \text{ where } \omega_o = \frac{1}{\sqrt{LC}}$$




$$Z(s) = Ls // \frac{1}{Cs} = \frac{s}{C(s^2 + 1/LC)}$$

$$Z(j\omega_o) = \infty \text{ where } \omega_o = \frac{1}{\sqrt{LC}}$$

**Figure 4.1** Ideal series and parallel resonant circuits.

*antiresonance*, occurs under the same conditions, but the circuit impedance looks like an open circuit. Figure 4.1 summarizes these facts.

## EXAMPLE 4.1

Determine the resonant frequencies in the circuit of Figure 4.2a.

### *Solution*

The 2 H–1/2 F combination produces a parallel resonance at $\omega_o = 1$ rad/s. Above this frequency the parallel combination looks capacitive and can produce series resonance with the (2/3)-H inductor. To identify where that occurs, an expression for $Z(s)$ is found by combining impedances, and a common denominator is obtained:

$$Z(s) = \frac{2}{3}s + \frac{(2s)(2/s)}{2s + 2/s} = \frac{\frac{4}{3}s^2 + \frac{4}{3} + 4}{2s + 2/s} = \frac{2}{3}\,\frac{s(s^2 + 4)}{s^2 + 1}$$

The usual checks should be made at $s = 0$ and at $s = \infty$ to show any discrepancies between the equation and the circuit (Figure 4.2b). Then we factor each polynomial to find the pole-zero form for $Z(s)$. In this case it can be done by inspection, with the

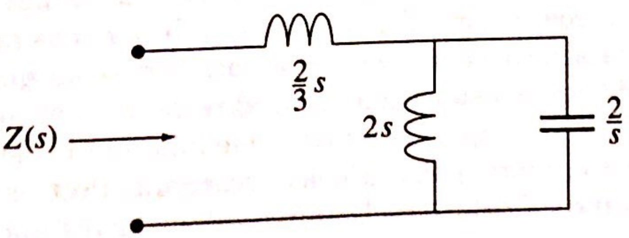



**Figure 4.2a**

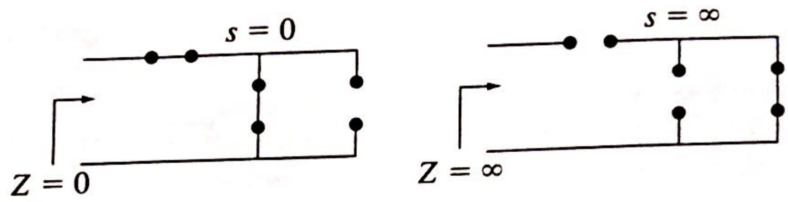



**Figure 4.2b** Sketches used to check the impedance expression at extreme frequencies.

$$Z(s) = \frac{2}{3}\,\frac{s(s + j2)(s - j2)}{(s + j)(s - j)}.$$

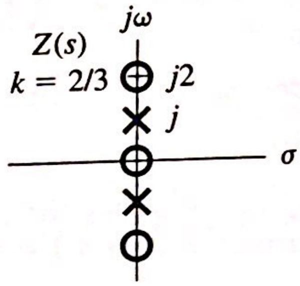



**Figure 4.2c** The pole-zero diagram of the circuit of Figure 4.2a.

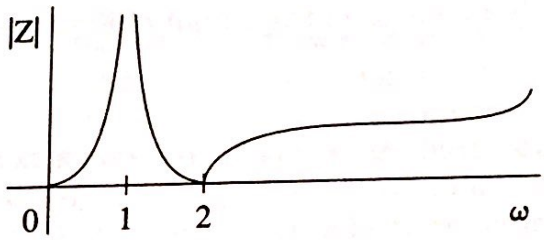



**Figure 4.2d** A sketch of the impedance's frequency response.

result as shown in Figure 4.2c. The impedance for sinusoids is shown in Figure 4.2d. It is the impedance along the $+j\omega$ axis.

Both the expression for $Z(s)$ and its graphical representation in the form of the pole-zero plot show that the series resonant effect occurs at $\omega = 2$ rad/s.

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Note that a circuit containing only ideal inductors and capacitors has poles and zeros along the $j\omega$ axis. This makes them easy to find, and we may intentionally idealize the components in order to get a first estimate of the circuit's resonant frequencies. The transfer functions of such circuits would be classified as marginally stable. Any initial disturbance would cause a sinusoidal natural response that continues forever.

All actual inductors and capacitors have some inherent resistance. Inductors are easily the least ideal, so a more realistic look at resonant effects would require including at least the resistance of inductors. Doing so will make the impedance expressions more complicated. The poles and zeros will be shifted off of, but still be close to, the $j\omega$ axis. Our visualization of a pole-zero plot as a topographical map suggests that the

$$F(s) = 3(s + a)$$

$$\underbrace{F(s)|_{s = b + jc}}_{foi} = 3(s + a)|_{s = b + jc}$$

$$= 3(b + a + jc)$$

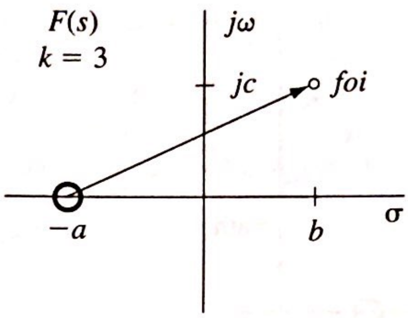



**Figure 4.3** $F(s)$ has a single zero and a multiplier of 3. Evaluating $F$ at any arbitrary value of $s$, called the *frequency of interest* (*foi*), is accomplished by substituting that $s$ into $F$. Equivalently, it may be accomplished by finding the magnitude and angle of the vector from the zero to the frequency of interest on a properly scaled pole-zero plot and then multiplying by $k$.

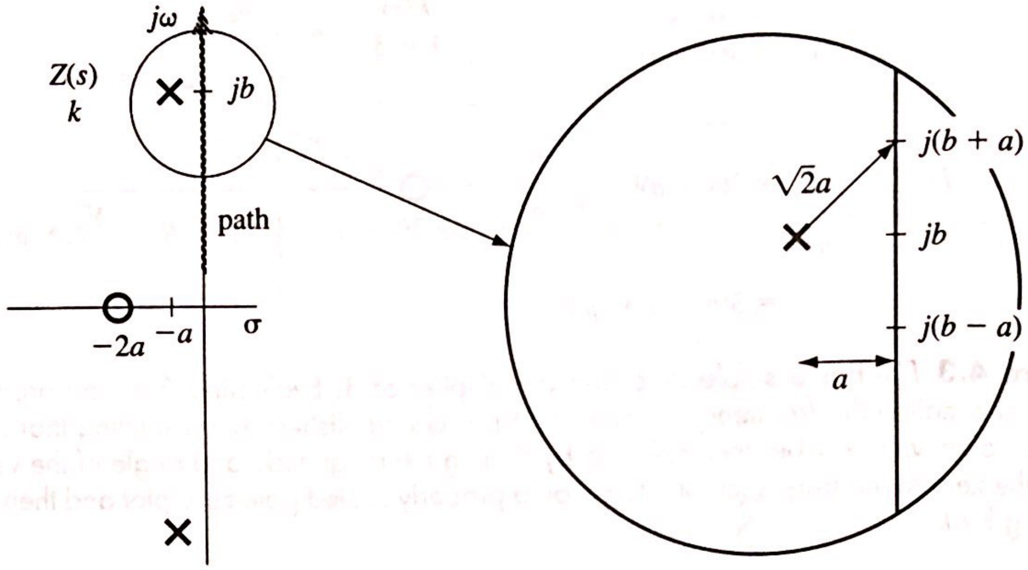
path $s = j\omega$ will take us up the mountain sides, but no longer to the infinite peaks or to zero elevation in the valleys. We would like to find a simple way to estimate how high or low we actually get and how steep the terrain is.

Consider the simple $F(s)$ and its corresponding pole-zero plot shown in Figure 4.3. With the multiplier indicated on the pole-zero plot, the plot contains all of the information available in the expression for $F(s)$. Given the plot, we can find $F$; given $F$, we can make the plot. If we want to evaluate $F$ at a particular $s_o$, called the *frequency of interest (foi)*, we can see that the value contributed by any root is exactly equal to the magnitude and angle of a vector drawn from that root to the frequency of interest. Generalizing this observation, the value of an $F(s)$ at any arbitrary frequency of interest may be found from its pole-zero diagram as

$$F(s_o) = k\,\frac{\text{product of vectors from the finite zeros to } foi}{\text{product of vectors from the finite poles to } foi} \tag{4.2}$$

By measuring magnitudes with a ruler and measuring angles with a protractor, we could rather quickly evaluate the impedance or transfer function of a circuit using this observation. Now, we are not going to seriously advocate this graphical calculation method in the computer age, but suppose all of the poles and zeros are close to the $j\omega$ axis. For a frequency of interest on the $j\omega$ axis, most of the vectors would have angles of nearly $\pm 90°$, and their magnitudes would be essentially their vertical components, which can be read directly on the frequency scale. In this case, no measuring tools are actually needed, and the pole-zero diagram does not need to be drawn to scale.

Consider the pole-zero diagram of the realistic parallel resonant circuit shown in Figure 4.4. It certainly seems clear that a sharp peaking could be expected as we move along the $j\omega$ axis and get close to the pole at $s = -a + jb$. Our first question is, How high does it get? Let's assume that the maximum occurs at our closest approach




$$Z(s) = \frac{(R + sL)(1/Cs)}{R + sL + 1/Cs} = \frac{s + R/L}{C\left(s^2 + s(R/L) + 1/LC\right)} = \frac{k(s + 2a)}{s^2 + 2as + a^2 + b^2}$$

**Figure 4.4** The pole-zero diagram for the impedance of a nonideal parallel resonant circuit. The inset shows the details of the frequency range around resonance.

point, $s = jb$. Using this as the *foi,* we mentally draw vectors from the poles and zeros to the $s = jb$ point. The vector from the zero has a much larger vertical component than a horizontal component. We will approximate the length of this vector by its larger component, which is $b$. This is within 2% of the correct value as long as one component is five times as long as the other. Applying the same criterion to the lower pole, it is even more accurate to approximate its magnitude by $2b$. The vector from the pole adjacent to the *foi* is an exception in that it has only a horizontal component, and we can estimate the maximum impedance value as

$$Z(jb) = k\frac{b\angle 90^\circ}{a\angle 0^\circ(2b\angle 90^\circ)} = \frac{k}{2a}\angle 0^\circ$$

The next question is, How steep is the response? A standard way to answer that question is to give the *bandwidth (B)* of the peak, defined as the frequency range over which the impedance stays at or above 0.7071 of its maximum value. To estimate that range, consider what happens to the vectors as the frequency moves up and down from $s = jb$, an amount equal to the distance of the pole from the $j\omega$ axis. As long as the pole is close to the axis, the percentage change in the length of the vector from the zero is small over that frequency range, and can be ignored for a first approximation. This is even more true for the vector from the lower pole, and since those two vectors change in the same direction, their ratio changes even less. The only vector that experiences a big change, in percentage terms, is the small vector from the pole whose peaking we are trying to evaluate. As the inset of Figure 4.4 shows, its length increases by a factor of $\sqrt{2}$ at the ends of the specified range, which is exactly the factor needed to reduce the impedance to 0.7071 of its maximum value. The conclusion is that the bandwidth is approximately $2a$, where $a$ is the distance of the pole from the $j\omega$ axis:

$$B \approx 2a \tag{4.3}$$

The equation in Figure 4.4 shows that the bandwidth is also the coefficient of the $s$ term in the quadratic causing the resonance.

To generalize these observations, how large a function gets at a frequency adjacent to one of its poles, or how low it is at a frequency adjacent to one of its zeros, can be estimated by evaluating the transfer function at that *foi*. In doing this, vectors having one component much larger than their other component may be approximated by their larger component with little loss in accuracy. The sharpness of the response may be measured by the bandwidth, which may be determined either by inspection of the coefficient of the $s$ term in the quadratic responsible for that pole or zero, or from $B = 2a$, where $a$ is the distance of that pole or zero from the $j\omega$ axis. This remains a good approximation if the only vector changing significantly in the vicinity of the pole or zero is the one from the pole or zero itself. For this to be true, other poles and zeros must be at least $10a$ away from the one whose resonance is being evaluated.

