

Assignment 8

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Problem 1

a)

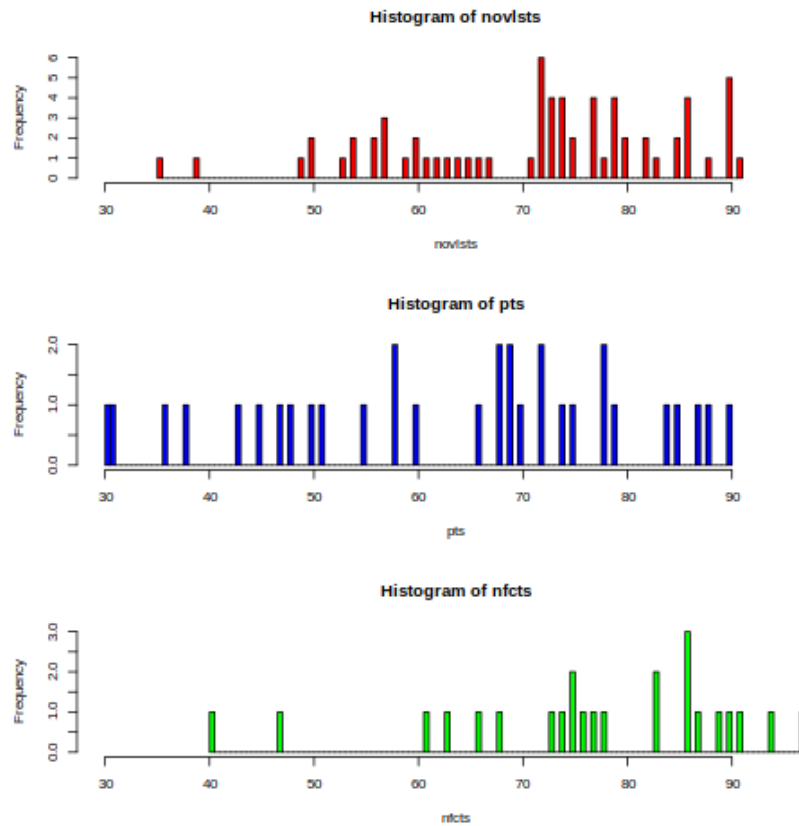
```
d <- read.csv("http://www.stat.yale.edu/~jtc5/238/data/cost-of-the-muse.csv")

ages <- d$Age

novlststs <- ages[d$Type == "Novels"]
pts <- ages[d$Type == "Poems"]
nfcts <- ages[d$Type == "Nonfiction"]

par(mfrow = c(3,1))
rg <- range(ages)

hist(novlststs, 100, xlim = rg, col="red")
hist(pts, 100, xlim = rg, col="blue")
hist(nfcts, 100, xlim = rg, col="green")
```



b)

```
lik <- function(th){
  mu1 <- th[1]; mu2 <- th[2]; mu3 <- th[3]
  s1 <- th[4]; s2 <- th[5]; s3 <- th[6]
  return(
    prod(dnorm(novlsts, mean=mu1, sd = s1)) *
    prod(dnorm(pts, mean=mu2, sd = s2)) *
    prod(dnorm(nfcts, mean=mu3, sd = s3))
  )
}

prior <- function(th){
  mu1 <- th[1]; mu2 <- th[2]; mu3 <- th[3]
  s1 <- th[4]; s2 <- th[5]; s3 <- th[6]
  return(prod(dunif(th, 0, 100)))
}
```

```

}

post <- function(th){
  s1 <- th[4]; s2 <- th[5]; s3 <- th[6]
  if((s1 < 0) | (s2 < 0) | (s3 < 0))return(0)
  return(prior(th) * lik(th))
}

MCMC <- function(reps, th){
  path <- matrix(0, nrow=reps, ncol=6)
  path[1, ] <- th
  for(i in 2:reps)
  {
    candidate <- th + rnorm(6)
    ratio <- post(candidate)/post(th)

    if(runif(1) < ratio) th <- candidate

    path[i, ] <- th
  }
  return(path)
}

reps <- 10000
strt <- c(65, 65, 65, 20, 20 , 20)

results <- MCMC(10000, strt)
mu1 <- results[, 1]
mu2 <- results[, 2]
mu3 <- results[, 3]
sd1 <- results[, 4]
sd2 <- results[, 5]
sd3 <- results[, 6]

qts <- c(0.025, 0.5, 0.975)

