Growth Model & Sensitivity Analysis - Sobol

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Growth Model & Sobol Sensitivity Analysis

This environmental model was completed as an assignment for the course, Environmental Data Science 230 | Environmental Science & Management: Modeling Environmental Systems. The goal of this assignment was to code a function to compute forest growth and conduct a sobol sensitivity analysis that explores how the estimated maximum and mean forest size varies. This assignment focuses on developing skills to create a model of forest growth and conduct a sobol sensitivity analysis.

Parameters

- K = carrying capacity (kgC)
- r = pre-canopy closure rate
- g = post-canopy closure rate
- threshold = canopy closure threshold (kgC)

Load Libraries

```
library(here)
library(tidyverse)
library(kableExtra)
library(deSolve)
library(sensitivity)
library(purrr)
library(viridis)
```

1. Implement a forest growth rate model

Forest size is measured in units of carbon (C)

```
# source the function

source(here("R","forestgrowthrate.R"))

dgrowthrate
```

```
## function (time, C, parms)
## {
##     if (C < parms$threshold) {
##         dC = parms$r * C
##     }
##     if (C >= parms$threshold) {
##         dC = parms$g * (1 - C/parms$K)
```

```
##     }
##     if (C > parms$K) {
##         dC = 0
##     }
##     return(list(dC))
## }
```

2. Run the model for 300 years (with ODE solver) and plot the result

Parameters for model

- K = 250 kgC (carrying capacity)
- r = 0.01 (exponential growth rate before before canopy closure)
- g = 2 kg/year (linear growth rate after canopy closure)
- threshold = 50 kgC (canopy closure threshold)

```
# create parameter list and specify the initial size and years to run the model

# set parameters
K = 250
r = 0.01
g = 2
threshold = 50

initialsize <- 10
years <- seq(from = 1, to = 300, by = 1)
parms <- list(K = K, r = r, g = g,  threshold = threshold)

#apply solver
results <- ode(initialsize, years, dgrowthrate, parms)

# convert results to data frame
results <- as.data.frame(results)

#add meaningful names to columns of results
colnames(results) = c("year", "C")
```

```
# view sample of df
results_sample <- head(results, n = 10)

results_table <- kable(results_sample,
                         caption = "Sample of Forest Growth ODE Model Results for 10 Years") %>%
  kable_styling(latex_options = "HOLD_position")

results_table
```

Table 1: Sample of Forest Growth ODE Model Results for 10 Years

year	C
1	10.00000
2	10.10050
3	10.20202
4	10.30455
5	10.40811
6	10.51271
7	10.61837
8	10.72508
9	10.83287
10	10.94175

```r
# plot results
model01_plot <- results %>%
  ggplot(aes(x = year, y = C)) +
  geom_point(color = "darkgreen") +
  geom_text(aes(x = 300, y = 190, label = "183.72"), stat = "unique",
size = 3, color = "slategrey") +
  labs(x = "Years", y = "Forest Size (kgC)",
       title = "Forest Growth Rate - 300 Years",
       subtitle = "K = 250 kgC, r = 0.01, g = 2 kg/yr, Canopy closure threshold = 50 kgC") +
  theme_minimal()

model01_plot
```

