Pricing S&P500 Binary Options on Kalshi

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1 Introduction

This paper examines the pricing dynamics of binary options on Kalshi's prediction market platform, focusing on contracts tied to the S&P 500 index's year-end closing value. These binary contracts, priced between $0 and $1, represent the market-implied probability of specific outcomes—whether the index will close within a designated range or above/below certain thresholds. A contract priced at $0.75 indicates a 75% probability of occurrence and offers a $0.25 profit if correct.

Unlike traditional options, binary options provide a fixed payout regardless of how far the underlying asset moves beyond the strike price. This distinctive feature creates unique pricing characteristics and potential market inefficiencies.

This research investigates: (1) the relationship between theoretical models and actual market prices, (2) how effectively adapted Black-Scholes models predict binary option prices, and (3) potential trading opportunities from identified mispricings. To provide a focused examination, I analyze two historical Kalshi contracts:

1. "Will the S&P 500 be above 5799.99 at the end of Dec 31, 2024?"
2. "Will the S&P 500 be between 4500 and 4699.99 at the end of Dec 29, 2023?"

These contracts were selected because they exemplify distinct pricing behaviors and volatility characteristics. The relevant market is available at: https://kalshi.com/markets/kxinxy/sp-500-yearly-range.

2 Data

For this analysis, I used three primary data sources:

- **S&P 500 Closing Prices:** Daily closing prices of the S&P 500 index, sourced from NASDAQ at https://www.nasdaq.com/market-activity/index/spx/historical.
- **Kalshi Trade History:** Historical trade data for the S&P 500 End of Year Close market on Kalshi is available at https://kalshi.com/markets/kxinxy/sp-500-yearly-range.
- **Interest Free Rate:** Market Yield on U.S. Treasury Securities at 1-Year Constant Maturity obtained from the Federal Reserve Economic Data (FRED) at https://fred.stlouisfed.org/series/DGS1.

3 Empirical Analysis

3.1 Price-Index Relationship

To understand how binary option prices respond to movements in the underlying index, I analyzed the relationship between S&P 500 values and corresponding contract prices.

3.1.1 Visual Representation of Contracts

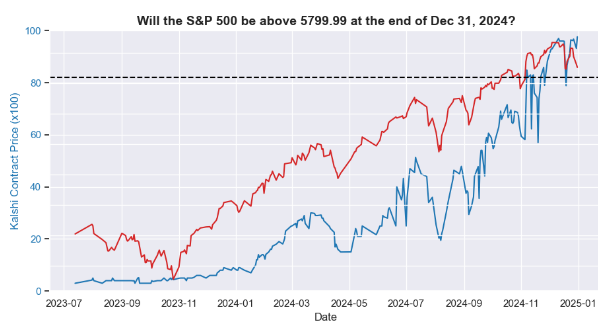

Figure 1: "Above" Contract: Will the S&P 500 close above 5799.99 on Dec 31, 2024?

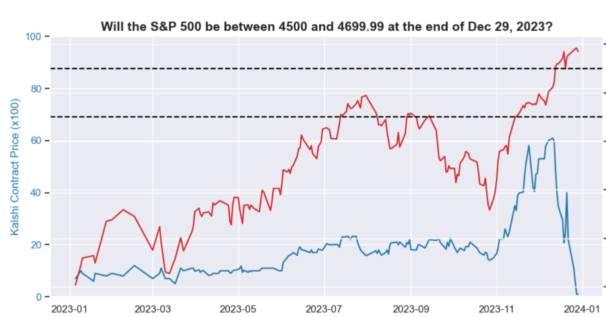

Figure 2: "Between" Contract: Will the S&P 500 close between 4500 and 4699.99 on Dec 29, 2023?