An alternate method of stating the steepness of the response is to specify the effective $Q$ of the resonance, where $Q$ is the ratio of resonant frequency to bandwidth. The higher the $Q$, the steeper the response curve:

$$Q = \frac{\omega_o}{B} \tag{4.4}$$

## EXAMPLE 4.2

Estimate the impedance and bandwidth of the resonant conditions in the circuit of Figure 4.5, given that both inductors have a resistance of 0.1 Ω.

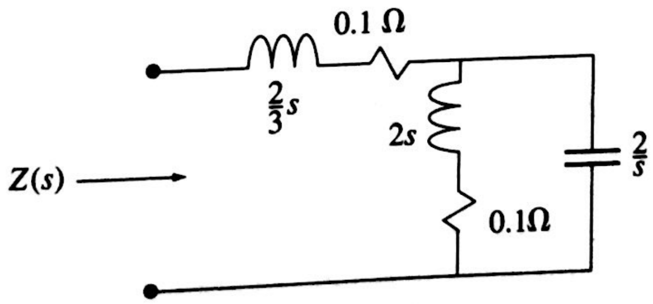



**Figure 4.5**

***Solution***

The derivation for the impedance starts off as before, but is more involved due to the presence of the resistive terms:

$$Z(s) = \frac{2}{3}s + 0.1 + \frac{(2s + 0.1)(2/s)}{2s + 0.1 + 2/s} = \frac{2}{3}s + 0.1 + \frac{2s + 0.1}{s^2 + 0.05s + 1}$$

Getting a common denominator and continuing to simplify to standard form gives

$$Z(s) = \frac{(2/3)s^3 + (0.4/3)s^2 + (8/3 + 0.005)s + 0.2}{s^2 + 0.05s + 1} = \frac{2}{3}\frac{(s^3 + 0.2s^2 + 4.008s + 0.3)}{s^2 + 0.05s + 1}$$

Now either we need a root-solving program, or we must find the one real root of the cubic by trial and error, divide that root out of the cubic to get the remaining quadratic, and so on. A sketch of the pole-zero diagram helps in visualizing the vectors, even though the critical values are taken from the result for $Z$:

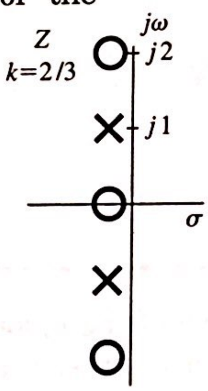
$$Z(s) = \frac{2}{3}\frac{(s + 0.075)(s^2 + 0.125s + 4)}{s^2 + 0.05s + 1} = \frac{2}{3}\frac{(s + 0.075)(s + 0.0625 \pm j1.9987)}{s + 0.025 \pm j0.9997}$$

Estimating the $|Z|$ and $B$ of the parallel resonance at $s = j1$,

$$|Z(j1)| \approx \frac{2}{3}\frac{(1)(3)(1)}{(0.025)(2)} = 40\ \Omega$$

$$B_1 \approx 0.05 \text{ rad/s}$$

$$Q_1 = \frac{\omega_o}{B_1} = \frac{1}{0.05} = 20$$




Estimating the $|Z|$ and $B$ of the series resonance at $s = j2$,

$$|Z(j2)| \approx \frac{2}{3}\frac{(2)(4)(0.0625)}{(1)(3)} = 0.111\ \Omega$$

$$B_2 \approx 0.125 \text{ rad/s}$$

$$Q_2 = \frac{\omega_o}{B_2} = \frac{2}{0.125} = 16$$

## 4.2 NONRESONANT SYSTEMS: *R-C* CIRCUITS

This section also applies to $R$-$L$ circuits, but anything that can be done with an $R$-$L$ circuit can also be done with a cheaper and smaller $R$-$C$ circuit. The poles and zeros for $R$-$C$ circuits all fall along the $\sigma$ axis in the $s$ plane. How such poles and zeros affect the terrain along the path $s = j\omega$ is much less dramatic and intuitive than for $L$-$C$

circuits, and a different approach is needed. After completing this section you will be able to:

- Estimate a frequency response using Bode straight-line approximations (SLAs).
- Use log-log graph paper for frequency response plots.
- Define and identify break frequencies.
- Explain the significance of low- and high-frequency asymptotes.
- Use symmetry to identify certain frequency response features.
- Explain the difference between wrapped and unwrapped phase.

A driving point or transfer function will take the general form

$$Z(s) = k\,s^n \frac{(s + a_1)\,(s + a_2)\cdots(s + a_q)}{(s + b_1)(s + b_2)\cdots(s + b_p)} \tag{4.5}$$

where $a$ and $b$ are real and $n$ is a positive or negative integer. Consider the factor $(s + a_1)$, where $s = j\omega$ and $a_1$ is real. As frequency varies, there will be times when $\omega$ is so low that it is negligible compared to $a_1$. On the other hand, when the frequency is very high, the $a_1$ value will be negligible compared to $\omega$. *The simplest possible approximation for this type of factor is just to keep the; larger number.* At low frequencies the factor is approximated by the constant, at high frequencies it is approximated by the $s$. We break from one approximation to the other when the two terms are the same size; consequently $a_1$ is called a *break frequency*. The approximation is least accurate at the break, where we would be approximating the factor by either $a_1$ or $ja_1$, when, in fact, its true magnitude and phase is $\sqrt{2}a_1\angle 45°$.

In any frequency range between break frequencies, some of the factors in Equation 4.5 will be approximated by their constants, and some will be approximated by their $s$. As a result, the expression for $Z(s)$ in any frequency range reduces to the form

$$Z(s) = Cs^m \tag{4.6}$$

where all those factors being approximated as constants combine with $k$ to produce the constant $C$ and all those factors being approximated as $s$ combine to produce a net $s^m$ term. Now we need to separate the magnitude and phase information in preparation for plotting:

$$\begin{aligned} Z(j\omega) &= C(j\omega)^m = C(\omega\, e^{j\pi/2})^m = C\omega^m \angle m90° \\ \angle Z(j\omega) &= m90° \end{aligned} \tag{4.7}$$

$$|Z(j\omega)| = C\,\omega^m \tag{4.8}$$

The phase is approximated as a constant over each frequency range between breaks, so plotting it does not present a problem. All that is needed is the power of the $s$ term for that particular frequency range. The magnitude is more complicated, but Equation 4.8

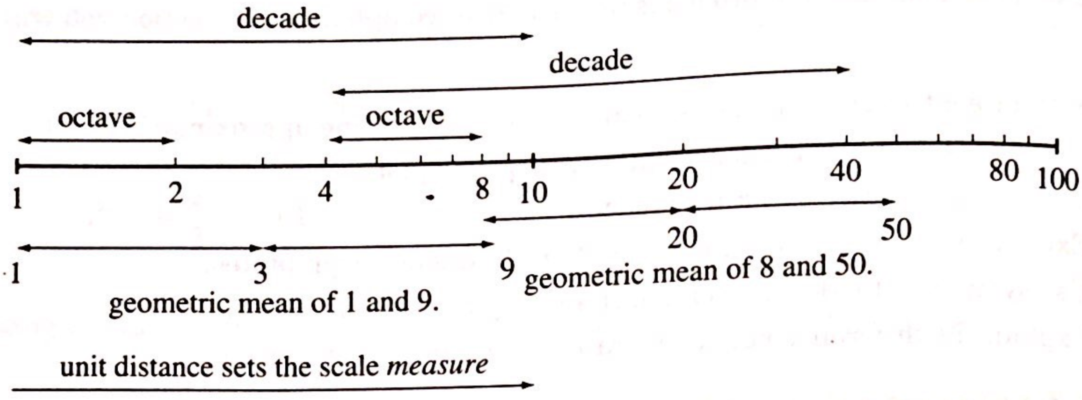



**Figure 4.6** Features of a logarithmic graph scale.

will look like a straight line if appropriate graph scales are used. If the log is taken of each side of $|Z(j\omega)|$, the result is

$$\underbrace{\log|Z(j\omega)|}_{y} = \underbrace{\log(C)}_{b} + m \underbrace{\log \omega}_{x} \tag{4.9}$$

which is the equation of a straight line with intercept $b$ and slope $m$ in the variables $\log|Z|$ and $\log \omega$. Commercial graph paper having this type of scale is called *log-log* paper. Frequency response curves drawn using these techniques are referred to as *Bode straight-line approximations* (SLAs).

For those unfamiliar with a log scale, the following information may be helpful. Refer to Figure 4.6 as necessary.

1. A $\log \omega$ scale has the numerical value of $\omega$ indicated on the scale, but its distance from the origin is proportional to $\log \omega$. So, for example, the frequency $\omega = 1$ is located $\log(1) = 0$ units from the origin (i.e., it is the origin), while the value $\omega = 10$ is located $\log 10 = 1$ unit from the origin. The distance between the number 10 and the number 1 sets the *measure* of the scale. Suppose we arbitrarily make that distance 4 inches. Then the number 3 should be placed $4 \times (\log 3) = 1.909$ inches from the number 1.

2. A ratio of 10:1 is called a *decade*, and a ratio of 2:1 is called an *octave*. The distance occupied by any constant ratio is a constant on log scales. In other words, the octave represented by the frequency range from $\omega = 1$ to $\omega = 2$ takes exactly the same number of inches as the octave from 2 to 4 or the octave from 3 to 6. Musically speaking, one octave is as important as another, and the log scale gives every octave equal emphasis. For this reason, the $\log \omega$ scale is preferred for all but resonant types of frequency response curves.

3. If frequency $\omega_2$ is a distance $\log \omega_2$ units from the origin, and frequency $\omega_1$ is a distance $\log \omega_1$ from the origin, then the frequency $\omega_o$ midway between them is a distance equal to the average of these two distances. Thus,

$$\log \omega_o = \frac{\log \omega_2 + \log \omega_1}{2} \qquad \text{or} \qquad \omega_o = (\omega_1\omega_2)^{1/2} = \sqrt{\omega_1\omega_2} \tag{4.10}$$

The midpoint number, $\omega_o$, is the geometric mean of the other two numbers.

4. The slope of our straight line is the integer $m$. On commercial graph papers, the vertical and horizontal scales usually have the same measure. With computer-generated log-log graphs, that is not necessarily true. A slope of +1 means that the line rises one decade (octave) vertically as it increases 1 decade (octave) horizontally.

## EXAMPLE 4.3

Sketch the frequency response of the impedance for the circuit in Figure 4.7a.

$Z_C = 2/s$

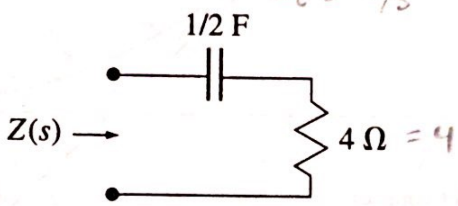



**Figure 4.7a**

### *Solution*

The expression for the impedance is

$$Z(s) = \frac{2}{s} + 4 = \frac{2 + 4s}{s} = 4\frac{(s + 1/2)}{s}$$

Comparing this to Equation 4.5, we see that $k = 4$, $n = -1$, and the only break frequency is at $\omega = 1/2$ rad/s.

At frequencies below the (1/2)-rad/s break frequency,

$$Z_{LF}(s) = \lim_{s \to 0} Z(s) = 4\frac{1/2}{s} = \frac{2}{s}$$

$$\therefore C = 2 \qquad \text{and} \qquad m = -1$$

$Z_{LF}$ is called the *low-frequency asymptote*. It will be a straight line with a slope of −1, and it will pass through 1 Ω when $\omega = 2$ rad/s, 20 Ω when $\omega = 0.1$ rad/s, and so on (just plug $\omega$ values into the equation for $|Z_{LF}|$).

