Projectile Motion and Conservation of Energy
Lab 2

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1 Introduction

One of the most important concept in introductory Physics is the concept of conservation of energy. The law of conservation is pivotal to understanding how and why processes in our world occur. The law of conservation states that energy cannot be created or destroyed, only transformed from one form to another.

$$K_1 + U_1 = K_2 + U_2 \tag{1}$$

Understanding this principal lets us view the world in a new lens and gives us the ability to predict the outcomes to certain events. One such event is an object moving in parabolic projectile motion under constant acceleration due to gravity. Assuming gravity is an internal force in our system and there is an absence of air resistance or other forces, we can predict the values of the potential and kinetic energies at different points along the parabolic path and thus predict the motion of the object. This importance of this powerful ability should not be undermined. Using this we can accurately guess the outcome of many real world events. One such use as a computer scientist could be using computers to predict the pathing of a golf ball or tennis ball after being hit and displaying that for viewers of a game using a Computer Graphics system.

In our experiment, we performed a series of calculations to estimate the landing position of a ball rolled down a tube. To do this, we used the equations listed below (2, 3, 4, and 5) in order to derive a formula to estimate the final displacement of an object after being launched.

$$K = K_{trans} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}mv^2 \tag{2}$$

$$\Delta K = W_f - \Delta U = mg\Delta h' - mg\Delta h = mg(h' - \Delta h) \tag{3}$$

$$x(t) = x_0 + v_{0,x}t \tag{4}$$

$$y(t) = y_0 + y_{0,y}t - \frac{1}{2}gt^2 \tag{5}$$

Equation 2 and 3 derive the formulas for the kinetic and potential energy at any point of the objects motion. The reason there are two terms in the potential energy calculation is because we need to account for some energy being lost due to friction while the ball is rolling down the ramp. We will use equations 4 and 5 to relate the energy of the object to its velocity and vertical and horizontal displacements. The results of the use of all of these equations is an estimate of where the object might land.

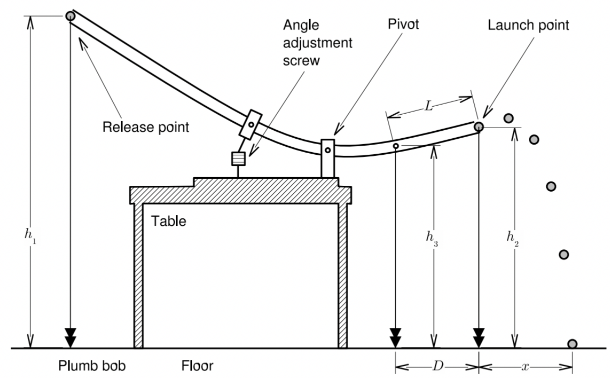

Figure 1: Diagram of Launching Apparatus

2 Methods

To calculate where the object was going to land after undergoing projectile motion, we use a ramp-like launch tube pictured in *Figure 1*. We use this type of apparatus because we will are able to measure the initial potential energy of the ball before it is released and we can make a estimate as to how much energy is lost while it is rolling down the ramp which we use to determine the kinetic energy. The kinetic energy can then be used to find the initial velocity which will allow us to calculate displacement.

The first step we performed was setting the ramp height so that the force of friction stops the ball just at the launch point. This will allow us to get an estimate for how much energy would leave the system due to friction and how much is being transformed into kinetic energy. After finding the height at which this occurred, we used a meter stick, which has a margin of error of ±0.001m, to find h'_1 and h'_2, or the starting heights. Then we adjusted the height to a height greater than h'_1 to ensure the ball would be launched off of the ramp this time. We then measured the height of the release point h_1, the launch point (h_2), and the ramp point (h_3) using the meter stick. We then calculated an estimate of where the ball would land using equations 6 and 7, which we derived as the equations to relate equations 2, 3, 4 and 5.

$$v_0 = \sqrt{\frac{10}{7}g([\Delta h] - [\Delta h'])} \tag{6}$$

$$\Delta x_{est} = v_{0,x}t = v_0\frac{D}{L}\frac{v_{0,y} + \sqrt{v_{0,y}^2 + 2gh_2}}{g} \tag{7}$$

Metal Ball		
Variables	Trial 1	Trial 2
h_1(m)	1.280 ± 0.001	1.257 ± 0.001
h_2(m)	1.160 ± 0.001	1.149 ± 0.001
h_3(m)	1.066 ± 0.001	1.072 ± 0.001
L(m)	0.298 ± 0.001	0.298 ± 0.001
D(m)	0.268 ± 0.001	0.299 ± 0.001

Table 1: Initial measurements for Metal Ball

After calculating our estimate of where the ball would land, we setup a piece of carbon paper on top of a piece of white paper with a crosshair on it Δx_{est} away from the launch point. We then launched the ball out of the tube onto the carbon paper which would leave an imprint onto the white paper which allowed us to analyze the results. We launched the ball 20 times then repeated these steps with a different height and a different ball material.