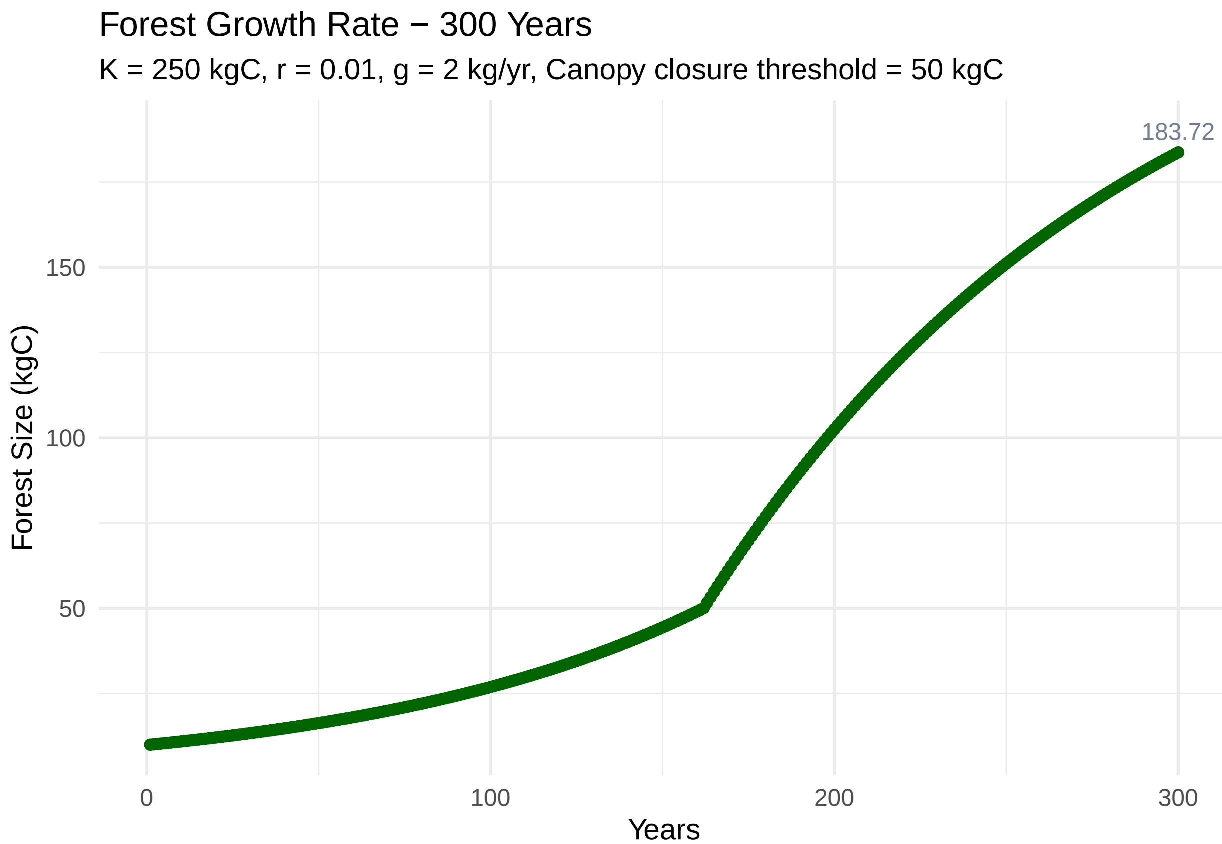

Figure 1: ODE solver results for forest growth rate model

3.A. Run a sobol sensitivity analysis that explores how the estimated maximum and mean forest size (e.g maximum and mean values of C over the 300 years) varies with the pre canopy closure growth rate (r) and post-canopy closure growth rate (g) and canopy closure threshold and carrying capacity(K)

