UNIVERSITY OF BIRMINGHAM

# A Comparison of Approaches to Combinatorial Optimisation for Multi-Day Route Planning

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xxxx Words

# Abstract

Write abstract

# Contents

# 1 Introduction

## 1.1 Motivation

Write about reasoning for this project, can copy a little bit over from presentation slides. Explain problem informally. Link into aims and objectives.

## 1.2 Aims and Objectives

Explin goals of the project, link in to methodology.

## 1.3 Methodology

Briefly explain how the project will be carried out. A more thorough description can be provided later on, i.e., when describing algorithms.

Mention code being available from github. Include a code segment and explain how figure captions denote filepath within the repository.

## 1.4 Summary

Explain what is in the rest of the report. "In this report I shall...", cover each section, etc.

# 2 Problem Formulation

## 2.1 Problem Description

Given a positive integer $d$ and a graph $G = (V, E)$, where $V$ is set of locations including a designated starting point $s$ and $E$ is a set of weighted edges linking every location to every other location, find a route that:

1. Visits all nodes $V \setminus s$ once.

2. Starts and finishes at $s$, having visited it $d$ times, without ever visiting consecutively.

3. Minimises both the cumulative edge weights in the route and the variance in cumulative weight between each visit to $s$.

## 2.2 Inputs, Outputs and Design Variables

Update so durations aren't in the graph, instead something like D, mapping a duration to each location.

Inputs:

- $d$: The number of times $s$ should be visited in a route. Contextually, $d$ represents the number of days a tourist will spend on their trip. $d \in \mathbb{Z}, d > 0$

- $G = (V, E)$: A pair comprising:

    - $V$: A set of nodes representing locations the tourist would like to visit. $v \in V, v = (x, y, t)$, a triple comprising:

    - $x$: Longitude, indicating the location's geographic east-west position on the earth $x \in \mathbb{Q}, -180 \leq x \leq 180$.
    - $y$: Latitude, indicating the location's geographic north-south position on the earth $y \in \mathbb{Q}, -90 \leq y \leq 90$.
    - $t$: Duration, in minutes, indicating how much time to spend at this location.

      $t \in \mathbb{Z}, t > 0$.
  - $E$: A set of edges $e \in E$ that connects every node to every other node, bidirectionally. $e = (v_1, v_2, w)$, a triple comprising:
    - $v_1$: A location representing the origin of the edge.

      $v_1 \in V$.
    - $v_2$: A location representing the destination of the edge.

      $v_2 \in V$.
    - $w$: A weight indicating the sum of the time it takes to travel from $v_1$ to $v_2$ and the time the tourist wishes to spent at $v_2$.

      $w \in Z, w > 0$.
- $s$: Starting point that should be visited $d$ times. Contextually, $s$ represents where the tourist is staying and will return to at the end of each day.

  $s \in V$.

Outputs:

- $R$: A valid route satisfying all constraints, represented as an ordered sequence of locations.

  $R = [r_1, r_2, \ldots, r_n], r_i \in V$.

## 2.3 Objective Function

As previously mentioned in the Problem Description, our goal is to find a route that minimises the cumulative weight and the variance in route weight between each visit to $s$. To accomplish this the following cost function is applied to each route:

$$Cost(R) = W/d \times (1 + \sigma^2) \tag{1}$$

Where $W$ is the sum of the weights of all edges traversed in the route and $\sigma^2$ is the variance of the sum of weights between each visit to $s$:

$$W = \sum_{i=0}^{n-1} w(r_i, r_{i+1}), r_i \in R \tag{2}$$

Where $w(r_i, r_{i+1})$ is the weight of the edge between $r_i$ and $r_{i+1}$.

$$\sigma^2 = \frac{\sum_{i=0}^{d}(x_i - \mu)}{d}, x_i \in X \tag{3}$$

Where $R$ is divided into sections between each visit to $s$ and $X$ is a list of the sum of weights within these sections.

$\mu$ is the mean cumulative weight of each $x_i$.

