FRANCESCA SPEZZANO

CS 321 Data Structures (Fall 2016)

Homework #3, final exam preparation

- **Q1(11 points): Running time, asymptotic notations and recurrence equations**

  (a)(3 pts)(Multiple choice) If an algorithm's running time can be expressed as a function $f(n) = \sqrt{n} + \log n^n$, then which one of the following is NOT a correct bound for the running time?

  1. $O(n^2)$
  2. $O(\sqrt{n})$
  3. $O(\log n^n)$
  4. $\Omega(n)$

  $f(n) = \sqrt{n} + n \log n$

  $n \log n > \sqrt{n} \Rightarrow O(\sqrt{n})$ IS NOT A CORRECT BOUND

  (b)(3 pts) Assume an algorithm runs in $\Theta(n^2)$, then which one of the following is a wrong asymptotic notation for it?

  1. $O(n^3)$
  2. $O(n^2)$
  3. $O(n)$
  4. $\Omega(n^2)$

  (c)(5 points) Please re-arrange (sort) the following functions based on their growth rates, from the least to the greatest. In addition, please underline those functions which have the same asymptotic bound.

  $$2^{n/2},\ \log_e n,\ n\log n,\ n!,\ 2^n,\ \log_2 n^2,\ \sqrt{n}$$

  $\underline{\log_e n}$ $\underline{\log_2 n^2}$ $\sqrt{n}$ $n \log n$ $2^{n/2}$ 2^n $n!$

- **Q2(19 points): Basic Data Structures**

  (a)(3 pts)(Multiple choice) Which one of the following is the FIRST true statement for linked lists (assume there is only one entry point to the list - Head)?

  1. It takes linear time to insert an element in the front of a list.
  2. It takes linear time to insert an element in the end of a singly circular list.
  3. It takes linear time to insert an element in the end of a doubly non-circular list.
  4. It takes linear time to insert an element in the end of a doubly circular list.

  The FIRST TRUE STATEMENT IS 2

  3 IS ALSO CORRECT

(b)(3 pts)(Multiple choice) If we read a sequence of items $< A, B, C, D, E, F >$ from a file and maintain some of the items in a stack by push and pop operations, which one of the following stack is possible? (Assume that each item can be pushed into the stack only once).

1. top $< D, E, F >$ bottom
2. top $< F, C, A >$ bottom
3. top $< C, D, E >$ bottom
4. top $< E, D, F >$ bottom

(c)(3 pts)(Multiple choice) For hashing by open addressing, we need to decide the table size m and some parameters so that the table can be fully utilized. What does that actually mean?

1. Allocate a table with size m equal to the number of elements stored.
2. Decide m and other parameters so that for any element, the initial m probe positions are distinct.
3. Decide m and other parameters to reduce the probability of collisions.
4. Decide m and other parameters so that no collision could happen.

(d)(10 pts) Suppose we would like to insert a sequence of numbers $< 22, 38, 7, 70, 46 >$ into a hash table with table size 8 using the three open addressing methods, with the primary hash function $h_1(k) = k \bmod 8$, the secondary hash function

