Optimizing Chromosome Length in Genetic Algorithms for the Lawn Mowing Problem

Jacob Biggs
Games Academy (of Aff.)
Falmouth University (of Aff.)
Cornwall, England
jb283210@falmouth.ac.uk
https://github.falmouth.ac.uk/Games-Academy-Student-Work-23-24/COMP213-2207976.git
https://github.com/JakeBiggs/GeneticLawnmowerAgents

***Abstract*—In this paper, an investigation is conducted into how chromosome length and thus search space complexity impact the efficacy of finding viable solutions in solving a novel variation of the well-known Lawn Mower Problem (LMP). LMP seeks to find for a given region covered by grass, the shortest path along which to move a lawn mower, such that all the grass is cut. An analysis of previous work helped guide a model that explored a 10x10 toroidal grid for a range of chromosome lengths to test how performance varied. Each test ran for 100 generations except the final test which ran for 500 generations, as the combinatorial complexity was much greater. Tournament selection methods were used to find parents for single point crossover and offspring were then mutated at 5%. It was found that combinatorial complexity increased exponentially with a linear increase in chromosome length. In the experiment, only chromosome lengths that were even factors of the total area of the search space were able to find optimal solutions. A chromosome length of 10 was found to be optimal in the search space of a 10x10 grid. Although some chromosome lengths were effective at finding viable solutions, the fine tuning required, in addition to processing power make simple genetic algorithms somewhat impractical in time sensitive applications. Future work could focus on dynamic chromosome lengths, accompanied with comparisons in performance to other optimisation techniques, such as Simulated Annealing or Particle Swarm Optimisation.**

***Index Terms*—Lawn Mowing Problem, Toroidal, Genetic Algorithms**

I. Introduction

The Lawn Mowing Problem (LMP) is a well-known problem in computational geometry [1], which seeks to find the tour in a given polygonal region which gets within a Euclidean distance of half of every point. The problem is NP-Hard (non-deterministic polynomial time) and has proven difficult to solve exactly, much like the Travelling Salesman Problem, of which LMP is a generalisation. [2]. NP-Hard problems, such as TSP and LMP, are characterised by the way that they can be transformed into other NP-hard problems. A problem is considered NP-hard if every problem in NP (the set of problems that can be solved in non-deterministic polynomial time) can be reduced to it in polynomial time [3]. This means if there was a fast solution for an NP-hard problem, it would be possible to solve all NP-hard problems quickly.

While the number of solutions are finite, it is challenging to realise optimal solutions mathematically [2]. Therefore, this paper's proposed solution is to undertake the problem heuristically, through employing the use of Genetic Algorithms (GAs) to converge on optimal solutions in a given problem space. Heuristics are commonly used in NP-Hard problems, as they provide a practical method for generating viable solutions, even if those solutions are not mathematically optimal [4]

This paper proposes a novel variation of LMP, which will be referred to as the Toroidal Grid-Based Lawn Mowing Problem (TGB-LMP). In the TGB-LMP, an agent is placed within a square toroidal grid of length and height n, representing our "lawn". The agent can move once per step in one of four directions: up, down, left or right. The agent is given n^2 steps to fill all n^2 cells of the grid

This paper aims to analyse how changing the length of an agent's chromosome impacts the efficacy of finding viable solutions in a given number of generations, and how the TGB-LMP is a practical example of how the power of heuristic methods are determined by the nature and complexity of the problem being solved.

II. Background

A. Literature Review

Genetic algorithms are a type of evolutionary algorithm inspired by the Darwinist theory of natural selection. They were introduced by John Holland in the 1970s and have since been widely used in optimization and search problems [5].

Holland's original work laid the foundation for the field. He introduced the concept of a 'schema' and demonstrated how GAs manipulate and recombine these schemas to find optimal solutions [6]

De Jong's 1975 dissertation, "An Analysis of the Behavior of a Class of Genetic Adaptive Systems", was one of the first comprehensive studies of GAs. He established a set of benchmark problems for GA research and proposed the first theoretical models of GA behavior [7].

More recently, research has focused on improving the efficiency and robustness of GAs. Techniques such as adaptive parameter tuning, hybridization with other algorithms, and the use of parallel and distributed computing environments have been explored [8].

The LMP was first described by Arkin et al. in 1993 [1], which proposed; for a given region covered by grass, find a short path along which to move a lawn mower, such that all the grass is cut. This work was a seminal introduction to the LMP, but lacked considerations for processing time in finding feasible solutions. Arkin et al. then later found the best current approximation algorithm for the problem in 2000. This new journal expanded upon the previous preliminary work and not only reduced the previous best factor by up to a factor of 2, but also significantly reduced running time [9]. [10]. It is a generalisation of the Travelling Salesman Problem (TSP), which is particularly well suited to the use of GAs. This is because heuristic methods such as GAs provide near-optimal solutions in a reasonable time, which is beneficial for NP-hard problems where finding an exact solution is computationally expensive [11]. While the TSP has proved to have exact methods of computation, even in a large problem space [12], the LMP has remained difficult to solve, with Fekete et al. finding the first practical breakthrough for provably good tours in 2023 [10]. The pioneering work, particularly that by Arkin and Fekete, has established the foundational principles and standards that underpin the research conducted in this paper. Their efforts in TSP, LMP and other NP-hard problems have provided a robust framework and well-defined pathway for guiding the investigation.

There are many practical applications to the LMP such as farming, where Bahnemann et al. presented a new path planner that achieves 14% lower path costs than previous traditional coverage planners. [13]. Oksanen and Visala also used the LMP in creating their agricultural field path planning algorithms [14].

Other practical applications of the LMP are manufacturing, where Arkin et al.explored optimisation problems with zigzag pocket machining [15], and even cleaning, where Bormann et al. proposed an autonomous robotic assistant for professional office cleaning [16]. These practical examples of LMP help contextualise the investigation conducted, and provide real-world uses for the research conducted.

However, most of these practical applications fall under the umbrella of robotic coverage problems and this topic has been explored thoroughly [14], [17]–[19]. For example, Howie Choset conducted a survey of robotic coverage techniques and found heuristics to be one of 4 main categories, highlighting the difficulty of solving shortest path problems. [17]. Furthermore, De Carufel et al. published a note on the unsolvability of the weighted region shortest path problem, which is similar to TSP, LMP and other optimisation problems [18].

The use of heuristics and approximate optimisation for NP-hard geometric computation problems is well documented. The work by Arkin et al. [4] provides a foundational contribution to the topic. The work introduced approximation algorithms for various geometric optimisation problems and helped gain an understanding of the trade-off between the quality of a solution and the computational complexity. Similarly Bern et al. [20] have made significant contributions to the field. Their work expanded upon Arkin et al. [4] to help better understand the design and analysis of efficient approximation algorithms. While these works do not directly discuss GAs, they provide valuable insights into the complexity and challenges of geometric computation problems and are helpful when guiding the design of the GA, particularly in terms of defining the fitness function and genetic operators for the model used in the investigation.

For the use of combinatorial optimisation, evolutionary algorithms, including GAs, have been effectively utilised since as early as 1987 [21]. A significant contribution to this field was made by Muhlenbein, Gorges-Schleuter and Kramer. They introduced a new genetic algorithm, implemented on an old parallel machine, where a solution was only allowed to mate with certain other solutions that were determined to be adequate, and an optimisation technique was applied to the offspring [21]. The technique was applied to the TSP, which is closely related to the problem explored in this paper, and is also NP-hard [2].

The use of GAs for target coverage problems is also documented [19]. Pehlivanoglu et al. created an enhanced GA for the path planning of autonomous UAVs, using the GA in conjunction with other artificial intelligent methods such as Ant Colony Optimisers, Vornoi diagrams and clustering methods. Although the problem-space of this work, with regards to combinatorial complexity, is much more advanced, it provides a invaluable insight into the ideation and design of an efficient GAs.

B. Genetic Algorithms

Genetic Algorithms aim to mimic the fundamentals of real-world evolution and are inspired by Darwin's theory [22]. They can be broken down into distinct procedures that help the algorithm create a modified, ideally better, copy of itself [22]. These different procedures include:

- **Fitness:** Fitness functions are a fundamental component of GAs. They provide a way to measure the quality of suitability of a given solution in the context of the problem.
- **Selection:**Selection is another crucial component of GAs. It is the process by which preferred individuals are chosen from the population create the next generation. The goal of selection is to prefer individuals with better fitness scores, while still allowing diversity to be maintained to avoid premature convergence.

  There are several selection methods used in GAs.

  Tournament selection operates by running multiple "tournaments" among a random sample of individuals from the population. The winner of each tournament, which is chosen by the highest fitness score, is then selected for crossover [23].

  Roulette Wheel selection, also known as Fitness Proportionate Selection, works by assigning each individual a probability that is proportional to the fitness. Then a random number is generated and the individual who falls within that number is selected, this is then repeated until the desired number of individuals have been selected [24].

Chromosome 1	0000	111111
Chromosome 2	2222	333333
Offspring 1	0000	333333
Offspring 2	2222	111111

Fig. 1. Single point crossover between two chromosomes. The crossover point would be considered 4.