## 2.4 Constraints

A valid solution must satisfy the following constraints:

- The route must visit every node $v \in \{V \setminus s\}$ exactly once:
  $\forall_{v \in \{V \setminus s\}}, |\{i \in \{1, \ldots, n\} : r_i = v\}| = 1$

- The route must visit $s$ exactly $d$ times:

  $|\{i \in \{1, \ldots, n\} : r_i = s\}| = d$

- The route must not visit $s$ consecutively:

  $\forall_{i \in \{1, \ldots, n-1\}}, r_i \neq r_{i+1}$

- The route must end at $s$:

  $r_n = s$

# 3 Literature Review

Plan (and write) literature review

Remember to write about the strengths and weaknesses of existing work. At the end of this chapter you can then give a summary of the gaps that you'll be trying to improve with your work, and on the strengths that you will be maintaining in your work.

This literature review aims to explore existing research and approaches to other combinatorial optimisation problems. There is extensive previous research on various combinatorial problems, for example, the Travelling Salesman Problem, Vehicle Routing Problem and Tourist Trip Design Problem. It is important to understand how these problems are similar to the one presented in this report, as well as where those similarities end. By gaining an understanding of the strengths and limitations of existing approaches to similar problems, we can make better informed decisions regarding which approaches to investigate, how they may be adapted to suit our specific constraints and how they might be implemented in practice. While the approaches taken to these problems may not be directly applicable to our own, it is likely we can adapt their techniques to suit the specific constraints of this problem.

The Travelling Salesman Problem (TSP) is perhaps one of the most studied optimisation problems. This extensive research on the problem has acted as an 'engine of discovery for general-purpose techniques' offering large contributions across a wide range of mathematics.

cite tsp: a computational study, pg 40–41

The TSP can be described simply as: Given a set of locations and the cost of travel between them, find the shortest route that visits each location and returns to the start

cite tsp: a computational study, pg 1

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There are clear similarities between the TSP and our own problem, both involve finding a route that visits every node, while attempting to minimise the time taken travelling said route. In fact, with an input of $d = 1$ our problem becomes the TSP, with greater values of $d$ introducing additional complexity. With this in mind we are able to investigate existing approaches to the TSP and consider how they can be applied to our problem. The TSP is proven to be NP-hard

cite introduction to algorithms, pg 1096–1097

, meaning that there are no known algorithms capable of solving the problem in polynomial time. Considering this, and the aforementioned similarities to our own problem, we can conclude that our problem is also NP-hard.

Discuss how knowledge of NP-hardness can be used to decide which algorithms to investigate.

Discuss a couple of solutions/types of solution

sentence about TSP variants, leads into paragraph about mTSP

## 3.1 Classical Traveling Salesman Problem (TSP)

- Definition and mathematical formulation
- Complexity analysis and NP-hardness
- Key solution approaches
- Relevance and limitations in relation to our specific problem

**Recommended Literature:**

- Applegate, D. L., Bixby, R. E., Chvátal, V., & Cook, W. J. (2006). *The Traveling Salesman Problem: A Computational Study.* Princeton University Press.
- Laporte, G. (1992). "The traveling salesman problem: An overview of exact and approximate algorithms." *European Journal of Operational Research*, 59(2), 231-247.
- Lin, S., & Kernighan, B. W. (1973). "An effective heuristic algorithm for the traveling-salesman problem." *Operations Research*, 21(2), 498-516.
- Helsgaun, K. (2000). "An effective implementation of the Lin–Kernighan heuristic." *European Journal of Operational Research*, 126(1), 106-130.

