UNIVERSITY OF BIRMINGHAM

A Comparison of Approaches to Combinatorial Optimisation for Multi-Day Route Planning

Jacob Luck

2338114

B.Sc. Computer Science with a Year in Industry

Project Supervisor - Leandro Minku

April 2025 xxxx Words

Abstract

Write abstract

Contents

1 Introduction **4**
- 1.1 Motivation 4
- 1.2 Aims and Objectives 4
- 1.3 Methodology 4
- 1.4 Summary 4

2 Problem Formulation **5**
- 2.1 Problem Description 5
- 2.2 Inputs, Outputs and Design Variables 5
- 2.3 Objective Function 6
- 2.4 Constraints 7

3 Literature Review **8**
- 3.1 Classical Traveling Salesman Problem (TSP) 9
- 3.2 Multiple Traveling Salesman Problem (mTSP) 9
- 3.3 Vehicle Routing Problem (VRP) and Variants 10
- 3.4 Tourist Trip Design Problem (TTDP) 10
- 3.5 Multi-Objective Optimization in Routing Problems 11
- 3.6 Balance-Oriented Routing Problems 12
- 3.7 Time-Dependent Routing Problems 12
- 3.8 Synthesis and Research Gaps 13
- 3.9 Conclusion 14

4 Algorithms Investigated **15**
- 4.1 Routing 15
  - 4.1.1 Brute Force 15
  - 4.1.2 Greedy Routing/Insertion 15
  - 4.1.3 Gift Wrapping 15
- 4.2 Clustering 15
  - 4.2.1 K-Means 16
- 4.3 Trip Generation 16
  - 4.3.1 Brute Force 16
- 4.4 Genetic Algorithms 16
  - 4.4.1 General Clustering 16
  - 4.4.2 Centroid-based Clustering 16
  - 4.4.3 Routing 16
  - 4.4.4 Trip Generation 16

5 Implementation **17**
5.1 Constraints 17

6 Evaluation and Comparison **18**

7 Conclusion and Future Work **19**
7.1 Project Reflection 19
7.2 Future work 19

8 Bibliography **20**

1 Introduction

1.1 Motivation

Write about reasoning for this project, can copy a little bit over from presentation slides. Explain problem informally. Link into aims and objectives.

1.2 Aims and Objectives

Explin goals of the project, link in to methodology.

1.3 Methodology

Briefly explain how the project will be carried out. A more thorough description can be provided later on, i.e., when describing algorithms.

1.4 Summary

Explain what is in the rest of the report. "In this report I shall...", cover each section, etc.

2 Problem Formulation

2.1 Problem Description

Given a positive integer d and a graph $G = (V, E)$, where V is set of locations including a designated starting point s and E is a set of weighted edges linking every location to every other location, find a route that:

1. Visits all nodes $V \setminus s$ once.
2. Starts and finishes at s, having visited it d times, without ever visiting consecutively.
3. Minimises both the cumulative edge weights in the route and the variance in cumulative weight between each visit to s.

2.2 Inputs, Outputs and Design Variables

Update so durations aren't in the graph, instead something like D, mapping a duration to each location.

Inputs:

- d: The number of times s should be visited in a route. Contextually, d represents the number of days a tourist will spend on their trip. $d \in \mathbb{Z}, d > 0$
- $G = (V, E)$: A pair comprising:
  - V: A set of nodes representing locations the tourist would like to visit. $v \in V, v = (x, y, t)$, a triple comprising:
    - x: Longitude, indicating the location's geographic east-west position on the earth[1].
      $x \in \mathbb{Q}, -180 \leq x \leq 180$.
    - y: Latitude, indicating the location's geographic north-south position on the earth[2].
      $y \in \mathbb{Q}, -90 \leq y \leq 90$.
    - t: Duration, in minutes, indicating how much time to spend at this location.
      $t \in \mathbb{Z}, t > 0$.

[1] While the coordinates of our locations are included in V, they are not directly tied to the weight of our edges E, which are based on time and not distance.

[2] See footnote 1

- E: A set of edges $e \in E$ that connects every node to every other node, bidirectionally. $e = (v_1, v_2, w)$, a triple comprising:
  - v_1: A location representing the origin of the edge.
    $v_1 \in V$.
  - v_2: A location representing the destination of the edge.
    $v_2 \in V$.
  - w: A weight indicating the sum of the time it takes to travel from v_1 to v_2 and the time the tourist wishes to spent at v_2.
    $w \in Z, w > 0$.
- s: Starting point that should be visited d times. Contextually, s represents where the tourist is staying and will return to at the end of each day.
  $s \in V$.

Outputs:

- R: A valid route satisfying all constraints, represented as an ordered sequence of locations.
  $R = [r_1, r_2, \ldots, r_n], r_i \in V$.