At frequencies above the (1/2)-rad/s break frequency,

$$Z_{HF}(s) = \lim_{s \to \infty} Z(s) = \frac{4s}{s} = 4$$

$$\therefore C = 4 \qquad \text{and} \qquad m = 0$$

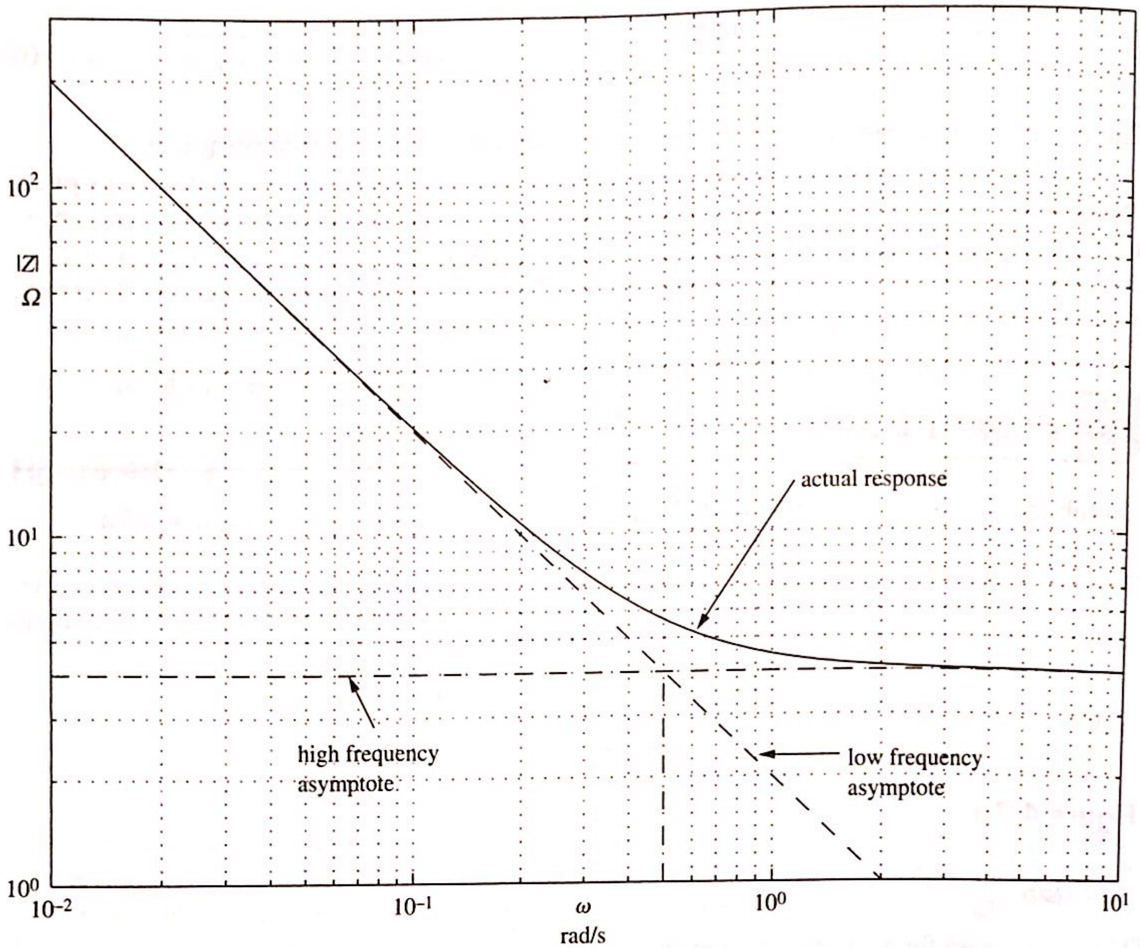



**Figure 4.7b** Bode SLA and actual response for the magnitude of the impedance of Example 4.3.

$Z_{HF}$ is called the *high-frequency asymptote*. In this case it is a horizontal line at an impedance of 4 Ω. The asymptotes and actual response for $|Z(s)|$ are shown in Figure 4.7b. Note that the actual impedance at the break is $\sqrt{2}(4) \approx 5.7$.

Since $m = -1$ at low frequencies, the phase is $-1(90°)$, and at high frequencies $m = 0$, so the phase is zero. The approximation for phase is cruder than for the magnitude plot, but it can be significantly improved in the special case where there is only one break frequency and it is caused by a real root. A ramp drawn from the low-frequency phase asymptote starting a decade below the break frequency and running to the high-frequency asymptote a decade above the break frequency provides a superior phase approximation (Figure 4.7c).

Now the results should be compared to what can be expected from the original circuit. At low frequencies the series capacitor blocks signals, producing a high-capacitive impedance. At high frequencies, the capacitor approaches a short circuit, and the circuit looks like the resistor. The changeover from a capacitive to a resistive circuit occurs at the frequency where the magnitudes of their impedances become equal.

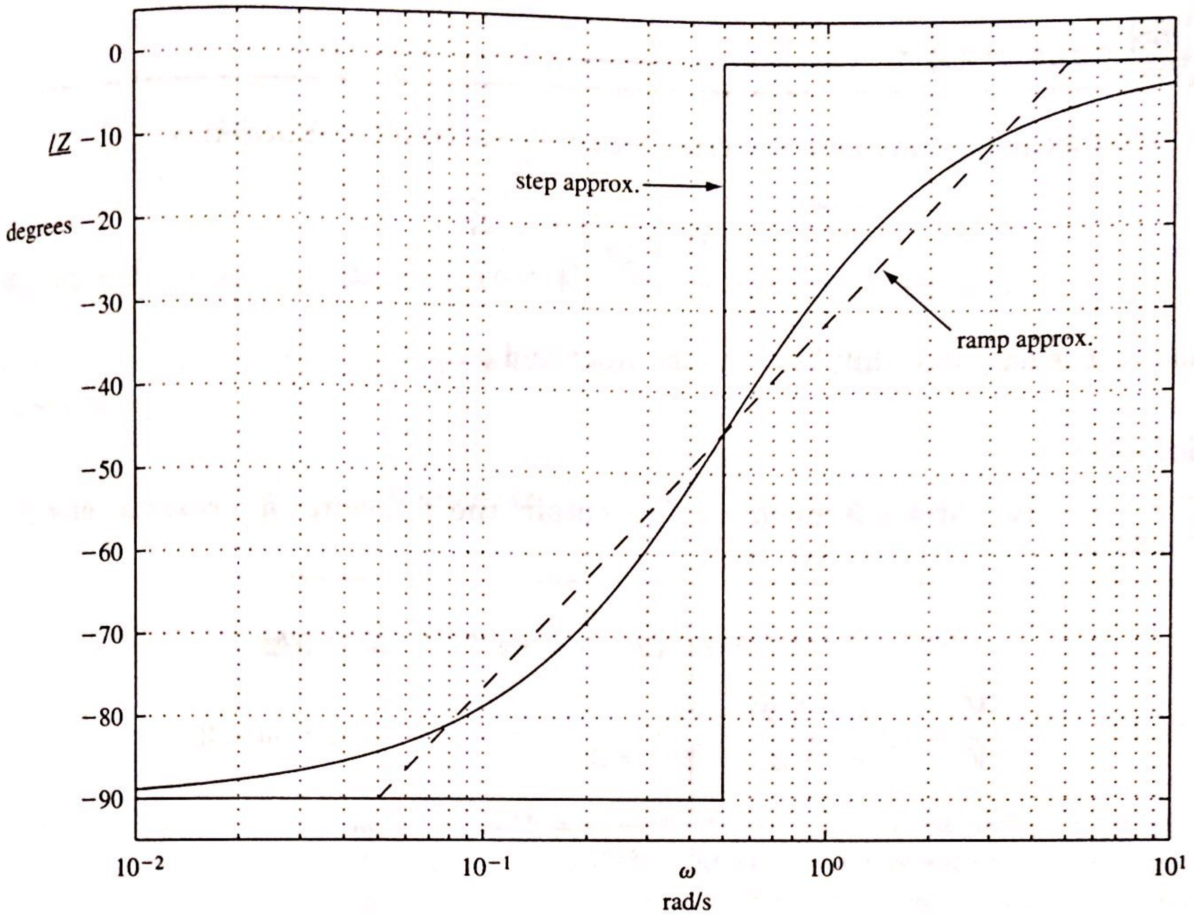



**Figure 4.7c** Bode step and ramp approximations for the phase of the impedance of Example 4.3.

The high- and low-frequency asymptotes are found from the same approximations used between break frequencies. They are especially important, however, because the actual response must ultimately approach them. The SLAs between break frequencies may be approached but never reached by the actual response. They may also be crossed through. They are essentially asymptotes that become overruled by the effects of subsequent break frequencies.

Before the computer era, the Bode SLA was a primary tool for obtaining frequency response information with a minimum of calculations. A variety of rules of thumb exist for estimating the deviation of the actual plot from the SLA at break frequencies. Using these rules, a few calculated points, and the best smooth curve through the SLAs, a surprisingly accurate response curve can be obtained. The value of this analysis technique today is more in its ability to indicate where, in frequency, special things will happen, so we can tell the computer where we want the actual response calculated. It is also useful in design when we can identify what components set the value of a particular break frequency or when symmetries in the SLA show practical limitations or other features we need to know about. Frequently a sketch of the response is all that is needed.

## EXAMPLE 4.4

At what frequency and gain does the circuit whose transfer function is

$$\frac{\vec{V}_o}{\vec{V}_i} = 12\frac{(s + 3/2)}{s(s + 3)}$$

have the least phase shift between its input and output?

### *Solution*

There are two break frequencies. We obtain the following approximations for the SLA:

$$\frac{\vec{V}_o}{\vec{V}_i} = 12\frac{(s + 3/2)}{s(s + 3)} = \begin{cases} 12\dfrac{3/2}{s(3)} = 6s^{-1} & \omega < 3/2 \\ 12\dfrac{s}{s(3)} = 4 & 3/2 < \omega < 3 \\ 12\dfrac{s}{s(s)} = 12s^{-1} & \omega > 3 \end{cases}$$

The low-frequency asymptote has a slope of $-1$ and passes through 4 at 1.5 rad/s. For $1.5 < \omega < 3$, the slope is 0 and the gain sits at 4. For $\omega > 3$, the slope returns to $-1$ and passes through 4 at $\omega = 3$. A sketch incorporating these facts is shown in Figure 4.8a. Since it is only a sketch, the axes have been labeled to indicate that a logarithmic measure is being used.