## 3.2 Multiple Traveling Salesman Problem (mTSP)

- Extension of the TSP with multiple agents

- Mathematical formulation differences from TSP
- Application to multi-day planning scenarios
- Connection to our requirement of visiting the starting point $d$ times

**Recommended Literature:**

- Bektas, T. (2006). "The multiple traveling salesman problem: an overview of formulations and solution procedures." *Omega*, 34(3), 209-219.
- Kara, I., & Bektas, T. (2006). "Integer linear programming formulations of multiple salesman problems and its variations." *European Journal of Operational Research*, 174(3), 1449-1458.
- Gavish, B., & Srikanth, K. (1986). "An optimal solution method for large-scale multiple traveling salesmen problems." *Operations Research*, 34(5), 698-717.
- Carter, A. E., & Ragsdale, C. T. (2006). "A new approach to solving the multiple traveling salesperson problem using genetic algorithms." *European Journal of Operational Research*, 175(1), 246-257.

## 3.3 Vehicle Routing Problem (VRP) and Variants

- Basic VRP definition and formulation
- Vehicle Routing Problem with Multiple Trips (VRPMT)
- Capacitated VRP and other variants

- Relevance to our balanced route planning requirement

**Recommended Literature:**

- Toth, P., & Vigo, D. (Eds.). (2002). *The Vehicle Routing Problem.* SIAM Monographs on Discrete Mathematics and Applications.
- Cattaruzza, D., Absi, N., & Feillet, D. (2016). "Vehicle routing problems with multiple trips." *4OR*, 14(3), 223-259.
- Brandão, J., & Mercer, A. (1997). "A tabu search algorithm for the multi-trip vehicle routing and scheduling problem." *European Journal of Operational Research*, 100(1), 180-191.
- Olivera, A., & Viera, O. (2007). "Adaptive memory programming for the vehicle routing problem with multiple trips." *Computers & Operations Research*, 34(1), 28-47.

## 3.4 Tourist Trip Design Problem (TTDP)

- Problem definition focusing on tourist-specific constraints
- Time-dependent considerations and point-of-interest selection
- Personalization aspects in tourist routing
- Direct applicability to our problem's tourism context

**Recommended Literature:**

- Vansteenwegen, P., Souffriau, W., & Van Oudheusden, D. (2011). “The orienteering problem: A survey.” *European Journal of Operational Research*, 209(1), 1-10.

- Gavalas, D., Konstantopoulos, C., Mastakas, K., & Pantziou, G. (2014). “A survey on algorithmic approaches for solving tourist trip design problems.” *Journal of Heuristics*, 20(3), 291-328.

- Souffriau, W., Vansteenwegen, P., Vanden Berghe, G., & Van Oudheusden, D. (2013). “The planning of cycle trips in the province of East Flanders.” *Omega*, 41(3), 522-531.

- Garcia, A., Vansteenwegen, P., Arbelaitz, O., Souffriau, W., & Linaza, M. T. (2013). “Integrating public transportation in personalised electronic tourist guides.” *Computers & Operations Research*, 40(3), 758-774.

## 3.5 Multi-Objective Optimization in Routing Problems

- Balancing competing objectives (like total weight vs. variance)
- Pareto optimality concepts
- Solution approaches for multi-objective routing
- Applicability to our dual-objective function

**Recommended Literature:**

- Jozefowiez, N., Semet, F., & Talbi, E. G. (2008). “Multi-objective vehicle routing problems.” *European Journal of Operational Research*, 189(2), 293-309.

- Paquete, L., & Stützle, T. (2006). “A study of stochastic local search algorithms for the biobjective QAP with correlated flow matrices.” *European Journal of Operational Research*, 169(3), 943-959.

- Laporte, G., Semet, F., Matl, P., & Voß, S. (2018). “Multi-objective vehicle routing problem.” *Operations Research Perspectives*, 5, 50-57.

- Coello, C. A. C., Lamont, G. B., & Van Veldhuizen, D. A. (2007). *Evolutionary Algorithms for Solving Multi-Objective Problems.* Springer.

## 3.6 Balance-Oriented Routing Problems

- Problems focusing on workload balancing
- Min-max objectives in routing
- Variance minimization approaches
- Connection to our goal of minimizing variance between trips

**Recommended Literature:**

- Jozefowiez, N., Semet, F., & Talbi, E. G. (2009). “An evolutionary algorithm for the vehicle routing problem with route balancing.” *European Journal of Operational Research*, 195(3), 761-769.