- **Crossover:** Crossover is also an essential part of GAs. It is a process where two parent solutions are combined to create new offspring solutions. The goal of crossover is to preserve good elements of the individual and ideally generate better offspring [22].
  There are many methods for performing crossover, each with different benefits and drawbacks. For the purposes of this experiment, both single-point and multi-point crossover were implemented.
  - **Single-point crossover** works by choosing a random point in a parent's genome, and swapping all the gene's after that point to the second parent to create new offspring. This method has the advantage of preserving sequences of moves that appear earlier in the solutions, which is beneficial when these represent good partial solutions [22].
  - **Multi-point crossover**, on the other hand, involves choosing multiple points in the parent genomes and swapping the genes in between those points. This method can potentially create more diverse offspring, but it also has a higher chance of breaking up beneficial sequences of genes [22].
- **Mutation:** The final of the genetic operations in this model is mutation. Drawing parallels with biological evolution [22], mutation in GAs introduces small random changes to genes. This operation helps to maintain diversity in the population and prevents the GA from prematurely converging to a sub-optimal solution [25].

III. Methodology

This section will discuss and detail the methodology used in the investigation. The approach involves the use of a Genetic Algorithm with a population of agents that aim to converge on a solution across generations, with genetic operators being applied in between each generation. These operators, along with the procedures for implementing them are described below.

A. Toroidal Grid-Based Lawn Mowing Problem

This section highlights the differences between the original LMP and the TGB-LMP used in the investigation. The TGB-LMP differs from the classical implementation in several key ways. While the LMP traditionally uses continuous geometry in a polygonal region, this variation has a discrete and finite problem space which reduces the problem's combinatorial complexity. Secondly, the TGB-LMP operates on a toroidal square grid. The unique wrap-around nature of the toroidal grid allows the agent to traverse the problem space without encountering true boundaries. Furthermore, the strict step limit adds a new layer of challenge, as the agent must cover the entire problem space within the optimal number of discrete steps

Despite these differences, the TGB-LMP retains the core objective of the LMP – efficient and complete coverage of a given environment. The TGB-LMP provides an insight into non-euclidean implementations of the LMP, that are more fitted for heuristic approaches such as genetic algorithms (GA). The finite and cyclical nature of the problem space allows for efficient encoding of solutions within a genome, which also facilitates the use of genetic operators such as selection, crossover and mutation.

B. The Agent

This section outlines how each agent/individual is constructed.

In the context of the study, each agent represents a series of moves the same length as the total size of the lawn; a 10x10 lawn is used, so each agent is granted 100 moves per generation. The goal of the agents is to find a tour across the problem space which covers all sectors in the minimum number of moves.

In the first generation, each agent makes a sequence of moves, with each move representing a gene. This sequence represents 1 chromosome (or genotype) and it is the optimal length of this chromosome that study aims to investigate. The moves are selected at random from the main cardinal directions until the chromosome is complete. Finally, this chromosome is then repeated, until the full genome has been populated.

Once the genome is populated, the next agent generates it's own move set encoded as a genome, until the entire population of agents have an initial genome. These move sets facilitate the use of the genetic operations that allow the agents to evolve.

C. Genetic Operations

This section will discuss each genetic operation, how it works, why it has been chosen for the investigation and the steps for replicating the experiment's methods.

- **Fitness:** Initially the fitness score was directly proportional to the number of unique cells covered.

$$fitness = \frac{mowedcells}{lawnsize} \times 100 \tag{1}$$

  This provided a baseline for guiding the agents, however later on this proved to not be specific enough. Agents were able to fill in approximately 80 percent of cells, but were unlikely to fill in any row or column entirely.
  This guided the second iteration of the fitness function, which not only takes into account the proportion of the environment covered but also the completeness of coverage in terms of rows and columns.

$$\text{fitness} = 100 \times \left(w_1 \times \frac{x}{y^2} + w_2 \times \frac{r}{y} + w_3 \times \frac{c}{y} \right)$$

Where:

- x represents the number of cells that have been mowed
- y represents one side of the square lawn
- r represents the number of rows completed
- c represents the number of columns completed

The weightings allow for fine adjustments to the importance of each factor. In the context of this investigation, x remains the most important factor, with c and r being less important, but still equally important as each other. Therefore $w_2 = w_3$ and $w_1 + w_2 + w_3 = 1$

- **Selection:** For this experiment, both tournament and roulette wheel selection where implemented, with tournament selection being chosen for the final experiment. This method provided a good balance between exploration and exploitation, while providing a constant selection pressure. This allows the GA to effectively search the solution space across generations. The tournament size for this experiment was set to $\frac{1}{20}$ of the population size.
- **Crossover:** Both single-point and multi-point crossover were implemented during testing.
  In the context of this experiment, it was found that single-point crossover worked better at preserving beneficial sequences of genes within the chromosome. Therefore single-point crossover was chosen for the final experiment, with the crossover point being selected at random each time the operation is performed.
- **Mutation:** In the context of this investigation, mutation was implemented by setting a mutation rate and iterating through every gene. If the current iteration's random value satisfies the mutation rate, the gene will change to another random direction. The mutation rate for this experiment is 5%.
- **Termination:** For each chromosome length being tested, the experiment will be considered a success if at least one complete solution is found within 100 generations. If a solution is found before 100 generations the experiment will continue gathering the metrics of the model for the remaining number of generations. For chromosome key lengths that do not achieve an optimal solution within the allotted 100 generations, the termination condition will extend to 500 generations to test if a viable solution can be found.

D. Experiment Design

This section will discuss what parameters are being varied and how results are measured and collected.

The main parameter being tested in this experiment is the chromosome length. As there are 100 moves, the maximum possible chromosome length would be 100, which would take up the whole genome.

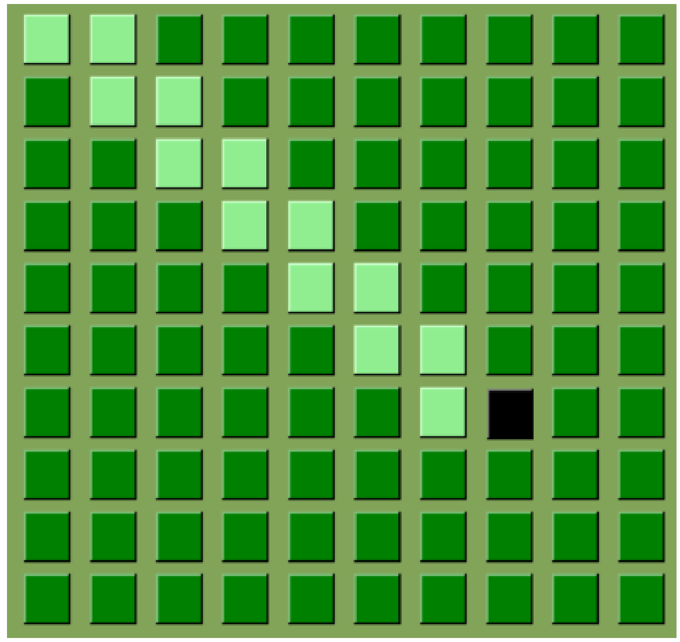

Fig. 2. A screenshot of an agent exploring the lawn used in the investigation

As there are 4 options in any direction for movement, with a chromosome length of 100, there would be 4^{100} different combinations of moves. As this is such a large search space, this experiment will test key chromosome lengths up to 20.

Due to the even squared nature of the search space and because the agent always starts on the same square, the parity (even or odd nature) of the chromosome length will determine if an agent can find an optimal solution. When an agent has an even chromosome length and starts on the first square (any corner) it is possible to complete a sequence of moves that, when repeated, do not retrace old steps. If a chromosome has an odd number of moves, as each agent's chromosome is iterated upon the agent will end up on alternating squares each time. In this case, the agent will alternate between a sub-optimal number of rows and columns – thus never breaking new ground.

Additionally, any chromosome that is not an even factor of 100 (2,4,10,20,50,100) will be much less likely to find an optimal solution. This is because the total number of moves will not divide evenly into the total number of cells in the grid, meaning the remainder of moves will have to still complete any remaining cells.

To test this parameter, the model will be ran with varying chromosome lengths to measure the viability of given solutions.

The chromosome lengths being used for this investigation are:

- $5, 10, 12, 14, 16, 18, 20$

With the key chromosome lengths being:

- $5, 10, 20$

Each length being tested is initially run for 100 generations, with 100 agents per generation, using singlepoint crossover, tournament selection and a 5% mutation rate. If a key chromosome length (factors of total area of search space) does not achieve an optimal solution within the allotted 100 generations, it will be extended to 500 to explore if finding a solution is feasible.

In order to effectively analyse the results, certain metrics are measured across generations, and these metrics are used to compare the performance of each varying chromosome length.

The metrics measured are:

- Average Fitness of all Agents
- Fitness score of best performing agent
- Diversity between agents (number of unique genomes within a population)

These metrics will help guide the conclusions gained from the investigation and will be used to analyse how the exploration and exploitation of the problem space varies in the various lengths.

IV. Results

The results gathered track best fitness, average fitness and population diversity (the number of unique genomes in a population) against the generation number, with the chromosome length being varied for each test.

A. *Analysis of Results:*

In the following section, an analysis of the results obtained from the investigation is presented.

The primary objective of this investigation was to better understand the relationship between chromosome length and performance in the context of GAs. It was hypothesised that, within the search space used, only chromosome lengths that were even factors of the total area of the search space and larger than one dimension of the search space would return an optimal result. This was found to be true, as only chromosome lengths of 10 and 20 returned complete solutions.

