

Math 1071
Semester 1, 2020

As you prepare for the final exam, it is strongly recommended that you learn the statements and the proofs of the following results to the point where you can reproduce them without using your notes or any other aids. At the final exam, you will be required to answer theoretical questions and to solve problems.

1. The squeeze theorem for sequences.
2. The theorem about the boundedness of a convergent sequence.
3. The theorem about the convergence of a bounded monotone sequence.
4. The theorem about the existence of a monotone subsequence.
5. The Bolzano-Weierstrass theorem.
6. The theorem about the limit of a sum, product and ratio of functions.
7. The proposition about the continuity of a differentiable function.
8. The theorem about the derivative of a sum, product and ratio.
9. Rolle's theorem.
10. The mean value theorem.
11. The fact that a function with vanishing derivative must be constant.
12. The theorem about the uniform continuity of a continuous function on an interval.
13. The theorem about the condition for integrability involving the inequality
$$U(f, P) - L(f, P) < \epsilon.$$
14. The theorem about the integrability of a function with at most finitely many discontinuities.
15. The theorem about the integrability of a monotone function.

16. The mean value theorem for integrals.
17. The theorem about the continuity of the function $F(x) = \int_a^x f(t) dt$.
18. The fundamental theorem of calculus.
19. The theorem about the substitution formula for definite integrals.
20. The theorem about integration by parts.
21. The integral test for the convergence of a series.
22. The limit comparison test for the convergence of a series.
23. The ratio test for the convergence of a series. (The proof from the lectures will suffice; i.e., you may assume $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ exists.)
24. The Leibniz test for the convergence of a series.
25. The theorem about the convergence of an absolutely convergent series.
26. The formulas (and their derivations) for the Maclaurin series of the following functions: e^x , $\sin x$, $\cos x$, $\frac{1}{1-x}$.
27. The proposition describing the set of solutions of an inhomogeneous linear system in terms of solutions to the associated homogeneous system.
28. The theorem about the relationship between the invertibility of a matrix and its null-space.
29. The theorem about the linearity of the determinant.
30. The theorem about the relationship between the invertibility of a matrix and its determinant. (You may use the fact that $\det(AB) = \det(A)\det(B)$ without proof.)