

Term Paper: A replication study of Optimal Financial Knowledge and Wealth Inequality

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1 Introduction

This is where the introduction goes

$$\alpha + \beta \cdot \sum a_i \tag{1}$$

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2 Model

2.0.1 State & Choice space

Our state space contains the following variables:

$$s = (a_t, f_t, \eta_t, e) \tag{2}$$

Where e is the education level which stays constant throughout the life cycle of an agent, a is the assets accumulated at time t , and f is the accumulated financial knowledge accumulated at period t . η_t is the sum of the transatory shock and the accumulated persistent shock.

We make a couple of simplifications compared to the original paper, mainly due to reducing the computational load when solving the model. An elaboration of our choice variables are presented below: We have 4 choice variables:

c, i, κ, ψ The consumption is denoted by c , and is made with a grid of 40 points. These 40 points are fractions of the assets accumulated at time t . This follows the paper exactly. The variable $i \in \{0, 1\}$ is a binary choice to whether or not to invest in capital. This contrasts the original paper, where the investment is considered to be a continuous variable, which are discretized into 25 points when implemented. The variable $\kappa_t \in \{0, 0.55\}$ is the share of assets to be invested in the risky asset conditional on having investing in the risky asset. This implies that $(1 - \kappa)$ will be invested in the risky asset. We assume in this implementation that κ is fixed. This is also a simplification compared to the original paper. The saving is denoted with ψ . The saving can be modelled as the residual i.e. we only need to consider the three choice variables:

$$\text{choice variables} = c, i, \kappa \tag{3}$$

2.0.2 Equations

$$V_t(s_t) = \max_{c_t, i_t, \kappa_t} u(c_t) + \beta \mathbb{E}[V_{t+1}(s_{t+1})] \quad (4)$$

$$a_{t+1} = \tilde{R}_t[a_t + y_t - c_t - \pi(i_t) - c_d I(k_t \neq 0)] \quad (5)$$

$$f_{t+1} = (1 - \delta)f_t + i_t \quad (6)$$

$$\tilde{R}_t = (1 - \kappa_t)R^{\text{riskfree}} + \kappa_t R^{\text{riskful}}(f_t) \quad (7)$$

$$R^{\text{riskful}}(f_{t+1}) = \bar{r} + r(f_t) + \sigma_\epsilon \epsilon_t \quad \sigma_\epsilon = 0.16 \quad (8)$$

$$r(f_t) = f_t^\alpha, \quad \alpha = 1, r(f_t) \in [0, 0.04] \quad (9)$$

$$\pi(i_t) = \omega \cdot i_t \quad (10)$$

$$\log y_t = g_{y,t}(t) + \eta_t \quad \nu_t \sim N(0, \sigma_y) \quad (11)$$

$$\eta_t = \mu_t + \nu_t \quad \nu_t \sim N(0, \sigma_\nu) \quad (12)$$

$$\mu_t = \rho \mu_{t-1} + \omega_t \quad \omega_t \sim N(0, \epsilon_\omega) \quad (13)$$

$$g_{y,t}(t) = \text{TO COME (Non parametric estimation)} \quad (14)$$

$$u(c_t) = n_t \cdot CRRA(c_t/n_t) \quad (15)$$

$$CRRA(c_t/n_t) = \frac{(c_t/n_t)^{1-\rho} - 1}{1 - \rho} \quad (16)$$

$$tr_t = \xi, \quad t > 65 \quad (17)$$

2.0.3 Modelling decisions & parameter choices

This model is to a large extent an extension of the paper **Optimal Financial Knowledge and Wealth Inequality**(REF), but we have made a number of modifications on the model. These modifications are primarily made, to make the model fit a danish context, and to reduce the size of the state space, that otherwise would slow down the model-solving computations significantly. The state space reductions and is discussed in the section 2.0.1.

Since we want to emulate the danish economy, we have made a set of choices.

1) We have removed the out-of-pocket expenditure, which is present in the original model. This is due to the fact, that the danish welfare system in

covers expenditures from illness, which is what the *oop* should model. 2) We have chosen to model government transfers as a constant, since that is how “folkepension” (which is the retirement benefit in Denmark) works.

Other simplifications and additions to the model is: We have chosen to model the function $r(f_t)$ as a potentially concave function. In the original paper, they model acquiring more financial knowledge increasingly more expensive, and have a linear $r(\cdot)$ function. We have chosen to model investing in financial knowledge, i_t , as a binary variable, where agents incur a fixed cost when investing, and it can therefore be argued, that $r(\cdot)$ in our setup should be concave.

3 Solving the model

To solve this model we need to solve the following problem: find the function φ that satisfies:

$$\varphi : (\mu_t, f_t, a_t, e) \mapsto (c_t^*, t_t^*, \kappa_t^*), V^*(\cdot) \quad (18)$$

In other words, for a given state space which action set/choice set should the agent take, if the agent wishes to maximize its cummulative utility during its lifecycle.

The bellman equation, is what allows to only consider two consecutive periods: $(t, t + 1)$. We can solve the model, by backwards induction. Solve the model in period T , where the model terminates.

4 Consumption preference

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