The asymptotic phase sits at $-90°$ except for the short region between the break frequencies. By symmetry, the phase will make its closest approach to zero at the geometric mean of the break frequencies. (See Figure 4.8b.) Symmetry

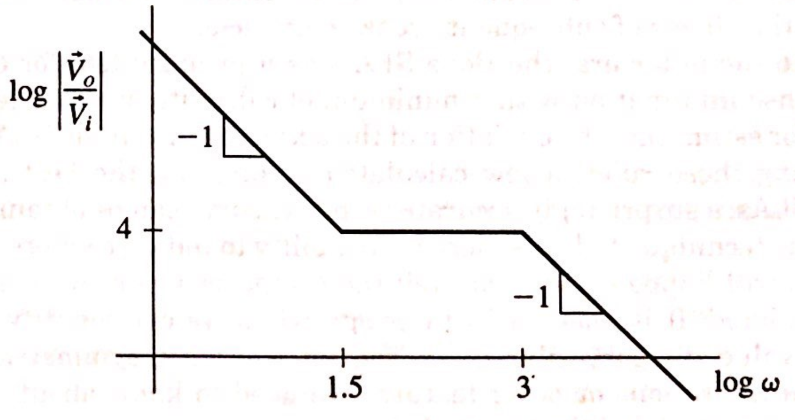



**Figure 4.8a** Bode SLA magnitude response for the transfer function of Example 4.4.

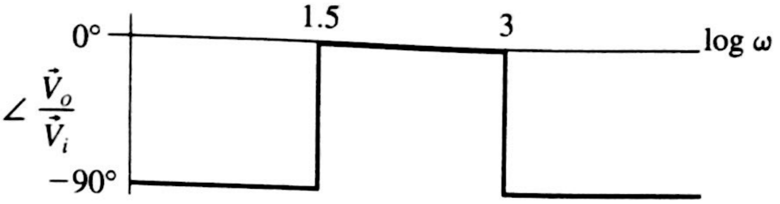



**Figure 4.8b** Bode SLA phase response for the transfer function of Example 4.4.

also indicates that the actual gain will pass through 4 at that frequency. A calculation shows

$$\frac{\vec{V}_o}{\vec{V}_i} = 4\angle -70.5° \,@\, \omega = \sqrt{(1.5)3} = 2.12 \text{ rad/s}$$

---

Transfer functions may occasionally also have zeros of the form

$$(s - a) = \begin{cases} -a & |s| < a \\ s & |s| > a \end{cases} \tag{4.11}$$

Such zeros are in the right half of the $s$ plane (RHP), and are approximated in exactly the same fashion as before, but the phase jump at the break requires a little extra thought. Specifically, should the factor be described as having an initial phase of $+180°$ or $-180°$? The issue is important in drawing the correct continuous phase curve. This is resolved by not allowing $s$ to vanish completely. If $s = j\varepsilon$, where $\varepsilon$ is a very small value, it becomes clear that the factor $-a + j\varepsilon$ has an angle in the second quadrant and is approaching 180° as a positive angle at low frequencies.

It is common to *wrap* phase information so that all angles are given as values between $+180°$ and $-180°$. This is justified by the fact that if only a single frequency sinusoid is available, it is impossible to tell the difference between a phase of $+270°$, $-90°$, and $-450°$, and so on. But these phase angles do represent different *delay times* for the sinusoid, and this can be measured if frequency can be varied or if more complicated waveforms are used. If the source frequency can be increased steadily, we could use a scope to observe the smooth transition of phase through its progression of values. The SLA phase plots provide an *unwrapped* phase, which properly represents the continuous phase shift produced by real circuits.

## 4.3 THE DECIBEL (dB)

The *bel* (after Alexander Graham Bell) is a unit that originated in studies on human hearing, but it has become used and abused in numerous applications where its logarithmic measure is the desired feature. After completing this section you will be able to:

- Define the decibel, and discuss its uses.
- Express Bode SLA gains in terms of a decibel scale.

The most common application of the decibel is for measuring power ratios. Power gains may be expressed in dB as

$$A_{P\,dB} = 10 \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \tag{4.12}$$

The decibel is also used to state an output power level against an input power standard. We could, for instance, express the output *power* of an amplifier in $\text{dB}_m$, instead of watts. The $m$ signifies that the dB scale is referenced to an input power of 1 mW. An output power of 1 watt could be given as 30 $\text{dB}_m$ using this procedure. $\text{dB}_W$ is another standard, using 1 watt as the input reference level. A thousand-watt amplifier delivers 30 $\text{dB}_W$ at full output. This usage makes some sense when one of the standard input power levels is part of a device's test specifications:

$$\text{dB}_m = 10 \log_{10}\left(\frac{P}{10^{-3}\,\text{W}}\right) \qquad \text{dB}_W = 10 \log_{10}\left(\frac{P}{1\,\text{W}}\right)$$

Assuming a circuit with equal input and load resistances, the dB measure can also be used to describe the circuit's voltage or current gains:

$$A_{v\,\text{dB}} = 10 \log_{10}\left(\frac{V_{\text{out}}^2}{V_{\text{in}}^2}\right) = 20 \log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) \tag{4.13}$$

In fact, by convention this is routinely done *even if the input and load resistances are known to be different.* We simply take Equation 4.13 as the definition of a voltage gain measured in decibels, and we do not care that it is not a true power ratio. The main advantage of doing this is that frequency response SLAs may be made on semilog paper, since the dB units provide a logarithmic measure for the vertical gain scale. Magnitude and phase plots can then be made on the same kind of graph paper. At least with current and voltage gains, the logarithm is taken of a dimensionless ratio. Carrying this practice to an extreme, transimpedances or transadmittances could be converted to a decibel measure by taking the logarithm, despite the fact that the ratios are not dimensionless. In these cases we will prefer to revert to log-log paper and give the proper units for the transfer function.

The key to making SLA plots was that the transfer function takes the form $Cs^m$ in any region between break frequencies. On log-log paper, this plots as a straight line with a slope of $m$. It rises $2m$ in an octave and $10m$ in a decade. On semilog paper with a dB vertical scale, the curve remains a straight line, but it rises at $(20 \log 2)m \approx 6m$ dB per octave or $(20 \log 10)m = 20m$ dB per decade. With this minor change in specifying slopes, the procedure used to draw SLA plots remains the same.

## EXAMPLE 4.5

Sketch the decibel magnitude plot and the phase plot for the transfer function.

$$A_v = \frac{\vec{V}_o}{\vec{V}_g} = \frac{2(s-2)}{(s+2)^2}$$

### Solution

The function has a single break frequency of 2 rad/s and a zero in the RHP:

$$A_v = \begin{cases} -1 = 1\angle{+180^\circ} & \omega < 2 \\ 2s^{-1} = \dfrac{2}{\omega}\angle{-90^\circ} & \omega > 2 \end{cases}$$

The magnitude plot is the same whether or not there are factors present in the RHP. In this case the low-frequency asymptote is horizontal, making it easy to position the plot vertically (Figure 4.9).

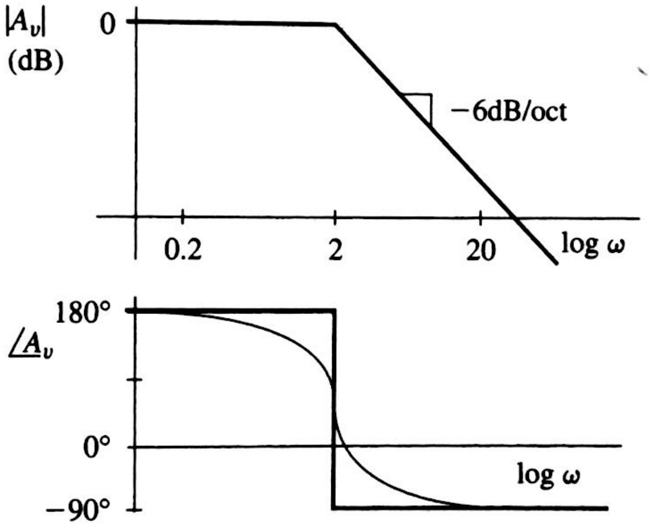



**Figure 4.9**

To check that we have the correct phase, calculate the transfer function at the break:

$$A_v(s=j2) = \frac{2(-2+j2)}{(2+j2)^2} = \frac{-1+j1}{(1+j)^2} = \frac{\sqrt{2}\angle 135^\circ}{2\angle 2(45^\circ)} = \frac{1}{\sqrt{2}}\angle 45^\circ$$

In measuring the bandwidth of resonances, we use a criterion based on the impedance being $\sqrt{2}$ or $1/\sqrt{2}$ of its minimum or maximum values, respectively. These numbers convert to $\pm(20 \log \sqrt{2}) = \pm 3$ dB. On this basis, we measure the bandwidth at a series resonance using the frequencies where the gain is 3 dB up from its minimum. Similarly, the bandwidth for a parallel resonance is determined from the frequencies where the gain is down 3 dB from the maximum.

## 4.4 GENERAL SYSTEMS: *R-L-C* CIRCUITS

In general, system polynomials will factor into a combination of real roots and complex roots but without a dominating resonant condition. In these cases, the SLA approach still provides the most useful information. After completing this section you will be able to:

- Apply the Bode SLA technique to systems with mild resonances.
- Define *damping ratio* and *natural resonant frequency*.

In circuits containing all three passive elements, and where the resistors are not just inherent inductor winding resistances, the circuit polynomials *may* contain some complex roots. When the polynomials have real coefficients, any complex root must appear along with its conjugate. Consider the pair of roots

$$(s + ce^{j\beta})(s + ce^{-j\beta})$$

If we use exactly the same rule as for real roots, we will just keep the larger number. So if $|s| < c$, we keep the constant term; otherwise we keep the $s$. Since the roots break together, we will be getting a $c^2$ if $|s| < c$, or we will be getting $s^2$ when $|s| > c$.

Multiplying out the conjugate roots gives the quadratic that created them. Note that using the standard rule of keeping the larger number in conjugate factors is equivalent to ignoring the $s$ term in the original quadratic.

$$(s + ce^{j\beta})(s + ce^{-j\beta}) = s^2 + (e^{j\beta} + e^{-j\beta})cs + c^2 = s^2 + (2c \cos \beta)s + c^2$$

We therefore adopt the following rule for SLA plots: *If a quadratic would factor into conjugate roots, leave the quadratic unfactored and ignore its center* ($s^1$) *term. Keep the larger of the two remaining numbers in* ($s^2 + c^2$).

A standard notation has been established for the quadratic based on the undamped natural resonant frequency, $\omega_n$ (which is also the break frequency), and the damping ratio, $\zeta$ (zeta):

$$s^2 + 2c \cos \beta s + c^2 = s^2 + 2\omega_n \zeta s + \omega_n^2$$

If the damping ratio is 1 or higher, the quadratic factors into real roots. For damping ratios less than 1, conjugate roots result. A damping ratio of zero puts the roots on the $j\omega$ axis. The damping ratio appears only in the center term; therefore it is ignored in obtaining the SLA. However, the actual value of the quadratic at the break is *exactly* the center term:

$$\left| \frac{\text{actual value @ } \omega_n}{\text{SLA value @ } \omega_n} \right| = 2\zeta \tag{4.14}$$

We conclude that when the standard SLA approximation is applied to complex roots, the actual curve will closely cling to the SLA, for $0.5 < \zeta < 1$, so that the SLA approximation for the quadratic is probably at least as good as, if not better than, the results obtained for real roots (Figure 4.10).

For $\zeta < 0.5$, complex roots show resonant peaking near the SLA corner instead of rounding it off. At the very least, the presence of such roots should serve as an alert that a few calculations are needed at or around the break to ensure an accurate plot. With those calculations, the SLA remains useful as a frequency response analysis tool, despite the presence of slight resonant effects.

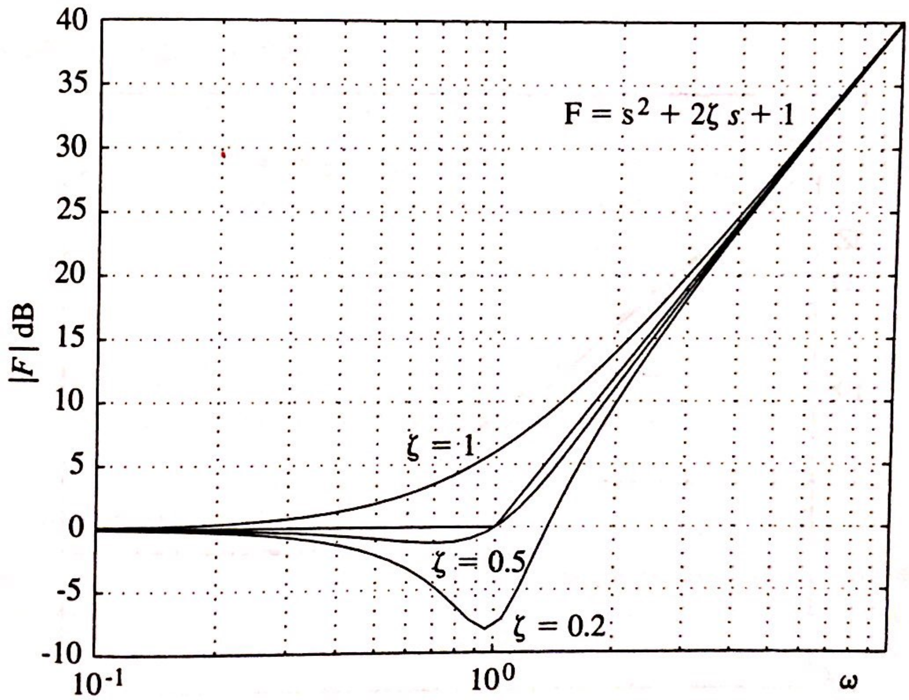



**Figure 4.10** Variation of actual and asymptotic response curves in the vicinity of the break frequency of a quadratic factor.