Figures 1 and 2 display the time series of both the S&P 500 index (red line) and the corresponding contract prices (blue line). The black horizontal lines indicate the boundaries for in-the-money conditions:

- For the "above" contract (Figure 1): The S&P 500 must close above the black line (5799.99) for a $1 payout
- For the "between" contract (Figure 2): The S&P 500 must close within the range marked by the two black lines (4500-4699.99) for a $1 payout

3.1.2 Contract Price Behavior

Above Contract. Figure 1 reveals a direct relationship between the S&P 500 level and the "above" contract price. As the index rises toward and beyond 5799.99, the contract price increases proportionally until the strike price is finally hit, reflecting the growing probability of an in-the-money expiration.

Between Contract. Figure 2 demonstrates a more complex relationship for the "between" contract. The contract price reaches its peak when the S&P 500 approaches the middle of the target range (4500-4699.99). As the index deviates from this range—whether moving too high or too low—the contract price decreases accordingly. This bidirectional sensitivity creates a distinctive price pattern compared to the "above" contract. When the index definitively exits the target range in the contract's final month, the price rapidly approaches zero, reflecting the near-certainty of expiring worthless.

4 Theoretical Pricing Model

4.1 Black-Scholes Adaptation for Binary Options

While traditional options are commonly priced using the Black-Scholes model, this framework requires adaptation for binary options due to their distinctive payout structure. This section develops a theoretical pricing approach based on risk-neutral probability principles.

4.1.1 Theoretical Foundation

The fundamental insight of Black-Scholes remains applicable: under risk-neutral valuation, the fair price of any option equals the expected discounted payoff. For binary options, this translates to:

$$\text{Price} = e^{-rt} \times P(\text{Option expires in-the-money}) \tag{1}$$

where r is the risk-free rate and t is the time to expiration in years.

4.1.2 Pricing Formula for "Above" Contracts

Figure 3: Theoretical vs. Market Prices: S&P 500 above 5799.99 (Dec 31, 2024)

For contracts that pay $1 when the underlying asset closes above a threshold K (e.g., "Will the S&P 500 be above 5799.99?"), the pricing formula is:

$$\text{Price}_{\text{above}} = e^{-rt} \times \mathcal{N}(d_2) \tag{2}$$

where $\mathcal{N}(d_2)$ is the cumulative distribution function of the standard normal distribution evaluated at d_2. In Black-Scholes terminology, d_2 represents the standardized distance to the strike price, adjusted for drift:

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{3}$$

where S is the current price of the underlying asset, K is the strike price, σ is the annualized volatility, r is the risk-free rate, and t is the time to expiration in years.

Figure 3 shows how the theoretical price (red line) compares with the actual market price (blue line) for the "above 5799.99" contract. The model demonstrates strong predictive power for this contract type, with a correlation coefficient (r) of 0.9735 and coefficient of determination (R^2) of 0.9477. This indicates that the adapted Black-Scholes model explains nearly 95% of the variation in market prices for this "above" contract.

4.1.3 Pricing Formula for "Between" Contracts

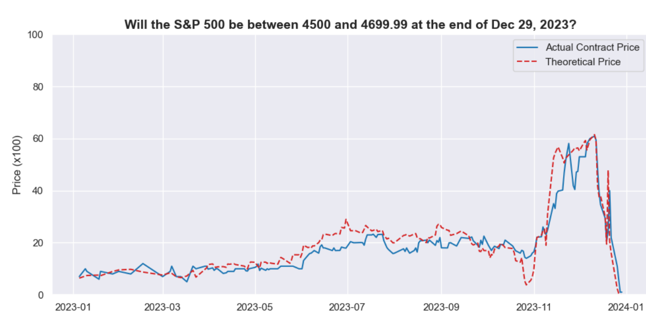

Figure 4: Theoretical vs. Market Prices: S&P 500 between 4500-4699.99 (Dec 29, 2023)

For contracts that pay \$1 when the underlying asset closes between two thresholds K_1 and K_2 (e.g., "Will the S&P 500 be between 4500 and 4699.99?"), we decompose the pricing into the difference between two cumulative probabilities:

$$\text{Price}_{\text{between}} = e^{-rt} \times \left[\mathcal{N}(d_2^{(2)}) - \mathcal{N}(d_2^{(1)})\right] \tag{4}$$

Where $\mathcal{N}(d_2^{(2)})$ represents the probability that the asset finishes below the upper bound K_2, and $\mathcal{N}(d_2^{(1)})$ represents the probability that the asset finishes below the lower bound K_1. Their difference yields the probability of the asset finishing between these two bounds.