Within 100 generations, only a chromosome of length 10 (*as seen in Fig. 6*) found viable solutions and outperformed all other lengths tested. This is inherently due to the squared nature of the problem space being explored and the implementation of encoding the genetic sequences. Each chromosome is repeated until 100 moves have occurred and the environment is a 10x10 toroidal grid. This means a gene sequence of 10 viable moves that avoids retracing previously visited paths can be encoded into a chromosome and repeated to form a complete solution and populate the individual's genome.

This became apparent when analysing the chromosome lengths of $12, 14, 16$ and 18 (*seen in Fig. 8, Fig. 9, Fig. 10 and Fig. 11*). Each chromosome length is effectively the "pattern length" of the moves. In this case, a chromosome length of 12, 14, 16, or 18 resulted in less optimal solutions, potentially due to the way these patterns interact with the 10x10 grid environment. For example, if the chromosome length is 12, after 8 full repeats (96 moves), there will be 4 moves left. These 4 moves will be the first 4 moves of the chromosome, repeated. Depending on the specifics of these moves and the agent's position after 96 moves, this resulted

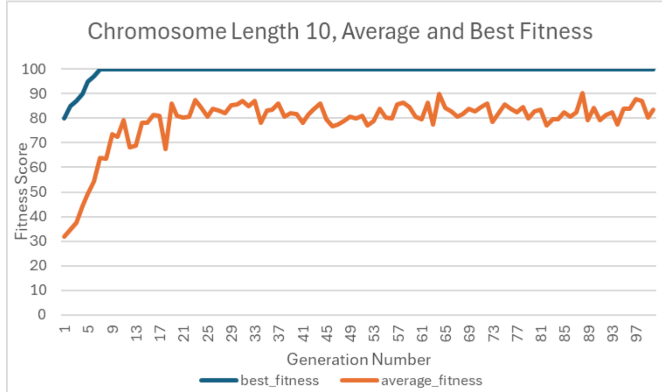

Fig. 3. Chromosome length 10, average and best fitness

in the agent missing some squares or revisiting others. There may be solutions of length 12, 14, 16 and 18, but the number of possible solutions, when considering the remainder, is much smaller and could not be found within 100 generations.

With these insights, it can be assumed that any chromosome length that is a factor of 100 would be more likely to find optimal solutions. However, this is incorrect. Upon testing the key chromosome length 5 (*seen in Fig. 4*) it was found that odd numbered chromosome lengths do not provide viable solutions. Moving in any of the four cardinal directions from a square in a grid will always change the parity of the position (from odd to even, or from even to odd). So, if the agent starts on an even square, after an odd number of moves it will always be on an odd square, and vice versa. Given the constraints of the problem, this limits the exploration of the agents. To complete a solution within the given 100 moves, an individual cannot revisit cells. Therefore, if an odd-numbered chromosome length agent performed what should be considered a series of good moves (i.e. does not traverse visited paths), it was found that the maximum coverage is limited to 50% of the total search space.

It is apparent that the number of viable chromosome lengths is much fewer than originally anticipated. Chromosome length 20 (*seen in Fig. 10*) did not return any optimal solutions within 100 generations. Interestingly, better fitness scores were seen for length 20 than in chromosome lengths that, in theory, have less combinatorial complexity. This highlights that the power of heuristic methods, especially regarding GAs, are determined by not just the complexity of the search space, but how the encoding of a solution interacts with that environment. Additionally, chromosome length 20 was ran to a further 500 generations (*seen in Fig. 11*), to showcase whether the model was able to achieve an optimal solution given enough time. It was found that, although the population became stuck in local optima more than once, the model was able to output an optimal solution by generation 179. Intriguingly, the model did not keep the fittest individual after finding a solution. This behaviour repeated to generation 203 and 206, before finally keeping optimal solutions within the population after generation 248. This is due to the way the tournament selection is implemented. As a random sample is taken from the fittest individuals, it is not guaranteed the best individuals will be selected for crossover. Although this may seem counterproductive, it was found to help maintain diversity within the population, which prevents the algorithm from prematurely converging, or being stuck in a local optimum.

With regard to diversity across the chromosome lengths tested, it was found that the number of unique individuals tended to decrease as the generations passed (*seen in Fig. 5, Fig. 7 and Fig. 12*). This could potentially be due to a number of reasons. Selection pressure means that the fittest individuals are more likely to be repeatedly selected for reproduction and as the algorithm progresses, it tends to converge towards a set of solutions that are similar. The stochastic nature of GAs may also contribute to certain sequences becoming more common. This decrease in diversity can lead to the algorithm getting stuck in local optima which, in future, could possibly be mitigated by running an island model with various populations or increasing the mutation rate.

It is shown that as the chromosome length (which is also representing the given search space) increases linearly, the complexity of the problem can increase exponentially. This is because each additional gene multiplies the number of possible combinations. As there are 4 choices of moves the number of combinations for any given chromosome length is 4^n, where n represents the length of the chromosome. This means that not only does the computational complexity increase with each extra gene per chromosome, it also increases at a greater rate the larger the chromosome length.

The techniques used, specifically the implementation of encoding solutions within a singular repeating fixed-length chromosome, may have impacted the results and limited the significance. Finding a pattern that satisfies all the conditions of the environment and can successfully be repeated is a computationally expensive task, and results may have been limited by the number of generations each length was ran for.

V. Conclusion and Future Work

This investigation has provided valuable insights into the relationship between chromosome length and the performance of GAs, especially in solving the the Lawn Mowing Problem. The findings align with existing research that suggests an exponential increase in complexity as the search space or chromosome length increases linearly [26]. This confirms that the power of heuristic methods are determined by the complexity of the given search space.

From a legal and professional standpoint, the use of GAs in such investigations must be conducted with due diligence, ensuring the methods used comply with all relevant regulations. This includes data protection and privacy laws if the GA is handling personal protected data. As the field of Artificial Intelligence (AI) develops more laws and regulations will apply [27].

Regarding sustainability, the use of GAs has both potential benefits and drawbacks. On the one hand, the efficient use of GAs for heuristically solving difficult problems, such as LMP, may lead to more energy efficient coverage solutions for a variety of uses, contributing to a global effort for environmental sustainability. However, the inefficient use of GAs should also be considered [27]. As proven in this paper, while larger search spaces in GAs become increasingly complex [26], other factors, such as the shape and size of the environment, may also impact performance. This means that if an unsuitable chromosome length is chosen, depending on the problem being solved, efforts and resources spent searching for an optimal solution could be ineffectual.

The results gained could also potentially be statistically insignificant. If the investigation were to be repeated, a G* Power analysis, or an Analysis of Variance test (ANOVA), would help gain a better understanding of the significance of the data. To move forward in the investigation taking a hybrid approach combining the performance of GAs with other optimisation techniques, such as Simulated Annealing (SA) [28] or Particle Swarm Optimisation (PSO) [29], could also help gain further insights into the viability of GAs for optimisation problems. PSO and SA use different search mechanisms compared to GAs. SA uses a probabilistic search that can escape local optima by allowing worse solutions in earlier stages of the search, whereas PSO uses a population where each individual adjusts itself based on the expierence of other individuals (particles).

There is significant research into hybridisation algorithms that combine the strentgths of all three methods [30]. Comparing GAs with SA and PSO could provide valuable insights into the strengths and weaknesses of these algorithms, and potentially inform the design of more effective solutions for hybrid algorithms with further applications [31]. To compare these techniques, first a control would be used, where a random search would attempt to find solutions for a fair baseline. Condition 1 would take a well-suited optimisation or search algorithm and attempt to find solutions. In condition 2, the best performing genetic model from this experiment (chromosome length 10) would also search for solutions. These results could then be compared to provide a better understanding of not only the performance of GAs, but how that performance compares with algorithms designed to solve the problem.

Future work could also explore the use of a variety of initial chromosomes, rather than a singular repeated pattern sequence, or alternative crossover, mutation and selection strategies. In conclusion, this investigation has not only contributed to the understanding of the performance of GAs, but also highlighted the importance of considering the wider implications of this technology. The insights gained help highlight how chromosome length impacts complexity, and how the nature of the search space being explored must be carefully considered when employing the use of GAs.

