

$$\begin{aligned}
 & \underbrace{P\left[\left|\frac{1}{n}\sum_{i=1}^n x_i - \mu\right| \geq \varepsilon\right]}_{\textcircled{1}} \\
 &= P\left[\left|\sum_{i=1}^n (x_i - \mu)\right| \geq n\varepsilon\right] \\
 &= P\left[\sum_{i=1}^n (x_i - \mu) \geq n\varepsilon\right] \times 2 \\
 &\stackrel{\textcircled{1}}{=} P\left[e^{t\sum_{i=1}^n (x_i - \mu)} \geq e^{tn\varepsilon}\right] \\
 &\leq \underbrace{E\left[e^{t\sum_{i=1}^n (x_i - \mu)}\right]}_{\textcircled{2}} / e^{tn\varepsilon}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} &= E\left[e^{t(x_1 - \mu)} \cdot e^{t(x_2 - \mu)} \cdots e^{t(x_n - \mu)}\right] \\
 &= E\left[\prod_{i=1}^n e^{t(x_i - \mu)}\right] \\
 &= \prod_{i=1}^n E\left[e^{t(x_i - \mu)}\right] = \left[E\left[e^{t(x - \mu)}\right]\right]^n \\
 &\leq e^{\frac{1}{8}n\varepsilon^2}
 \end{aligned}$$

$$\therefore \textcircled{1} \leq 2 \cdot e^{\frac{1}{8}n\varepsilon^2} / e^{tn\varepsilon}$$

$$= 2e^{(\frac{1}{8}n\varepsilon^2 - n\varepsilon t)}$$

$$\begin{aligned}
 f(t) &= \frac{1}{8}n\varepsilon^2 - n\varepsilon t & \therefore \textcircled{1} &\leq 2e^{(2n\varepsilon^2 - 4n\varepsilon^2)} \\
 0 = \frac{\partial f}{\partial t} &= \frac{1}{4}n\varepsilon - n\varepsilon & &= 2e^{-2n\varepsilon^2}
 \end{aligned}$$

$$\therefore \frac{1}{4}t = \varepsilon \therefore t = 4\varepsilon$$