Figure 4 illustrates how the theoretical price (orange line) compares with the actual market price (blue line) for the "between 4500-4699.99" contract. While still showing strong alignment, the model performs slightly less precisely for this contract, with a correlation (r) of 0.9290 and R^2 of 0.8631. This suggests that approximately 86% of the price variation in "between" contracts can be explained by my adapted model.

4.2 Interest Rate Adjustment

A key implementation detail concerns the appropriate risk-free rate to use in the model. Starting October 10, 2024, Kalshi began providing APY on account balances. To account for this platform-specific feature, I adjusted the interest rate parameter:

$$r_{\text{effective}} = r_{\text{treasury}} - r_{\text{kalshi}} \tag{5}$$

Based on empirical observation, the difference between Kalshi's APY and the 1-year Treasury yield averages approximately 0.25% (25 basis points). For example, when Kalshi offers 3.75% APY while the 1-year Treasury yields 4.00%, the effective risk-free rate used in the model is 0.25%.

5 Mispricing Analysis

To investigate potential arbitrage opportunities, I analyzed differences between theoretical and market prices, examining whether pricing discrepancies exhibit mean-reverting behavior and autocorrelation patterns.

5.1 Statistical Properties of Price Differentials

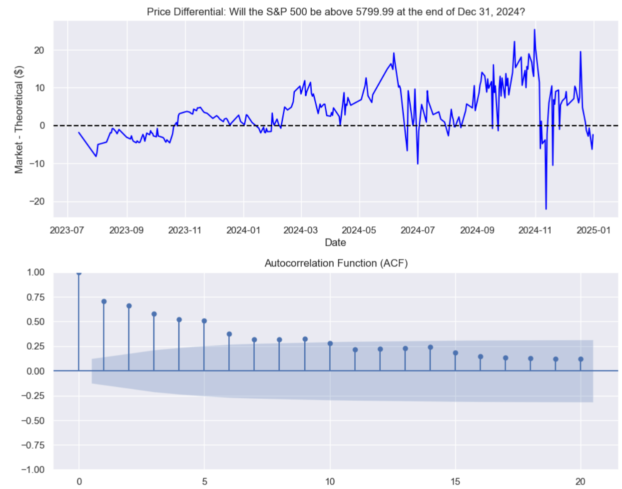

Figure 5: Price Differential and Autocorrelation: Above 5799.99 Contract

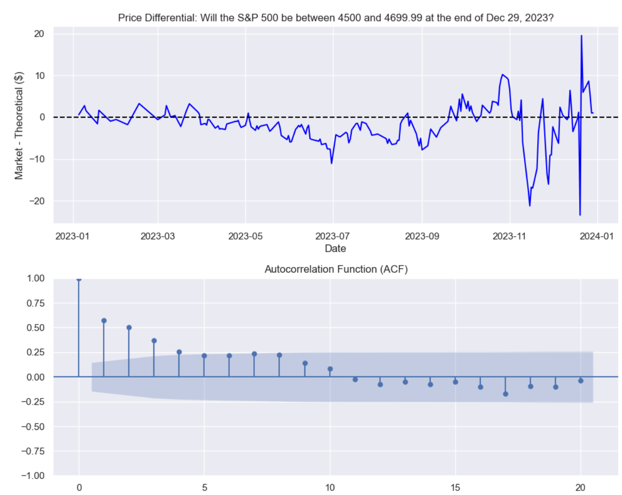

Figure 6: Price Differential and Autocorrelation: Between 4500-4699.99 Contract

The stationarity analysis revealed an interesting contrast between the two contracts. The "above" contract sits at the borderline of stationarity with an ADF test statistic of -2.786 (p=0.060) – just missing the conventional 0.05 threshold while satisfying the 10% significance level. The "between" contract, however, showed much clearer mean-reverting behavior with a test statistic of -4.669 (p=0.0001), strongly rejecting the non-stationarity hypothesis.