```

0.025

0.5

0.975

Novelist μ credible intervals (0.025, 0.5, 0.97, respectively)

`quantile(mu1, qts)`

68.279693490428

71.4616761425772

74.8208908432986

Novelist SD credible intervals (0.025, 0.5, 0.97, respectively)

`quantile(sd1, qts)`

11.3638852594947

13.2705816035933

15.8373466476875

Poets μ credible intervals (0.025, 0.5, 0.97, respectively)

`quantile(mu2, qts)`

57.1944221434365

63.2242358562457

69.0383701320644

Novelist SD credible intervals (0.025, 0.5, 0.97, respectively)

`quantile(sd2, qts)`

14.025471795276

17.9889332578746

24.1268678417386

Nonfiction writers μ credible intervals (0.025, 0.5, 0.97, respectively)

`quantile(mu3, qts)`

70.8826163141118

76.6621468998993

82.2454072795531

Nonfiction writers SD credible intervals (0.025, 0.5, 0.97, respectively)

`quantile(sd3, qts)`

11.2462241539382

14.8611998181229

20.5064952415497

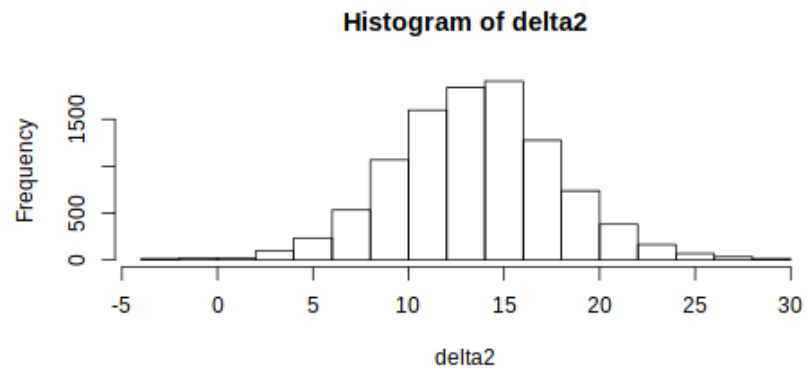
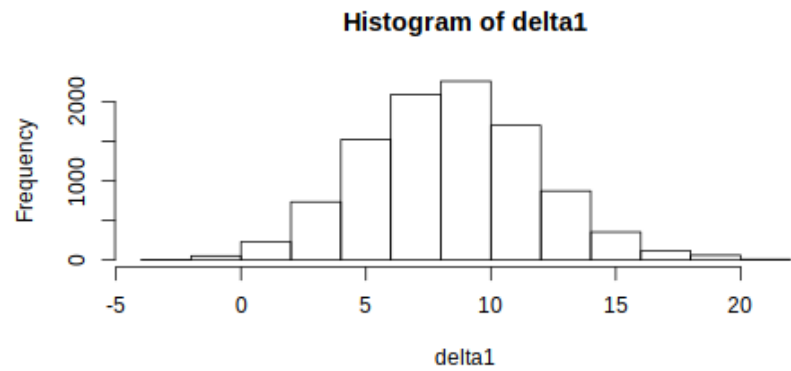
c)

```
df <- (as.data.frame(results))
names(df) <- c("mu1", "mu2", "mu3", "sd1", "sd2", "sd3")

# novelist - poets
delta1 <- df$mu1 - df$mu2
# nonfiction writers - poets
delta2 <- df$mu3 - df$mu2

par(mfrow = c(2,1))

hist(delta1)
hist(delta2)
```



Novelist - poets credible intervals (0.025, 0.975)

```
quantile(delta1, c(0.025,0.975))
```

```
1.82649328592878  
15.4865634533126
```