Assume that parameters are all normally distributed with means as given above and standard deviation of 10% of mean value

```
# set the number of parameters
np = 200

K = rnorm(mean = K, sd = K*0.10, n = np)
r = rnorm(mean = r, sd = r*0.10, n = np)
g = rnorm(mean = g, sd = g*0.10, n = np)
threshold = rnorm(mean = threshold, sd = threshold*0.10, n = np)

X1 = cbind.data.frame(r = r, K = K, g = g, threshold = threshold)

# repeat to calculate second set of samples
np = 200
```

```
K = rnorm(mean = K, sd = K*0.10, n = np)
r = rnorm(mean = r, sd = r*0.10, n = np)
g = rnorm(mean = g, sd = g*0.10, n = np)
threshold = rnorm(mean = threshold, sd = threshold*0.10, n = np)

X2 = cbind.data.frame(r = r, K = K, g = g, threshold = threshold)
```

```
# create sobol object and get parameters
sens_forest <- sobolSalt(model = NULL, X1, X2, nboot = 300)

sens_forestSize_df <- as.data.frame(sens_forest$X)

sens_forestSize_df <- sens_forestSize_df %>%
  rename(r = "V1",
         K = "V2",
         g = "V3",
         threshold = "V4")
```

```
p_wrapper = function(threshold, r, g, K, initialsize, years, func) {

  parms <- list(threshold = threshold, r = r, g = g, K = K)

  forest_sensitivity <- ode(func = dgrowthrate, y = initialsize, times = years,
                            parms = parms)

  forest_sensitivity <- as.data.frame(forest_sensitivity)

  colnames(forest_sensitivity) = c("years","C")

  # get metrics

  max_carbon <- max(forest_sensitivity$C)
  mean_carbon <- mean(forest_sensitivity$C)

  return(list(max_carbon=max_carbon, mean_carbon=mean_carbon))
}
```

```
allresults = sens_forestSize_df %>%
  pmap(p_wrapper, initialsize = initialsize, years = years,
       func = dgrowthrate)

allres = allresults %>%
  map_dfr(`[`,c("max_carbon","mean_carbon"))

all_results <- pivot_longer(allres, cols = c(max_carbon, mean_carbon), names_to = "name", values_to = "
```

3.B. Graph the results of the sensitivity analysis as a box plot of maximum forest size and a plot of the two Sobol indices (S and T).

```
results_plot <- all_results %>%
  ggplot(aes(y = carbon, x = name, fill = name)) +
  geom_boxplot() +
  scale_fill_viridis(discrete = TRUE, alpha=0.6) +
```

```
  labs(x= "", y = "Carbon (kg)",
       title = "Sobol Analysis - Forest Growth",
       subtitle = "Max & mean values of Carbon over 300 years") +
  theme_minimal() +
  theme(legend.position="none")

results_plot
```