What's particularly interesting is that despite these stationarity differences, both contracts display pronounced autocorrelation patterns. Looking at the ACF values, the "above" contract shows remarkably strong persistence – 0.709 at lag 1 and still maintaining 0.510 by lag 5. The "between" contract starts with substantial correlation at 0.571 for lag 1, but decays more quickly to 0.216 by lag 5. These patterns suggest that pricing discrepancies aren't merely random noise.

5.2 Trading Strategy Implications

The statistical properties of these price differentials provide compelling evidence that we are on the right methodological track. Our adapted Black-Scholes model demonstrates strong predictive performance with high R^2 values, closely tracking actual market prices. However, several observations suggest refinements are needed before implementation:

- The differential stationarity results between contracts (borderline for the "above" contract vs. strong for the "between" contract) likely indicates limitations in our model rather than fundamentally different market dynamics.
- The strong autocorrelation patterns, particularly pronounced in the "above" contract, suggest systematic elements of price formation that our current specification does not fully capture.
- The persistence of these patterns across multiple lags points to structured rather than random deviations, reinforcing that we are capturing fundamental pricing mechanisms while missing some nuances.
- Critical considerations before trading implementation include:
  - Accounting for transaction costs, particularly Kalshi fees and bid-ask spreads, which are currently absent from our analysis
  - Refining our volatility modeling approach, which represents the most significant opportunity for improvement

The most promising direction for model enhancement lies in volatility specification. Our current one-month rolling average approach fails to capture the term structure of volatility. A more sophisticated implementation would involve extracting implied volatilities from corresponding traditional options on the same underlying with matching expiration dates, enabling construction of a proper volatility surface that reflects market expectations across different strike prices and time horizons.

Overall, the close alignment between theoretical and market prices confirms the validity of our approach, while the identified statistical patterns offer a clear roadmap for model refinement before deploying any trading strategy based on these apparent pricing discrepancies.

6 Conclusion

This paper examined the pricing dynamics of binary options on Kalshi's prediction market platform, focusing on contracts tied to the S&P 500 index's year-end closing value. Using an adaptation of the Black-Scholes model, I successfully approximated market prices with correlation coefficients exceeding 0.9 for both "above" and "between" contract types.

With over $40 million in total trading volume on S&P 500 year-end contracts, Kalshi provides sufficient liquidity for meaningful analysis and potential trading implementation, making it a viable platform for applying these findings.

This project provided an excellent opportunity to synthesize concepts from multiple courses. I applied the options pricing knowledge gained from Natenberg's "Volatility and Options Pricing" and my Intro to Financial Engineering course, while implementing statistical methods from Statistics for Financial Engineering. The combination of exploratory data analysis, time series analysis, interest rate modeling, and returns analysis created a comprehensive framework for evaluating market efficiency in this emerging asset class.

What made this project particularly engaging was the opportunity to apply theoretical concepts to a real-world trading platform with actual market data. The process of adapting established financial models to a relatively new market structure proved both challenging and rewarding, feeling more like an independent research project than a typical assignment.

References

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[2] Nielsen, L. T. (1992). *Understanding N(d_1) and N(d_2): Risk-adjusted probabilities in the Black-Scholes model*. INSEAD. https://financetrainingcourse.com/education/wp-content/uploads/2011/03/Understanding.pdf

Appendix

My Python code and all data sources can be found here: https://github.com/Jake0826/kalshi-binary-option-pricing