CS 261 – Data Structures

Big-Oh and Execution Time: A Review

Big-Oh: Purpose

A machine-independent way to describe execution time

Purpose of a Big-Oh characterization is: to describe change in execution time relative to change in input size in a way that is independent of issues such as machine times or compilers

Big-Oh: Algorithmic Analysis

We want a method for determining the relative speeds of algorithms that:

- doesn’t depend upon hardware used (e.g., PC, Mac, etc.)
- the clock speed of your processor
- what compiler you use
- even what language you write in

Algorithmic Analysis

- Suppose that algorithm ***A*** processes *n* data elements in time ***T***.

- Algorithmic analysis attempts to estimate how ***T*** is affected by changes in *n*. In other words, ***T*** is a function of *n* when we use ***A***.

Define Big-O on board…

A Simple Example

- Suppose we have an algorithm that is O(n) (e.g., summing elements of array)
- Suppose to sum 10,000 elements takes 32 ms.
- How long to sum 20,000 elements?
- If the size doubles, the execution time doubles

Non-Linear Times

- Suppose the algorithm is $O(n^2)$ (e.g., sum elements of a 2-D array)
- Suppose size doubles, what happens to execution time?
- It goes up by 4
- Why 4?
- Need to figure out how to do this …

The Calculation

The ratio of the big-Oh sizes should equal the ratio of the execution times

$$\frac{{n_1}^2}{{n_2}^2} = \frac{t_1}{t_2}$$

We increased n by a factor of two:

$$\frac{n^2}{(2n)^2} = \frac{t}{x}$$

then solve for x

A More Complex Problem

- Acme Widgets uses a merge sort algorithm to sort their inventory of widgets
- If it takes 66 milliseconds to sort 4096 widgets, then approx. how long will it take to sort 1,048,576 widgets?

(Note: merge sort is O(n log n), 4096 is 2^{12}, and 1,048,576 is 2^{20}, and)

A More Complex Problem (cont.)

Setting up the formula:

$$\frac{n_1 \log n_1}{n_2 \log n_2} = \frac{t_1}{t_2} \longrightarrow \frac{2^{12} \log 2^{12}}{2^{20} \log 2^{20}} = \frac{66\text{ ms}}{x}$$

Solve for x (remember $\log 2^y$ is just y)

Determining Big Oh: Simple Loops

For simple loops, ask yourself how many times loop executes as a function of input size:

- Iterations dependent on a variable *n*
- Constant operations within loop

```
double minimum(double data[], int n) {
//  Pre: values has at least one element.
// Post: returns the smallest value in collection.

  int    i;
  double min = data[0];

  for(i = 1; i < n; i++)
    if(data[i] < min) min = data[i];

  return min;
}
```

O(*n*)

Determining Big Oh: Not-So-Simple Loops

Not always simple iteration and termination criteria

- Iterations dependent on a function of n
- Constant operations within loop
- Possibility of early exit:

```
int isPrime(int n) {
  int i;
  for(i = 2; i * i < n; i++) {     // If i is a factor,
    if (n % i == 0) return 0;      //   then not prime.
  }                                // If loop exits without finding
  return 1;                        //   a factor → n is prime.
}
```

$O(\sqrt{n})$

But what happens if it exits early?

Best case?

Average case?

Worst case?

Determining Big Oh: Nested Loops

Nested loops (dependent or independent) multiply:

```
void insertionSort(double arr[], unsigned int n) {
  unsigned i, j;
  double   elem;

  for(i = 1; i < n; i++) {                n - 1 times

    // Move element arr[i] into place.
    elem = arr[i];
    for (j = i - 1; j >= 0 && elem < arr[j]; j--) {
      arr[j+1] = arr[j];  j--;          // Slide old value up.
    }
    arr[j+1] = elem;
  }
}
```

Worst case (reverse order): $1 + 2 + \ldots + (n\text{-}1) = (n^2 - n) / 2 \rightarrow O(n^2)$

8	7	6	5	4	3	2	1

→

2	3	4	5	6	7	8	1

Determining Big Oh: Recursion

For recursion, ask yourself:

(a) How many times will function be executed?

(b) How much time does it spend on each call?

Multiply these together

```
double exp(double a, int n) {
  if (n < 0) return 1 / exp(a, -n);
  if (n = 0) return 1;
  return a * exp(a, n - 1);
}
```

Not always as simple as above example:

Often easier to think about algorithm instead of code

Determining Big Oh: Logarithmic

- I'm thinking of a number between 0 and 1024, after each guess I'll tell you if it's higher or lower.
- How many guesses do you need to find my number?
- Answer: approximately $\log 1024 = \log 2^{10} = 10$
- In algorithmic analysis, the log of n is the number of times you can split n in half (binary search, etc)

Summation and the Dominant Component

- A method's running time is sum of time needed to execute sequence of statements, loops, etc. within method
- For algorithmic analysis, the largest component dominates (and constant multipliers are ignored)
  - Function $f(n)$ dominates $g(n)$ if there exists a constant value n_0 such that for all values of $n > n_0$, $f(n) > g(n)$

Example: analysis of a given method shows its execution time as

$$\underbrace{8n}_{\text{Single loop}} + \underbrace{3n^2}_{\text{Nested loop}} + \underbrace{23}_{\substack{\text{Constant-time}\\ \text{statements.}}}$$