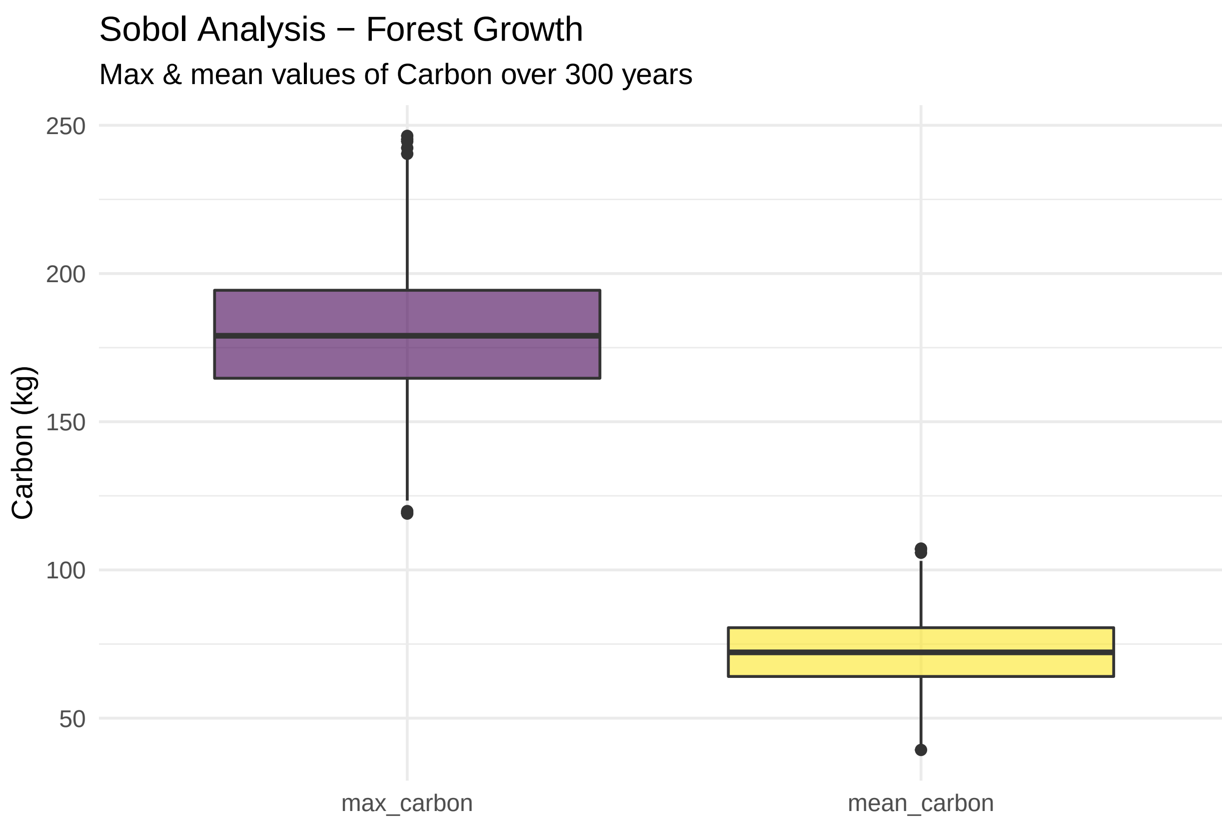

Figure 2: Sobol sensitivity analysis results for forest growth rate model maximum and mean forest size

```
sense_mean <- sensitivity::tell(sens_forest, allres$mean_carbon)

sense_mean_S <- as.data.frame(sense_mean$S)

sense_mean_S <- sense_mean_S %>%
  rowid_to_column(var = "parms")

sense_mean_S[1,1] <- "threshold"
sense_mean_S[2,1] <- "r"
sense_mean_S[3,1] <- "g"
sense_mean_S[4,1] <- "K"

sense_mean_T <- as.data.frame(sense_mean$T)

sense_mean_T <- sense_mean_T %>%
  rowid_to_column(var = "parms")
```

```
sense_mean_T[1,1] <- "threshold"
sense_mean_T[2,1] <- "r"
sense_mean_T[3,1] <- "g"
sense_mean_T[4,1] <- "K"
```

```
sense_max <- sensitivity::tell(sens_forest, allres$max_carbon)

sense_max_S <- as.data.frame(sense_max$S)

sense_max_S <- sense_max_S %>%
  rowid_to_column(var = "parms")

sense_max_S[1,1] <- "threshold"
sense_max_S[2,1] <- "r"
sense_max_S[3,1] <- "g"
sense_max_S[4,1] <- "K"

sense_max_T <- as.data.frame(sense_max$T)

sense_max_T <- sense_max_T %>%
  rowid_to_column(var = "parms")

sense_max_T[1,1] <- "threshold"
sense_max_T[2,1] <- "r"
sense_max_T[3,1] <- "g"
sense_max_T[4,1] <- "K"
```

```
plotSmean <- sense_mean_S %>%
  ggplot(aes(x = original, y = parms, fill = parms)) +
  geom_col() +
  scale_fill_manual(values = c("goldenrod2", "indianred1","slateblue3", "darkolivegreen"), labels = c("
  labs(fill = "Parameter",
       x = "Sobol Sensitivity Value",
       y = "Parameter",
       title = "First Order Sensitivity Index for Mean Forest Growth") +
  theme_minimal()

plotSmean
```