VI. Appendices

A. Chromosome Length 5 Graphs:

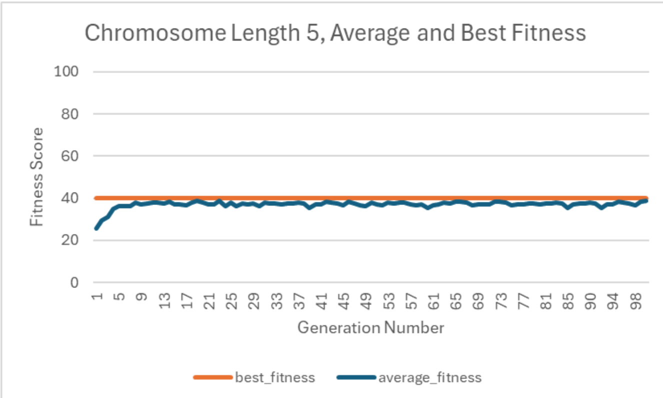

Fig. 4. Chromosome length 5, average and best fitness

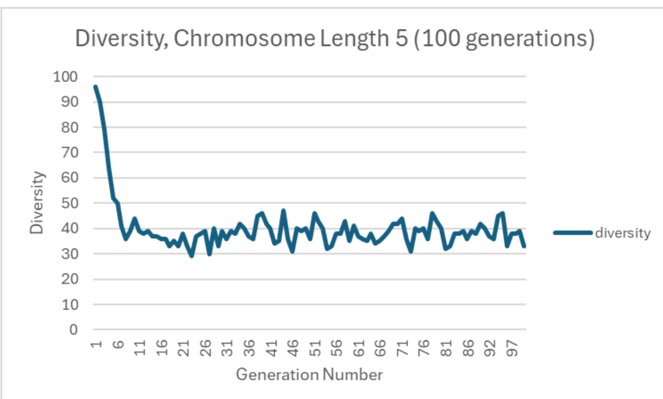

Fig. 5. Chromosome length 5, diversity across generations

B. Chromosome Length 10 Graphs:

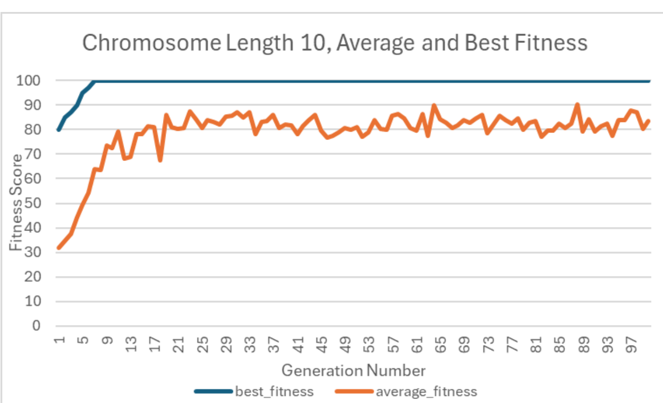

Fig. 6. Chromosome length 10, average and best fitness

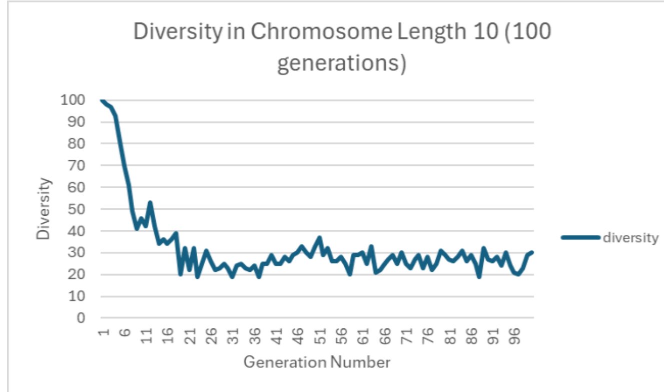

Fig. 7. Chromosome length 10, diversity across generations

C. Chromosome Lengths 12,14,16,18 Graphs:

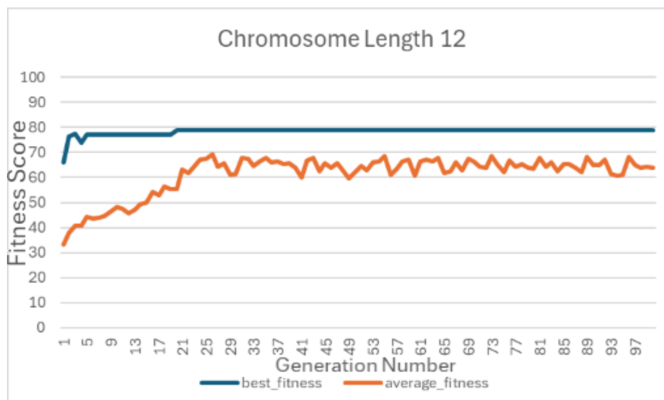

Fig. 8. Chromosome length 12, average and best fitness

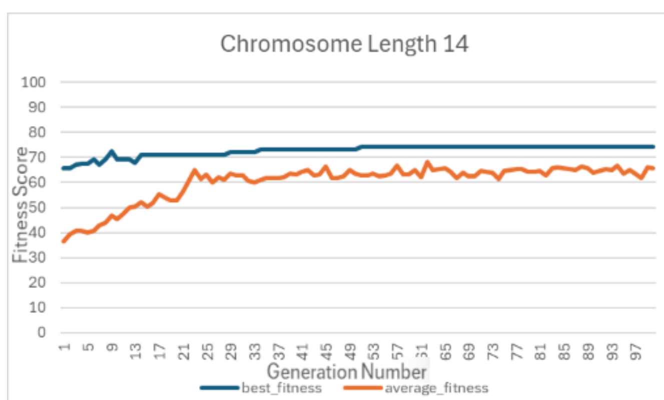

Fig. 9. Chromosome length 14, average and best fitness

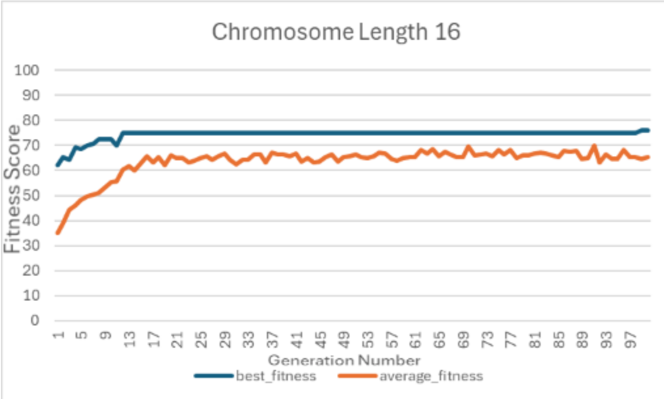

Fig. 10. Chromosome length 16, average and best fitness

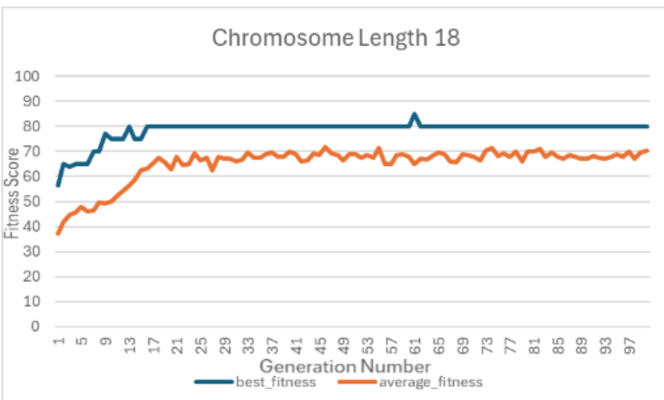

Fig. 11. Chromosome length 18, average and best fitness

D. Chromosome Length 20 Graphs:

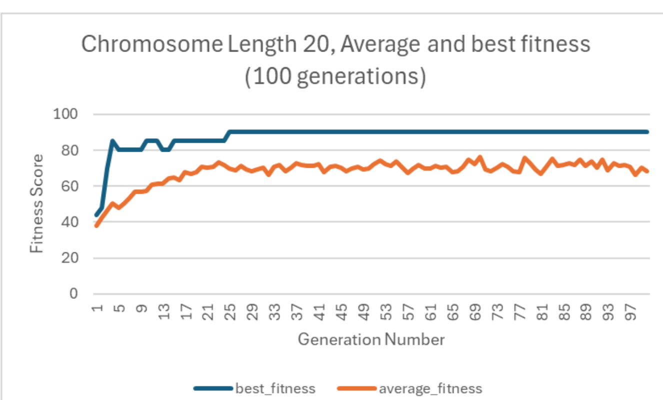

Fig. 12. Chromosome length 20 to 100 generations, average and best fitness

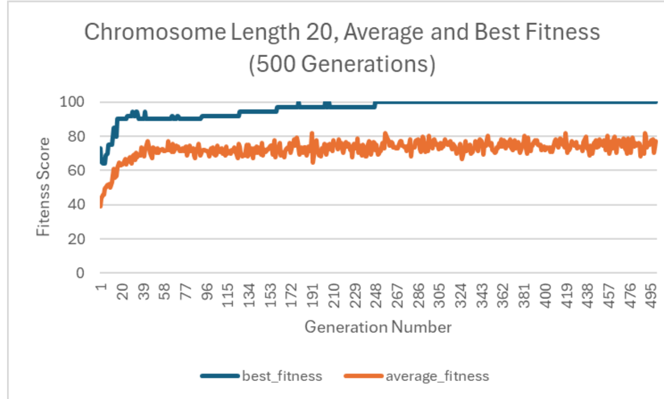

Fig. 13. Chromosome length 20 to 500 generations, average and best fitness

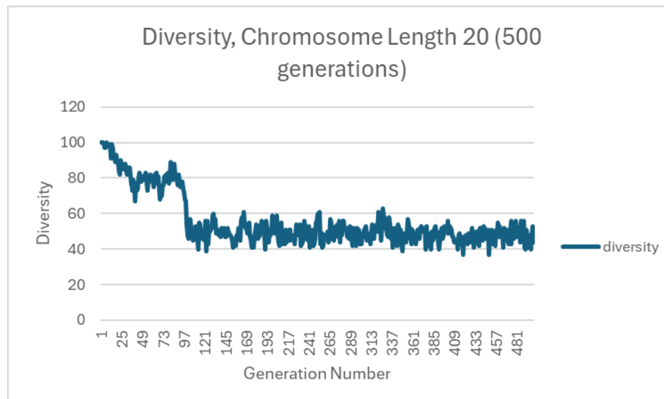

Fig. 14. Chromosome length 20 to 500 generations, showing diversity across generations