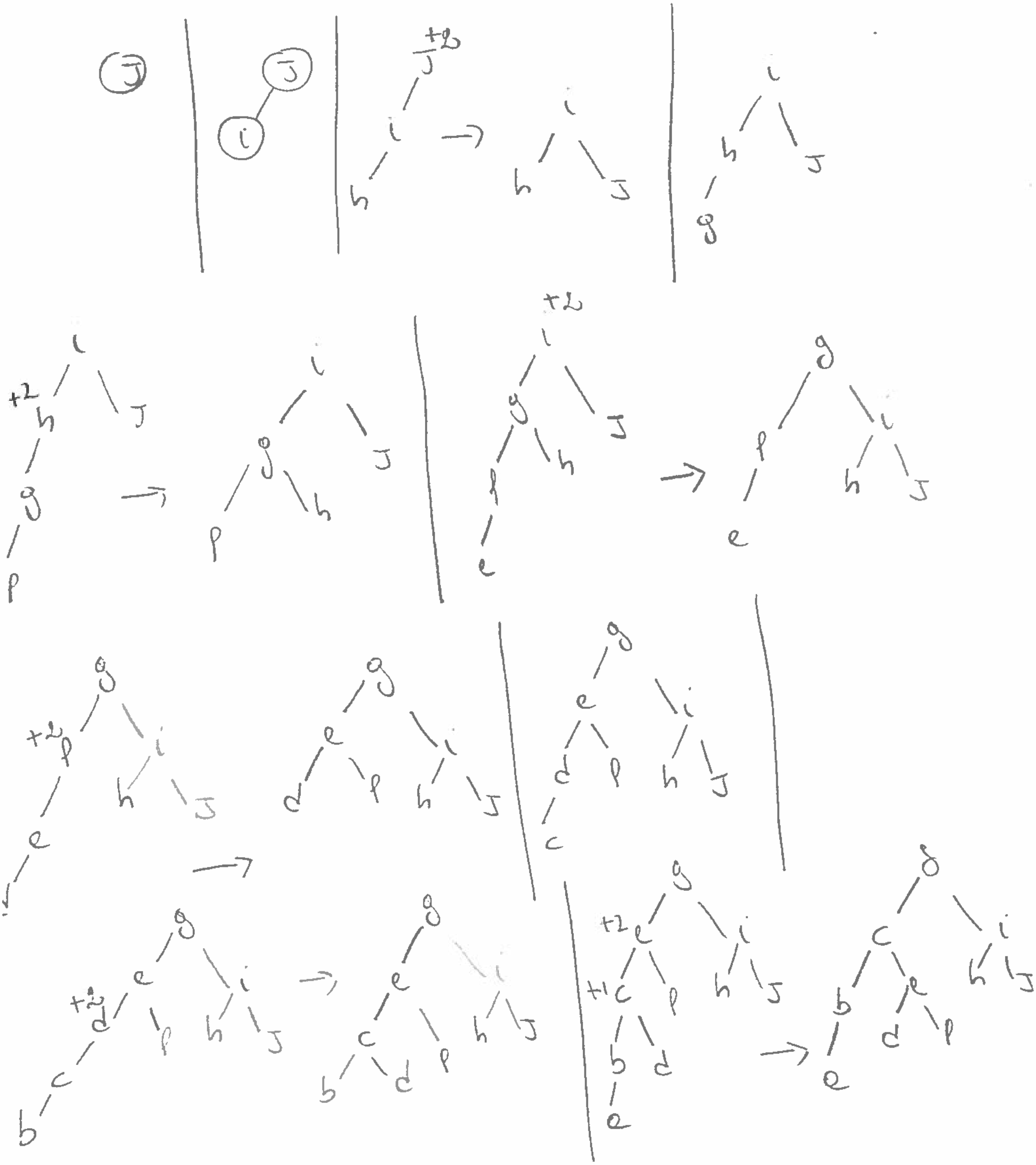
$$h_2(k) = \begin{cases} 1 + (k \bmod 7) & \text{if } k \bmod 7 \text{ is even} \\ k \bmod 7 & \text{if } k \bmod 7 \text{ is odd} \end{cases}$$

and the constants $c_1 = c_2 = 1/2$ (in quadratic probing). Please insert numbers into the tables below.

index	linear	quadratic	double
0	7	7	7
1	70	70	38
2	46		
3			46
4		46	
5			
6	22	22	22
7	38	38	7