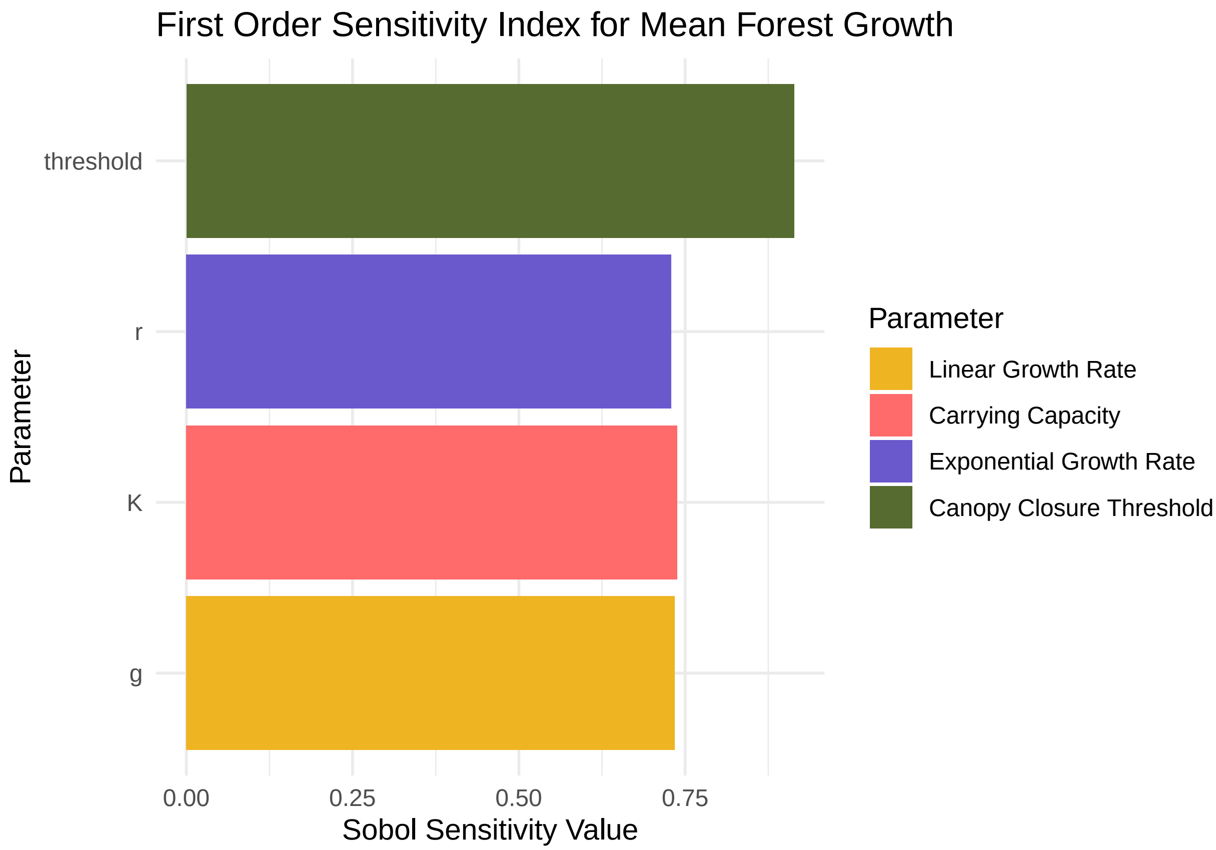

Figure 3: Mean First Order Effect Sensitivity Index of forest growth

```
plotSmax <- sense_max_S %>%
  ggplot(aes(x = original, y = parms, fill = parms)) +
  geom_col() +
  scale_fill_manual(values = c("goldenrod2", "indianred1","slateblue3", "darkolivegreen"),
                    labels = c("Linear Growth Rate", "Carrying Capacity", "Exponential Growth Rate", "C
  labs(fill = "Parameter",
       x = "Sobol Sensitivity Value",
       y = "Parameter",
       title = "First Order Sensitivity Index for Maximum Forest Growth") +
  theme_minimal()

plotSmax
```

