e/m of the Electron
Lab 4

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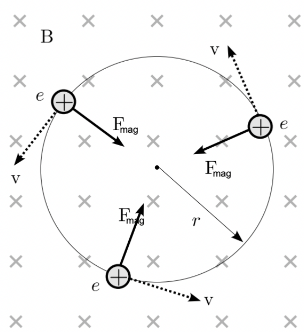

Figure 1: Centripetal motion of a charged particle in a magnetic field

1 Introduction

In this experiment, we recreate one of of most important experiments in history. The mass to charge ratio of an electron was something physicists speculated on for years before 1897 when Thomas discovered it in his "cathode rays" experiment. In this lab, we recreated his experiment using modern technology. The importance of this experiment, once again, should not be undermined. The resulting quantity we will be calculating would theoretically allow us to have a quantity for the mass of an electron if we knew the charge, which at the time physicists had also been speculating on. The mass of the electron has been critical in understanding the interactions of the atomic particles and has allowed us to dive even deeper.

In our experiment we will use equations derived from other experiments with charged particles. The first equation we will be using is listed below.

$$\vec{F_{mag}} = e\vec{v} \times \vec{B} \tag{1}$$

In this equation, we can get the value for a force on a moving charged particle in a magnetic field, $\vec{B}$ with a charge e and velocity $\vec{v}$. This is similar to the equations we used on the previous lab relating the current in a current carrying wire in a magnetic field. In the case where the velocity of the charged particle and the magnetic field are perpendicular to each other, the charged particle ends up moving in a circular path (refer to Figure 1). Because of this, the magnetic field is in principle acting a centripetal force on the charged particle and we can equate equation 1 to the equation for centripetal force.

$$\frac{mv^2}{r} = evB \tag{2}$$

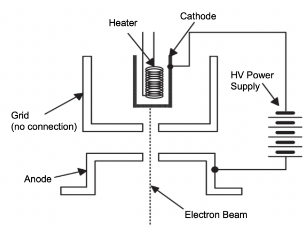

Figure 2: Electron gun in sealed tube

Unfortunately direct measurement of the velocity of a tiny charged particle is quite difficult, however if we accelerate the charged particle from rest with a potential difference, we can calculate the velocity using conservation of energy.

$$\frac{1}{2}mv^2 = eV \tag{3}$$

Then we can shuffle around the variables in equation 3 and substitute a solution for velocity into equation. This will allow us to have measurable quantities for every variable besides m and e and thus we can find a value for the charge to mass ratio.

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{4}$$

2 Methods

With equation 4, we know what quantities we will need to find the charge to mass ratio of an electron. We will be using a device that abstracts all of these quantities to users internally, but I will explain how I performs this. First, we will need to have a potential difference. To create a potential difference, the device uses an electron gun in a vacuum tube. An indirectly heated cathode, will be used to accelerate electrons with a potential difference of up to 500 V. The electrons will be fired in a narrow beam and shot into a magnetic field produced by Helmholtz coils. The magnetic field produced by the Helmholtz is given by the equation below.

$$B_{loop} = \frac{\mu_0 R^2 I}{2(R^2 + x^2)^{3/2}} \tag{5}$$

By substituting constants given to us in the Lab manual and multiplying the previous equation by the number of coils in our electromagnet we can get the equation shown below.

$$B_I = \left(\frac{4\pi \times 10^{-7}(132)}{(0.1475)(1 + 1/4)^{3/2}}\right)I = 8.05 \times 10^{-4} I \tag{6}$$

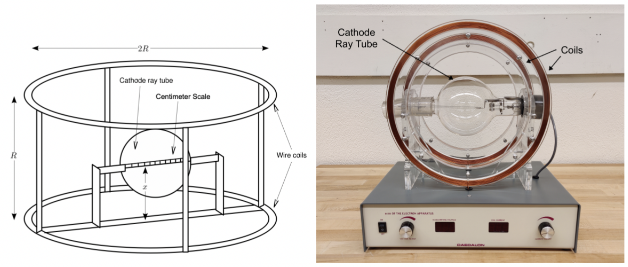

Figure 3: Helmholtz coils and magnetic field

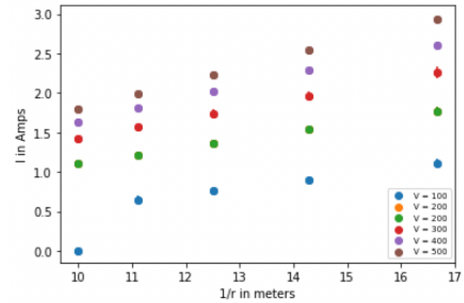

Figure 4: Graph of data from all of the trials

With this, we will be able to calculate the magnetic field produced by our Helmholtz coils using an applied current by an electromagnet.

