

A stylized model of housing and agglomeration

Introduction

This model aim to present a stylized model that allows users to easily reason about housing policy. Especially, in a time where YIMBY/NIMBY debates have become ever more present.

The Static Model

We consider a world where there exist two types of dwellings; city and rural. They differ by the fact that city have agglomeration effects, which can be important to consider in when considering housing policies. We denote city and rural, C and R respectively. We additionally, consider that there already is a stock of housing Q_R, Q_C which is fixed (in the short run at least). There are L households in the economy, that can either live in the city or in rural areas, denoted by L_R, L_C , and we easily conclude that $L = L_R + L_C$, and trivially we assume the stock of housing exceeds the number of individuals $L < Q_R + Q_C$.

We can consider the utility functions of the individual households (which we for simplicity let correspond to their income). The income for family i depends on whether they live in the city or in a rural area:

$$Y_{Ci} = \underbrace{K_C + \tilde{L}_C^\alpha + e_i}_{W_C} - P_C$$
$$Y_{Ri} = W_R - P_R$$

I.e. the utility is the wage (W) minus the price of housing. K_C is a constant, \tilde{L}_C is the agglomeration effect and e_i is just some small wage perturbation, so the aggregated demand slopes down. In other words, in this formulation, since we are mostly interested in housing in the cities, capture the relevant dynamics. P_R, P_C is the price of housing in rural areas and cities respectively. We are interested in how this clears the economy. Finally a note about \tilde{L}_C this can be thought of households in the city last period (myopic agents), or the expected number of households in the city (shared by all agents). Just note, that L_C and \tilde{L}_C does not represent the same. This model yields P_C, P_R, L_C, L_R given \tilde{L}_C (and the other parameters).

We consider the agents to be utility maximizing and they chose the city if $Y_C > Y_R$ and vice versa. Using this we can derive the aggregate demand (for simplicity we let $e \sim \text{Uniform}(\underline{e}, \bar{e})$, such that the demand curve is linear. We can derive an expression for the aggregate demand for city housing as function of relative prices **maybe better expressed as** $Y_C > Y_R$:

$$Y_C - Y_R =$$
$$K_C + \tilde{L}_C^\alpha + e_i - P_C - (W_R - P_R) \geq 0$$
$$\Rightarrow e_i \geq \Delta P + B(\tilde{L}_C)$$

Where $\Delta P \equiv P_C - P_R$, and B is just a generic function of \tilde{L}_C , i.e. $K_C + \tilde{L}_C^\alpha$. We use that expression of the density for uniform distributions and write:

$$L_C = D_C(\Delta P) = \frac{\bar{e} + B(\tilde{L}) - \Delta P}{\bar{e} - \underline{e}}$$

With the inverse demand:

$$\Delta P = \bar{e} + B(\tilde{L}) - L_C(\bar{e} - \underline{e})$$

We also need to consider the supply side of the housing. Since capital in the short run is fixed, we could think of the supply to be mirrored “L” shaped, however, we model it a bit more flexible as:

$$S_C = \min(a_C + b_C L_C, Q_C)$$

$$S_R = \min(a_R + b_R L_R, Q_R)$$

This is to say the supply is upward sloping (with slope b), and vertical when reaching the maximum Q .

We can plot these separately (see first plot), but it makes reasoning easier when we plot them against each other (next plot). If we consider that $L = L_C + L_R \Rightarrow L_C = L - L_R$. In other words we can plot the two supply functions on the same axis but opposite directions L_C as we normally would while L_R is mirrored. We now the total length of the x-axis is L since that is the total number of households in the economy. The flipped supply $\underline{S}_R = \max(L - a_r - b_r(L - L_C), L - Q_R)$.

Underneath this plot we draw a second plot, which corresponds to $Z(L_C) \equiv S_C - \underline{S}_R$ and $D_C(L_c)$, with ΔP on the Y -axis and L_C on the X -axis.

These two plots allow us to reason about demand and supply and how supply shocks translate into other prices and even longer run adjustments through agglomeration.

