Комп’ютерна алгебра

Лабораторна робот: базис Гребнера

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Варіант 3

Алгоритм Гребнера:

Input: A finite set *B* of polynomials

Output: A finite Gröbner basis *G* equivalent to *B*

1 *G*:=*B*

2 *C*:=*G*×*G*

3 while *C*≠∅ do

4 Choose a pair (*f*,*g*) from *C*

5 *C*:=*C*∖{(*f*,*g*)}

6 *h*:=RED(SPOL(*f*,*g*),*G*)

7 if *h*≠0 then

8 *C*:=*C*∪(*G*×{*h*})

9 *G*:=*G*∪{*h*}

10 return *G*

Результат:

Редукований:

i = y\*z + z + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = x\*z + (-1)\*x + 2\*y^2 + (-1)\*y,

S remainder = (-1)\*x + 2\*y^2 + y + (-1)\*1

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z]

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i = y\*z + z + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = x\*z + (-1)\*x + 2\*y^2 + (-1)\*y,

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1]

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i = x\*y\*z + y\*z + z , j = x\*z + (-2)\*y + 1

S polynomial = 2\*y^2 + y\*z + (-1)\*y + z,

S remainder = 2\*y^2 + (-1)\*y + 1

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1]

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i = y\*z + z + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = x\*z + (-1)\*x + 2\*y^2 + (-1)\*y,

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1]

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i = x\*y\*z + y\*z + z , j = x\*z + (-2)\*y + 1

S polynomial = 2\*y^2 + y\*z + (-1)\*y + z

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i = x\*y\*z + y\*z + z , j = y\*z + z + (-1)\*1

S polynomial = (-1)\*x\*z + x + y\*z + z

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = (-2)\*y^2\*z + (-1)\*y\*z + 2\*y + z + (-1)\*1

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = y\*z + z + (-1)\*1

S polynomial = (-1)\*x\*z + x + (-2)\*y^3\*z + (-1)\*y^2\*z + y\*z

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = x\*y\*z + y\*z + z

S polynomial = (-2)\*y^3\*z + (-1)\*y^2\*z + (-1)\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = x\*z + (-2)\*y + 1

S polynomial = (-1/2)\*x\*y\*z + 1/2\*x\*z + 2\*y^3 + (-1)\*y^2

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i = 2\*y^2 + (-1)\*y + 1 , j = y\*z + z + (-1)\*1

S polynomial = (-3/2)\*y\*z + y + 1/2\*z,

S remainder = y + 2\*z + (-3/2)\*1

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1]

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i = y\*z + z + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = x\*z + (-1)\*x + 2\*y^2 + (-1)\*y,

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1, y + 2\*z + (-3/2)\*1]

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i = x\*y\*z + y\*z + z , j = x\*z + (-2)\*y + 1

S polynomial = 2\*y^2 + y\*z + (-1)\*y + z

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i = x\*y\*z + y\*z + z , j = y\*z + z + (-1)\*1

S polynomial = (-1)\*x\*z + x + y\*z + z

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = (-2)\*y^2\*z + (-1)\*y\*z + 2\*y + z + (-1)\*1

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = y\*z + z + (-1)\*1

S polynomial = (-1)\*x\*z + x + (-2)\*y^3\*z + (-1)\*y^2\*z + y\*z

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = x\*y\*z + y\*z + z

S polynomial = (-2)\*y^3\*z + (-1)\*y^2\*z + (-1)\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = x\*z + (-2)\*y + 1

S polynomial = (-1/2)\*x\*y\*z + 1/2\*x\*z + 2\*y^3 + (-1)\*y^2

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i = 2\*y^2 + (-1)\*y + 1 , j = y\*z + z + (-1)\*1

S polynomial = (-3/2)\*y\*z + y + 1/2\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = x\*y\*z + y\*z + z

S polynomial = (-1/2)\*x\*y\*z + 1/2\*x\*z + (-1)\*y^2\*z + (-1)\*y\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = (-1)\*x + 2\*y^2 + y + (-1)\*1

S polynomial = (-1/2)\*x\*y + 1/2\*x + 2\*y^4 + y^3 + (-1)\*y^2

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i = y + 2\*z + (-3/2)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = 2\*x\*z^2 + (-3/2)\*x\*z + 2\*y^2 + (-1)\*y

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i = y + 2\*z + (-3/2)\*1 , j = y\*z + z + (-1)\*1

S polynomial = 2\*z^2 + (-5/2)\*z + 1,

S remainder = 2\*z^2 + (-5/2)\*z + 1

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1, y + 2\*z + (-3/2)\*1]

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i = y\*z + z + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = x\*z + (-1)\*x + 2\*y^2 + (-1)\*y,

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1, y + 2\*z + (-3/2)\*1, 2\*z^2 + (-5/2)\*z + 1]