### EXAMPLE 4.6

Plot the SLA of the impedance magnitude for the circuit in Figure 4.11a.

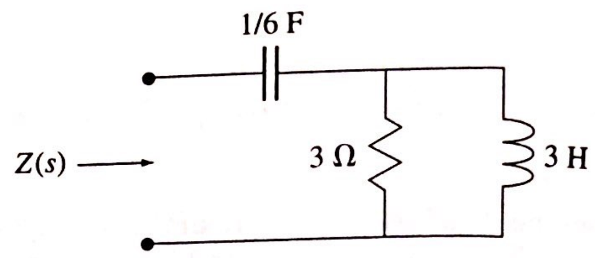



**Figure 4.11a**

***Solution***

The impedance is found as

$$Z(s) = \frac{6}{s} + \frac{3(3s)}{3 + 3s} = \frac{18 + 18s + 9s^2}{3s(s + 1)} = 3\frac{s^2 + 2s + 2}{s(s + 1)}$$

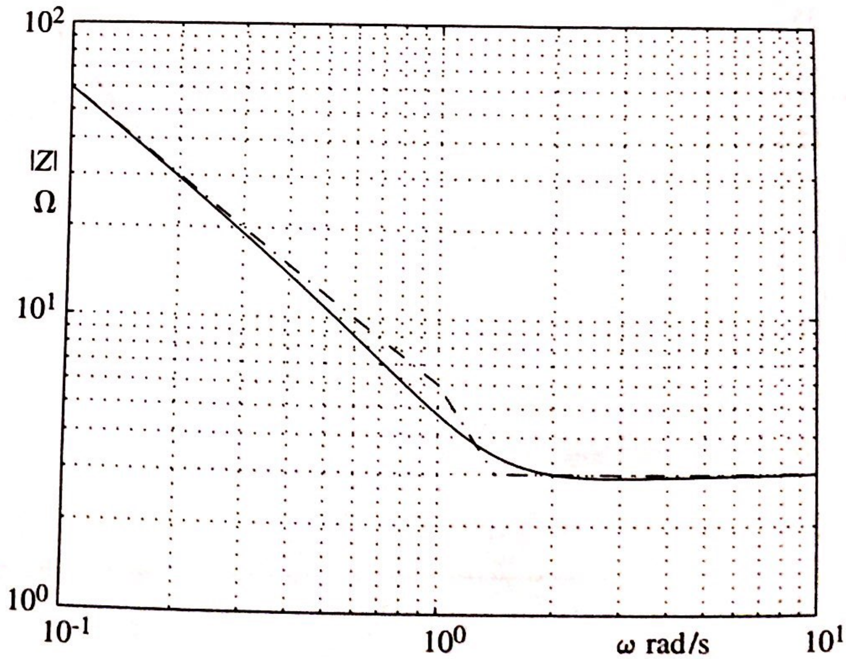



**Figure 4.11b** Comparison of actual and Bode SLA magnitude response for the impedance of Example 4.6.

The numerator quadratic produces zeros when

$$s = \frac{-2 \pm \sqrt{4 - 4(2)}}{2} = -1 \pm j$$

so we leave the quadratic unfactored and note that

$$2\zeta\omega_n = 2, \qquad \omega_n = \sqrt{2}, \qquad \therefore \zeta = 0.707$$

This value of damping ratio suggests that the SLA will be a good predictor of the actual response. No extra calculations are necessary for this value of $\zeta$.

There are two break frequencies, 1 rad/s and $\sqrt{2}$ rad/s (Figure 4.11b):

$$Z(s) \approx 3\frac{(s^2 + 2)}{s(s + 1)} = \begin{cases} 6s^{-1} & \omega < 1 \\ 6s^{-2} & 1 < \omega < \sqrt{2} \\ 3s^{0} & \omega > \sqrt{2} \end{cases}$$

---

Both pole-zero plots and Bode SLAs can be useful in estimating a system's frequency response in appropriate circumstances. Each technique can also provide some insight that could be valuable in design problems. Still, when it comes down to a final design, the ability to obtain a computer-generated frequency response plot is indispensable.

## 4.5 MATLAB LESSON 4

This lesson will concentrate attention on those MATLAB commands most often used for obtaining the frequency response of continuous-time signals and for graphing this information. After completing this section you will be able to:

- Use the MATLAB frequency response tools.
- Compare SLA and actual response curves.
- Recognize and correct unrealistic decibel scale ranges.
- Capture important data from a graph or from a vector.

---

### MATLAB EXAMPLES

We saw several ways to evaluate a function along some path in a complex plane in Chapters 1 and 2. If the path is the $j\omega$ axis, and the simplified polynomial notation is used, special commands are available to handle this task:

**freqs** **bode**

We will demonstrate by plotting the frequency response for the impedance of Example 4.2 in the vicinity of its pole.

```
> num=(2/3)*[1 .2 4.008 .3];     % defines the numerator polynomial
> den=[1 .05 1];                 % defines the denominator polynomial
> w=linspace(.95,1.05);          % defines a frequency range of about twice
                                   the expected bandwidth around parallel
                                   resonance
> z=freqs(num,den,w);            % calculates Z over the specified frequency
                                   range if s = jw
> plot(w,abs(Z))                 % compare |Z|max and B with estimates from
                                   the example
```

freqs(num,den,w) is the equivalent of polyval(num,j*w)./(polyval(den,j*w), where a range of w has been defined. It allows us to calculate a driving point or transfer function over any desired frequency range, and lets us select among appropriate plotting options.

```
> freqs(num,den,w) % this is easier, but allows less control over the
                     graphs
> bode(num,den,w)  % this apparently does not allow graph changes
```

Now let's plot the magnitude and phase angle for the transfer function of Example 4.4 and compare it to Figures 4.8a and b. The range of frequencies used should usually extend *about* a decade below the lowest break frequency and *about* a decade above the highest break frequency. This ensures that the response will nearly reach the high- and low-frequency asymptotes and that the trend of the graph is not misleading.

```
> num=12*[1 3/2];          % defines the numerator polynomial 12(s + 1.5)
> den=[1 3 0];             % defines the denominator polynomial s^2 + 3s
> w = logspace(-1,1);      % picks 50 (default) equally spaced frequency
                             points along a log scale from 10^-1 to 10^1
> Av=freqs(num,den,w);     % calculates the values of Av over the frequency
                             range specified
```

**loglog semilogx semilogy**

```
> loglog(w,abs(Av))              % produces a plot of |Av| vs. ω on
                                   log-log axes
> semilogx(w,180*angle(Av)/pi)   % produces a plot of ∠Av in degrees
                                   vs. log ω
> AvDB=20*log10(abs(Av));        % calculates Av in decibels
> semilogx(w,AvDB)               % plots |Av| in dB vs. log ω
> bode(num,den,w)                % plots |Av| in dB and ∠Av in degrees
                                   vs. log ω
```

Computers tend to be nonjudgmental about decibel scales. *Any scale exceeding a range of 120 dB should be regarded with skepticism*. Use the **axis** command to limit the axis range to realistic values.

Suppose you have a graph and want to extract numerical information from it. There are various techniques available. The first, and simplest, is to put the graph data in paired points and print it to the screen.

```
> w=linspace(.9, 1.1);
> num=[1 0];
> den = [1 .08 1.1];          % defines a resonant response s/(s^2 + .08s + 1.1)
> z=freqs(num,den,w);         % calculate the impedance (or whatever)
> plot(w,abs(z))              % you should have a graph with a peak of about
                                12.5 @ ω = 1.05
```

Both $w$ and $z$ are row vectors. The notation $w.'$ converts the $w$ row vector into a column vector. This is called *transposing* the vector, and is discussed more fully in Chapter 6. Transposing $z$ as well allows us to form an array of paired data points.

```
> a=[w.' abs(z).']      % print an array of paired values of w and |z|
```

By scrolling through the array we find the calculated maximum is 12.4982, and it occurs at $\omega = 1.0495$. The impedance would be down to 0.7071 of its maximum when $|z| = 8.838$, which occurs at an $\omega$ of about 1.0091 and 1.0899. The bandwidth consequently is 0.0808.

## max min

If you don't want to search manually, you can find the maximum or minimum element in an array:

```
> [zmax,row]=max(a(:,2))    % search all rows of column 2 in the array a
                              for a maximum
> wmax=w(row)               % knowing the row allows you to find the
                              corresponding frequency
```

With a little ingenuity, you can also find the bandwidth:

```
> b=abs(1./(zmax*.7071 - abs(z)));    % generate a curve that has a max
                                        at the condition desired
> [acc,k]=max(b)                      % find index of half power
                                        frequency. acc could be used
                                        to estimate accuracy
> B=abs(wmax-w(k))*2                  % calculate bandwidth assuming
                                        equal deviation about wmax
```

## zoom ginput

You also can just read data off of a graph. If you want, you can zoom in by positioning the mouse cursor at a point on a graph and using the **zoom** command:

```
> zoom on
  % click (left) mouse button to establish region to zoom around and start
    zooming; click until done
> zoom off
> help zoom   % for all the options
```

The zoom feature has to be turned on and off so MATLAB will know what command the mouse click goes with. Another command using the mouse click is **ginput**. It allows you to pick up graphical data. If necessary, get back the impedance information.

```
> w=linspace(.9, 1.1);              % these four steps have already been
                                    done unless starting over
> num=[1 0];                        %
> den = [1 .08 1.1];                %
> z=freqs(num,den,w);               % you may already have everything to
                                    here
> zmax=max(abs(z));                 % determine the maximum
> plot(w,abs(z),w,.7071*zmax)       % plot |z|, and .7071 of max. on same
                                    graph
> [freq,ohms]=ginput(3)             % cursor changes to crosshair and
                                    graph comes forward
```

Click on the intersection of the curves and at the maximum. You have specified three clicks. After you have made three clicks, return to the Command Window to see the results. *Warning: ginput* collects the coordinates at which the clicks were made. It is up to you to click on the curve and, when necessary, to keep track of the order in which the clicks were made. Now the variables *freq* and *ohms* contain the information needed. If, for instance, you clicked on the desired points in succession going left to right, you could calculate bandwidth as B = freq(3)-freq(1).

## EXAMPLE 4.7

Use MATLAB to plot the current gain transfer function given by

$$\frac{\vec{I}_o}{\vec{I}_i} = 20\frac{s-4}{s^2+4s+16}$$

Determine the maximum gain in decibels and the frequency where the phase is zero.