Nonfiction writers - poets credible intervals (0.025, 0.975)

```
quantile(delta2, c(0.025,0.975))
```

```
5.03124181787953  
22.2888167376865
```

d)

```
delta3 <- delta2 - delta1  
mean(delta3 > 0)
```

```
0.9403
```

Problem 2

```
library('rjags')  
mymodel <- "  
  model {  
    for(i in 1:length(novlstst)){  
      novlstst[i] ~ dnorm(mu1 , tau1)  
    }  
    for(i in 1:length(pts)){  
      pts[i] ~ dnorm(mu2 , tau2)  
    }  
    for(i in 1:length(nfctst)){  
      nfctst[i] ~ dnorm(mu3 , tau3)  
    }  
    mu1 ~ dunif(0, 100)  
    mu2 ~ dunif(0, 100)  
    mu3 ~ dunif(0, 100)  
    sig1 ~ dunif(0, 100)  
    sig2 ~ dunif(0, 100)  
    sig3 ~ dunif(0, 100)  
    tau1 <- 1/(sig1 ^2)  
    tau2 <- 1/(sig2 ^2)
```

```

    tau3 <- 1/(sig3 ^2)
  }
  "
jm <- jags.model (file = textConnection ( mymodel ),
                  data=list(novlstst=novlstst ,
                             pts=pts,
                             nfctst=nfctst),
                  inits=list(mu1 =65, sig1 =20,
                             mu2 =65, sig2 =20,
                             mu3 =65, sig3 =20))
cs <- coda.samples (jm , c('mu1','sig1','mu2','sig2', 'mu3', 'sig3'), 100000)
s <- as.data.frame (cs [[1]])

qts <- c(0.025, 0.5, 0.975)
quantile(s$mu1, qts)

          0.025  68.2467778031878
          0.5   71.4358081522129
          0.975  74.6465196684498

qts <- c(0.025, 0.5, 0.975)
quantile(s$mu2, qts)

          0.025  56.8408695246123
          0.5   63.1785840942802
          0.975  69.5388634163273

qts <- c(0.025, 0.5, 0.975)
quantile(s$mu3, qts)

          0.025  14.032656853907
          0.5   17.7715410431306
          0.975  23.4585322202547

```

```
0.025 70.8154910038315
0.5    76.8770028433609
0.975 82.9686162380408
```

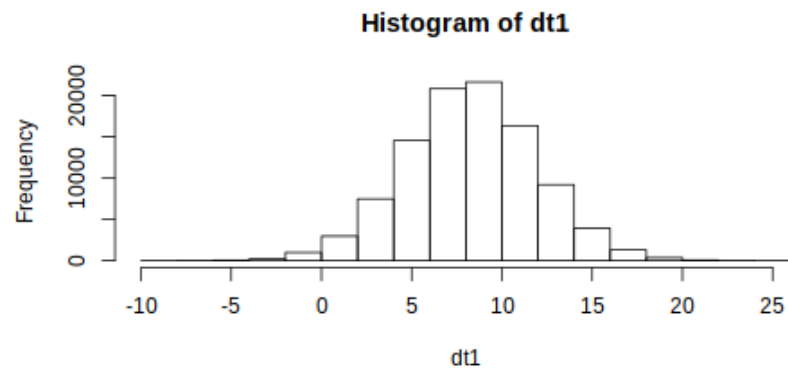
```
qts <- c(0.025, 0.5, 0.975)
quantile(s$sig3, qts)
```

```
0.025 11.1451597067461
0.5    14.6521626614445
0.975 20.4100336403886
```

```
dt1 <- s$mu1 - s$mu2
dt2 <- s$mu3 - s$mu2
```

```
par(mfrow = c(2,1))
```

```
hist(dt1)
hist(dt2)
```

```
dt3 <- dt2 - dt1
mean(dt3 > 0)
```

0.93957

Problem 3

a)

```
xTrt <- 56
xCtrl <- 84
modelHeart <- "
model{
  xTrt ~ dbin(pTrt, 2051)
  xCtrl ~ dbin(pCtrl, 2030)
```