- Dell'Amico, M., Monaci, M., Pagani, C., & Vigo, D. (2007). "Heuristic approaches for the fleet size and mix vehicle routing problem with time windows." *Transportation Science*, 41(4), 516-526.

- Lee, T. R., & Ueng, J. H. (1999). "A study of vehicle routing problems with load-balancing." *International Journal of Physical Distribution & Logistics Management*, 29(10), 646-657.

- Liu, R., Xie, X., Augusto, V., & Rodriguez, C. (2013). "Heuristic algorithms for a vehicle routing problem with simultaneous delivery and pickup and time windows in home health care." *European Journal of Operational Research*, 230(3), 475-486.

## 3.7 Time-Dependent Routing Problems

- Integration of visit durations into routing decisions
- Time windows and scheduling constraints
- Solution approaches for time-dependent problems
- Relevance to our edge weight definition that incorporates visit duration

**Recommended Literature:**

- Ichoua, S., Gendreau, M., & Potvin, J. Y. (2003). "Vehicle dispatching with time-dependent travel times." *European Journal of Operational Research*, 144(2), 379-396.

- Donati, A. V., Montemanni, R., Casagrande, N., Rizzoli, A. E., & Gambardella, L. M. (2008). "Time dependent vehicle routing problem with a multi ant colony system." *European Journal of Operational Research*, 185(3), 1174-1191.

- Hashimoto, H., Yagiura, M., & Ibaraki, T. (2008). "An iterated local search algorithm for the time-dependent vehicle routing problem with time windows." *Discrete Optimization*, 5(2), 434-456.
- Figliozzi, M. A. (2012). "The time dependent vehicle routing problem with time windows: Benchmark problems, an efficient solution algorithm, and solution characteristics." *Transportation Research Part E: Logistics and Transportation Review*, 48(3), 616-636.

## 3.8 Synthesis and Research Gaps

- Comparison of problem characteristics across reviewed literature
- Key methodological approaches applicable to our problem
- Identification of research gaps in addressing our specific problem constraints
- Potential directions for adaptation of existing methodologies

**Recommended Literature:**

- Laporte, G. (2009). "Fifty years of vehicle routing." *Transportation Science*, 43(4), 408-416.
- Cordeau, J. F., Gendreau, M., Laporte, G., Potvin, J. Y., & Semet, F. (2002). "A guide to vehicle routing heuristics." *Journal of the Operational Research Society*, 53(5), 512-522.

- Eksioglu, B., Vural, A. V., & Reisman, A. (2009). "The vehicle routing problem: A taxonomic review." *Computers & Industrial Engineering*, 57(4), 1472-1483.

- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2013). "Heuristics for multi-attribute vehicle routing problems: A survey and synthesis." *European Journal of Operational Research*, 231(1), 1-21.

## 3.9 Conclusion

- Summary of most relevant approaches
- Recommendation for methodological direction
- Justification for selected approach based on literature findings

**Recommended Literature:**

- Gendreau, M., Potvin, J. Y., Bräumlaysy, O., Hasle, G., & Løkketangen, A. (2008). "Metaheuristics for the vehicle routing problem and its extensions: A categorized bibliography." In *The vehicle routing problem: Latest advances and new challenges* (pp. 143-169). Springer.

- Bräysy, O., & Gendreau, M. (2005). "Vehicle routing problem with time windows, Part I: Route construction and local search algorithms." *Transportation Science*, 39(1), 104-118.

- Glover, F., & Kochenberger, G. A. (Eds.). (2003). *Handbook of Metaheuristics.* Springer.

- Talbi, E. G. (2009). *Metaheuristics: From Design to Implementation*. John Wiley & Sons.

# 4 Algorithms Investigated

There should be a link either here or at end of literature which forms the basic for different methods (clustering, routing, trip generation).

Paragraph describing different types of algorithm used (Routing then cluster, Cluster then Routing, Genetic, etc.)

