Quantitative analysis of the complexity of dynamical systems*

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Abstract

Usage: Secondary publications and information retrieval purposes.

Structure: The paper presents a comprehensive framework for quantifying complexity in chaotic systems.

I. INTRODUCTION

The Lorenz attractor arises from a simplified model of atmospheric convection developed by Edward N. Lorenz in 1963. It is also studied in information theory, complexity science, and geometric analysis.
It is given by three coupled differential equations. The solution to these equations traces a beautiful butterfly-shaped trajectory in xyz-space. By simply observing this trajectory, we recognize that it is not a simple system. Yet quantifying this complexity remains difficult. There are some defined ways to measure complexity such as Kolmogorov complexity, López-Ruiz-Mancini-Calbet (LMC) complexity, etc. We can compute existing complexity measures, but this immediately raises a vital question: "Are these measures even sufficient for capturing the true complexity of this system? If not, what alternative approach can we discover?" If we succeed in establishing a meaningful new quantity for measuring complexity, it must carry genuine physical significance.
Simultaneously, we can explore other measurements that can describe the system like entropy, as entropy in general quantifies the unpredictability and the lack of information. One finds several entropies such as Boltzmann entropy, Shannon entropy, Kolmogorov–Sinai entropy (KS entropy), approximate entropy, sample entropy, permutation entropy, and many more. With so many entropy definitions available, we must ask: Do all entropies tell the same story, or do they reveal fundamentally different aspects? Moreover, we could seek novel kinds of entropy that might describe this system more effectively.
Furthermore, the Lorenz attractor is an example of deterministic chaos. The term deterministic chaos means that the system originates from a set of deterministic equations, yet produces behavior that is unpredictable in the long term although it is predictable in the short term. Given its chaotic nature, we might also calculate quantities like Lyapunov exponents, which explain how sensitive a dynamical system is to initial conditions, and multifractal spectrum (need to write why we use it), etc. Beyond dynamics, we may also look for geometric properties – extracted not only from the trajectory itself, but also from phase space, velocity space, and beyond. (need to write what kind of geometry we may find)
In order to calculate all these, we have three time series (x, y, z). One may study them statistically or using information theory. In either case, each time series can be studied individually, but we must understand their interdependence. Therefore, these series demand mutual analysis.
To uncover relationships between key quantities, we can employ tools like mutual information, transfer mutual information, etc. Even though the system is chaotic, we can search for periodicity using autocorrelation which measures how a signal relates to itself over time. Moreover, we need to understand how we can find geometric properties from these time series.

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II. LITERATURE SURVEY

A. Shannon Entropy

Shannon entropy (SE), introduced by Claude Shannon (site) that quantifies the average uncertainty of a random variable. For a discrete variable X with outcome x_i, is defined as:

$$H(X) = -\sum_i p(x_i)\log_2 p(x_i) \tag{1}$$

where $p(x_i)$ is the probability of outcome x_i.where,

$$H(X) \geq 0$$

SE calculates the entropy of entropy of symbolic sequences. High SE indicates high chaoticity. However, SE ignores geometric structure. As a result same SE values can be arrived from geometrically different structure. Besides it can not capture directional information flow.

B. Kolmogorov-Sinai Entropy

The Kolmogorov-Sinai (KS) entropy, introduced independently by Kolmogorov (1958) and Sinai (1959). It measure the rate of information production in deterministic dynamical systems which refers to the rate at which a system generates uncertainty about its future state. KS entropy is defined as:

$$h_u = \sup_{\mathcal{P}} \lim_{n\to\infty} \frac{1}{n} h(P_n) \tag{2}$$

C. Citations and References

1. Citations

a. Syntax

b. The options of the cite command itself

2. Example citations

3. References

4. Example references

D. Footnotes

III. METHODOLOGY

$$\chi_+(p) \lesssim [2|\mathbf{p}|(|\mathbf{p}|+p_z)]^{-1/2} \begin{pmatrix} |\mathbf{p}|+p_z \\ px+ip_y \end{pmatrix}, \tag{3}$$

$$\left\{ 1\!\!1234567890abc123\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2} \right\}. \tag{4}$$

A. Multiline equations

$$\begin{aligned}\mathcal{M} =& ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}\delta_{\sigma_1,-\sigma_2}(g_{\sigma_2}^e)^2\chi_{-\sigma_2}(p_2) \\ &\times[\epsilon_j l_i \epsilon_i]_{\sigma_1}\chi_{\sigma_1}(p_1),\end{aligned} \tag{5}$$

$$\begin{aligned}\sum |M_g^{\text{viol}}|^2 &= g_S^{2n-4}(Q^2)\ N^{n-2}(N^2-1) \\ &\times \left(\sum_{i<j}\right) \sum_{\text{perm}} \frac{1}{S_{12}} \frac{1}{S_{12}} \sum_\tau c_\tau^f\,.\end{aligned} \tag{6}$$

$$\begin{aligned}\sum |M_g^{\text{viol}}|^2 &= g_S^{2n-4}(Q^2)\ N^{n-2}(N^2-1) \\ &\times \left(\sum_{i<j}\right) \left(\sum_{\text{perm}} \frac{1}{S_{12}S_{23}S_{n1}}\right) \frac{1}{S_{12}}\,.\end{aligned}$$

$$g^+g^+ \rightarrow g^+g^+g^+g^+\dots\,,\quad q^+q^+ \rightarrow q^+g^+g^+\dots\,. \tag{2.6$'$}$$

$$\begin{aligned}\mathcal{M} =& ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}(g_{\sigma_2}^e)^2\chi_{-\sigma_2}(p_2) \\ &\times[\epsilon_i]_{\sigma_1}\chi_{\sigma_1}(p_1).\end{aligned} \tag{7a}$$

1. Wide equations

The equation that follows is set in a wide format, i.e., it spans the full page. The wide format is reserved for long equations that cannot easily be set in a single column:

$$\mathcal{R}^{(\mathrm{d})} = g^e_{\sigma_2} \left(\frac{[\Gamma^Z(3,21)]_{\sigma_1}}{Q^2_{12} - M^2_W} + \frac{[\Gamma^Z(13,2)]_{\sigma_1}}{Q^2_{13} - M^2_W} \right) + x_W Q_e \left(\frac{[\Gamma^\gamma(3,21)]_{\sigma_1}}{Q^2_{12} - M^2_W} + \frac{[\Gamma^\gamma(13,2)]_{\sigma_1}}{Q^2_{13} - M^2_W} \right) . \tag{8}$$

IV. RESULTS AND DISCUSSION

V. CONCLUSION

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Appendix A: Technical Details

The appendix provides additional mathematical details omitted from the main text for readability.

$$\mathcal{R} = \sum_{i=1}^{N} \frac{x_i^2}{2\sigma^2}. \tag{A1}$$