2.3 Objective Function

As previously mentioned in the Problem Description, our goal is to find a route that minimises the cumulative weight and the variance in route weight between each visit to s. To accomplish this the following cost function is applied to each route:

Update equation for cost, talk to leandro about how to handle changing this.

$$Cost(R) = W \times (1 + \sigma) \tag{1}$$

Where W is the sum of the weights of all edges traversed in the route and σ is the standard deviation of the sum of weights between each visit to s:

$$W = \sum_{i=0}^{n-1} w(r_i, r_{i+1}), r_i \in R \tag{2}$$

Where $w(r_i, r_{i+1})$ is the weight of the edge between r_i and r_{i+1}.

$$\sigma = \sqrt{\frac{\sum_{i=0}^{d}(x_i - \mu)}{d}}, x_i \in X \tag{3}$$

Where R is divided into sections between each visit to s and X is a list of the sum of weights within these sections.
μ is the mean cumulative weight of each x_i.

2.4 Constraints

A valid solution must satisfy the following constraints:

- The route must visit every node $v \in \{V \setminus s\}$ exactly once:
  $\forall_{v \in \{V \setminus s\}}, |\{i \in \{1, \dots, n\} : r_i = v\}| = 1$
- The route must visit s exactly d times:
  $|\{i \in \{1, \dots, n\} : r_i = s\}| = d$
- The route must not visit s consecutively:
  $\forall_{i \in \{1, \dots, n-1\}}, r_i \neq r_{i+1}$
- The route must end at s:
  $r_n = s$

3 Literature Review

Plan (and write) literature review

Remember to write about the strengths and weaknesses of existing work. At the end of this chapter you can then give a summary of the gaps that you'll be trying to improve with your work, and on the strengths that you will be maintaining in your work.

This literature review aims to explore existing research and approaches to other combinatorial optimization problems. There is extensive previous research on various combinatorial problems, for example, the Travelling Salesman Problem (TSP), Vehicle Routing Problem (VRP) and Tourist Trip Design Problem (TTDP). It is important to understand how these problems are similar to the one presented in this report, as well as where those similarities end. By gaining an understanding of the strengths and limitations of existing approaches to similar problems, we can make better informed decisions regarding which approaches to investigate, how they may be adapted to suit our specific constraints and how they might be implemented in practice. While the approaches taken to these problems may not be directly applicable to our own, it is likely we can adapt their techniques to suit the specific constraints of this problem.

Since its formulation in the 1930s, The Traveling Salesman Problem (TSP) has become one of the most extensively studied problems in combinatorial optimization. In its simplest form, TSP can be stated as follows: given a set of cities and the distances between each pair of cities, find the shortest possible route that visits each city exactly once and returns to the origin city [**?**]. The simplicity of this formulation belies the computational complexity of finding optimal solutions, particularly as the number of cities increases

The [problem1] was first defined in [paper1] as [definition], since its introduction [problem1] has been widely studied, including many variants of the original problem. blah blah blah. This is similar to our problem because [reason]. Another problem is [problem1], which is defined by [paper2] as [definition]. blah blah blah. This is similar to our problem because [reason]. [paper3] proposes a solution to [problem] using [approach]. blah blah blah, pros, cons, etc.//

READ, YOU NEED TO READ THE PAPERS stop faffing about and read the papers

3.1 Classical Traveling Salesman Problem (TSP)

- Definition and mathematical formulation
- Complexity analysis and NP-hardness
- Key solution approaches
- Relevance and limitations in relation to our specific problem

Recommended Literature:

- Applegate, D. L., Bixby, R. E., Chvátal, V., & Cook, W. J. (2006). *The Traveling Salesman Problem: A Computational Study.* Princeton University Press.
- Laporte, G. (1992). "The traveling salesman problem: An overview of exact and approximate algorithms." *European Journal of Operational Research*, 59(2), 231-247.
- Lin, S., & Kernighan, B. W. (1973). "An effective heuristic algorithm for the traveling-salesman problem." *Operations Research*, 21(2), 498-516.
- Helsgaun, K. (2000). "An effective implementation of the Lin–Kernighan heuristic." *European Journal of Operational Research*, 126(1), 106-130.

3.2 Multiple Traveling Salesman Problem (mTSP)

- Extension of the TSP with multiple agents
- Mathematical formulation differences from TSP
- Application to multi-day planning scenarios
- Connection to our requirement of visiting the starting point d times

Recommended Literature:

- Bektas, T. (2006). "The multiple traveling salesman problem: an overview of formulations and solution procedures." *Omega*, 34(3), 209-219.

- Kara, I., & Bektas, T. (2006). "Integer linear programming formulations of multiple salesman problems and its variations." *European Journal of Operational Research*, 174(3), 1449-1458.