Chromosome Length 5 Data:

Generation Number	Best Fitness	Mean Fitness	Diversity
1	40	25.78	96
2	40	29.30	90
3	40	31.36	79
4	40	34.83	64
5	40	36.17	52
6	40	36.07	50
7	40	36.08	41
8	40	37.80	36
9	40	37.11	39
10	40	37.47	44
11	40	37.81	39
12	40	37.81	38
13	40	37.59	39
14	40	38.41	37
15	40	36.97	37
16	40	36.91	36
17	40	36.56	36
18	40	37.85	33
19	40	38.64	35
20	40	37.92	33
21	40	37.27	38

Generation Number	Best Fitness	Mean Fitness	Diversity
22	40	37.16	33
23	40	38.84	29
24	40	36.39	37
25	40	38.00	38
26	40	36.41	39
27	40	37.37	30
28	40	37.13	40
29	40	37.67	33
30	40	36.14	39
31	40	37.92	36
32	40	37.46	39
33	40	37.41	38
34	40	37.04	42
35	40	37.35	40
36	40	37.47	37
37	40	38.12	36
38	40	37.46	45
39	40	35.34	46
40	40	37.01	42
41	40	37.14	40
42	40	38.36	34
43	40	37.94	35
44	40	37.58	47
45	40	36.85	36
46	40	38.30	31
47	40	37.70	40
48	40	36.68	39
49	40	36.36	40
50	40	37.95	36
51	40	37.09	46
52	40	36.65	43
53	40	37.88	40
54	40	37.62	32
55	40	37.79	33
56	40	38.08	38
57	40	37.09	38
58	40	36.56	43
59	40	37.29	35
60	40	35.44	41
61	40	36.76	37
62	40	37.18	36
63	40	38.00	35
64	40	37.35	38
65	40	38.47	34
66	40	38.38	35
67	40	37.94	37
68	40	36.69	39
69	40	37.22	42
70	40	37.08	42
71	40	36.96	44
72	40	38.47	36
73	40	38.38	31
74	40	37.94	40
75	40	36.69	39
76	40	37.22	40
77	40	37.08	36
78	40	37.46	46
79	40	37.41	43
80	40	37.04	40
81	40	37.35	32
82	40	37.47	33
83	40	38.12	38
84	40	37.46	38
85	40	35.34	39
86	40	37.04	36
87	40	37.35	39
89	40	37.47	38
90	40	38.12	42
91	40	37.46	40
92	40	35.34	37
93	40	37.01	36
94	40	37.14	45
95	40	38.36	46
96	40	37.94	33
97	40	37.58	38
98	40	36.85	38
99	40	38.30	39
100	40	38.68	33

Chromosome Length 10 Data:

Generation Number	Best Fitness	Mean Fitness	Diversity
1	80	31.69	100
2	85	34.62	98
3	87	37.54	97
4	90	44.42	93
5	95	49.26	81
6	97	54.14	70
7	100	63.94	61
8	100	63.56	49
9	100	73.46	41
10	100	72.57	46
11	100	79.38	42
12	100	68.35	53
13	100	68.90	42
14	100	77.99	34
15	100	78.31	36
16	100	81.44	34
17	100	81.04	36
18	100	67.37	39
19	100	86.07	20
20	100	81.13	32
21	100	80.14	22
22	100	80.57	32
23	100	87.30	19
24	100	84.16	25
25	100	80.82	31
26	100	83.86	26

Generation Number	Best Fitness	Mean Fitness	Diversity
27	100	83.10	22
28	100	82.08	23
29	100	85.25	25
30	100	85.64	23
31	100	86.90	19
32	100	84.80	24
33	100	87.12	25
34	100	78.22	23
35	100	83.16	22
36	100	83.52	24
37	100	85.92	19
38	100	80.56	25
39	100	82.18	25
40	100	81.62	29
41	100	78.22	25
42	100	81.43	25
43	100	83.86	28
44	100	86.14	26
45	100	79.50	29
46	100	76.76	30
47	100	77.58	33
48	100	78.86	30
49	100	80.54	28
50	100	79.96	33
51	100	81.14	37
52	100	77.15	29
53	100	78.84	32
54	100	83.76	26
55	100	80.32	26
56	100	80.10	28
57	100	85.80	25
58	100	86.50	20
59	100	84.44	29
60	100	80.70	29
61	100	79.60	30
62	100	86.31	25
63	100	77.56	33
64	100	89.96	21
65	100	84.14	22
66	100	82.80	25
67	100	80.50	27
68	100	81.85	29
69	100	83.68	25
70	100	82.74	30
71	100	84.42	25
72	100	86.04	23
73	100	78.50	27
74	100	82.12	29
75	100	85.68	23
76	100	83.74	28
77	100	82.52	22
78	100	84.44	25
79	100	80.08	31
80	100	82.78	29
81	100	83.63	27
82	100	77.15	26
83	100	79.72	28
84	100	79.70	31
85	100	82.54	26
86	100	80.62	29
87	100	82.45	25
88	100	90.30	19
89	100	79.34	32
90	100	84.16	27
91	100	79.36	26
92	100	81.26	28
93	100	82.36	24
94	100	77.58	30
95	100	83.68	24
96	100	83.80	21
97	100	87.91	20
98	100	87.18	23
99	100	80.16	29
100	100	83.60	22

Chromosome	**Length**	**12**	**Data:**
Generation Number	Best Fitness	Mean Fitness	Diversity
1	66.2	33.21	100
2	76.4	37.85	99
3	77.4	40.65	96
4	74	40.79	97
5	77	44.27	95
6	77.2	43.72	99
7	77.2	43.99	93
8	77.2	44.67	96
9	77.2	46.32	93
10	77.2	48.19	93
11	77.2	47.54	84
12	77.2	45.84	86
13	77.2	47.08	86
14	77.2	49.13	79
15	77.2	50.53	72
16	77.2	54.26	68
17	77	52.98	59
18	77	56.46	53
19	77	55.49	57
20	78.8	55.49	52
21	78.8	63.36	38
22	78.8	61.84	29
23	78.8	64.77	30
24	78.8	67.24	24
25	78.8	67.35	26
26	78.8	69.31	24
27	78.8	64.30	33
28	78.8	65.52	33
29	78.8	61.13	34
30	78.8	61.33	29

Generation Number	Best Fitness	Mean Fitness	Diversity
31	78.8	67.87	30
32	78.8	67.49	22
33	78.8	64.77	29
34	78.8	66.51	27
35	78.8	67.72	23
36	78.8	65.99	32
37	78.8	66.27	31
38	78.8	65.43	31
39	78.8	65.54	36
40	78.8	63.81	30
41	78.8	59.81	34
42	78.8	66.66	29
43	78.8	67.66	24
44	78.8	62.32	33
45	78.8	65.60	35
46	78.8	63.75	28
47	78.8	65.7	29
48	78.8	62.64	32
49	78.8	59.71	33
50	78.8	62.12	33
51	78.8	64.52	28
52	78.8	62.98	35
53	78.8	65.88	30
54	78.8	66.48	28
55	78.8	68.6	28
56	78.8	61.21	31
57	78.8	63.59	31
58	78.8	66.29	28
59	78.8	67.19	29
60	78.8	60.63	31
61	78.8	66.41	29
62	78.8	67.22	30
63	78.8	66.33	39
64	78.8	67.82	36
65	78.8	61.84	30
66	78.8	62.51	34
67	78.8	66.21	25
68	78.8	62.79	30
69	78.8	67.32	22
70	78.8	66.34	33
71	78.8	64.1	35
72	78.8	64.07	32
73	78.8	68.62	28
74	78.8	65.15	30
75	78.8	62.19	33
76	78.8	66.83	29
77	78.8	64.25	27
78	78.8	65.46	32
79	78.8	64.07	37
80	78.8	63.45	33
81	78.8	67.84	30
82	78.8	64.41	26
83	78.8	66.02	29
84	78.8	62.44	31
85	78.8	65.37	29
86	78.8	65.45	29
87	78.8	63.95	25
88	78.8	61.95	35
89	78.8	68.2	29
90	78.8	64.84	28
91	78.8	64.96	32
92	78.8	67.2	27
93	78.8	61.29	33
94	78.8	60.77	33
95	78.8	60.91	33
96	78.8	68.08	31
97	78.8	64.83	26
98	78.8	64.02	28
99	78.8	64.14	26
100	78.8	63.74	32

Chromosome Length 14 Data:

Generation Number	Best Fitness	Mean Fitness	Diversity
1	65.8	36.58	100
2	65.8	39.48	99
3	67.5	40.62	100
4	67.6	40.74	99
5	67.4	40.07	99
6	69.4	40.59	99
7	67.5	42.87	98
8	69.4	43.87	99
9	72.6	46.83	94
10	69.4	45.53	97
11	69.4	47.64	98
12	69.4	49.98	93
13	67.8	50.22	87
14	71.2	52.12	85
15	71.2	50.43	91
16	71.2	51.68	82
17	71.2	55.37	83
18	71.2	54.06	85
19	71.2	53.00	84
20	71.2	52.78	77
21	71.2	56.53	66
22	71.2	60.17	50
23	71.2	64.97	29
24	71.2	61.48	36
25	71.2	63.18	34
26	71.2	60.09	41
27	71.2	62.10	40
28	71.2	61.12	43
29	72.2	63.47	35
30	72.2	62.83	37
31	72.2	62.83	34
32	72.2	60.60	48
33	72.2	59.81	50
34	73.2	60.92	44