### *Solution*

```
> num=20*[1 -4];den=[1 4 16];      % define the transfer function
> w=logspace(-1,2,100);             % run ω from 0.1 to 100 (both
                                    breaks are @ 4)
> Ai=freqs(num,den,w);              % calculate current gain
> semilogx(w,20*log10(abs(Ai)));    % plot gain in dB
> grid
> xlabel('frequency in radians/sec.')
> ylabel('current gain in dB')
```

The first plot is shown in Figure 4.12a. If you wish to, you could add the Bode SLA version to this graph for comparison. Remember that **plot** interpolates between

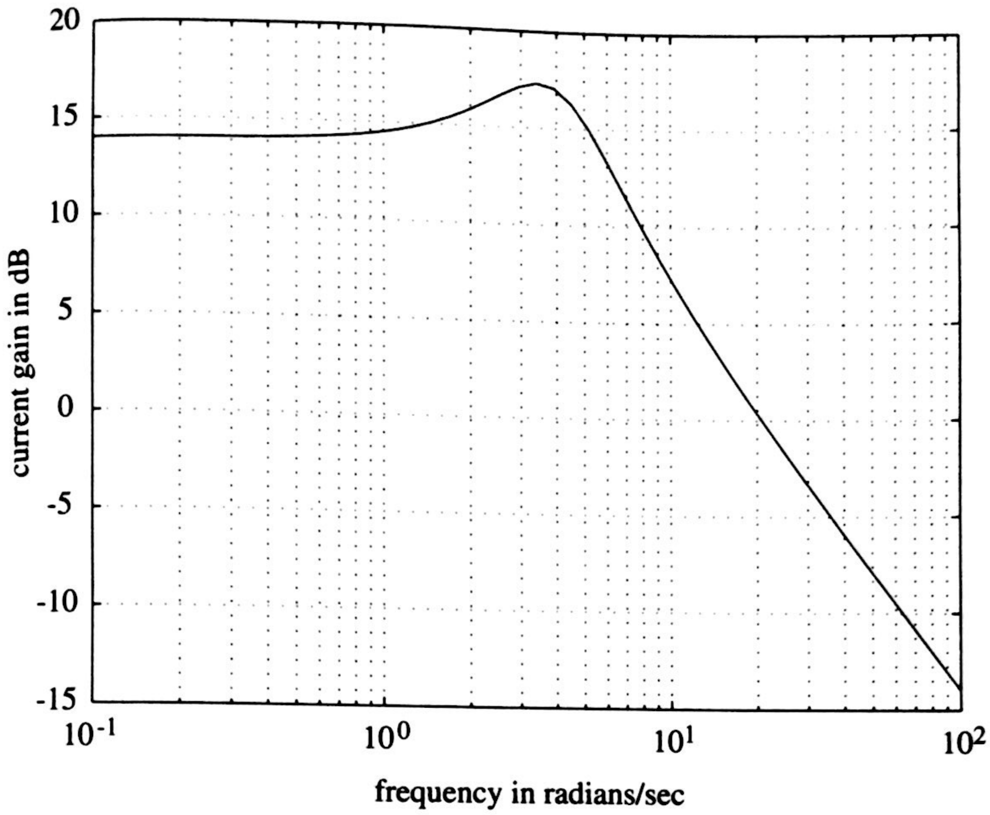



**Figure 4.12a**

successive data points to give a smooth curve. By giving the starting and ending points only, **plot** joins them with a straight line.

In this curve the numerator and denominator both break at $\omega = 4$, so we have only the high and low frequency asymptotes to plot.

```
> hold                                                  % retain actual
                                                          plot
> x=[0.1 4];y=[20*log10(5) 20*log10(5)];plot(x,y,'--')  % plot LF
                                                          asymptote
> x=[4 400];y=[20*log10(5) 20*log10(5)-40];plot(x,y,'--') % plot HF
                                                          asymptote
> axis([0.1 100 -15 20])                                % restore axes
> hold

> semilogx(w,180*angle(Ai)/pi)                          % plot phase)
> grid
> xlabel('frequency in radians/sec')
> ylabel('phase in degrees')
```

The second plot is given in Figure 4.12b.

```
> maxgain=max(20*log10(abs(Ai)))     % find the maximum gain in dB
maxgain = 17.3055
> phase=abs(180*angle(Ai)/pi);       % find phase magnitude
```

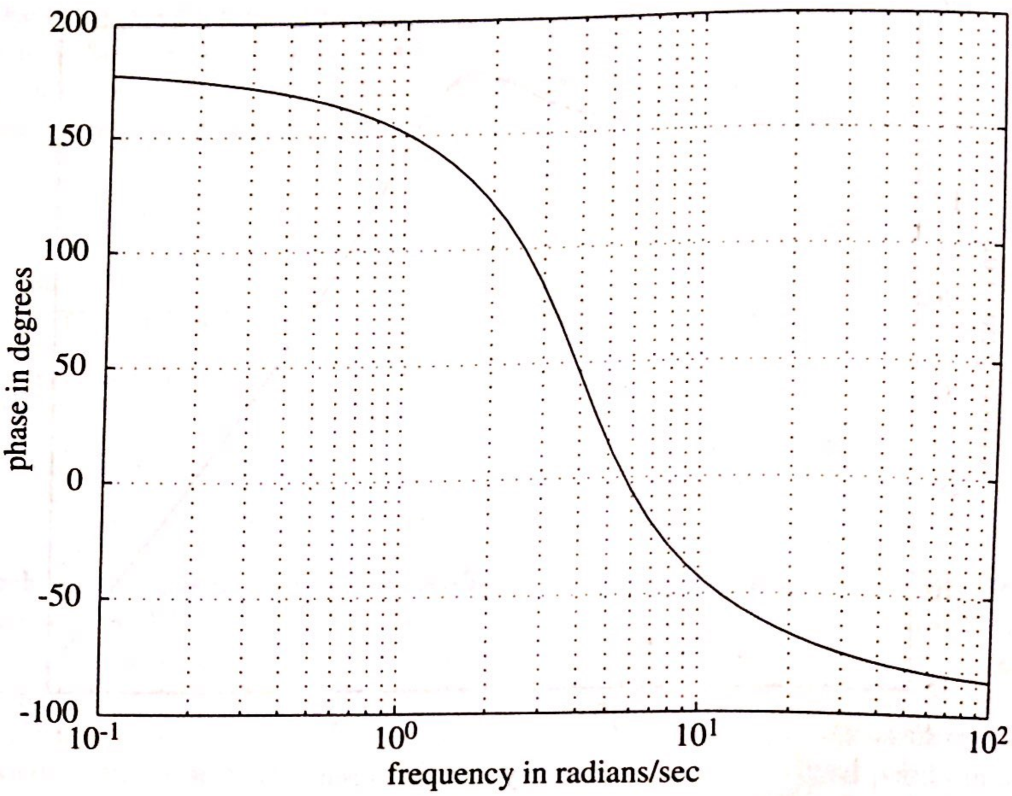



**Figure 4.12b**

```
> [ang,row]=min(phase)          % find the min. absolute phase
ang = 1.2345                    % to be 1.2345 degrees at the 59th freq.
row = 59
> w(59)
ans = 5.7224                    % phase data closest to zero is at
                                  5.72 rad/s.
```

## CHAPTER SUMMARY

Evaluating an $F(s)$ along, the $+j\omega$ axis reveals its frequency response, which is a standard way of characterizing the capabilities of many systems. Having a software tool for providing a frequency response plot is essential to anyone working in the electrical technologies.

For highly resonant systems, the pole-zero diagram is often useful in predicting the main features of the system response, namely, its maximum or minimum amplitude and its bandwidth.

For systems without dominating resonance effects, using logarithmic graph scales allows an overall picture of a response to be developed with few if any calculations. The frequency response can be represented by a series of straight-line segments on logarithmic graph scales. Often the dB unit is used to give a linear gain scale a logarithmic measure. These Bode SLAs can frequently be generalized to provide essential design information.

## PROBLEMS

### Section 4.1

1. For the circuits of Figures P4.1a and b, find $Z(s)$ in pole-zero form and sketch its magnitude vs. $\omega$.

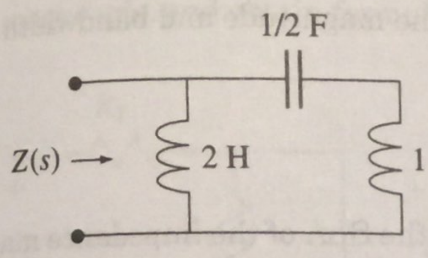



Figure P4.1a

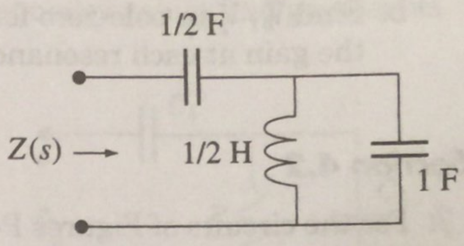



Figure P4.1b

2. For the circuit of Figure P4.2:
   a. Find $Z(s)$ in pole-zero form and sketch its magnitude vs. $\omega$.
   b. Find $\vec{I}_o/\vec{I}_g$ and sketch its magnitude vs. $\omega$.

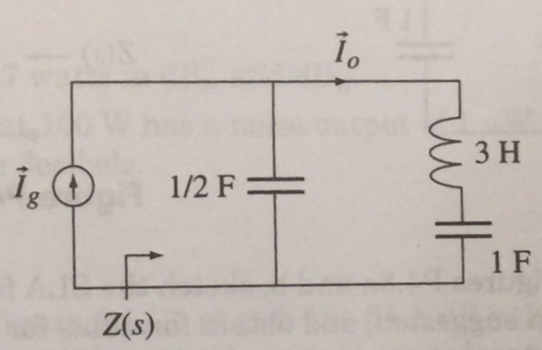



Figure P4.2

3. For the circuit of Figure P4.3:
   a. Find $\vec{V}_o/\vec{V}_g$ and sketch its magnitude vs. $\omega$.
   b. Find $Z(s)$ in pole-zero form and sketch its magnitude vs. $\omega$.

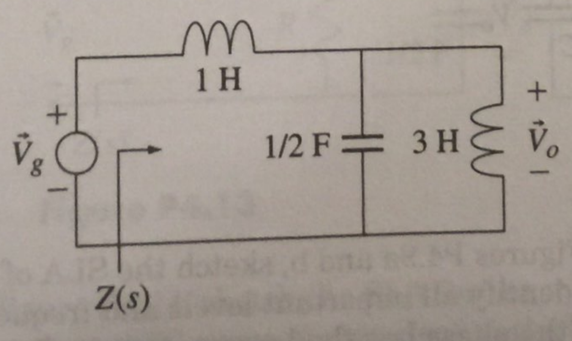



Figure P4.3

4. For the circuit of Figure P4.1b, assume the inductor has an inherent resistance of 0.05 Ω. Find $Z(s)$ in pole-zero form, and estimate the magnitude and bandwidth of the impedance at each resonance.
5. For the circuit of Figure P4.2, assume the inductor has an inherent resistance of 0.1 Ω. Find $Z(s)$ in pole-zero form, and estimate the magnitude and bandwidth of the impedance at each resonance.
6. For the circuit of Figure P4.3, assume each inductor has an inherent resistance of 0.1 Ω.
   a. Find $Z(s)$ in pole-zero form, and estimate the magnitude and bandwidth of the impedance at each resonance.
   b. Find $\vec{V}_o/\vec{V}_g$ in pole-zero form, and estimate the magnitude and bandwidth of the gain at each resonance.

### Section 4.2

7. For the circuits of Figures P4.7a and b, sketch the SLA of the impedance magnitude and phase. Identify all important levels and frequencies.

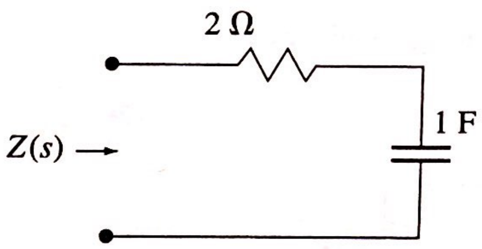



**Figure P4.7a**

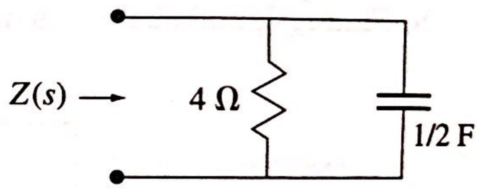



**Figure P4.7b**

8. For the circuits of Figures P4.8a and b, sketch the SLA for the magnitude of the transfer function suggested, and obtain formulas for the break frequency values and gain levels.

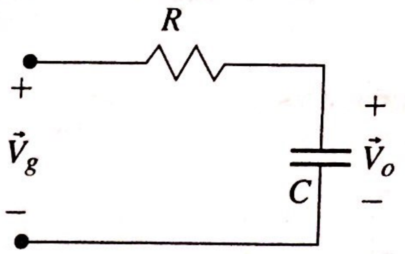



**Figure P4.8a**

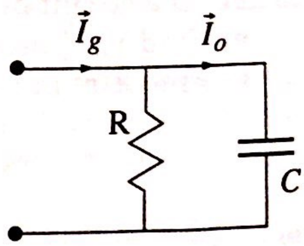



**Figure P4.8b**

9. For the circuits of Figures P4.9a and b, sketch the SLA of the impedance magnitude and phase. Identify all important levels and frequencies. Identify the frequency at which the phase has the largest magnitude.