- Gavish, B., & Srikanth, K. (1986). "An optimal solution method for large-scale multiple traveling salesmen problems." *Operations Research*, 34(5), 698-717.

- Carter, A. E., & Ragsdale, C. T. (2006). "A new approach to solving the multiple traveling salesperson problem using genetic algorithms." *European Journal of Operational Research*, 175(1), 246-257.

3.3 Vehicle Routing Problem (VRP) and Variants

- Basic VRP definition and formulation
- Vehicle Routing Problem with Multiple Trips (VRPMT)
- Capacitated VRP and other variants
- Relevance to our balanced route planning requirement

Recommended Literature:

- Toth, P., & Vigo, D. (Eds.). (2002). *The Vehicle Routing Problem.* SIAM Monographs on Discrete Mathematics and Applications.

- Cattaruzza, D., Absi, N., & Feillet, D. (2016). "Vehicle routing problems with multiple trips." *4OR*, 14(3), 223-259.

- Brandão, J., & Mercer, A. (1997). "A tabu search algorithm for the multi-trip vehicle routing and scheduling problem." *European Journal of Operational Research*, 100(1), 180-191.

- Olivera, A., & Viera, O. (2007). "Adaptive memory programming for the vehicle routing problem with multiple trips." *Computers & Operations Research*, 34(1), 28-47.

3.4 Tourist Trip Design Problem (TTDP)

- Problem definition focusing on tourist-specific constraints
- Time-dependent considerations and point-of-interest selection
- Personalization aspects in tourist routing

- Direct applicability to our problem's tourism context

Recommended Literature:

- Vansteenwegen, P., Souffriau, W., & Van Oudheusden, D. (2011). "The orienteering problem: A survey." *European Journal of Operational Research*, 209(1), 1-10.
- Gavalas, D., Konstantopoulos, C., Mastakas, K., & Pantziou, G. (2014). "A survey on algorithmic approaches for solving tourist trip design problems." *Journal of Heuristics*, 20(3), 291-328.
- Souffriau, W., Vansteenwegen, P., Vanden Berghe, G., & Van Oudheusden, D. (2013). "The planning of cycle trips in the province of East Flanders." *Omega*, 41(3), 522-531.
- Garcia, A., Vansteenwegen, P., Arbelaitz, O., Souffriau, W., & Linaza, M. T. (2013). "Integrating public transportation in personalised electronic tourist guides." *Computers & Operations Research*, 40(3), 758-774.

3.5 Multi-Objective Optimization in Routing Problems

- Balancing competing objectives (like total weight vs. variance)
- Pareto optimality concepts
- Solution approaches for multi-objective routing
- Applicability to our dual-objective function

Recommended Literature:

- Jozefowiez, N., Semet, F., & Talbi, E. G. (2008). "Multi-objective vehicle routing problems." *European Journal of Operational Research*, 189(2), 293-309.
- Paquete, L., & Stützle, T. (2006). "A study of stochastic local search algorithms for the biobjective QAP with correlated flow matrices." *European Journal of Operational Research*, 169(3), 943-959.
- Laporte, G., Semet, F., Matl, P., & Voß, S. (2018). "Multi-objective vehicle routing problem." *Operations Research Perspectives*, 5, 50-57.

- Coello, C. A. C., Lamont, G. B., & Van Veldhuizen, D. A. (2007). *Evolutionary Algorithms for Solving Multi-Objective Problems.* Springer.

3.6 Balance-Oriented Routing Problems

- Problems focusing on workload balancing
- Min-max objectives in routing
- Variance minimization approaches
- Connection to our goal of minimizing variance between trips

Recommended Literature:

- Jozefowiez, N., Semet, F., & Talbi, E. G. (2009). "An evolutionary algorithm for the vehicle routing problem with route balancing." *European Journal of Operational Research*, 195(3), 761-769.
- Dell'Amico, M., Monaci, M., Pagani, C., & Vigo, D. (2007). "Heuristic approaches for the fleet size and mix vehicle routing problem with time windows." *Transportation Science*, 41(4), 516-526.
- Lee, T. R., & Ueng, J. H. (1999). "A study of vehicle routing problems with load-balancing." *International Journal of Physical Distribution & Logistics Management*, 29(10), 646-657.
- Liu, R., Xie, X., Augusto, V., & Rodriguez, C. (2013). "Heuristic algorithms for a vehicle routing problem with simultaneous delivery and pickup and time windows in home health care." *European Journal of Operational Research*, 230(3), 475-486.

3.7 Time-Dependent Routing Problems

- Integration of visit durations into routing decisions
- Time windows and scheduling constraints
- Solution approaches for time-dependent problems
- Relevance to our edge weight definition that incorporates visit duration

Recommended Literature:

- Ichoua, S., Gendreau, M., & Potvin, J. Y. (2003). "Vehicle dispatching with time-dependent travel times." *European Journal of Operational Research*, 144(2), 379-396.