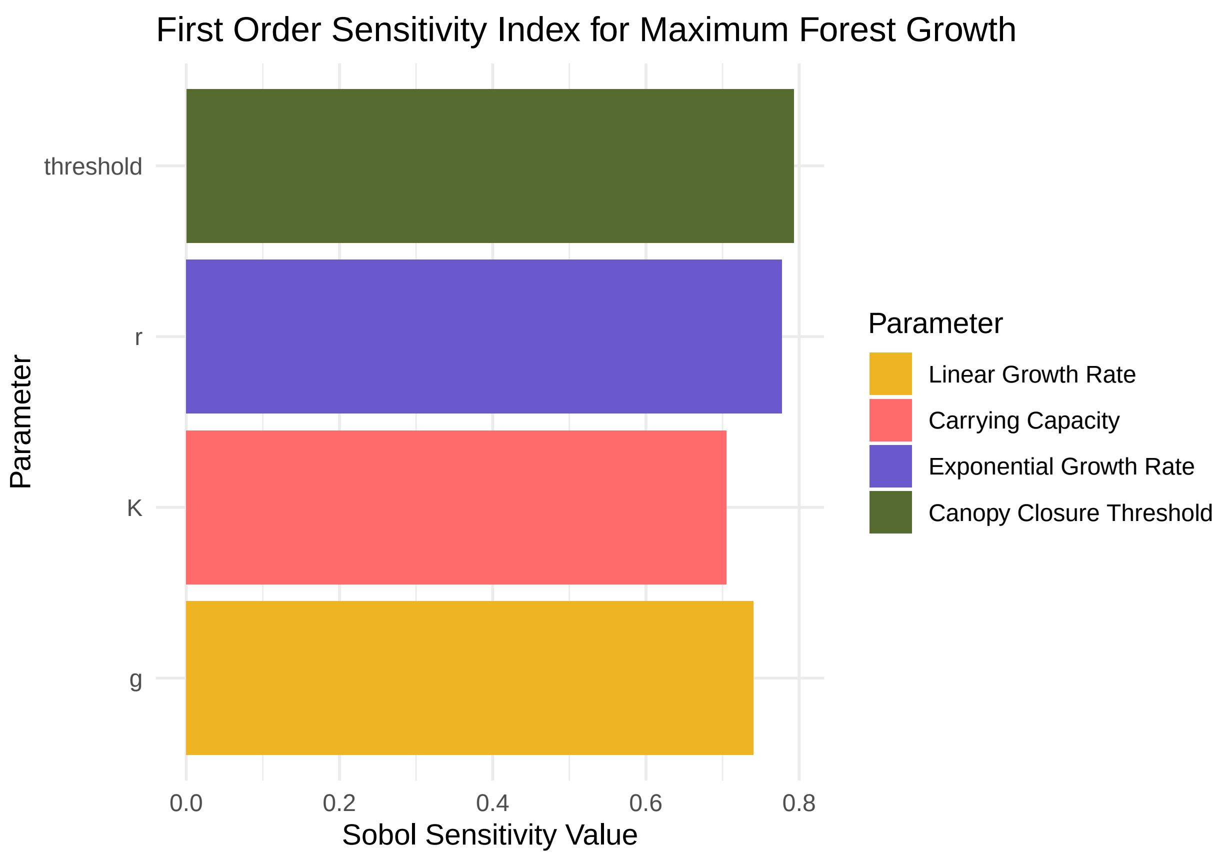

Figure 4: Maximum First Order Effect Sensitivity Index of forest growth

```
plotTmean <- sense_mean_T %>%
  ggplot(aes(x = original, y = parms, fill = parms)) +
  geom_col() +
  scale_fill_manual(values = c("goldenrod2", "indianred1","slateblue3", "darkolivegreen"), labels = c("
  labs(fill = "Parameter",
       x = "Sobol Sensitivity Value",
       y = "Parameter",
       title = "Total Effect Sensitivity Index for Mean Forest Growth") +
  theme_minimal()

plotTmean
```