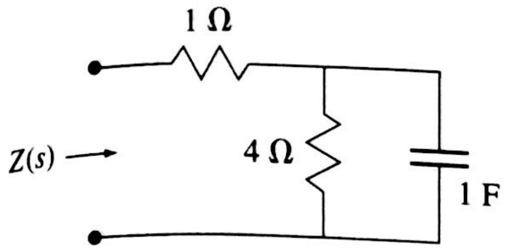



Figure P4.9a

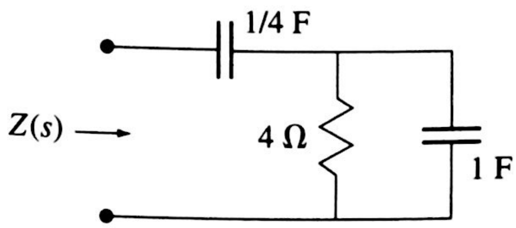



Figure P4.9b

10. For the circuits of Figures P4.10a and b, sketch the SLA of the voltage gain magnitude, and obtain formulas for the break frequency values and gain levels.

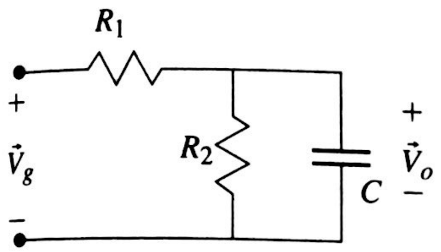



Figure P4.10a

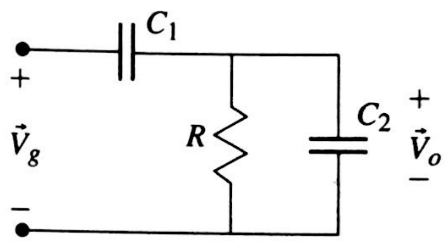



Figure P4.10b

## Section 4.3

11. Express a power of 7 watts in $dB_m$ and $dB_W$.
12. An amplifier rated at 100 W has a noise output of 1 $\mu$W. Express the output to noise power ratio in decibels.

## Section 4.4

13. For the circuit of Figure P4.13, sketch the SLA dB voltage gain given that $R = 1\ \Omega$. Do you expect the actual curve to stay close to the SLA at the break frequency? Why?

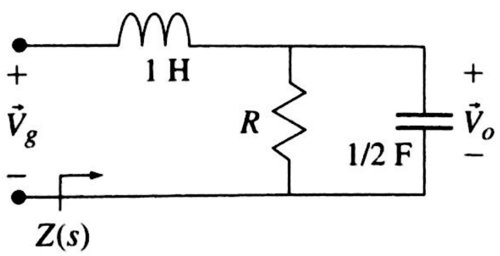



Figure P4.13

14. For the circuit of Figure P4.14, sketch the SLA for the magnitude of the impedance. Do you expect the actual curve to stay close to the SLA? Why?

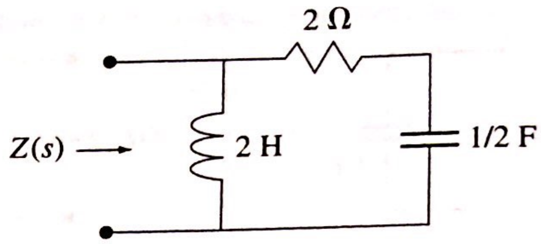



**Figure P4.14**

15. Sketch the SLA, magnitude and phase, for the gain $G(s)$. Calibrate the gain scale in decibels and the phase scale in degrees. Determine the actual maximum gain achieved and the frequency where the phase is zero in each case.
a. $G(s) = \dfrac{12s}{(s+2)(s+8)}$ b. $G(s) = \dfrac{12s}{(s^2+2s+16)}$ c. $G(s) = \dfrac{12s}{(s+4)^2}$

16. Each of the functions $F$, $G$, and $H$ have the same SLA plots.
   a. Addressing the magnitude response curves, which will have an actual response closest to the SLA? Which will have an actual response farthest from the SLA?
   b. Addressing the phase response curves, which will have an actual response closest to the SLA? Which will have an actual response farthest from the SLA?

$$F(s) = \frac{4}{s^2 + 4s + 4} \qquad G(s) = \frac{4}{s^2 + 2s + 4} \qquad H(s) = \frac{4}{s^2 + 4}$$

### Section 4.5

17. For the circuit of Figure P4.17, obtain an actual frequency response plot for the magnitude of $Z(j\omega)$ on log-log paper. Use a frequency range of 0.01–10 rad/s. Superimpose the SLA on top of the plot.

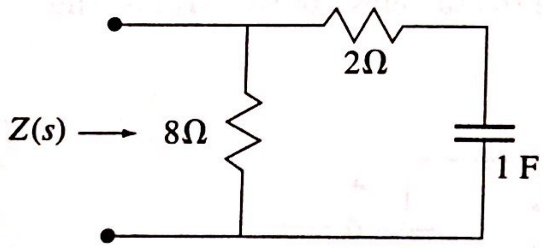



**Figure P4.17**

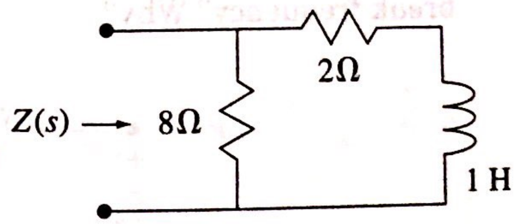



**Figure P4.18**

18. For the circuit of Figure P4.18, obtain an actual frequency response plot for the magnitude of $Z(j\omega)$ on log-log paper. Use a frequency range of 1–100 rad/s. Superimpose the SLA on top of the plot.

19. Obtain the actual impedance magnitude and phase responses for the circuit of Figure P4.13 for $R = 2\ \Omega$. Draw the SLA for each on top of the actual response curves. What is the value of the damping ratio for the complex root?

20. A graph of a transfer function has been generated over the frequency range of 0.1–10 rad/s using the MATLAB command > freqs(num, den,w). The gain magnitude needs to be limited to a range from 1 to 10. Show the MATLAB command sequence required to accomplish this.

## Additional Problems

21. A scope using a properly compensated $\times 10$ probe is shown in Figure P4.21. Sketch the input impedance (magnitude and phase) of the scope as a function of frequency. Determine the break frequency in hertz.

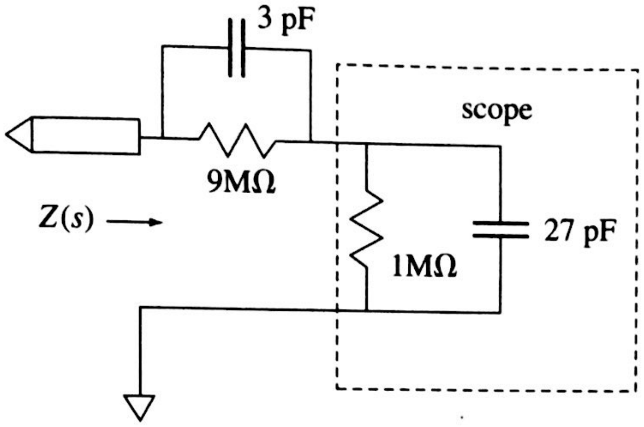



**Figure P4.21**

22. Given that $A_v = \dfrac{40s^2}{(s+2)^2(s+10)}$, use MATLAB to produce the actual and Bode SLA plots of $|A_v|$ on the same graph. Use a frequency range of 0.1 to 100 rad/s.
23. The circuit of Figure P4.23 is used to provide phase-lead compensation in control systems. Derive its voltage transfer function and sketch its magnitude and phase SLA. Determine each break frequency in terms of $R_1$, $R_2$, and $C$.
24. The circuit of Figure P4.24 is used to provide phase-lag compensation in control systems. Derive its voltage transfer function and sketch its magnitude and phase SLA. Determine each break frequency in terms of $R_1$, $R_2$, and $C$.

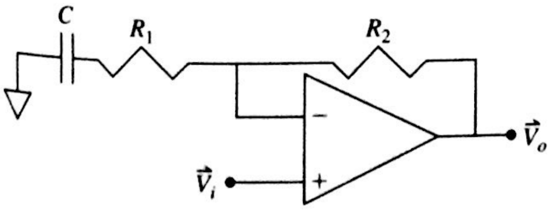



**Figure P4.23**

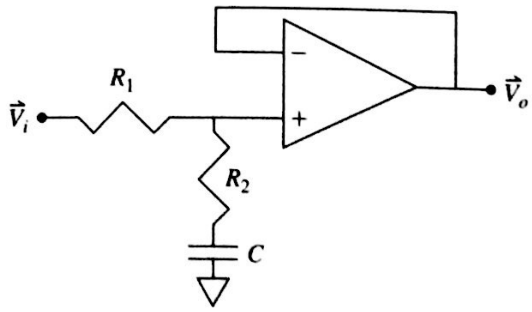



**Figure P4.24**

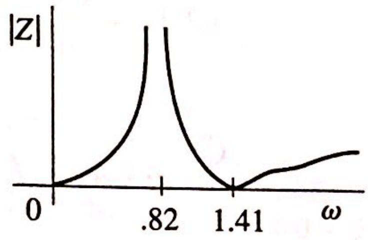
3.25
```
>subplot(2,1,1); plot(w,abs(H))
>title('|H| vs. w')
>subplot(2,1,2); plot(w,angle(H))
>title('angle of H vs w')
```

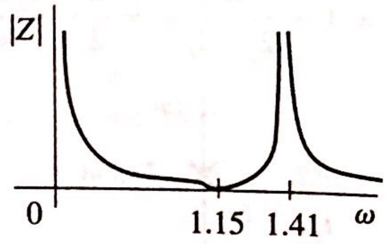
3.27
```
a) x = 0 1 0 2 0 3 0 4 0 5 0 6 0 7 0 8 0 9 0 10
b) x = 2 1 0 -1 -2
```

3.29 $H \le 0.01222$ $1/H \ge 81.8$

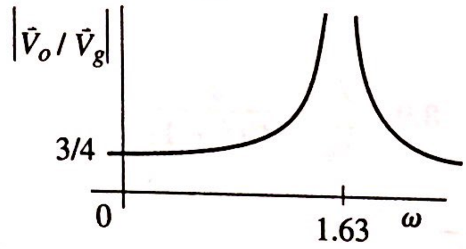
## Chapter 4

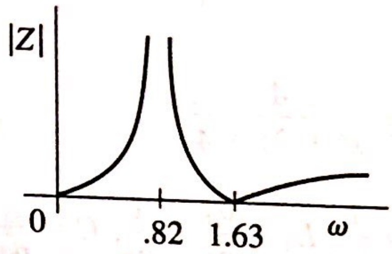
4.1 a) $Z(s) = \dfrac{2s(s \pm j1.4142)}{3(s \pm j0.8165)}$ b) $Z(s) = \dfrac{3(s \pm j1.1547)}{s(s \pm j1.4142)}$




4.3 a) $\dfrac{\vec{V}_o}{\vec{V}_g} = \dfrac{2}{s^2 + 8/3}$ b) $Z(s) = \dfrac{s(s \pm j1.633)}{(s \pm j0.8165)}$



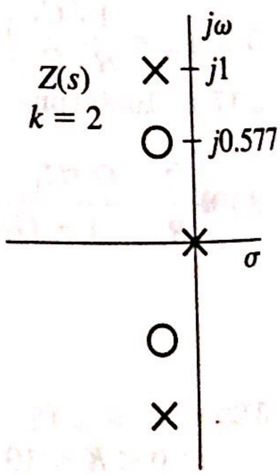

4.5 $Z(s) = \dfrac{2(s + 0.0167 \pm j0.577)}{s(s + 0.0167 \pm j)}$

a) $|Z(j1)| = \dfrac{2(0.423)(1.577)}{1(2)(0.0167)} = 39.9\ \Omega$;

