
Financial Literacy: A Dynamic Model

Replication study of Optimal Financial Knowledge and Wealth Inequality (JPE)

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April 9, 2019

Original Model

$$\begin{aligned} V_d(s_t) &= \max_{c_t, i_t, \kappa_t} n_{e,t} u(c_t/n_{e,t}) \\ &\quad + \beta p_{e,t} \int_{\varepsilon} \int_{\eta_y} \int_{\eta_o} V(s_{t+1}) dF_e(\eta_o) dF_e(\eta_y) dF(\varepsilon), \\ a_{t+1} &= \tilde{R}_{\kappa}(f_{t+1})[a_t + y_{e,t} + \text{oop}_{e,t} + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0)], \\ f_{t+1} &= (1 - \delta)f_t + i_t, \\ \tilde{R}_{\kappa}(f_{t+1}) &= (1 - \kappa_t)\bar{R} + \kappa_t \tilde{R}(f_t), \end{aligned}$$

- Income Process (OOP similar)

$$\begin{aligned} \log y_{e,t} &= g_{y,e}(t) + \mu_{y,t} + \nu_{y,t}, \\ \mu_{y,t} &= \rho_{y,e} \mu_{y,t-1} + \varepsilon_{y,t}, \\ \varepsilon_{y,t} &\sim N(0, \sigma_{y,\varepsilon}^2), \quad \nu_{y,t} \sim N(0, \sigma_{y,v}^2) \end{aligned}$$

First step: Simplifications to make it run

- ▶ State space: $s_t = (\eta_{y,t}, \eta_{o,t}, e, f_t, a_t)$. Control variables: c_t, i_t, κ_t .
- ▶ OOP process removed (also potentially irrelevant for Danish case, as this includes medical expenditures etc.)
- ▶ Remove permanent income shock $AR(1)$ process $\mu_{y,t}$ to make integration easier
 - ▶ If we remove it, the income process follows

$$\log y_{e,t} = g_{y,e}(t) + \nu_{y,t}, \nu_{y,t} \sim N(0, \sigma_{y,\nu}^2)$$

where Gauss-Hermite is feasible to implement. We are not sure yet how to integrate out the sum of an $AR(1)$ process and a normally distributed random variable.

- ▶ Discretize investment in financial knowledge i_t and share of wealth invested in high-return asset κ_t to make control variable space smaller.

Current Progress and to-do's

- ▶ At the given formulation, we believe our model can be expressed in similar ways as what we have done in class for finite DP models
 - ▶ Thus, we solve the model by backwards induction starting at age $T = 90$ with no bequest motives.
- ▶ We have set up all relevant functions and parameters and are ready to solve the model.
- ▶ After having made the simplified model work, we want to gradually incorporate the omitted parts bit by bit (starting with e.g. the mortality risk $p_{e,t}$).
- ▶ Depending on how far we get, it would be interesting to take the model to Danish data.
 - ▶ We don't intend to estimate the model - the authors do not do this either - but it would be interesting to compare moments of the model with Danish data.