Remember to justify the choice of algorithms. You may also need to explain how to adopt these algorithms in your work. A figure showing the ralationship between different components of your work may also help.

## 4.1 Clustering

Clustering as a concept can be described as 'the unsupervised classification of patterns (observation, data items, or feature vectors) into groups (clusters)'

cite Data Clustering: A review, A.K. Jain

. In our problem, clustering will be used to group locations together to form an itinerary for each day of the trip. These clusters (or days) will then be used as an input for some routing algorithm, which will try and find an optimal route for each day. These routes can then be combined to form a complete route for the trip, which can be evaluated using our cost function. The goal of our clustering algorithms is to find a set of clusters that, when combined with some routing algorithm, will produce a route that minimises the cost function.

The clustering algorithms implemented in this project are: K-Means, Genetic Clustering and Genetic Centroid-based Clustering.

### 4.1.1 K-Means

K-Means is an iterative clustering algorithm that defines its clusters using a set of centroids (means) which are given a location in the input space. The algorithm starts by initialising random centroids and iteratively improving the clustering from there, continuing until the algorithm converges (on a local optimum) or an iteration limit is reached. While this algorithm may not find the best solution, it is rather quick, with a time complexity of $O(kni)$, where $k$ is the number of clusters, $n$ is the number of locations and $i$ is the maximum allowed number of iterations.

In our implementation of K-Means we will initialise our centroids by generating random geographic coordinates in a similar area to the locations in our input. We will be using the coordinates of our locations to calculate the Euclidean distances between locations and centroids, these locations will be assigned to the cluster of the closest centroid.

```python
# Gets a matrix of distances from each location to each centroid
distances = np.linalg.norm(
    coordinates[:, np.newaxis, :2] - centroids[:, :2], axis=2)

# For each location (index in distance matrix) gets the index for the
# centroid with the smallest distance
clusters = np.argmin(distances, axis=1)
return clusters
```

Figure 4.1: Clustering._assign_nodes_to_centroid in algorithms\clustering.py

After assignment, the centroids are recalculated such that their coordinates are the average of all locations assigned to their cluster.

```python
computed_means = np.empty((num_days, 2))

for i in range(num_days):
    cluster = coordinates[coordinates[:, 2] == i, :2]
    computed_means[i] = cluster.mean(axis=0)
return computed_means
```

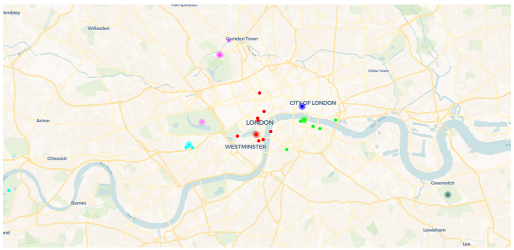
Figure 4.2: KMeans._compute_means in algorithms\clustering.py

These steps of cluster assignment and centroid recalculation are repeated until either a maximum allowed number of iterations is reached, or until the algorithm converges on an optimum solution. Our convergence criterion is that the centroids stop changing between iterations, i.e., the centroids are the same as the previous iteration.

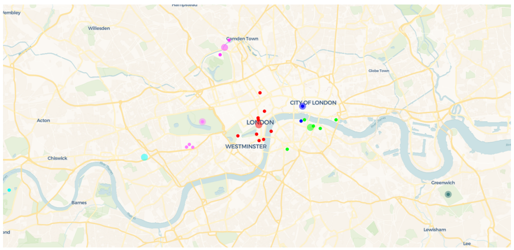
```python
for _ in range(self.maximum_iterations):
    cluster_assignments = self._assign_nodes_to_centroid(coordinates,
                                                          means)
    coordinates[:, 2] = cluster_assignments

    if np.array_equal(cluster_assignments, previous_clusters):
        break
    previous_clusters = np.array(cluster_assignments, copy=True)

    means = self._compute_means(coordinates, num_days)

return cluster_assignments
```

Figure 4.3: KMeans.find_clusters in algorithms\clustering.py

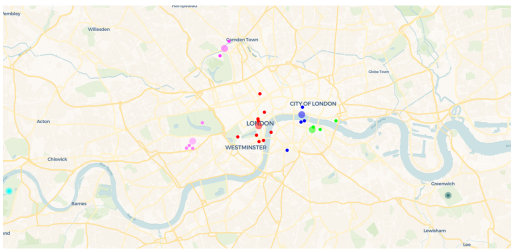
Below is an example of the K-Means algorithm run on an input of 25 points of interest around London over seven days.