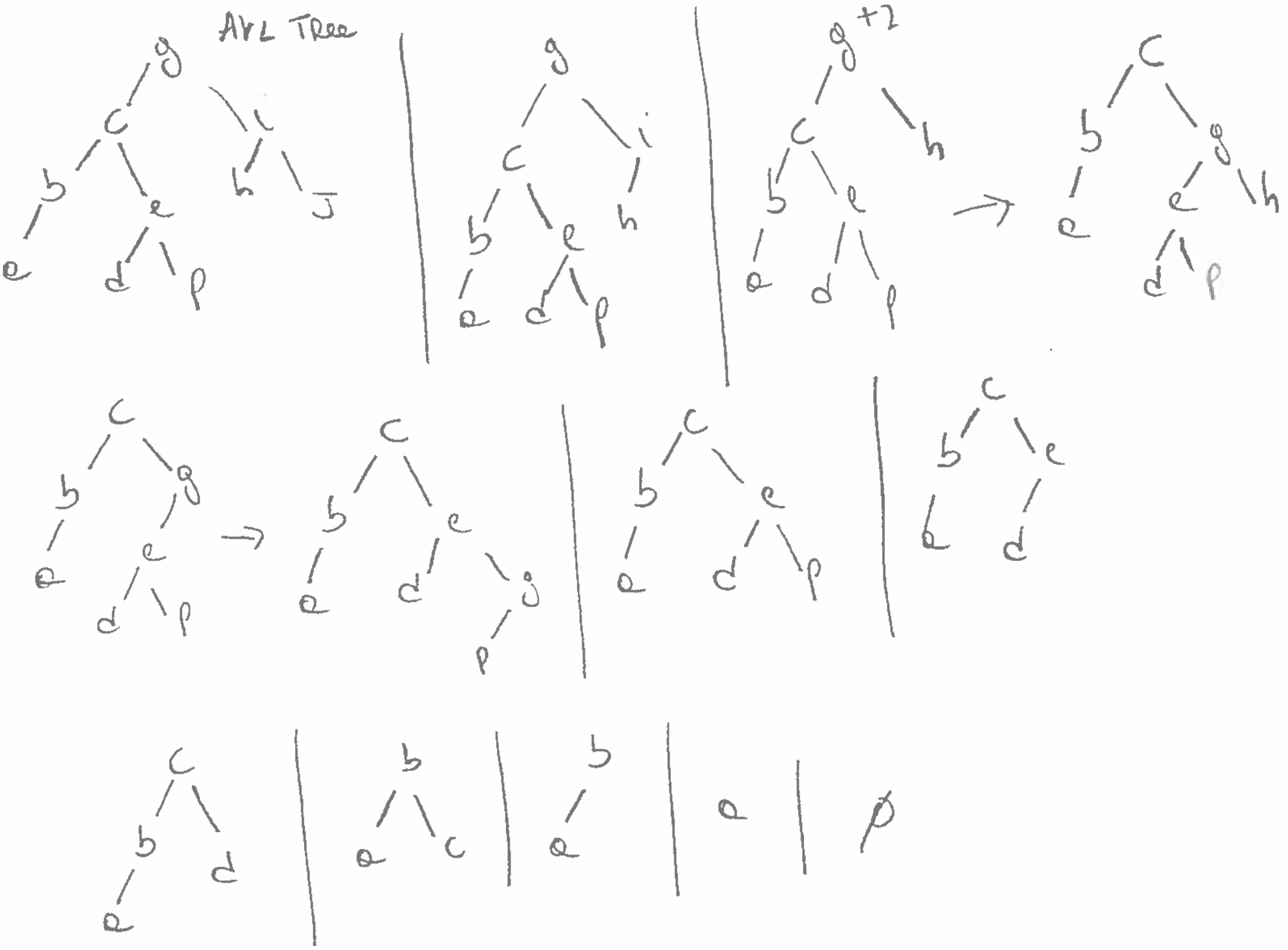
- **Q3(18 points): AVL Trees**

  (a)(9 pts) What are the sequence of AVL tress after inserting each character in the list $< j, i, h, g, f, e, d, c, b, a >$ to an initially empty AVL tree? Note: please draw only one tree after each insertion.



~~j~~, ~~i~~, ~~h~~, ~~g~~, ~~f~~, ~~e~~, ~~d~~, ~~c~~, ~~b~~, ~~a~~

(b)(9 pts) From the AVL tree you have built in part (b), what are the sequence of trees after deleting each character in the list (i.e., delete j , delete i, ...)? Note: please draw only one tree after each deletion.



