1 Model reformulated in Buffer Stock notation

In order to keep track of the permanent and transitory shocks, we reformulate the model in Buffer Stock notation.

$$V_t(s_t) = \max_{c_t, i_t, \kappa_t} n_{e,t} u(c_t/n_{e,t}) + \beta \rho_{e,t} E_t[V_{t+1}(s_{t+1})]$$

such that

$$\begin{aligned}
a_t &= m_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0) \\
m_{t+1} &= \tilde{R}_\kappa(f_{t+1}) a_t + Y_{t+1} + tr_{t+1} \\
Y_{t+1} &= \xi_{t+1} p_{y,t+1} + g_{y,e}(t+1) \\
p_{y,t+1} &= G p_{y,t} \psi_{t+1} \\
f_{t+1} &= (1-\delta) f_t + i_t \\
\tilde{R}_\kappa(f_{t+1}) &= (1-\kappa_t)\bar{R} + \kappa_t \tilde{R}(f_{t+1}) \\
\log \tilde{R}(f_{t+1}) &= \bar{r} + r(f_t) + \epsilon_t
\end{aligned}$$

where ψ_t, ξ_t are log-normally distributed and ϵ_t normally distributed. The state space s_t is given by $s_t = (m_t, p_t, f_t, e)$. We discretize the choice space with $\kappa_t \in \{0, 0.55\}$, $i_t \in \{0, 1\}$, $c_t \in \mathbb{R}^+$, and we change the dynamics of the model such that investing in financial knowledge i_t affects the next period's expected assets (and not period $t+2$). The growth rate G should also vary with time, which should be looked in to.

1.1 Choice-specific value functions

1.1.1 Case $\kappa_t = 0.55$ and $i_t = 1$

In this case, we have

$$\tilde{R}_\kappa(f_{t+1}) = (1-0.55)\bar{R} + 0.55 \times \exp(\bar{r} + r(f_t) + \sigma_\epsilon \epsilon_t)$$

and thus

$$m_{t+1} = \tilde{R}_\kappa(f_{t+1}) \left[m_t - c_t - \pi(1) - c_d \right] + Y_{t+1} + tr_{t+1}$$

where all stochastic elements can be integreated out via Gauss-Hermite integration. Thus, given an education level e, for each point in the state space (m_t, p_t, f_t) we evaluate 4 discrete choices and optimize consumption given these choices. Then, we save this solution $(c_t^*, i_t^*, \kappa_t^*)$.

1.2 Notes

One interesting reformulation to the Danish context would be to make pension tranfers fixed after age 65 as a function of the wealth at retirement.