3 Results & Analysis

To start analyzing the data, I compiled it all into *Table 1* and *Table 2*. The only part I left out of the table was the $\Delta h'$ we calculated for friction which were 0.020 ± 0.001 (m) and 0.060 ± 0.001 (m) for the metal and plastic balls respectively. To calculate the error and estimated displacements of all of the spheres, I used wrote a python program that would use all of the data in the tables and equations 6 and 7 to perform the calculates and compound the error. I tested my program by running the example in the lab manual and calculated the same displacement that was said to be correct. After performing the calculation on our data set for each of the trials, we got the displacement displayed in *Table 3*. After calculating the averages and error for the actual trials, we got the data in *Table 4* and *Table 5*. The prediction for the x displacement in trial 1 with the metal ball and the observed average distance agrees with our predication as $\bar{x}$ lies within the uncertainty of Δx_{est}. Trial 2 for the metal ball is close however it is outside of our range of uncertainties. Trial 1 and 2 for the plastic ball however are significantly out of range of our uncertainties. We believe the reason for this is because our calculations for the estimation are off. Three reasons our predictions could be significantly off for the plastic ball are air resistance, wind, and our moment of inertia. Since the plastic ball was lighter than the metal ball, it is more susceptible to smaller forces that might not effect the heavy ball such as air resistance or wind forces. There may have been an air resistance that was slowing the ball in the air reducing the distance it would have traveling or a draft or disturbance in the launching mechanism throwing off the projectile more significantly since it is so light. We did not account for either of these possibilities when predicting where the plastic ball would land. The last possibility is that the moment of inertia is not the same as the metal ball. The mass in the plastic ball could be distributed differently which would cause the ball to have less translational kinetic energy when it is launched. We also did not account for this possibility.

Futhermore, we will look at the differences in the actual displacement from our estimated displacement using histograms. The histograms will display the occurrences and the distance from the estimated values. We will continue to analyze x from this viewpoint and additionally look at the z values we found as well.

Plastic Ball		
Variables	Trial 1	Trial 2
h_1(m)	1.283 ± 0.001	1.273 ± 0.001
h_2(m)	1.132 ± 0.001	1.143 ± 0.001
h_3(m)	1.056 ± 0.001	1.064 ± 0.001
L(m)	0.298 ± 0.001	0.298 ± 0.001
D(m)	0.323 ± 0.001	0.334 ± 0.001

Table 2: Initial measurements for Plastic Ball

Trials	Metal Ball	Plastic Ball
Trial 1	0.559 ± 0.007	0.617 ± 0.007
Trial 2	0.564 ± 0.006	0.558 ± 0.005

Table 3: Estimated Displacement Δx_{est} (m)

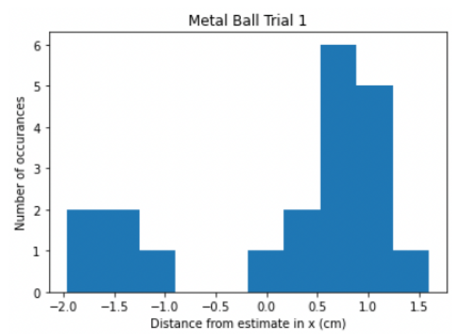

Figure 2: Difference between estimated and actual values of x for first trial with the metal ball

To continue the discussion from earlier, we can observe the histograms (*Figure 2* and *Figure 3*) for the metal ball trials. While trial 1 does not show a gaussian distribution, we can see that all of our actual values were within 2 centimeters from our estimation which is close considering that we have to make many assumptions to get to this point. Trial 2 displays a gaussian or normally distributed curve however, all of the points are at least 2 centimeters away with most of the distances being 3.2 cm. A possible reason for this could be a measurement in one of our values or a miss-measurement in the location we placed our cross-hair on our blank paper. We had difficulties aligning the paper to the launching point of the tube, however this may not account for all of the error here. More of this can be seen in *Figure 4* and *Figure 5* when