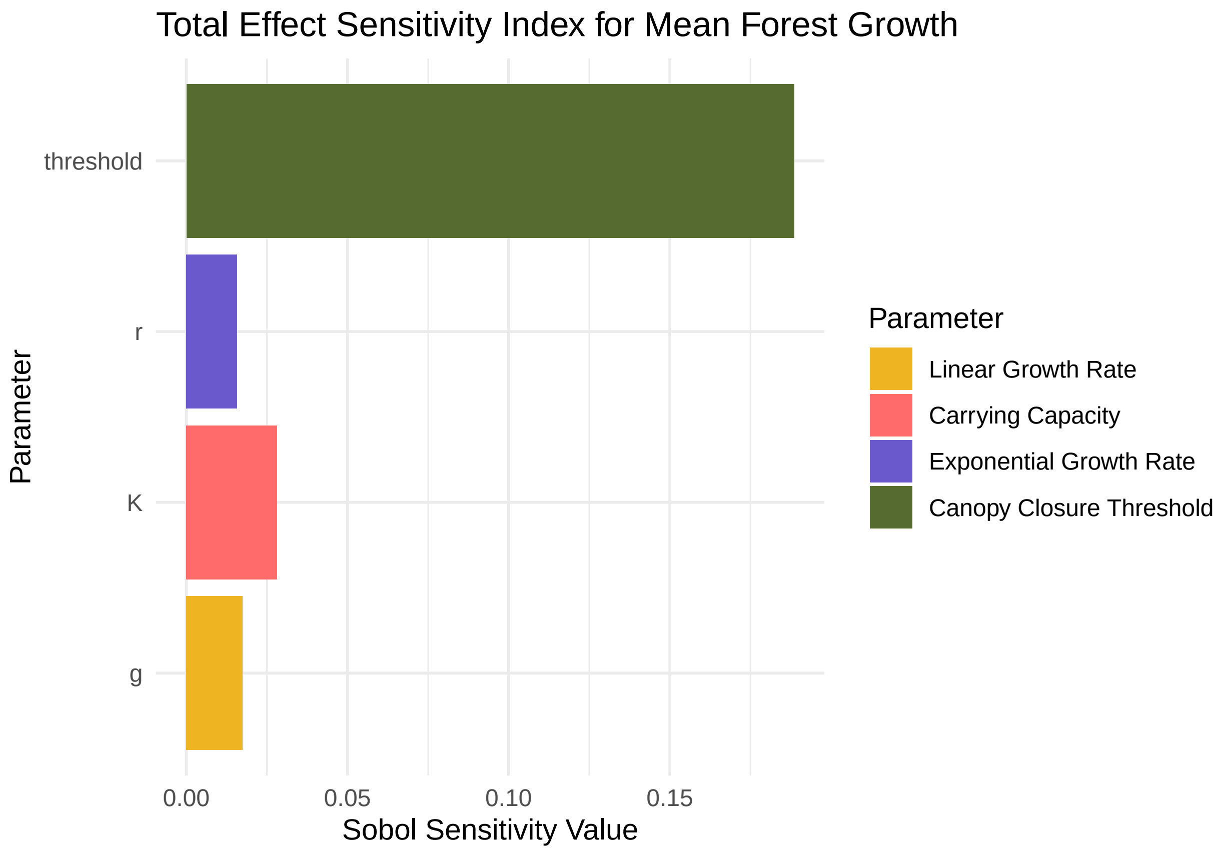

Figure 5: Mean Total Effect Sensitivity Index of forest growth

```
plotTmax <- sense_max_T %>%
  ggplot(aes(x = original, y = parms, fill = parms)) +
  geom_col() +
  scale_fill_manual(values = c("goldenrod2", "indianred1","slateblue3", "darkolivegreen"),
                    labels = c("Linear Growth Rate", "Carrying Capacity", "Exponential Growth Rate", "Ca
  labs(fill = "Parameter",
       x = "Sobol Sensitivity Value",
       y = "Parameter",
       title = "Total Effect Sensitivity Index for Maximum Forest Growth") +
  theme_minimal()

plotTmax
```