This whole process is occurring in a sealed glass tube, however, a small amount of helium was added inside the tube. This allows for the electrons to leave a trail as they are fired due to the ionization of the helium atoms. This will allow us to read off of a small ruler inside the tube as to how far the electrons traveled and what the radius of curvature was. The device used is shown in figures 2 and 3.

To start the experiment. We first used a compass to align our apparatus to Earth's magnetic field. This is so there are no stray ambient magnetic field lines that could be affecting the path of the electrons as they travel in the magnetic field inside the apparatus. By aligning the device to the ambient magnetic field we ca make sure it has not affect on the pathing. We aligned it by angling it according to the direction a compass was pointing.

Next we turned on the device and waited for it to perform a warming up procedure and self test. After this was done, we started collecting data. We first dialed the potential difference to be 100 V. We decided to try to make our radius of curvatures we tested constant across all of our trials. We changed the current being applied to the magnetic field until the the path of the electrons had a radius of 6 cm, 7 cm, 8 cm, 9 cm, and 10 cm. After performing these 5 trials, we increased the potential difference by 100 V and did this until 500 V. At each radius, we increased and decreased our current slightly find how many steps of current it would take to change the radius, this will be our error when performing analysis.

3 Results & Analysis

Before we start analyzing our results. We can derive a formula for a linear relationship between the radius and the current which will allow us to find the rate of change of a line of best fit to find the charge to mass ratio. Using equations 6 and 4. We can derive the expression bellow.

$$I = \left(\frac{1}{8.05 \times 10^{-4}}\sqrt{\frac{2V}{e/m}}\right)\frac{1}{r} \tag{7}$$

To find this equation, we plugged the result for the magnetic field into equation 4 and solved for I. We can graph a linear relationship between I and $\frac{1}{r}$ to find the charge to mass ratio.

In *Figure 4*, we can see the graphed relationship between $1/r$ and I. Clearly there is a very strong linear relationship between the two variables. Another thing to note is it seems the current required for the magnetic field that bends the path of the electrons increases as the potential difference increases. This makes sense because the electrons are being fired with a higher potential difference, thus the kinetic energy that is transformed is larger and the velocity is larger and thus more of a centripetal force is required to start circular motion. One thing to note that we will see come up when discussing each individual trial is that in the first trial we had issues reaching the last data point for the radius. The reason for this is because the testing apparatus became extremely inaccurate and measurements were not readable since the line of electrons was changing so much. I believe the reasoning for this to be the fact that the electrons are given too little energy in the electron gun and curving a condensed path with a larger radius was difficult to perform since the spread of electrons was so largely spread out. Another thing we noticed was when we increased the potential difference and decreased the radius of curvature, the line of electrons thinned and was more precise. This is probably because they are fired with such high energy they are guaranteed to follow the path we are trying to impose on them. This is why when we look at our data for each individual trial, there is a trend of error decreasing as the potential difference increases. Additionally, when the radius of curvature is small, the current and magnetic fields are large and have a stronger affect on the electrons while the further points have a weaker effect and might not perfectly curve every electron. This is why measuring the current got more difficult as we increased the radius.

To discuss more about our data, we performed a weighted linear least squares fit or weighted linear regression to our data and graphed the resulting slope and intercept. The error of every point was plotted but most of the errors are quite small so they are difficult to display on the plots, but they are there.

In our first trial, the results were pretty consistent, but with a large error. This was up until the 5th test with the lowest current and largest radius we had difficulties measuring the point at which the current created a magnetic field that would curve the electron beam to the 10 cm marker of the apparatus. In this trial, the slope of our regression was 0.214 ± 0.06 and the y intercept is -2.13 ± 0.01. While the error is not that large, the data is most definitely skewed because of the first point. After the first trial, there is a trend of increasing slope from 0.1 and the first trials slope is higher than every other trial, but looking at the data it would have been around the

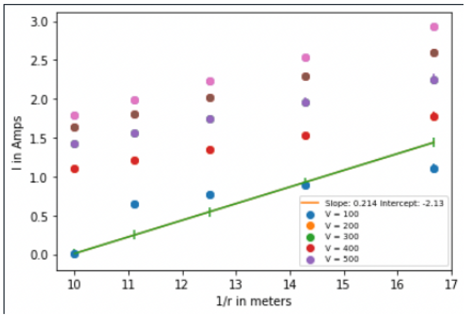

Figure 5: 100 V Potential Difference Trial

correct slope if the first point had not been there.

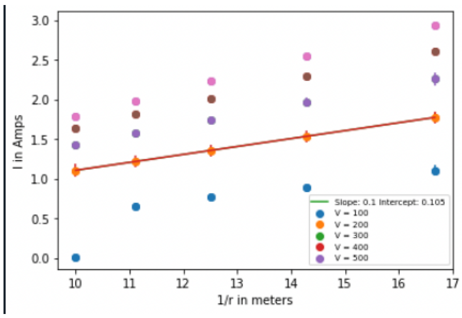

Figure 6: 200 V Potential Difference Trial

In the second trial in *Figure 6*, there was not anything that was significant that could have changed the results. The slope was 0.10 ± 0.08 and the intercept was 0.105 ± 0.006.