Generation Number	Best Fitness	Mean Fitness	Diversity
35	73.2	61.66	50
36	73.2	61.76	46
37	73.2	61.76	47
38	73.2	62.17	42
39	73.2	63.47	37
40	73.2	63.22	35
41	73.2	64.26	31
42	73.2	64.83	30
43	73.2	62.96	36
44	73.2	63.34	32
45	73.2	66.47	28
46	73.2	61.84	36
47	73.2	61.62	38
48	73.2	62.55	40
49	73.2	65.00	29
50	73.2	63.57	31
51	74.2	62.69	41
52	74.2	62.78	37
53	74.2	63.71	38
54	74.2	62.36	42
55	74.2	62.72	38
56	74.2	63.57	38
57	74.2	66.73	28
58	74.2	63.32	30
59	74.2	63.10	35
60	74.2	64.97	29
61	74.2	62.14	41
62	74.2	68.24	25
63	74.2	64.81	30
64	74.2	65.20	34
65	74.2	65.69	29
66	74.2	64.20	30
67	74.2	61.80	42
68	74.2	63.78	41
69	74.2	62.55	37
70	74.2	62.34	43
71	74.2	64.63	36
72	74.2	64.18	38
73	74.2	63.96	31
74	74.2	61.48	38
75	74.2	64.64	39
76	74.2	65.03	29
77	74.2	65.41	28
78	74.2	65.26	29
79	74.2	64.34	32
80	74.2	64.42	31
81	74.2	64.45	33
82	74.2	62.87	37
83	74.2	65.76	30
84	74.2	66.05	33
85	74.2	65.85	31
86	74.2	65.17	36
87	74.2	65.09	34
88	74.2	66.39	31
89	74.2	65.51	27
90	74.2	64.07	41
91	74.2	64.74	35
92	74.2	65.26	34
93	74.2	65.05	32
94	74.2	66.63	29
95	74.2	63.51	37
96	74.2	65.05	30
97	74.2	63.43	33
98	74.2	61.68	41
99	74.2	65.93	28
100	74.2	65.83	28

Chromosome	**Length**	**16**	**Data:**
Generation Number	Best Fitness	Mean Fitness	Diversity
1	62.2	34.92	100
2	65.4	39.14	99
3	64.4	44.19	98
4	69.2	45.97	100
5	68.6	48.18	99
6	70.0	49.73	98
7	70.8	50.28	95
8	72.6	50.97	92
9	72.6	53.33	90
10	72.6	55.51	89
11	70	55.75	81
12	75	60.36	63
13	75	61.76	57
14	75	60.02	59
15	75	62.70	53
16	75	65.70	45
17	75	63.25	49
18	75	65.45	43
19	75	62.26	52
20	75	66.20	45
21	75	65.05	47
22	75	65.02	48
23	75	63.20	47
24	75	64.04	51
25	75	65.03	46
26	75	65.77	39
27	75	64.20	53
28	75	65.70	45
29	75	66.80	41
30	75	64.39	44
31	75	62.59	50
32	75	64.29	54
33	75	64.26	53
34	75	66.45	48
35	75	66.27	47
36	75	63.32	60
37	75	67.09	46
38	75	66.23	47

Generation Number	Best Fitness	Mean Fitness	Diversity
39	75	66.28	48
40	75	65.54	54
41	75	66.92	52
42	75	63.50	54
43	75	64.99	47
44	75	63.35	51
45	75	63.46	57
46	75	65.18	52
47	75	66.51	51
48	75	63.50	59
49	75	65.22	60
50	75	65.72	55
51	75	66.44	46
52	75	65.36	57
53	75	65.09	55
54	75	65.74	53
55	75	67.09	52
56	75	66.64	49
57	75	64.74	55
58	75	63.83	55
59	75	64.80	64
60	75	65.26	59
61	75	65.49	55
62	75	68.34	51
63	75	66.72	52
64	75	68.52	47
65	75	65.86	56
66	75	67.50	47
67	75	66.51	49
68	75	65.37	60
69	75	65.24	56
70	75	69.64	39
71	75	65.94	51
72	75	66.46	48
73	75	66.73	45
74	75	65.52	51
75	75	68.32	44
76	75	66.40	53
77	75	68.19	41
78	75	65.05	53
79	75	66.15	49
80	75	66.04	50
81	75	66.82	50
82	75	67.03	45
83	75	66.79	46
84	75	65.98	48
85	75	65.27	51
86	75	67.87	45
87	75	67.30	46
88	75	67.68	43
89	75	64.47	51
90	75	65.13	49
91	75	70.06	36
92	75	63.35	53
93	75	66.51	53
94	75	64.58	53
95	75	64.56	56
96	75	68.10	44
97	75	65.47	58
98	75	65.49	51
99	76	64.55	56
100	76	65.33	57

Chromosome	**Length**	**18**	**Data:**
Generation Number	Best Fitness	Mean Fitness	Diversity
1	56.4	37.23	100
2	65	41.69	100
3	64	44.58	96
4	65	45.78	100
5	65	47.82	99
6	65	46.11	98
7	69.8	46.55	97
8	69.8	49.76	97
9	77	49.42	93
10	75	49.87	95
11	75	52.01	97
12	75	54.36	96
13	80	56.51	90
14	75	58.66	84
15	75	62.48	70
16	80	63.13	67
17	80	65.4	71
18	80	67.42	63
19	80	65.66	64
20	80	62.75	67
21	80	67.99	55
22	80	64.65	55
23	80	64.82	61
24	80	69.24	51
25	80	66.53	57
26	80	67.36	60
27	80	62.54	73
28	80	67.83	63
29	80	67.21	70
30	80	67.23	63
31	80	65.92	65
32	80	66.81	62
33	80	69.75	63
34	80	67.48	62
35	80	67.41	59
36	80	68.99	62
37	80	69.77	54
38	80	67.69	68
39	80	67.96	59
40	80	69.83	60
41	80	69.06	57
42	80	66.04	58

Generation Number	Best Fitness	Mean Fitness	Diversity
43	80	66.54	59
44	80	69.33	54
45	80	68.49	53
46	80	71.6	45
47	80	69.11	48
48	80	68.48	54
49	80	66.46	57
50	80	68.84	57
51	80	68.75	58
52	80	67.61	65
53	80	68.66	53
54	80	67.42	60
55	80	71.42	52
56	80	64.94	62
57	80	65.07	64
58	80	68.46	63
59	80	68.86	64
60	80	67.78	73
61	85	65.05	73
62	80	67.13	65
63	80	66.62	57
64	80	68.28	55
65	80	69.46	54
66	80	68.84	59
67	80	66.17	59
68	80	65.73	66
69	80	68.96	62
70	80	68.38	60
71	80	67.94	59
72	80	66.37	68
73	80	70.36	57
74	80	71.56	50
75	80	68.02	68
76	80	69.27	64
77	80	67.78	71
78	80	69.94	66
79	80	66.11	76
80	80	69.86	65
81	80	69.99	62
82	80	71.08	58
83	80	67.74	68
84	80	69.63	70
85	80	67.65	78
86	80	67.09	73
87	80	68.44	66
88	80	67.71	69
89	80	67.09	74
90	80	67.26	69
91	80	68.26	70
92	80	67.38	73
93	80	67.21	73
94	80	67.70	68
95	80	68.76	65
96	80	67.71	63
97	80	70.04	55
98	80	67.26	63
99	80	69.74	57
100	80	70.17	57

Chromosome	**Length**	**20**	**Data:**
Generation Number	Best Fitness	Mean Fitness	Diversity
1	73	38.94	100
2	65	43.72	100
3	64	45.79	100
4	64	46.05	100
5	64	49.31	99
6	69	50.22	97
7	70	51.35	97
8	75	51.66	100
9	75	52.03	98
10	75	49.91	98
11	75	53.04	98
12	80	57.06	99
13	85	60.73	91
14	85	55.84	99
15	80	56.55	97
16	90	61.33	92
17	90	63.25	91
18	90	64.49	89
19	90	63.07	93
20	90	63.79	89
21	90	63.36	89
22	90	64.68	84
23	90	66.81	82
24	90	63.64	90
25	91.6	65.52	86
26	91.6	64.97	88
27	91.6	67.95	85
28	91.6	65.48	86
29	91.6	63.99	88
30	94.4	68.49	85
31	91.6	69.27	82
32	91.6	66.17	86
33	94.4	70.21	84
34	94.4	67.82	86
35	90	69.26	81
36	90	69.40	76
37	90	68.98	79
38	90	73.46	73
39	90	71.27	79
40	90	68.38	77
41	94.4	72.48	67
42	90	73.41	75
43	90	77.07	73
44	90	74.62	74
45	90	71.95	80
46	90	70.08	83