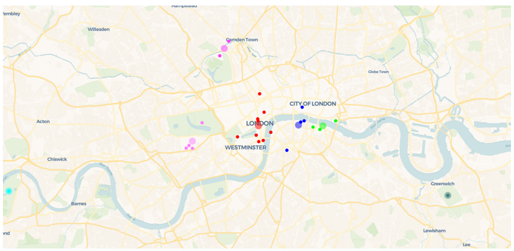
(a) Step1



(b) Step2



(c) Step3



(d) Step4

### 4.1.2 Genetic Centroid-based Clustering

Explain genetic centroid-based clustering and how it differs from general clustering.

### 4.1.3 Genetic Clustering

Explain genetic clustering

## 4.2 Routing

Explain purpose of routing/goal of algorithms.

### 4.2.1 Brute Force

Write brute force explanation

The brute force algorithm is an exhaustive algorithm that tries every possible route to find one with the least cost. By checking every route it is guaranteed to find the optimal route, however, its computational cost becomes impractical as input size grows, having a time complexity of $O(n!)$.

Maybe cite time complexity of brute force?

Considering we will be comparing algorithms based on their speed and the quality of their results, brute force is a useful benchmark, providing a lower bound for speed and an upper bound for quality.

In our brute force implementation, where n is the number of locations in the route, we will be generating all $n - 1!$ permutations of the set

$\{1, 2, \ldots, n-1\}$, with each permutation representing the order of locations visited in a route. Each route will be evaluated according to our optimisation function, and the route with the lowest cost will be returned. We only need to consider $n-1!$ permutations because all our routes will start and end at the same location.

### 4.2.2 Greedy Routing

Explain greedy routing algorithm

Greedy routing

### 4.2.3 Gift Wrapping

Explain gift wrapping algorithm

Something like: "Once gift wrapping has found a convex hull, a greedy insertion algorithm is used to find the optimal route within the convex hull."

### 4.2.4 Genetic Routing

Explain genetic routing

## 4.3 Route Insertion

Explain route insertion, how it is used in route planning and the goal of our algorithms.

### 4.3.1 Brute Force

Explain how brute force algorithm can be modified for route insertion.

### 4.3.2 Greedy Insertion

Explain how greedy algorithm can be modified for route insertion.

## 4.4 Trip Generation

Explain trip generation, how it is used in route planning and the goal of our algorithms.

### 4.4.1 Brute Force

Explain how brute force algorithm can be modified for trip generation.

### 4.4.2 Genetic Trip Generation

Explain genetic trip generation

# 5 Evaluation and Comparison

## 5.1 Methodology

### 5.1.1 Constraints

## 5.2 Results & Analysis

Gather data for London and Birmingham with POI

Write paragraph about experiment process. Comparison based on computation time and route evaluation. Describe how route is evaluated. Describe data being tested on.

Present comparison of different combinations of algorithms on different inputs.

Reflect about the questionns you are trying to answer with your evaluation. You can have onne subsection for each question that you are trying to answer. It's also important to justify your choices when it comes to the methodology used for evaluation.

# 6 Conclusion and Future Work

Write conclusion, discuss results, comparison of algorithms, etc.

## 6.1 Project Reflection

Reflect on the project, what went well, what didn't go well, what I would do differently. This should lead into future work.

## 6.2 Future work

Discuss further work, what I will be doing to improve the project

Discuss spectral clustering

Discuss Neural Networks

# 7 Bibliography