$$\underbrace{P\left[\left|\frac{1}{n}\sum_{i=1}^{n} x_i - \mu\right| \geq \varepsilon\right]}_{⓪}$$

$$= P\left[\left|\sum_{i=1}^{n}(x_i-\mu)\right| \geq n\varepsilon\right]$$

$$= \underbrace{P\left[\sum_{i=1}^{n}(x_i-\mu) \geq n\varepsilon\right]}_{①} \times 2$$

$$① = P\left[e^{t\Sigma(x_i-\mu)} \geq e^{tn\varepsilon}\right]$$

$$\leq \underbrace{E\left[e^{t\Sigma(x_i-\mu)}\right]}_{②} / e^{tn\varepsilon}$$

$$② = E\left[e^{t(x_1-\mu)} \cdot e^{t(x_2-\mu)} \cdots e^{t(x_n-\mu)}\right]$$

$$= E\left[\prod_{i=1}^{n} e^{t(x_i-\mu)}\right]$$

$$= \prod_{i=1}^{n} E\left[e^{t(x_i-\mu)}\right] = \left[E\left[e^{t(x-\mu)}\right]\right]^{n}$$

$$\leq e^{\frac{1}{8}nt^2}$$

$$\therefore ⓪ \leq 2 \cdot e^{\frac{1}{8}nt^2} / e^{tn\varepsilon}$$

$$= 2e^{\left(\frac{1}{8}nt^2 - n\varepsilon t\right)}$$

$$f(t) = \frac{1}{8}nt^2 - n\varepsilon t$$

$$0 = \frac{\partial f}{\partial t} = \frac{1}{4}nt - n\varepsilon$$

$$\therefore \frac{1}{4}t = \varepsilon \quad \therefore t = 4\varepsilon$$

$$\therefore ⓪ \leq 2e^{\left(2n\varepsilon^2 - 4n\varepsilon^2\right)}$$

$$= 2e^{-2n\varepsilon^2}$$