THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Student Number |__|__|__|__|__|__|__|__|

Family Name ____________________

First Name ____________________

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School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2020

MATH1071 Advanced Calculus and Linear Algebra I

This paper is for St Lucia Campus students.

Examination duration: 120 minutes (+ additional 30 mins encompassing reading time and time to scan and upload solutions). You must commence your exam at the time listed in your personalised timetable. The exam will remain open **only** for the duration shown.

Materials permitted while completing the exam: No materials permitted. You may not make use of any material. This includes web-sites, books and software.

Instructions to students: This exam has 10 questions worth a total of 100 marks. Answer all questions. Show all working. Questions carry marks indicated.

You can print the exam and write on the exam paper, or write your answers on blank paper, or write electronically on a suitable device. Scan or photograph your work if necessary and upload your answers to Blackboard as a single pdf file.

Whom to contact: Since students may not all undertake the online exam at the same time, or in the same time zone, and that some questions may be randomised, responding to student queries and/or relaying corrections to exam content during the exam will not be feasible. Course coordinators will not be able to respond to academic queries during the exam.

If you have any concerns or queries about a particular question, or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to 'raise your hand' in an examination room.

If you experience any technical difficulties during the exam, contact the Library AskUs service via the Live Chat or Phone for advice (open 7:00am – 10:00pm AEST every day during the final exam period). You should ask the library staff for an email documenting the advice provided so you can provide this to the course coordinator.

Certification (must be signed before submission):

I certify that my submitted answers are entirely my own work and that I have neither given nor received any unauthorised assistance on this assessment item.

Signed:______________________________Date:_____________________

1. (15 points) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function such that

$$\lim_{x \to 0} f(x) = l \neq 0.$$

Prove that

$$\lim_{x \to 0} \frac{1}{f(x)} = \frac{1}{l}.$$

(You must use an ϵ-δ argument, not just refer to a theorem from the lecture notes.)

(question 1 continued)

(question 1 continued)

2. (a) (2 points) State the definition of a peak point of a sequence.

(b) (4 points) Give an example of a sequence of real numbers that has no peak points (you do *not* need to prove non-existence of peak points).

(c) (4 points) Give an example of a sequence of real numbers that has precisely three peak points. Identify the peak points (you do *not* need to prove that the points you identified are peak points and that all the other points are not).

3. (10 points) Let $(a_n)_{n=1}^{\infty}$ be a Cauchy sequence of real numbers. Suppose the subsequence $(a_{2k})_{k=1}^{\infty}$ converges to 0. Prove that $(a_n)_{n=1}^{\infty}$ itself converges to 0.

(question 3 continued)

4. (10 points) Let $f\colon \mathbb{R} \to \mathbb{R}$ be a continuous function such that $\lim_{x\to 0} f(x) = 2$. Prove that there exists a $\delta > 0$ such that, on the interval $(-\delta, \delta)$, the function f is bounded.

(question 4 continued)

5. (8 points) Compute the derivative of the function

$$f(x) = \begin{cases} x^3 \cos \frac{1}{x^2}, & x \neq 0, \\ 0, & x = 0, \end{cases}$$

at $x = 0$.

(question 5 continued)

6. (10 points) Let the function $f : [0, 1] \to \mathbb{R}$ be defined by $f(x) = x$. Find a partition P of the interval $[0, 1]$ such that the upper sum $U(f, P)$ and the lower sum $L(f, P)$ satisfy the inequality

$$U(f, P) - L(f, P) < \frac{1}{3}.$$

To get full credit, you must show how you compute or estimate $U(f, P) - L(f, P)$.

(question 6 continued)

7. Find the following. Show your work.

   (a) (5 points)

$$\int_{1}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

(b) (5 points)

$$\frac{d}{dx}\int_{x}^{2x} e^{\cos t}\, dt \qquad \text{for } x > 0.$$

8. Determine whether the following series converge or diverge. Show your work.

   (a) (5 points)

$$\sum_{n=1}^{\infty} \frac{n}{(n-1)!}$$

(b) (5 points)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$$

9. (10 points) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}.$$

(question 9 continued)

10. (7 points) Let A and B be square matrices. Assume that $\det(AB) = 4$ and $\det(A^2B^3) = 16$. Find $\det A$ and $\det B$.

(question 10 continued)

End of examination