Trials	Metal Ball	Plastic Ball
Trial 1	0.561 ± 0.002	0.438 ± 0.006
Trial 2	0.591 ± 0.001	0.438 ± 0.009

Table 4: Actual Displacement in the x direction $\bar{x}$ (m)

Trials	Metal Ball	Plastic Ball
Trial 1	-0.015 ± 0.006	-0.008 ± 0.002
Trial 2	-0.0120 ± 0.0004	0.014 ± 0.004

Table 5: Actual Displacement in the z direction $\bar{z}$ (m)

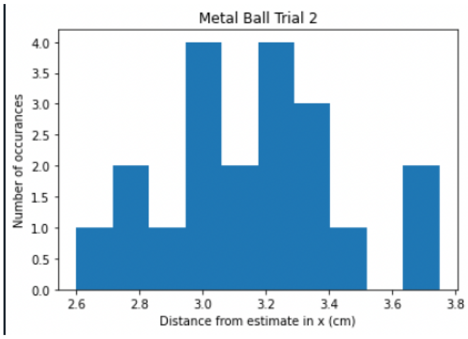

Figure 3: Difference between estimated and actual values of x for second trial with the metal ball

we can observe our z values. In both of our trials, our z values were pretty far from centered. This is due to our difficulty in aligning the paper to the launching point as I previously mentioned. One thing to note about our results is that they are mostly consistent. There is an effectively normal distribution in both the x and z components of trial 2. This is due to the higher consistency of using the metal uniform sphere as a projectile. It was less likely to be affected by outside and unaccounted for forces or properties. The outliers for the first trial's z distance, could be due to some movement in the paper or table miss-aligning our launching mechanism or paper that effected the rest of the launches.

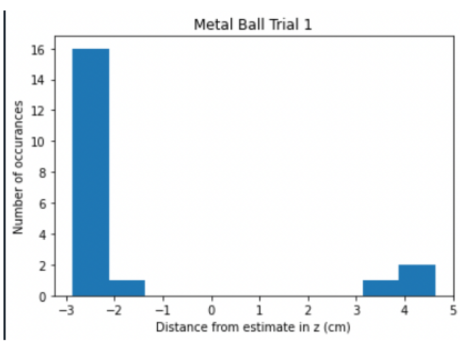

Figure 4: Difference between estimated and actual values of z for first trial with the metal ball

Now we will look at the data we collected for the plastic ball trials. Upon looking at the histograms (*Figure 5 & 6 & 7 & 8*), we can see that the data is not as normally distributed as the data for the metal ball trials. There are more outliers and the distances from the predicted values are all significantly higher. We discussed earlier some of the reason for the differences from what we were expecting. Because of the inconsistency of the plastic ball, the spread in our graphs is significantly larger than the metal ball trials.

The spread of the data is clearly not the same for the metal

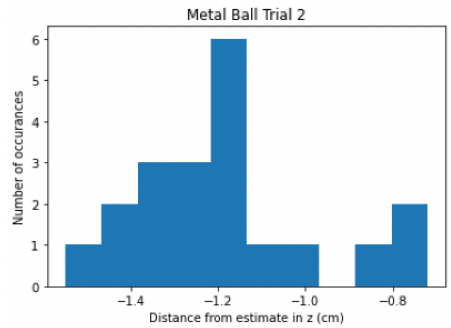

Figure 5: Difference between estimated and actual values of z for second trial with the metal ball

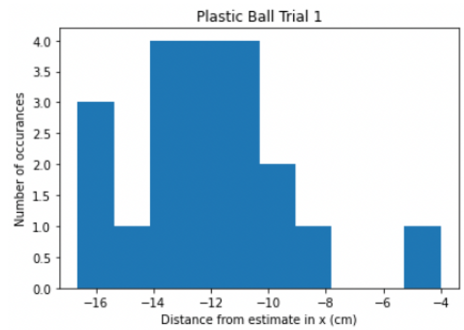

Figure 6: Difference between estimated and actual values of x for first trial with the plastic ball

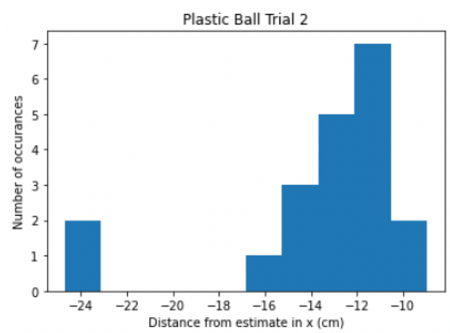

Figure 7: Difference between estimated and actual values of x for second trial with the plastic ball