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i = x\*y\*z + y\*z + z , j = x\*z + (-2)\*y + 1

S polynomial = 2\*y^2 + y\*z + (-1)\*y + z

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i = x\*y\*z + y\*z + z , j = y\*z + z + (-1)\*1

S polynomial = (-1)\*x\*z + x + y\*z + z

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = (-2)\*y^2\*z + (-1)\*y\*z + 2\*y + z + (-1)\*1

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = y\*z + z + (-1)\*1

S polynomial = (-1)\*x\*z + x + (-2)\*y^3\*z + (-1)\*y^2\*z + y\*z

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i = (-1)\*x + 2\*y^2 + y + (-1)\*1 , j = x\*y\*z + y\*z + z

S polynomial = (-2)\*y^3\*z + (-1)\*y^2\*z + (-1)\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = x\*z + (-2)\*y + 1

S polynomial = (-1/2)\*x\*y\*z + 1/2\*x\*z + 2\*y^3 + (-1)\*y^2,

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1, y + 2\*z + (-3/2)\*1, 2\*z^2 + (-5/2)\*z + 1]

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i = 2\*y^2 + (-1)\*y + 1 , j = y\*z + z + (-1)\*1

S polynomial = (-3/2)\*y\*z + y + 1/2\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = x\*y\*z + y\*z + z

S polynomial = (-1/2)\*x\*y\*z + 1/2\*x\*z + (-1)\*y^2\*z + (-1)\*y\*z

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i = 2\*y^2 + (-1)\*y + 1 , j = (-1)\*x + 2\*y^2 + y + (-1)\*1

S polynomial = (-1/2)\*x\*y + 1/2\*x + 2\*y^4 + y^3 + (-1)\*y^2

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i = y + 2\*z + (-3/2)\*1 , j = x\*z + (-2)\*y + 1

S polynomial = 2\*x\*z^2 + (-3/2)\*x\*z + 2\*y^2 + (-1)\*y

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i = y + 2\*z + (-3/2)\*1 , j = y\*z + z + (-1)\*1

S polynomial = 2\*z^2 + (-5/2)\*z + 1

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i = y + 2\*z + (-3/2)\*1 , j = x\*y\*z + y\*z + z

S polynomial = 2\*x\*z^2 + (-3/2)\*x\*z + (-1)\*y\*z + (-1)\*z

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i = y + 2\*z + (-3/2)\*1 , j = (-1)\*x + 2\*y^2 + y + (-1)\*1

S polynomial = 2\*x\*z + (-3/2)\*x + 2\*y^3 + y^2 + (-1)\*y

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i = y + 2\*z + (-3/2)\*1 , j = 2\*y^2 + (-1)\*y + 1

S polynomial = 2\*y\*z + (-1)\*y + (-1/2)\*1

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i = 2\*z^2 + (-5/2)\*z + 1 , j = x\*z + (-2)\*y + 1

S polynomial = (-5/4)\*x\*z + 1/2\*x + 2\*y\*z + (-1)\*z

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i = 2\*z^2 + (-5/2)\*z + 1 , j = y\*z + z + (-1)\*1

S polynomial = (-5/4)\*y\*z + 1/2\*y + (-1)\*z^2 + z

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i = 2\*z^2 + (-5/2)\*z + 1 , j = x\*y\*z + y\*z + z

S polynomial = (-5/4)\*x\*y\*z + 1/2\*x\*y + (-1)\*y\*z^2 + (-1)\*z^2

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i = 2\*z^2 + (-5/2)\*z + 1 , j = (-1)\*x + 2\*y^2 + y + (-1)\*1

S polynomial = (-5/4)\*x\*z + 1/2\*x + 2\*y^2\*z^2 + y\*z^2 + (-1)\*z^2

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i = 2\*z^2 + (-5/2)\*z + 1 , j = 2\*y^2 + (-1)\*y + 1

S polynomial = (-5/4)\*y^2\*z + 1/2\*y^2 + 1/2\*y\*z^2 + (-1/2)\*z^2

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i = 2\*z^2 + (-5/2)\*z + 1 , j = y + 2\*z + (-3/2)\*1

S polynomial = (-5/4)\*y\*z + 1/2\*y + (-2)\*z^3 + 3/2\*z^2

polynoms: [x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1, y + 2\*z + (-3/2)\*1, 2\*z^2 + (-5/2)\*z + 1]

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Result:

[x\*z + (-2)\*y + 1, y\*z + z + (-1)\*1, x\*y\*z + y\*z + z, (-1)\*x + 2\*y^2 + y + (-1)\*1, 2\*y^2 + (-1)\*y + 1, y + 2\*z + (-3/2)\*1, 2\*z^2 + (-5/2)\*z + 1]

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