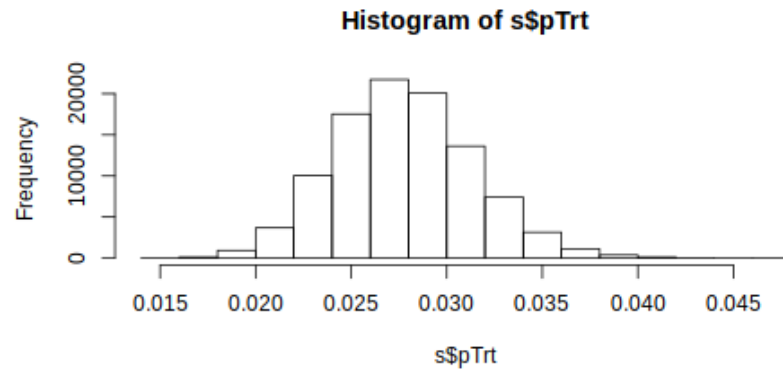
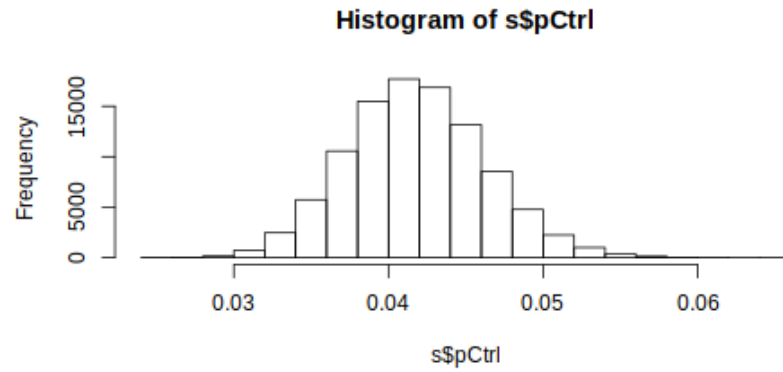
```

        pTrt ~ dunif(0, 1)
        pCtrl ~ dunif(0, 1)
    }
"

jm <- jags.model (file = textConnection ( modelHeart ),
                  data=list(xTrt=xTrt, xCtrl=xCtrl),
                  )
cs <- coda.samples (jm , c("pTrt", "pCtrl"), 100000)
s <- as.data.frame (cs [[1]])

par(mfrow = c(2,1))
hist(s$pCtrl)
hist(s$pTrt)

```



b)

```
qt <- c(0.025, 0.975 )
quantile(s$pCtrl, qt)

0.025  0.0335144495945179
0.975  0.0508070951553886

quantile(s$pTrt, qt)

0.025  0.0211767269637845
0.975  0.0353057663606585
```

c)

```
quantile( 100 * (s$pCtrl - s$pTrt)/s$pCtrl, qt)
```

```
0.025  7.90152955146893
0.975  52.4127972295888
```

4)

a)

$P(\theta_{trt}) = P(\theta_{ctrl}) = U(0, 1) = \text{Beta}(1, 1)$
 $L(\theta_{trt}) = \text{Beta}(56, 1995), L(\theta_{ctrl}) = \text{Beta}(84, 1946)$
 Thus the post. dist. for θ_{trt} is: $B(56 + 1, 1995 + 1)$ and the post dist for θ_{ctrl} is: $B(84 + 1, 1946 + 1)$

b)

Credible intervals for θ_{trt}

```
i1 <- qbeta(.025, 57, 1996)
i2 <- qbeta(.975, 57, 1996)
```

```
sprintf("The credible interval is [%s, %s]", i1, i2)
```

```
The credible interval is [0.021105000440969, 0.0352939682359724]
```

Credible intervals for θ_{ctrl}

```
i1 <- qbeta(.025, 85, 1947)
i2 <- qbeta(.975, 85, 1947)
```

```
sprintf("The credible interval is [%s, %s]", i1, i2)
```

```
The credible interval is [0.0335636928570328, 0.0509513571991438]
```

c)

```
sTrt <- rbeta(10000, 57, 1996)
sCtrl <- rbeta(10000, 85, 1947)
```

```
reduc <- (100* ( sCtrl - sTrt))/sCtrl
quantile(reduc, c(0.025, 0.975))
```

```
8.61584056106628
52.7245199349204
```