In the third trial in *Figure 7*, the error seemed to increase for some reason, however the data looks pretty much identical to that of the second trial. The reason for the error potentially increasing could be to us having difficulty discerning what counted as lighting up a point on the ruler and our group switching readers mid-trial. One thing that we can begin to notice by looking at our data is that the earlier points seem to be getting closer vertically and the slope is increasing. So there is a less significant effect on the first points when increasing the potential difference. The slope of this trial was 0.125 ± 0.12 and the intercept was 0.174 ± 0.010.

In our fourth our in *Figure 8*, the trend we noticed in trial three continues. The error additionally decreased in this trial, which makes sense because of the more concentrated beam of

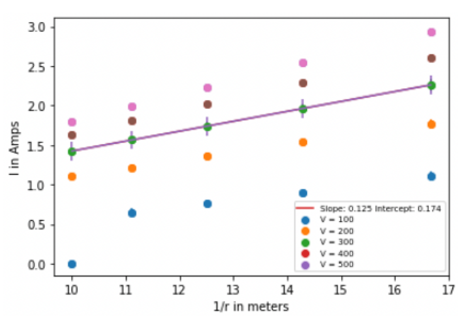

Figure 7: 300 V Potential Difference Trial

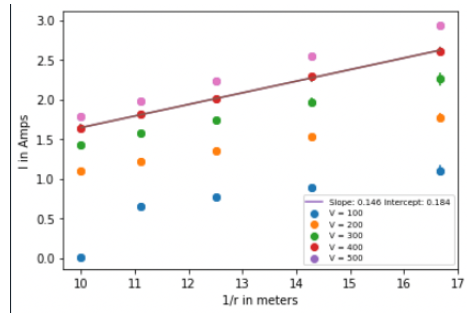

Figure 8: 400 V Potential Difference Trial

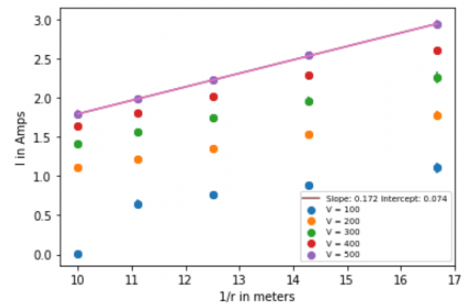

Figure 9: 500 V Potential Difference Trial

electrons being fired like we mentioned earlier. The slope of this trial was 0.146 ± 0.046 and the intercept was 0.184 ± 0.004.

In the fifth and final trial in *Figure 9*, the trends mentioned in trials three and four continued once again. The slope is the largest out of all of the trials excluding trial one at 0.172 ± 0.052 and the intercept was 0.074 ± 0.004.

Using these five slope quantities, we can calculate and estimate for the charge to mass ratio. Since there are no error in the quantities within our slope, we do not have to propagate any error besides the one we got from our weighted linear regression. In the end our charge ratios were $\{6.74 \times 10^9 \pm 0.8, 6.17 \times 10^{10} \pm 0.08, 5.93 \times 10^{10} \pm 0.07, 5.79 \times 10^{10} \pm 0.05, 5.22 \times 10^{10} \pm 0.05\}$. After averaging all of our values we got a value of $4.76 \times 10^{10} \pm 0.06$ as our estimate for the charge to mass ratio of the electron. Unfortunately, is pretty far off from the actual value of 1.758×10^{11} and is definitely not within statistical error in the end, however it is very possible some statistical error along the way may have caused us to be far off. We we're one order off on all of our final calculations, one misstep along the way to calculating the final value could have thrown it off for all of our data points.

Some systematic error that could have thrown off our calculations could be the ambient magnetic field of the earth. While we tried our hardest to avoid any out magnetic field, it is very possible that it still affected the results of our experiment whether it be slightly or significantly enough for us to not get the same results. Another thing is that our apparatus behaved oddly sometimes, such as the time where it was impossible to read the current for the 5th distance in the first trial. This could have been a sign of systematic error in the testing apparatus that was out of our control. Another possible error could have come from us reading the data off of the ruler incorrectly, it was hard to pinpoint at times what point each number was lit up.

4 Conclusion

Overall in this experiment we were able to make an estimate for one of the most important quantities in modern physics, the charge to mass ratio. We got to see how almost all of the subjects and fields of classical physics were able to be used to solve a pivotal problem for physicists. While our estimate was pretty far off from the correct value, and way out of range when accounting for error, it was a interesting opportunity and the experiment itself was fun to perform. In the future, to calculate a more accurate value, the experiment should be performed in a region where there is guaranteed to be no outside ambient magnetic field. Additionally, the testing apparatus should be rigorously tested with to ensure that it will work with all of the potential values experimenters will try and provide an easy and accurate reading.