2 Problem Formulation

2.1 Problem Description

Given a positive integer d and a graph $G = (V, E)$, where V is set of locations including a designated starting point s and E is a set of weighted edges linking every location to every other location, find a route that:

1. Visits all nodes $V \setminus c$ once.
2. Starts and finishes at s, having visited it d times, without ever visiting consecutively.
3. Minimizes both the cumulative edge weights in the route and the variance in cumulative weight between each visit to s.

2.2 Inputs and Outputs

Inputs:

- d: The number of times s should be visited in a route. Contextually, d represents the number of days a tourist will spend on their trip. $d \in \mathbb{Z}, d > 0$
- $G = (V, E)$: A pair comprising of:
  - V: A set of nodes representing locations the tourist would like to visit. $v \in V, v = (x, y, t)$, a triple comprising of:
    - x: Longitude, indicating the location's geographic east-west position on the earth[1].
      $x \in \mathbb{Q}, -180 \leq x \leq 180$.
    - y: Latitude, indicating the location's geographic north-south position on the earth[2].
      $y \in \mathbb{Q}, -90 \leq y \leq 90$.
    - t: Duration, in minutes, indicating how much time to spend at this location.
      $t \in \mathbb{Z}, t > 0$.
  - E: A set of edges $e \in E$ that connects every node to every other node, bidirectionally. $e = (v_1, v_2, w)$, a triple comprising of:

[1] While the coordinates of our locations are included in V, they are not directly tied to the weight of our edges E, which are based on time and not distance.

[2] See footnote 1

  - v_1: A location representing the origin of the edge.
    $v_1 \in V$.
  - v_2: A location representing the destination of the edge.
    $v_2 \in V$.
  - w: A weight indicating the sum of the time it takes to travel from v_1 to v_2 and the time the tourist wishes to spent at v_2.
    $w \in Z, w > 0$.

- s: Starting point that should be visited d times. Contextually, s represents where the tourist is staying and will return to at the end of each day.
  $s \in V$.

Outputs:

- R: A valid route satisfying all constraints, represented as an ordered sequence of locations.
  $R = [r_1, r_2, \ldots, r_n]$, $r_i \in V$.

2.3 Optimisation

As previously mentioned in the Problem Description, our goal is to find a route that minimises the cumulative weight and the variance in route weight between each visit to s. To accomplish this the following cost function is applied to each route:

$$Cost(R) = W \times (1 + \sigma) \tag{1}$$

Where W is the sum of the weights of all edges traversed in the route and σ is the standard deviation of the sum of weights between each visit to s:

$$\sigma = \sqrt{\frac{\sum_{i=0}^{d}(x_i - \mu)}{d}}, x_i \in X \tag{2}$$

Where R is divided into sections between each visit to s and X is a list of the sum of weights within these sections. μ is the mean cumulative weight of each x_i.