Capacitance and the Oscilloscope
Lab 8

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1 Introduction

In this experiment, we explored the intricacies and properties of RC, or Resistance-Capacitance, circuits. RC Circuits are critical to our modern day technology and can be found from cars, to cameras and traffic lights. It is crucial to understand how they affect the current and voltage in a circuit to understand their use cases and to find the best places to use them.

In an RC circuit there are two significant components we need besides a battery and wires, a resistor and a capacitor. Resistors are devices which cause a drop in voltage across it and have a resistance property R. Capacitors are devices composed of 2 adjacent conducting plates, which when pumped with voltage will create a potential difference and accumulate charge and energy. Capacitors have a capacitance property C which determines how much charge can be stored.

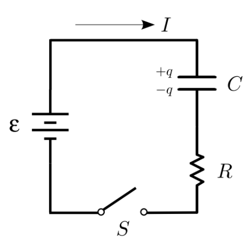

Figure 1: RC Circuit with charging capacitor

To investigate RC circuits we will be looking at two scenarios. One is where there is a charging capacitor in series with a resistor. *Figure 1* shows a circuit schematic of what this might look like. We can use Kirchhoff's loop rule to create an equation for the circuit shown below.

$$\Delta V_{battery} + \Delta V_{capacitor} + \Delta V_{resistor} = 0 \tag{1}$$

We know according to Kirchhoff's Law's that a closed loop circuit has to have a total potential difference of zero. The equation relating resistance and voltage is given below where V is the voltage, I is the current across the device and R is the resistance.

$$V = IR \tag{2}$$

If we are able to determine the current, which we can measure using an ammeter, and know the resistance of the device, we can find the potential difference across it. We can also find the relationship between the potential difference across a capacitor and its capacitance below where Q is the maximum charge on the plates of the capacitor.

$$V = \frac{Q}{C} \tag{3}$$

While we cannot measure the charge itself, we know that the current is the change in charge with respect to time. So if we are able to find an equation for the charge on the capacitor at any given moment in time, we can take the integral to find the equation for current in the circuit. We can use these facts and equations 2 and 3 to create a differential equation we can solve for the charge and current below where ϵ is the potential difference across the battery.

$$\epsilon - R\frac{di}{dt} - \frac{Q}{C} = 0 \tag{4}$$

When can then use this equation to solve for the current with respect to time and the charge with respect to time below.

$$I(t) = \frac{\epsilon}{R}e^{-t/RC} \tag{5}$$

$$q(t) = Q\left(1 - e^{-t/RC}\right) \tag{6}$$

The current in the circuit when charging a capacitor exponentially approaches zero while the charge exponentially approaches the maximum charge.

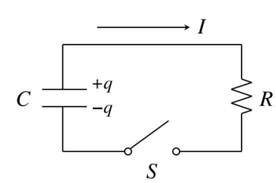

Figure 2: RC Circuit with discharging capacitor

We can also use the loop rule to create an equation for a discharging capacitor. An example circuit schematic of a discharging capacitor can be seen in *Figure 2*. Since the capacitor is pumping current into the circuit this time we will not use a battery to provide a potential difference. Thus, our equation is just going to need to account for the resistor and capacitor. We can create the loop equation below.

$$\frac{Q}{C} + IR = 0 \tag{7}$$

Then we can solve the differential equation, created by substituting I in with dq/dt, similarly to the charging capacitor to find the current and charge time functions below.

$$I(t) = -\frac{\epsilon}{R} e^{-t/RC} \tag{8}$$

$$q(t) = Q e^{-t/RC} \tag{9}$$

When discharging a capacitor, we can see that the charge will exponentially decrease over time and the current will also decrease over time exponentially however, this time the current will be moving in the opposite direction, indicated by the negative sign. This is because the capacitor is discharging so it would make sense the charge over time is negative.

Knowing all of this, we can now begin talking about the experimental procedure and how we used these equations to investigate the properties of RC circuits.

2 Methods

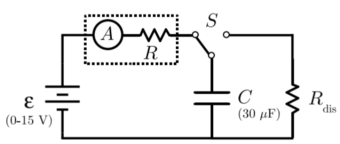

Figure 3: RC Circuit with charging capacitor and ammeter

To start the experiment, we first setup the circuit shown in *Figure 3* to start charging a capacitor. The ammeter we used to measure the current at any moment while the capacitor was charging had an known internal resistance of $10,000\Omega$. We were able to measure the current over time with three different capacitance values since our capacitor had a switch allowing us increase the capacitance in steps of $10\mu F$ from $10 - 30$ by adding additional capacitors in parallel (the capacitance of capacitors in parallel is the sum of their capacitance values). We set our power source to pump $5V$ in the circuit and we made sure to be very careful when handling the equipment to avoid being electrically shocked.