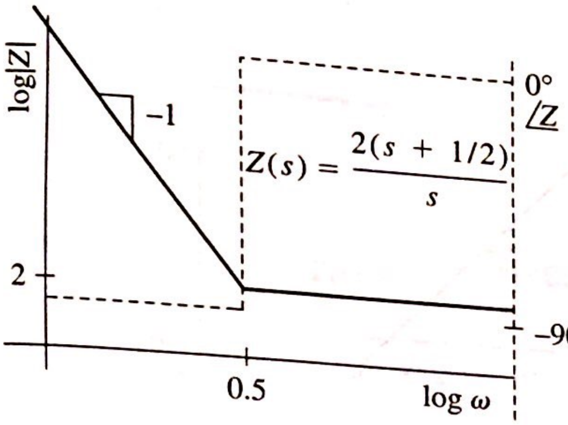
$B \approx 2(0.0167) = 0.0334$ rad/s

b) $|Z(j0.577)| = \dfrac{2(0.0167)(1.154)}{0.423(1.577)(0.577)} = 0.10\ \Omega$

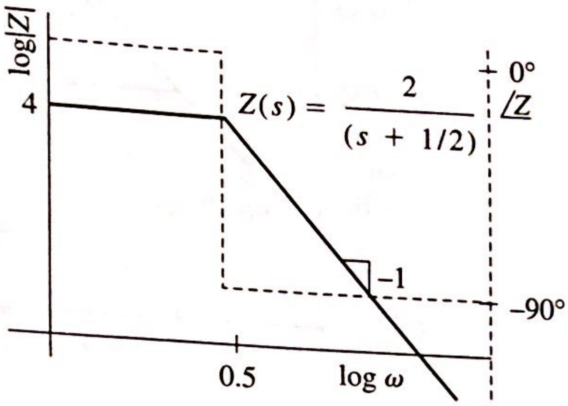
$B \approx 0.0334$ rad/s



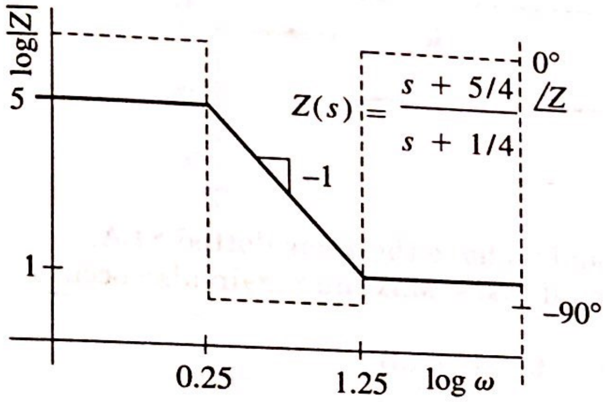
4.7 a)



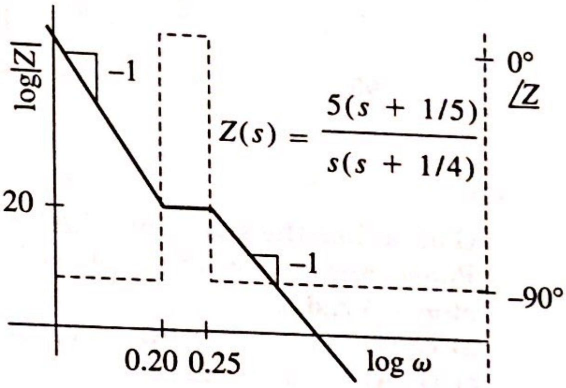

b)




4.9 a)




b)




$$\omega = \sqrt{0.25(1.25)} = 0.559 \text{ rad/s}$$

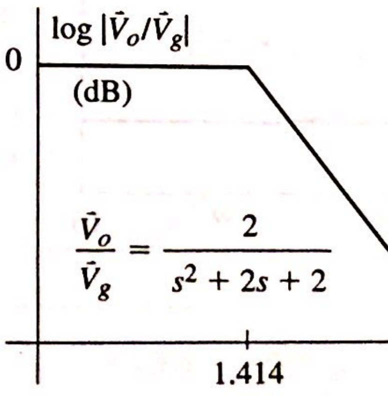
for max capacitance

$$\omega = \sqrt{0.20(0.25)} = 0.224 \text{ rad/s}$$

for min capacitance

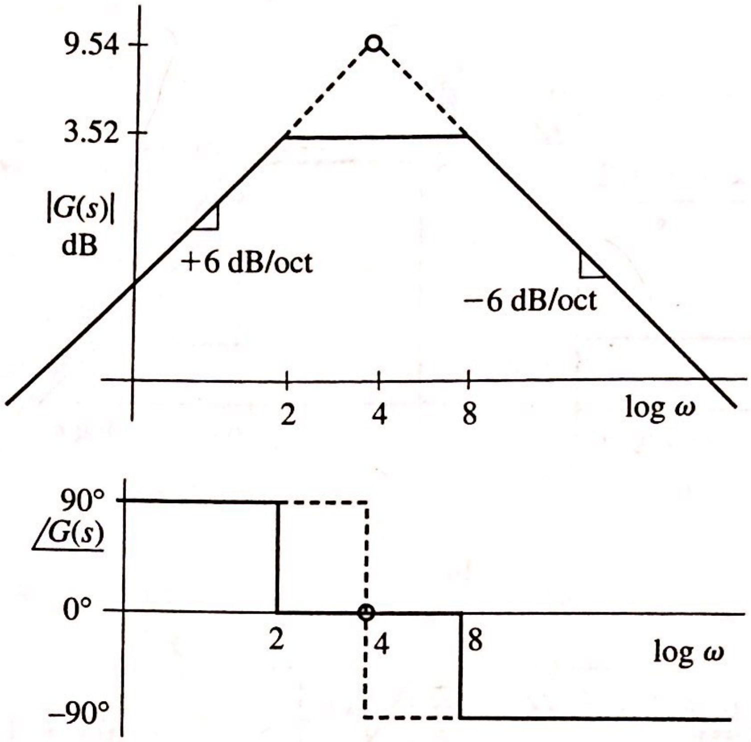
4.11 38.45 dBm = 8.451 dBW

4.13




SLA and actual curve will agree at least as well as for real roots because the damping ratio is 0.707

4.15

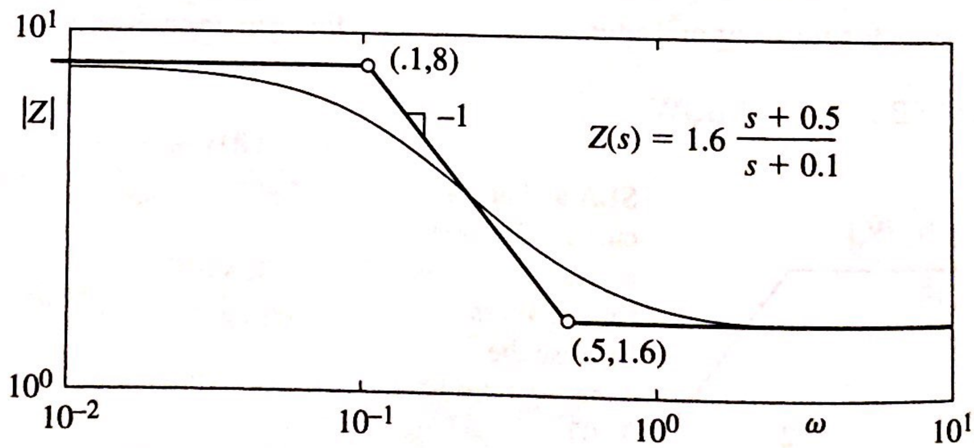



G of (a) has the solid line SLA, (b) and (c) have the same dotted SLA. Phase crosses zero at $\omega = 4$ rad/s in all cases. Maximum gain also occurs at $\omega = 4$ rad/s.

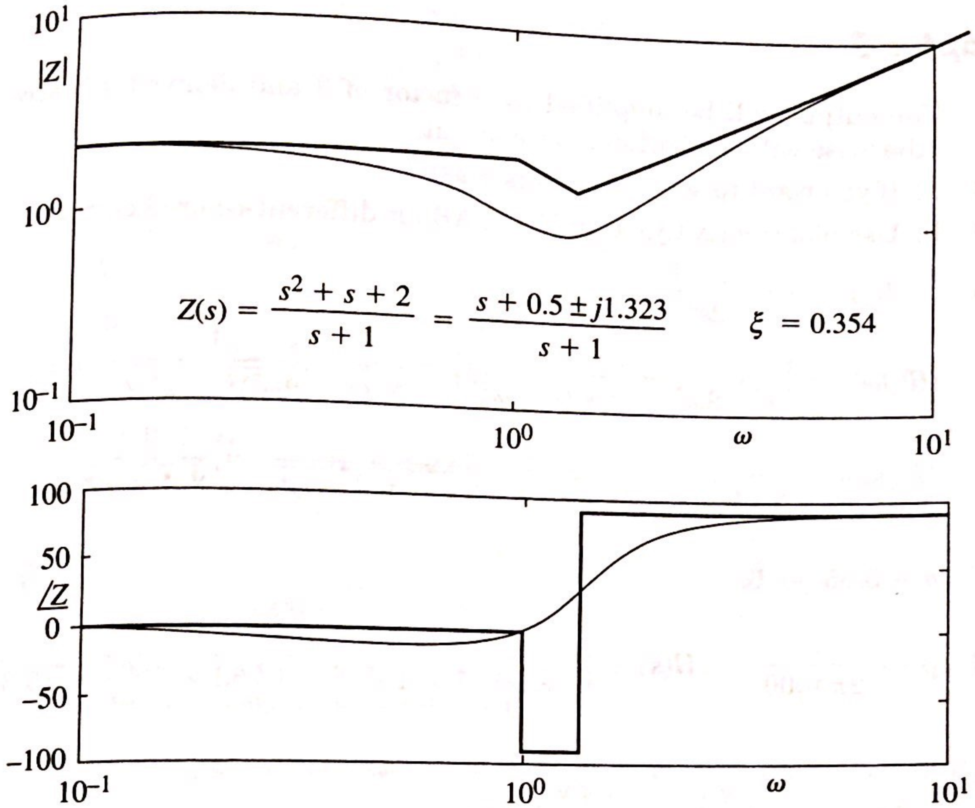
a) $G(j4) = 1.2 = 1.58$ dB  b) $G(j4) = 6 = 15.6$ dB

c) $G(j4) = 1.5 = 3.52$ dB

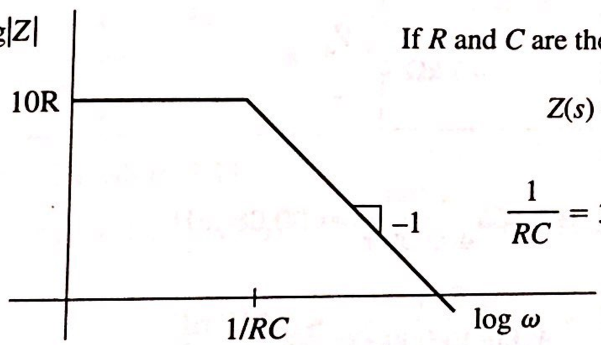
4.17



4.19




4.21




If $R$ and $C$ are the input of the scope itself,

$$Z(s) = \frac{10}{C(s + 1/RC)}$$

$$\frac{1}{RC} = 37 \text{ krad/s} \rightarrow 5.90 \text{ kHz!}$$

4.23 $\dfrac{V_o}{V_i} = 1 + \dfrac{Z_f}{Z_i} = 1 + \dfrac{R_2}{R_1 + 1/sC} = \dfrac{(R_1 + R_2)Cs + 1}{R_1 Cs + 1}$