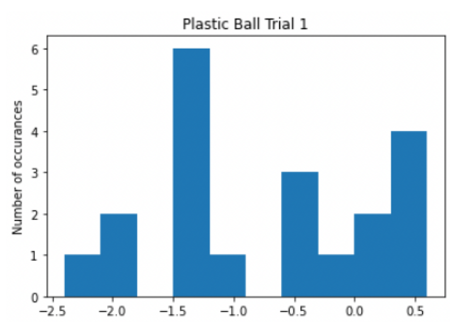

Figure 8: Difference between estimated and actual values of z for first trial with the plastic ball

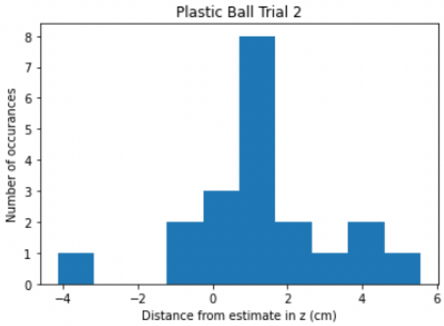

Figure 9: Difference between estimated and actual values of z for second trial with the plastic ball

and plastic spheres and it shouldn't be. In the cases of the plastic sphere there is a lot more uncertainty in our measurements as we have mentioned. Because the plastic ball is lighter, it could have been affected by slight changes in the draft in the air throwing off its projectile motion and our predictions. In the room we performed our experiments in, there are a variety of factors such as subtle movements of the table we shared with other students or changes in the air that could have messed up the spread. In the future, to minimize the spread, we could do the experiment in a much more controller area where we know the conditions will be the same every launch. Additionally, our calculations may not have been right because the moment of inertia's of the two spheres could be different like we mentioned earlier as well. We did not inspect our spheres for imperfections on the outer surface but that could have been another space for error. If we assume the plastic sphere was hollow and had a larger moment of inertia because its mass was distributed further from its radius than a uniform sphere, then maybe our estimation would have been more accurate. If the moment of inertia was larger, the amount of energy transformed into rotational kinetic energy would have been higher in our calculations, which would have in turn lowered our expected translational kinetic energy and initial velocity.

Another note to mention is our calculation of friction was probably inaccurate. The coefficient of friction, and thus, the force of friction increases when the speed of an object increases. The friction we found when the ball perfectly stopped at the launching point was not necessarily the same amount of friction during our launch trials. Other unpreventable ways the friction could have been effected was dust or particles collecting in the tube, or the spheres outer surfaces wearing down from multiple trials. While these aren't specifically likely, it is something that could have thrown off our estimates.

We can try to calculate an estimate for the friction value assuming it is constant over the length of the tube (which is not actually the case but assumption we are making). We can use Equation 8 to try to make our estimation. For the length of the tube, I estimate it to be $6 * 0.298$ (m). I made this estimation using the length of the launching mechanism L in our diagram and multiplied is by 6 as a prediction for its actual value. Using these values and solving for f using Equations 3 and the values for h' calculated in trial 1, I got a prediction of 0.006 Newtons. Assuming that the sphere has a mass of 50 grams, the force of gravity is 0.500 Newtons. Thus, our force of friction seems to not be significant if this is the case, however this is after making a lot of assumptions. The assumption that the friction is same over the whole length of the motion is most likely the cause of the friction being so low. If the friction increased in force as the ball traveled the length as it would do in real life, it would be a lot more significant than I predicted it to be.

$$W = f_f l \tag{8}$$

When we derived our equations, we assume that everything is under perfect conditions. We assume that the object we are considering is a perfectly uniformly distributed sphere and friction is acting consistently the whole time the ball is rolling down the tube. We also assume there is no air resistance or other forces that could be acting on the object while it is moving. It is very possible that movements in the floor or wall could have slightly moved the launching apparatus. These assumptions compound which make all of our predictions off from what actually occurs.

4 Conclusion

In this experiment we were able to showcase how the law of conservation of energy can be applied to estimate a projectiles path. While some of our predictions were not necessarily accurate, we are able to infer as to where the discrepancies in our results may have come from whether it be due to friction, or slight changes in the air pattern or assumptions we make about the distribution of mass in the object. For future experiments testing this law, experimenters should try to be able to control as many of the conditions that may affect the results as possible and try to make as little assumptions as possible when analyzing the results. In real life, when trying to estimate the path of a projectile, all of the variables should be known and accounted for to estimate the exact path and landing position of a projectile.