- Donati, A. V., Montemanni, R., Casagrande, N., Rizzoli, A. E., & Gambardella, L. M. (2008). "Time dependent vehicle routing problem with a multi ant colony system." *European Journal of Operational Research*, 185(3), 1174-1191.

- Hashimoto, H., Yagiura, M., & Ibaraki, T. (2008). "An iterated local search algorithm for the time-dependent vehicle routing problem with time windows." *Discrete Optimization*, 5(2), 434-456.

- Figliozzi, M. A. (2012). "The time dependent vehicle routing problem with time windows: Benchmark problems, an efficient solution algorithm, and solution characteristics." *Transportation Research Part E: Logistics and Transportation Review*, 48(3), 616-636.

3.8 Synthesis and Research Gaps

- Comparison of problem characteristics across reviewed literature
- Key methodological approaches applicable to our problem
- Identification of research gaps in addressing our specific problem constraints
- Potential directions for adaptation of existing methodologies

Recommended Literature:

- Laporte, G. (2009). "Fifty years of vehicle routing." *Transportation Science*, 43(4), 408-416.

- Cordeau, J. F., Gendreau, M., Laporte, G., Potvin, J. Y., & Semet, F. (2002). "A guide to vehicle routing heuristics." *Journal of the Operational Research Society*, 53(5), 512-522.

- Eksioglu, B., Vural, A. V., & Reisman, A. (2009). "The vehicle routing problem: A taxonomic review." *Computers & Industrial Engineering*, 57(4), 1472-1483.

- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2013). "Heuristics for multi-attribute vehicle routing problems: A survey and synthesis." *European Journal of Operational Research*, 231(1), 1-21.

3.9 Conclusion

- Summary of most relevant approaches
- Recommendation for methodological direction
- Justification for selected approach based on literature findings

Recommended Literature:

- Gendreau, M., Potvin, J. Y., Bräumlaysy, O., Hasle, G., & Løkketangen, A. (2008). "Metaheuristics for the vehicle routing problem and its extensions: A categorized bibliography." In *The vehicle routing problem: Latest advances and new challenges* (pp. 143-169). Springer.
- Bräysy, O., & Gendreau, M. (2005). "Vehicle routing problem with time windows, Part I: Route construction and local search algorithms." *Transportation Science*, 39(1), 104-118.
- Glover, F., & Kochenberger, G. A. (Eds.). (2003). *Handbook of Metaheuristics*. Springer.
- Talbi, E. G. (2009). *Metaheuristics: From Design to Implementation.* John Wiley & Sons.

4 Algorithms Investigated

Paragraph describing different types of algorithm used (Routing then cluster, Cluster then Routing, Genetic, etc.)

Remember to justify the choice of algorithms. You may also need to explain how to adopt these algorithms in your work. A figure showing the ralationship between different components of your work may also help.

4.1 Routing

Explain purpose of routing/goal of algorithms.

4.1.1 Brute Force

Write brute force explanation

4.1.2 Greedy Routing/Insertion

Explain greedy routing algorithm

Explain greedy insertion algorithm

4.1.3 Gift Wrapping

Explain gift wrapping algorithm

Something like: "Once gift wrapping has found a convex hull, a greedy insertion algorithm is used to find the optimal route within the convex hull."

4.2 Clustering

Explain purpose of clustering, how it is used in route planning and the goal of our algorithms.

4.2.1 K-Means

Explain k-means algorithm

4.3 Trip Generation

Explain trip generation, how it is used in route planning and the goal of our algorithms.

4.3.1 Brute Force

Explain how brute force algorithm can be modified for trip generation.

4.4 Genetic Algorithms

Explain basics of genetic algorithms, include how by considering different genomes, the algorithm can be used for routing, clustering or trip generation.

4.4.1 General Clustering

Explain genetic clustering

4.4.2 Centroid-based Clustering

Explain genetic centroid-based clustering and how it differs from general clustering.

4.4.3 Routing

Explain genetic routing

4.4.4 Trip Generation

Explain genetic trip generation

5 Implementation

5.1 Constraints

6 Evaluation and Comparison

Write paragraph about experiment process. Comparison based on computation time and route evaluation. Describe how route is evaluated. Describe data being tested on.

Present comparison of different combinations of algorithms on different inputs.

Reflect about the questionns you are trying to answer with your evaluation. You can have onne subsection for each question that you are trying to answer. It's also important to justify your choices when it comes to the methodology used for evaluation.

7 Conclusion and Future Work

Write conclusion, discuss results, comparison of algorithms, etc.

7.1 Project Reflection

Reflect on the project, what went well, what didn't go well, what I would do differently. This should lead into future work.

7.2 Future work

Discuss further work, what I will be doing to improve the project

8 Bibliography

Fill in bibliography