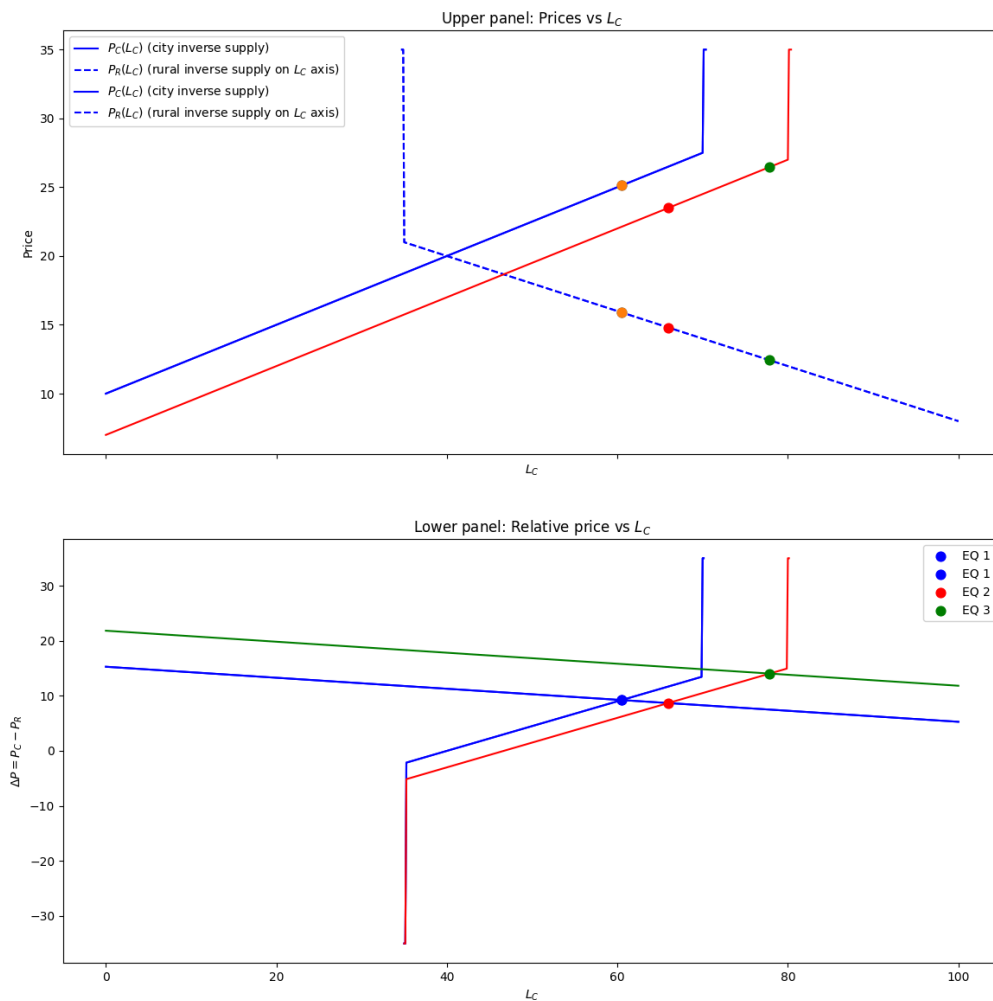


Figure 1: Supply and Demand example

How to read the figure

What's on each axis

- Horizontal (both panels): the number of households living in the city. Moving right means more people in the city and fewer in rural areas.
- Top panel (Prices): the city housing price curve and the rural housing price curve, both drawn against city population so you can compare them on one axis.
- Bottom panel (Relative price): the price gap between city and rural (city price minus rural price). Two curves live here: a demand curve and a supply-implied curve. Demand slopes down, supply slopes up.

The example above shows how a very often considered thought experiment of building more housing in the city. Before going into this specific example, let us summarize the reasoning:

1. We begin in EQ 1 where we have a price on houses in both rural and city areas.
2. Policy makers increase the supply of housing in the cities, which starts an adjustment of the supply curve (from blue to red), which is both reflected in top and bottom plot. Basically we see

(in this instance), the vertical right most line moves right and the upward sloped full (not dashed) line moves down. A classic example of increased supply. In the lower panel this is also reflected in Z -curve that captures the supply as expressed only by the number of households that end in the city L_C . We are now in EQ 2. which imply more people are now living in the city, fewer are living in rural areas, and the price of housing in both city and rural areas have decreased.

3. Due to more people living in the city, agglomeration effects begin to kick in, and demand adjusts (long term adjustment). This is reflected in the green demand curve moving up/right in the bottom figure. Which implies even more people are now living in the city, which is reflected in prices in the city now increasing, while the prices in rural areas are now even more decreased.

