11. Conclusions

- main ideas of the course
- importance of modeling in optimization
- what we didn't cover. . .

Modeling

mathematical optimization

- problems in engineering design, machine learning, statistics, economics, . . . , can often be expressed as mathematical optimization problems
- techniques exist to take into account multiple objectives or uncertainty in the data

tractability

- roughly speaking, tractability in optimization requires convexity
- algorithms for nonconvex optimization find local (suboptimal) solutions, or are very expensive
- surprisingly many applications can be formulated as convex problems

Theoretical consequences of convexity

- local optima are global
- extensive duality theory
  - systematic way of deriving lower bounds on optimal value
  - necessary and sufficient optimality conditions
  - certificates of infeasibility
  - sensitivity analysis
  - sometimes dual has intuitive interpretation
- solution methods with polynomial worst-case complexity theory

Practical consequences of convexity

(most) convex problems can be solved globally and efficiently

- basic algorithms (gradient descent, Newton, barrier method, . . .) are easy to implement and work well for small-medium size problems (larger problems if structure is exploited)
- high-level modeling tools like CVX make modeling and problem specification easier (especially for prototyping)

How to use convex optimization

to use convex optimization in some applied context

- use approximate modeling
  - start with simple models, small problem instances, inefficient solution methods
  - if you don't like the results, no need to spend effort on more accurate models or efficient algorithms

- work out, simplify, interpret optimality conditions and dual

- even if the problem is nonconvex, you could still use convex optimization
  - as relaxations
  - in subproblems, *e.g.*, to find search direction
  - by repeatedly forming and solving a convex approximation at the current point

What's next?

some topics we didn't cover:

- more application fields where key problems rely on convexity
- **algorithms:** unconstrained minimization (chap 9), equality constrained minimization (chap 10), barrier methods (chap 11)
- localization (cutting plane, ellipsoid), subgradients
- methods for very large scale problems; numerical linear algebra
- distributed convex optimization
- **convex relaxations:** special problem classes such as sparse signal recovery, matrix rank minimization,. . .
- many more applications

some of these will be covered in special topics project-oriented sequel course in ECE (I will teach this in Spr 2025).
also check out Math 581 (in Math, by D. Drusvyatskiy), and CSE 535 (CS theory viewpoint, used to be taught by Y. T. Lee)).

Acknowledgements

many thanks to:

- TA: Natalia Pavlasek, Grader: Wenzheng Zhao
- S. Boyd, L. Vandenberghe, and M. Grant (for creating this course, the book, CVX,. . .)
- ee578 students: for your hard work!