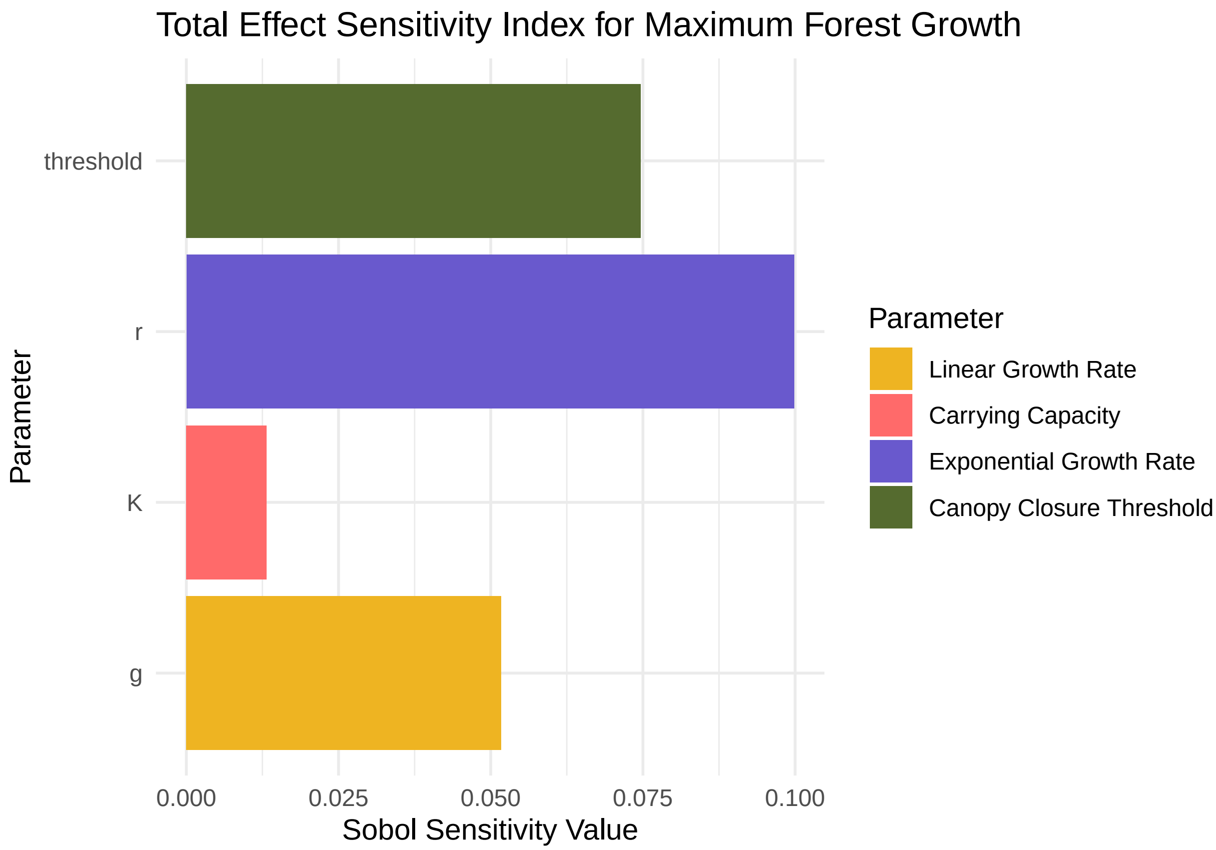

Figure 6: Maximum Total Effect Sensitivity Index of forest growth

3.C. Discuss what the results of your simulation might mean for climate change impacts on forest growth

Based on the boxplot of the sensitivity analysis we can see that the max carbon ranges from ~170-190kg with in the IQR. With max and min values of ~245kg and ~120kg respectively. The mean carbon ranging from ~60-80kg within the IQR and with max and min values of ~115kg and ~30kg. This figure and values give insight to understanding how the model will react for mean and max carbon when providing a range of inputs.

The mean first order index shows canopy closure threshold having the greatest sensitivity to change. The maximum first order index follows a similar trend of parameter sensitivity but with diminished differences between parameters. Based on first order index sensitivity we know canopy closure threshold and exponential growth rate hold the most weight in affecting the model.

The mean total effect index indicates that canopy closure threshold will have the greatest impact on model sensitivity. The Max total effect index indicates that canopy closure threshold and exponential growth rate will impact the model the greatest. These trends are similar to the first order index.

When thinking about these results in regards to climate change, we might hypothesis that as average temperatures increase we could see decreased growth in forests. This would affect the canopy closure threshold and growth rate, thus leading to potentially a greater decrease in forest carbon storage.