Summarizing effect of $Q_R \uparrow$

- in the city: $P_C \downarrow \Rightarrow L_C \uparrow \Rightarrow \text{agglomeration} \uparrow \Rightarrow L_C \uparrow \Rightarrow P_C \uparrow$
- in rural areas: $L_R \downarrow \Rightarrow P_R \downarrow$ Then agglomeration happens such that more people want to live in city. substitution implies: $L_R \downarrow \Rightarrow P_R \downarrow$

Importantly, even though we know for sure that the effect will be lower prices in rural areas and more people living in the city, the price effect is more ambiguous. Yes, we can know for sure that the prices will be lower in rural areas, however, we cannot know for sure that the agglomeration effect is dominated by the supply shock. This depends entirely on the models parameters, or rather on the underlying preferences of people in the real world.

case 2: Demand shock with multiple types

Before demand was defined the following way:

$$Y_{Ci} = K_C + \tilde{L}_C^\alpha + e_i - P_C$$

and

$$Y_{Ri} = K_R - P_R$$

We now consider two types A and B where B now gets a benefit when they live in the city. We can consider some government benefit to parents. So we can think of K_{CA} and $K_{CB} = K_{CA} + \rho$ assume there is κ and $1 - \kappa$ of each type. We can now derive the demand, using the same logic as previously: Agents are utility maximizing and they chose the city if $Y_C > Y_R$ and vice versa. Using this we can derive the aggregate demand (for simplicity we let $e \sim \text{Uniform}(\underline{e}, \bar{e})$, such that the sub-demand curve is linear. We can derive an expression for the aggregate demand for city housing as function of relative prices **maybe better expressed as $Y_C > Y_R$** :

$$\begin{aligned} Y_C - Y_R &= \\ K_C + \tilde{L}_C^\alpha + e_i - P_C - (K_R - P_R) &\geq 0 \\ \Rightarrow e_i &\geq \Delta P + B(\tilde{L}_C) \end{aligned}$$

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Two-Type City Demand with a City-Specific Benefit

Assume two agent types, A and B , with population shares κ and $1 - \kappa$.

Preferences:

$$Y_{Ci} = K_C + \tilde{L}_C^\alpha + e_i - P_C, \quad Y_{Ri} = K_R - P_R.$$

Type B receives an additional city benefit ρ (so $K_{CB} = K_{CA} + \rho$). Idiosyncratic taste $e_i \sim \text{Uniform}([e_{\min}, e_{\max}])$.

Define relative price and baseline term:

$$\Delta P = P_C - P_R, \quad B(\tilde{L}_C) = -(K_C - K_R) - \tilde{L}_C^\alpha.$$

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Individual choice

An individual chooses the city iff $Y_C \geq Y_R$, which is equivalent to a threshold:

- Type A: $e_i \geq \tau_A = \Delta P + B(\tilde{L}_C)$
- Type B: $e_i \geq \tau_B = \Delta P + B(\tilde{L}_C) - \rho$

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Sub-demand shares

With $e_i \sim \text{Uniform}([e_{\min}, e_{\max}])$, the share living in the city is

$$s_t = \text{clip}\left(\frac{e_{\max} - \tau_t}{E}, 0, 1\right), \quad E = e_{\max} - e_{\min}.$$

So

$$s_A = \frac{e_{\max} - \Delta P - B(\tilde{L}_C)}{E},$$

$$s_B = \frac{e_{\max} - \Delta P - B(\tilde{L}_C) + \rho}{E},$$

in the interior case ($e_{\min} < \tau_t < e_{\max}$).

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Aggregate demand

Total city demand:

$$L_{C(\Delta P, \tilde{L}_C)} = \kappa s_A + (1 - \kappa) s_B.$$

Inverse demand (interior)

$$L_C = \frac{e_{\max} - \Delta P - B(\tilde{L}_C) + (1 - \kappa)\rho}{E}$$

Invert:

$$\Delta P = e_{\max} - B(\tilde{L}_C) + (1 - \kappa)\rho - EL_C.$$

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Sub-demands by type

$$\Delta P = e_{\max} - B(\tilde{L}_C) - \frac{E}{\kappa} L_C^A$$

$$\Delta P = e_{\max} - B(\tilde{L}_C) + \rho - \frac{E}{1 - \kappa} L_C^B$$

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Corner cases

If $\tau_t \leq e_{\min}$ then $s_t = 1$. If $\tau_t \geq e_{\max}$ then $s_t = 0$. Always: $L_C \in [0, 1]$.

Observations:

Now we realize, for a given clearing price delta ΔP , we have two prices, one for rural P_R and one for the city P_C . Holding say P_R constant (as is the case in a given equilibrium, we can plot the sub demands for type A and type B as a function of L_C , i.e. for given ΔP we can have a plot of P_C vs L_{CA} and L_{CB} . In this way we can compare before and after a benefit of size ρ is introduced.

Concretely:

We start by $\rho = 0$, and find the equilibrium in the three relevant plots. Then we change ρ . This corresponds to a demand shock (in bottom right corner plot). This yields new prices (upper right plot), which then allow us to plot the new L_{CA} and L_{CB} in the upper left corner plot.