When we turned on the power and closed the switch allowing for the capacitor to start charging, we used an iPhone camera to record the current over time, since we had no other way of measuring it besides eyeballing it in the moment at specific time intervals. We felt recording the data using a video would make recording data a lot easier since we can always look back and review it in case we might have missed something.

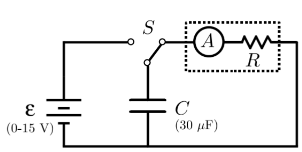

Figure 4: RC Circuit with discharging capacitor and ammeter

After measuring the current over time for the charging capacitor, we moved the ammeter and rewired the circuit to allow for the capacitors to discharge into a resistor after being completely charged at different capacitance values. This can be seen in *Figure 4* For this part, we also used an iPhone to record a video of the current over time.

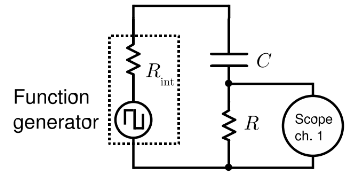

Figure 5: RC Circuit with function wave generator

When the time constant is really small, it would have been very difficult to manually measure the current over time. The time constant is the time it takes for a charging capacitor to reach 63% of its maximum charge or 37% of a discharging capacitors total charge given by the equation below.

$$\tau = RC \tag{10}$$

It is an arbitrary value that lets us characterize the circuit. Because of this, for the second part of the experiment we used an oscilloscope to measure almost instantaneous changes in the current and voltage of the circuit. First we experimented with a voltage wave generator and plugged it into the oscilloscope to understand how it works. Then we connected the function wave generator to a circuit containing a resistor of $10,000\Omega$ resistance and a capacitor with .082 mF capacitance, this setup can be seen in *Figure 5*. We then connected the circuit to the oscilloscope so that it could observe the voltage and current over time. The wave generator would generate a voltage that would oscillate over time and we could see how that affected the circuit. We ended up generating a square wave so that the voltage would be either on or off discretely. After generating the wave in the circuit, we could view the voltage over time and the current over time in the oscilloscope and measure the difference in time from different points on the curve of the graph. We measured the time it takes for the voltage to drop in 37% of its initial value and tested this with 5 different frequencies in the wave generator.

3 Results & Analysis

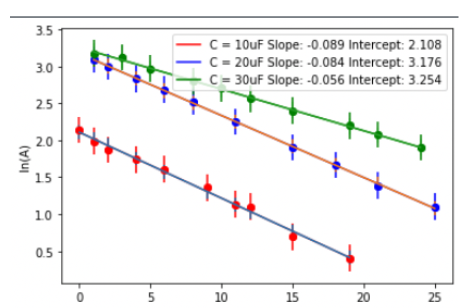

Figure 6: Charging Capacitor ln(I) over time

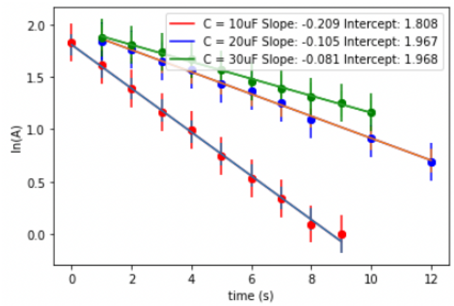

Figure 7: Discharging Capacitor ln(I) over time

For the first part of our experiment, I plotted a scatter plot of the natural log of the current over time in the circuit. The reason I used the natural log is because the time variable is part of an exponential function and we can use natural log to linearize the data. The linearization of the results can be seen in *Figure 6* and *Figure 7*. The slope of the linearization can be used to find the time constant of our circuit. The relationship can be shown in the equation below where I_0 is equal to ϵ/R.

$$\ln I = \ln I_0 - \frac{1}{RC}t \tag{11}$$

From this equation, we can conclude that the y intercept of the graphs is the natural log of the initial current in the circuit and the inverse of the slope is the time constant of the circuit. The error for each of our three trials was $\pm 0.094, \pm 0.098, \pm 0.107$ for the charging capacitor and $\pm 0.106, \pm 0.114, and \pm 0.123$ for our discharging capacitor. With this error, the respective slopes of our charging and discharging capacitor trials are within error of each other. This makes sense since the time constant of a circuit with the same capacitance and resistance should be the same when charging or discharging it. The estimated time constants we get for all of our trials are $11.24, 11.90, 17.86$ for the charging capacitor and $4.78, 9.52, 12.35$.