Generation Number	Best Fitness	Mean Fitness	Diversity	Generation Number	Best Fitness	Mean Fitness	Diversity
47	90	67.43	80	100	91.6	72.76	54
48	90	73.70	78	101	91.6	71.67	48
49	90	73.14	81	102	91.6	73.33	46
50	90	71.47	79	103	91.6	69.68	51
51	90	70.38	80	104	91.6	68.55	57
52	90	72.91	81	105	91.6	69.23	52
53	90	70.43	83	106	91.6	74.69	47
54	90	71.37	78	107	91.6	72.38	45
55	90	71.71	73	108	91.6	70.78	46
56	90	72.52	79	109	91.6	72.59	48
57	90	72.63	78	110	91.6	73.02	47
58	90	71.46	82	111	91.6	69.88	53
59	90	71.63	80	112	91.6	69.97	51
60	90	71.69	81	113	91.6	68.97	45
61	90	72.11	79	114	91.6	73.92	40
62	90	76.98	75	115	91.6	69.97	55
63	90	71.74	82	116	91.6	71.59	54
64	90	72.54	77	117	91.6	70.46	46
65	91.6	71.14	83	118	91.6	70.05	48
66	91.6	75.40	77	119	91.6	71.81	47
67	90	70.83	81	120	91.6	71.41	51
68	90	73.22	75	121	91.6	73.32	45
69	90	74.70	68	122	91.6	70.89	56
70	91.6	72.84	73	123	91.6	70.18	39
71	91.6	73.85	70	124	91.6	67.19	56
72	90	71.29	76	125	91.6	75.51	43
73	90	72.52	76	126	94.4	73.86	50
74	90	71.28	81	127	94.4	70.37	50
75	90	72.84	80	128	94.4	68.29	51
76	90	71.50	78	129	94.4	68.99	52
77	90	73.30	82	130	94.4	68.57	59
78	90	73.81	80	131	94.4	69.02	60
79	90	68.99	83	132	94.4	74.94	54
80	90	72.36	77	133	94.4	68.45	57
81	90	74.55	80	134	94.4	74.88	49
82	90	71.26	89	135	94.4	69.48	50
83	90	72.46	86	136	94.4	73.01	51
84	90	73.12	79	137	94.4	71.15	48
85	90	68.95	83	138	94.4	67.48	49
86	90	67.45	88	139	94.4	72.98	47
87	90	72.71	82	140	94.4	70.47	52
88	90	73.08	79	141	94.4	74.13	48
89	90	75.56	79	142	94.4	70.46	50
90	90	71.73	76	143	94.4	73.44	52
91	90	68.68	82	144	94.4	73.03	47
92	91.6	71.40	78	145	94.4	69.12	47
93	91.6	72.17	75	146	94.4	76.28	48
94	91.6	72/00	76	147	94.4	68.84	52
95	91.6	71.68	78	148	94.4	72.68	48
96	91.6	71.59	74	149	94.4	71.10	50
97	91.6	71.53	72	150	94.4	71.25	48
98	91.6	70.39	68	151	94.4	72.55	48
99	91.6	68.17	67	152	94.4	72.51	47

Generation Number	Best Fitness	Mean Fitness	Diversity	Generation Number	Best Fitness	Mean Fitness	Diversity
153	94.4	74.01	45	206	100	72.20	44
154	94.4	74.55	41	207	97.2	75.57	42
155	94.4	74.60	44	208	97.2	73.72	51
156	94.4	69.28	46	209	97.2	71.47	55
157	94.4	77.48	42	210	97.2	77.62	43
158	94.4	67.72	48	211	97.2	75.73	47
159	94.4	70.54	46	212	97.2	74.94	44
160	97.2	73.51	52	213	97.2	75.15	43
161	97.2	74.08	50	214	97.2	72.35	51
162	97.2	75.91	45	215	97.2	78.05	44
163	97.2	69.04	56	216	97.2	75.86	45
164	97.2	70.63	58	217	97.2	76.17	49
165	97.2	69.34	55	218	97.2	72.53	51
166	97.2	67.67	61	219	97.2	70.24	51
167	97.2	71.18	54	220	97.2	72.96	45
168	97.2	72.22	52	221	97.2	76.69	47
169	97.2	73.36	52	222	97.2	74.25	48
170	97.2	72.57	49	223	97.2	75.72	46
171	97.2	76.11	54	224	97.2	75.38	48
172	97.2	70.56	55	225	97.2	71.92	43
173	97.2	74.50	47	226	97.2	74.15	49
174	97.2	75.11	45	227	97.2	67.99	54
175	97.2	78.19	41	228	97.2	72.51	53
176	97.2	74.37	47	229	97.2	75.81	48
177	97.2	79.40	41	230	97.2	72.78	49
178	97.2	76.79	47	231	97.2	73.10	54
179	100	73.02	50	232	97.2	78.54	42
180	97.2	69.13	54	233	97.2	70.37	43
181	97.2	73.53	50	234	97.2	74.61	44
182	97.2	72.24	46	235	97.2	70.17	49
183	97.2	70.54	53	236	97.2	69.06	56
184	97.2	73.68	47	237	97.2	75.18	46
185	97.2	73.58	48	238	97.2	71.94	53
186	97.2	69.05	53	239	97.2	68.26	49
187	97.2	71.11	56	240	97.2	68.11	45
188	97.2	73.77	54	241	97.2	73.90	53
189	97.2	71.31	52	242	97.2	77.34	41
190	97.2	74.58	49	243	97.2	76.23	43
191	97.2	81.64	40	244	97.2	69.21	47
192	97.2	64.79	56	245	97.2	71.71	51
193	97.2	72.50	52	246	97.2	76.82	42
194	97.2	74.29	49	247	97.2	76.00	43
195	97.2	77.42	44	248	100	71.74	46
196	97.2	72.03	51	249	100	74.91	55
197	97.2	68.67	48	250	100	69.46	56
198	97.2	71.05	49	251	100	69.82	60
199	97.2	68.27	59	252	100	71.20	59
200	97.2	73.22	53	253	100	71.63	61
201	97.2	72.28	50	254	100	76.31	43
202	97.2	75.48	49	255	100	75.23	48
203	100	70.95	49	256	100	73.10	49
204	97.2	69.16	59	257	100	82.08	41
205	97.2	74.90	48	258	100	79.17	42

Generation Number	Best Fitness	Mean Fitness	Diversity	Generation Number	Best Fitness	Mean Fitness	Diversity
259	100	78.37	46	312	100	76.68	43
260	100	77.32	48	313	100	75.23	49
261	100	74.63	48	314	100	74.85	54
262	100	76.55	47	315	100	77.01	49
263	100	76.26	50	316	100	74.17	46
264	100	73.62	44	317	100	72.66	52
265	100	74.18	46	318	100	75.69	53
266	100	72.88	57	319	100	73.81	54
267	100	72.97	55	320	100	72.37	50
268	100	73.09	46	321	100	69.71	61
269	100	74.33	46	322	100	71.69	58
270	100	77.19	45	323	100	72.73	52
271	100	77.31	50	324	100	75.67	45
272	100	74.43	53	325	100	70.07	53
273	100	74.29	50	326	100	66.90	63
274	100	73.27	54	327	100	70.98	58
275	100	74.16	47	328	100	73.70	58
276	100	71.62	56	329	100	77.32	52
277	100	73.80	44	330	100	71.51	53
278	100	74.91	55	331	100	72.71	51
279	100	78.29	48	332	100	74.72	48
280	100	68.24	56	333	100	75.01	47
281	100	73.53	56	334	100	69.86	58
282	100	73.32	51	335	100	70.91	54
283	100	73.49	51	336	100	72.98	46
284	100	77.09	52	337	100	73.21	52
285	100	77.54	47	338	100	77.34	41
286	100	78.18	51	339	100	73.59	43
287	100	77.19	46	340	100	71.45	52
288	100	72.62	53	341	100	77.40	50
289	100	70.72	52	342	100	78.26	45
290	100	79.95	45	343	100	77.10	42
291	100	78.59	42	344	100	74.88	54
292	100	74.61	52	345	100	69.48	52
293	100	70.98	46	346	100	74.87	49
294	100	76.00	44	347	100	74.54	51
295	100	71.52	51	348	100	78.31	42
296	100	80.25	42	349	100	80.19	39
297	100	74.81	52	350	100	76.10	48
298	100	75.97	51	351	100	74.66	49
299	100	72.92	51	352	100	76.22	46
300	100	77.91	43	353	100	71.82	44
301	100	78.19	44	354	100	71.93	50
302	100	75.30	43	355	100	70.16	57
303	100	75.55	49	356	100	70.84	54
304	100	74.67	47	357	100	73.45	46
305	100	72.21	51	358	100	73.16	52
306	100	76.03	52	359	100	76.10	43
307	100	73.23	53	360	100	73.94	51
308	100	74.39	45	361	100	78.10	48
309	100	74.10	49	362	100	74.18	46
310	100	78.32	47	363	100	77.56	44
311	100	75.39	49	364	100	75.72	43