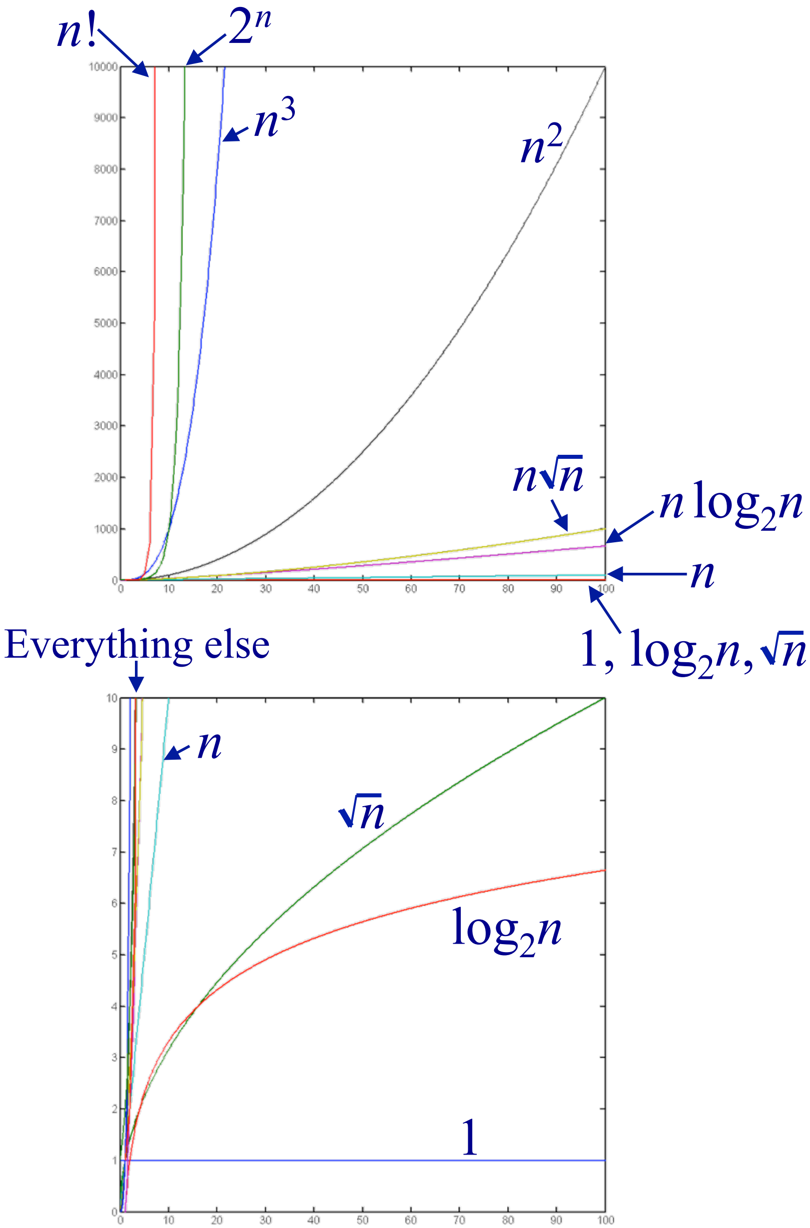
Don't write $O(8n + 3n^2 + 23)$ or even $O(n + n^2 + 1)$, but just $O(n^2)$

Best Case or Worst Case ?

```
void insertionSort (double v [ ], int n)
 {
   for (int i = 1; i < n; i++) {
      double element = v[i];
      int j = i - 1;
      while (j >= 0 && element < v[j])
         { v[j+1] = v[j]; j = j - 1; }
      v[j+1] = element;
   }
} // normally we use worst case time
```

Computation Complexities

Function	Common Name
$n!$	Factorial
2^n (or c^n)	Exponential
n^d, $d > 3$	Polynomial
n^3	Cubic
n^2	Quadratic
$n\sqrt{n}$	
$n \log n$	
n	Linear
$\sqrt{n}$	Root-n
$\log n$	Logarithmic
O(1)	Constant



Benchmarking

- Algorithmic analysis is the first and best way, but not the final word
- What if two algorithms are both of the same complexity?
- Example: bubble sort and insertion sort are both O(n^2)
  - So, which one is the "faster" algorithm?
  - Benchmarking: run both algorithms on the same machine
  - Often indicates the constant multipliers and other "ignored" components
  - Still, different implementations of the same algorithm often exhibit different execution times – due to changes in the constant multiplier or other factors (such as adding an early exit to bubble sort)

Let's Practice: What is the O(??)

```
int countOccurrences (double [ ] data, double testValue) {
   int count = 0;
   for (int i = 0; i < data.length; i++) {
      if (data[i] == testValue)
         count++;
   }
   return count;
}
```

O(??) in terms of n (worst case)

```
int isPrime (int n) {
  for (int i = 2; i * i <= n; i++) {
    if (0 == n % i) return 0;
  }
  return 1; /* 1 is true */
}
```

Worst case O(??)

```
void printPrimes (int n) {
  for (int i = 2; i < n; i++) {
    if (isPrime(i))
      printf(“Value %d is prime\n”, i);
  }
}
```

Nested Loops - O(??)

```
void matMult (int [][] a, int [][] b, int [][] c) {
   int n = n; // assume all same size
   for (int i = 0; i < n; i++)
      for (int j = 0; j < n; j++) {
         c[i][j] = 0;
         for (k = 0; k < n; k++)
            c[i][j] += a[i][k] * b[k][j];
      }
}
```

Less obvious O(??)

```
void selectionSort (double storage [ ], int n) {
    for (int p = n – 1;   p > 0; p--) {
      int indexLargest = 0;
      for (int i = 1; i <= p; i++) {
        if (storage[i] > storage[indexLargest])
          indexLargest = i;
      }
      if (indexlargest != p)
        swap(storage, indexLargest, p);
    }
  }
}
```