Something we should expect to see is that when the capacitance is 30uf, the time constant should be 3 times the time constant when the capacitance is 10uf or the ratio should be 3:1 or 3:2 when the time constant is 20uf. With our data, we can see that if we account for error, this would hold almost all of the cases in both or charging and discharging capacitor, but it is significantly more obvious in our discharging capacitor data. For the charging capacitor our $\tau_{30/20} = 1.5 \pm 0.9$, $\tau_{30/10} = 1.59 \pm 0.90$, and $\tau_{20/10} = 1.05 \pm 0.96$ and our discharging capacitor the values were $\tau_{30/20} = 1.30 \pm 0.93$, $\tau_{30/10} = 2.58 \pm 0.86$, and 1.99 ± 0.93. All of our values are within range of the expected values except for the $\tau_{30/10}$ for the charging capacitor, the reason this value is off is because it appears our data for this trial is off. Since the y intercept is our initial current, it should be the same. This was our first trial and I believe the issue comes down to how we were recording data with a video and started the video too late on the first trial since we were doing it for the first time. If the current started from the normal initial current for the other trials, the slope would be steeper resulting in the outcome we would have expected.

For our second part with our oscilloscope we can calculate our expected time constant to be 824. Our resulting time constants we calculated with different frequencies were 860, 840, 840, 840, 880. All of them were slightly off from our expected results. This was our first time using this equipment and it was quite difficult to use so that might be a source of error. Another plausible source of error is the circuit equipment we used. The equipment can be off by a factor of 10% which is huge. If our resistor was 10% off of its original value of $10,000\Omega$, our time constant decreased by 82 seconds. Knowing this it is very possible our equipment's recorded values are incorrect and we are within error.

On the oscilloscope we saw the graph of the voltage over time. The graph of the voltage across the resistor was what we were expecting it to look like. We saw an exponential decaying function that was mirrored across the x axis when the capacitor was discharging. This makes sense because the voltage would decrease exponentially as the current does since the resistance in the circuit is staying the same. Voltage and current are in a proportional relation according to equation 2, which also backs our thinking here. We can apply this reasoning to make a prediction about what the voltage across the capacitor would look like. I believe that it will be the mirror across the x axis of the voltage across the resistor graph. The reason it will have the same shape is because the charge on the capacitor is also an exponential function with the same time constant. Unlike the current, when the capacitor is charging the charge starts at 0 and increases exponentially while the current decreases exponentially. The opposite occurs when the capacitor is discharging. So the voltage across the capacitor will also always be the opposite of the voltage across the resistor since voltage also proportional to the charge on the capacitor according to equation 3.

Since the voltage across the resistor and the capacitor should be opposites and the circuit is a closed loop, the voltage across both elements should be zero.

Our oscilloscope is in parallel with our resistor with an internal resistance of $1 * 10^6\Omega$. If we calculate a new equivalent resistance knowing this we get $9.9 * 10^3\Omega$. If we recalculate the time constant knowing this we get a value of 812, which isn't significantly different from the other value we calculated. If our resistor we used was also $1 * 10^6\Omega$, our equivalent resistance would be $5 * 10^5\Omega$. This would significantly decrease the time constant to 41.

If we tried using significantly larger frequencies in our test we would have issues calculating the time constant because the capacitor would not have enough time to discharge before it starts charging again due to the wave generator. We can figure our a reasonable range of frequencies to use because frequency is the inverse of time. A reasonable amount of time that we would want before creating a new wave would be the amount of time it takes to charge and then discharge the capacitor. So if we are able to find that amount of time and take the inverse of it, we can get an upper bound for the range of frequencies. There doesn't have to be a lower bound since any lower frequency would allow for more time between charging and discharging.

4 Conclusion

In this lab we were able to investigate the properties of RC circuits. We also got to learn about oscilloscopes and wave generators and how we can use them to observe these properties. We calculated our time constants with a resistance of $10,000\Omega$ and a capacitance of 10μF, 20μF, and 30μF respectively $11.24, 11.90, 17.86$ for the charging capacitor and $4.78, 9.52, 12.35$ when discharging. Within error these values agree with what we were expecting. We also saw that our ratios of our time constants also agree with the ratios we were expecting, except for the first trial where we had systematic error calculating our current over time which started the initial current lower than it was supposed to start at. The error was large enough such that the time constants were in agreement while the ratio was slightly out of range of error. When testing our oscilloscope we got results that matched what we were expecting as well. The graph had an exponential shape which matched with the shape we would have expected from the equation relating voltage and current and the equation of current over time. This lab was a great way of learning about how the oscilloscope works and testing its functions.