Generation Number	Best Fitness	Mean Fitness	Diversity	Generation Number	Best Fitness	Mean Fitness	Diversity
365	100	73.01	48	418	100	73.55	47
366	100	76.59	49	419	100	82.06	37
367	100	75.58	51	420	100	70.25	47
368	100	74.72	48	421	100	74.79	46
369	100	78.32	46	422	100	72.02	48
370	100	73.35	52	423	100	72.86	45
371	100	72.08	53	424	100	77.13	43
372	100	75.22	50	425	100	75.03	49
373	100	77.85	51	426	100	76.06	45
374	100	73.1	50	427	100	74.79	44
375	100	74.19	52	428	100	76.64	47
376	100	76.47	40	429	100	73.12	52
377	100	72.68	53	430	100	73.60	48
378	100	70.15	45	431	100	74.02	41
379	100	73.61	49	432	100	73.73	44
380	100	75.48	42	433	100	73.85	49
381	100	76.23	42	434	100	71.63	50
382	100	80.35	40	435	100	76.84	47
383	100	72.49	50	436	100	78.55	44
384	100	69.59	45	437	100	79.60	42
385	100	79.27	47	438	100	72.89	50
386	100	73.71	52	439	100	74.64	52
387	100	71.57	53	440	100	68.85	52
388	100	75.09	49	441	100	74.29	49
389	100	74.41	50	442	100	71.25	45
390	100	76.28	44	443	100	70.09	53
391	100	77.75	40	444	100	76.05	52
392	100	75.87	45	445	100	76.44	46
393	100	77.66	48	446	100	77.68	48
394	100	75.06	52	447	100	73.39	48
395	100	74.39	46	448	100	74.00	51
396	100	75.14	48	449	100	78.22	37
397	100	71.82	53	450	100	75.08	48
398	100	72.85	50	451	100	76.61	44
399	100	74.07	51	452	100	74.39	51
400	100	72.12	50	453	100	77.72	43
401	100	72.25	56	454	100	76.74	48
402	100	73.46	49	455	100	71.73	46
403	100	73.23	49	456	100	80.36	42
404	100	73.26	53	457	100	76.32	51
405	100	71.11	52	458	100	69.96	48
406	100	73.56	51	459	100	71.99	55
407	100	71.03	47	460	100	76.90	47
408	100	75.28	47	461	100	72.89	53
409	100	77.11	44	462	100	78.63	49
410	100	78.27	44	463	100	79.99	47
411	100	76.30	45	464	100	70.27	48
412	100	73.70	45	465	100	75.92	41
413	100	76.08	40	466	100	77.73	52
414	100	78.49	48	467	100	77.67	45
415	100	78.02	49	468	100	75.97	48
416	100	75.01	43	469	100	73.86	51
417	100	76.78	47	470	100	76.94	50

Generation Number	Best Fitness	Mean Fitness	Diversity
471	100	78.47	43
472	100	72.27	47
473	100	72.54	51
474	100	69.73	56
475	100	78.71	43
476	100	72.34	53
477	100	70.56	54
478	100	76.05	46
479	100	79.26	50
480	100	74.02	56
481	100	76.15	52
482	100	74.66	47
483	100	75.20	52
484	100	75.89	52
485	100	75.51	44
486	100	68.49	56
487	100	74.28	52
488	100	70.98	54
489	100	69.78	56
490	100	81.84	41
491	100	79.95	40
492	100	73.57	51
493	100	76.16	46
494	100	75.30	41
495	100	75.46	48
496	100	77.43	44
497	100	78.28	40
498	100	70.54	48
499	100	73.38	53
500	100	77.22	44

References

[1] E. M. Arkin, "The lawnmower problem," in *Proc. 5th Canadian Conf. Computational Geometry Waterloo, Ontario, Canada*, 1993.

[2] S. P. Fekete, D. Krupke, M. Perk, C. Rieck, and C. Scheffer, "The lawn mowing problem: From algebra to algorithms," *arXiv preprint arXiv:2307.01092*, 2023.

[3] R. E. Ladner, "On the structure of polynomial time reducibility," *Journal of the ACM (JACM)*, vol. 22, no. 1, pp. 155–171, 1975.

[4] E. M. Arkin and R. Hassin, "Approximation algorithms for the geometric covering salesman problem," *Discrete Applied Mathematics*, vol. 55, no. 3, pp. 197–218, 1994.

[5] J. H. Holland, "Genetic algorithms," *Scientific american*, vol. 267, no. 1, pp. 66–73, 1992.

[6] J. R. Sampson, "Adaptation in natural and artificial systems (john h. holland)," 1976.

[7] K. A. De Jong, *An analysis of the behavior of a class of genetic adaptive systems.* University of Michigan, 1975.

[8] S. Katoch, S. S. Chauhan, and V. Kumar, "A review on genetic algorithm: past, present, and future," *Multimedia tools and applications*, vol. 80, pp. 8091–8126, 2021.

[9] E. M. Arkin, S. P. Fekete, and J. S. Mitchell, "Approximation algorithms for lawn mowing and milling. a preliminary version of this paper was entitled "the lawnmower problem" and appears in the proc. 5th canad. conf. comput. geom., waterloo, canada, 1993, pp. 461–466.," *Computational Geometry*, vol. 17, no. 1, pp. 25–50, 2000.

[10] S. P. Fekete, D. Krupke, M. Perk, C. Rieck, and C. Scheffer, "A closer cut: Computing near-optimal lawn mowing tours," in *2023 Proceedings of the Symposium on Algorithm Engineering and Experiments (ALENEX)*, pp. 1–14, SIAM, 2023.

[11] S. Chatterjee, C. Carrera, and L. A. Lynch, "Genetic algorithms and traveling salesman problems," *European Journal of Operational Research*, vol. 93, no. 3, pp. 490–510, 1996.

[12] M. Trick, "The traveling salesman problem: A computational study," *Operations Research Letters*, vol. 36, no. 2, pp. 276–277, 2008.

[13] R. Bähnemann, N. Lawrance, J. J. Chung, M. Pantic, R. Siegwart, and J. Nieto, "Revisiting boustrophedon coverage path planning as a generalized traveling salesman problem," in *Field and Service Robotics: Results of the 12th International Conference*, pp. 277–290, Springer, 2021.

[14] T. Oksanen and A. Visala, "Coverage path planning algorithms for agricultural field machines," *Journal of field robotics*, vol. 26, no. 8, pp. 651–668, 2009.

[15] E. M. Arkin, M. Held, and C. L. Smith, "Optimization problems related to zigzag pocket machining," *Algorithmica*, vol. 26, no. 2, pp. 197–236, 2000.

[16] R. Bormann, J. Hampp, and M. Hägele, "New brooms sweep clean - an autonomous robotic cleaning assistant for professional office cleaning," in *2015 IEEE International Conference on Robotics and Automation (ICRA)*, pp. 4470–4477, 2015.

[17] H. Choset, "Coverage for robotics–a survey of recent results," *Annals of mathematics and artificial intelligence*, vol. 31, pp. 113–126, 2001.

[18] J.-L. De Carufel, C. Grimm, A. Maheshwari, M. Owen, and M. Smid, "A note on the unsolvability of the weighted region shortest path problem," *Computational Geometry*, vol. 47, no. 7, pp. 724–727, 2014.

[19] Y. V. Pehlivanoglu and P. Pehlivanoglu, "An enhanced genetic algorithm for path planning of autonomous uav in target coverage problems," *Applied Soft Computing*, vol. 112, p. 107796, 2021.

[20] M. Bern and D. Eppstein, "Approximation algorithms for geometric problems," *Approximation algorithms for NP-hard problems*, pp. 296–345, 1997.

[21] D. Kobler, *Evolutionary algorithms in combinatorial optimizationEvolutionary Algorithms in Combinatorial Optimization*, pp. 950–959. Boston, MA: Springer US, 2009.

[22] P. Kora and P. Yadlapalli, "Crossover operators in genetic algorithms: A review," *International Journal of Computer Applications*, vol. 162, no. 10, 2017.

[23] T. Blickle, "Tournament selection," *Evolutionary computation*, vol. 1, pp. 181–186, 2000.

[24] F. Yu, X. Fu, H. Li, and G. Dong, "Improved roulette wheel selection-based genetic algorithm for tsp," in *2016 International conference on network and information systems for computers (ICNISC)*, pp. 151–154, IEEE, 2016.

[25] A. Hassanat, K. Almohammadi, E. Alkafaween, E. Abunawas, A. Hammouri, and V. S. Prasath, "Choosing mutation and crossover ratios for genetic algorithms—a review with a new dynamic approach," *Information*, vol. 10, no. 12, p. 390, 2019.

[26] J. R. Koza *et al.*, "Genetic evolution and co-evolution of computer programs," *Artificial life II*, vol. 10, pp. 603–629, 1991.

[27] M. Ferro, G. D. Silva, F. B. de Paula, V. Vieira, and B. Schulze, "Towards a sustainable artificial intelligence: A case study of energy efficiency in decision tree algorithms," *Concurrency and Computation: Practice and Experience*, vol. 35, no. 17, p. e6815, 2023.

[28] P. J. Van Laarhoven, E. H. Aarts, P. J. van Laarhoven, and E. H. Aarts, *Simulated annealing*. Springer, 1987.

[29] D. Wang, D. Tan, and L. Liu, "Particle swarm optimization algorithm: an overview," *Soft computing*, vol. 22, pp. 387–408, 2018.

[30] E. Mirsadeghi and S. Khodayifar, "Hybridizing particle swarm optimization with simulated annealing and differential evolution," *Cluster Computing*, vol. 24, pp. 1135–1163, 2021.

[31] D. Yang, B. Xu, B. Xu, T. Lu, and X. Wang, "Application of particle swarm optimization with simulated annealing in mit regularization image reconstruction," *Symmetry*, vol. 14, no. 2, p. 275, 2022.