- **Q4(7 points): Priority Queues**

  Using a Max-heap to implement a priority queue requires a procedure to decide the relative order of events in queue. An event a has a higher priority than another event b if a's priority value $>$ b's priority value. If both a and b have the same priority, then the event with earlier arrival time has a higher priority. Two events cannot arrive at the same time. Write a Compare procedure to decide the relative order of two events.

```
Compare(a, b)     // both a and b are events
// a.priority, b.priority: their priority values.
// a.time, b.time: arrival time (earlier time has a smaller numeric value)
// return 1 if a has a higher priority than b; Otherwise, return -1
{
    if (a.priority > b.priority)
        return +1;
    else if (a.priority < b.priority)
        return -1;
    else if (a.time < b.time) return +1;
    else return -1;
}
```

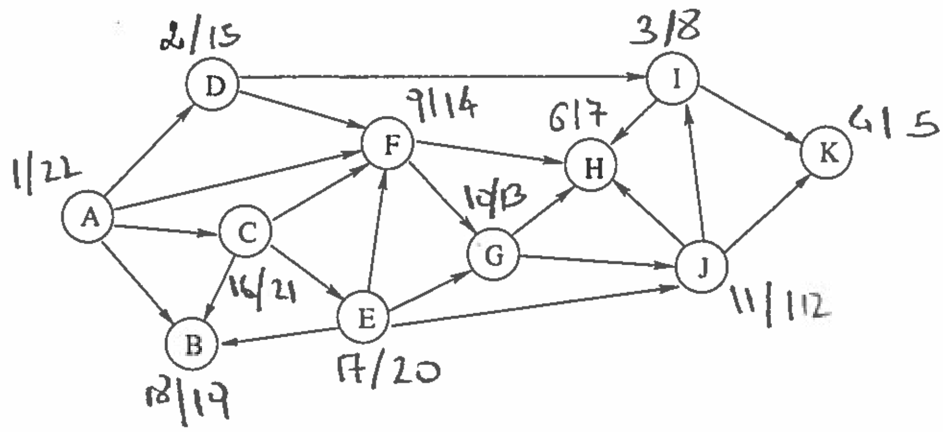
- **Q5(7 points): Recursion in Tree Structures**

  Given a binary tree, please write a pseudocode to calculate the weight of the tree (sum of the weights of all edges in the tree). Let $w(u, v)$ denote the weight of the edge (u, v), where u is the parent of another node v.

```
Tree-Weight(x)     // x is the root of the tree
{
    if (x.left == null && x.right == null)
        return 0;
    if ( x.left == null) return w(x, x.right) + Tree_weight(x.right);
    else if ( x.right == null) return w(x, x.left) + Tree_weight(x.left);
    else return w(x, x.right) + w(x, x.left) +
            Tree_weight(x.right) + Tree_weight(x.left);
}
```

- **Q6(18 points): Graph Algorithms**

  A weighted and directed graph is given below.



(a)(4 pts) Based on the DAG above, please find a (any) discovering sequence of vertices in a BFS search if A is the source vertex.

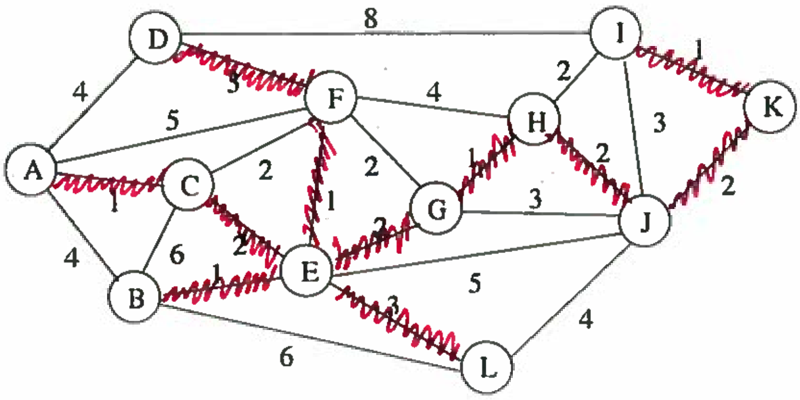
A D F C B I H G E J K

(b)(4 pts) Based on the DAG above, please find a (any) discovering sequence of vertices in a DFS search.

A D I K H F G J C E B

(c)(4 pts) Based on the DAG above, please find a (any) topological sequence of vertices.

A → C → E → B → D → F → G → J → I → H → K (d)(6 pts) Please find a (any) minimum spanning tree of the graph below



Weight = 19