Assignment 1

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Problem 1

a)

Solution

The Answer is:

$$\frac{\binom{4}{1}\binom{13}{2}\binom{3}{3}13^3}{\binom{52}{5}} = 0.2637455 \tag{1}$$

Breakdown

Given that there are 5 cards and only 4 suits, we are guaranteed to have a repeat. $\binom{4}{1}$ represents the number of ways to choose the repeated suit. $\binom{13}{2}$ represents the number of ways to choose the ranks of the repeated suit cards *without* overcounting. $\binom{3}{3}$ represents the number of ways to choose the rest of the ranks (just one way, as the last three cards must each be one of the last three ranks). 13^3 represents the ways to choose the ranks for the rest of the cards. And finally we divide by the number of possible 5 card hands.

b)

The Answer is:

$$\frac{\binom{4}{1}\binom{13}{3}\binom{3}{3}13^3 + \binom{4}{2}\binom{13}{2}^2\binom{2}{2}13^2}{\binom{52}{6}} = 0.4264821 \tag{2}$$

Breakdown

The two multiplicative sequences (separated by addition) can be seen to represent two different "cases" which I will outline below.

- Case 1: Three cards share the same rank

  $\binom{4}{1}$ represents the number of ways of choosing the suit for the three cards. $\binom{13}{3}$ represents the number of ways of choosing the ranks for these 3 cards of the same suit. $\binom{3}{3}$ represents the number of ways to choose the rest of the ranks (just one way, as the last three cards must each be one of the last three ranks). 13^3 represents the ways to choose the ranks for the rest of the cards. And finally we divide by the number of possible 5 card hands.

- Case 2: Two sets of two cards each share the same rank

  $\binom{4}{2}$ represents the number of ways of choosing the two suits for the four cards. $\binom{13}{2}^2$ represents the number of ways of choosing the ranks for these 2 sets of cards of the same suit. $\binom{2}{2}$ represents the number of ways to choose the rest of the ranks (just one way, as the last two cards must each be one of the last three ranks). 13^2 represents the ways to choose the ranks for the rest of the cards. And finally we divide by the number of possible 6 card hands.

Problem 2: simulations

```
probAllSuits <- function(reps, handsize){

# Initialize the deck
  H <- rep("h", 13)
  S <- rep("s", 13)
  C <- rep("c", 13)
  D <- rep("d", 13)
  deck <- c(H,S,C,D)



  count <- 0 # Tracks number of hands w/ all ranks
  for (i in 1:reps){
  # samples handsize number of cards from the deck
    hand <- sample(deck, handsize, replace=FALSE)
    # if all ranks are present, increment counter
    if(length(table(hand)) == 4){
      count = count + 1
    }
```

```
  }

  sprintf("Probability of all suits appearing in %s-card
             hand after %s reps: %s" ,
             handsize,
             reps,
             count / reps)

}
```

```
probAllSuits(10000,5)
```

Probability of all suits appearing in 5-card
hand after 10000 reps: 0.2625

```
probAllSuits(10000,6)
```

Probability of all suits appearing in 6-card
hand after 10000 reps: 0.4321

Problem 3

a)

To me this sounds like the following:

$$P(A\Delta B) = P(A \cup B) - P(A \cap B) \tag{3}$$

From the notes we have:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4}$$

Therefore:

$$P(A\Delta B) = P(A) + P(B) - 2P(A \cap B) \tag{5}$$

b)

From the problem we have:

$$P(A \mid B) > P(A) \tag{6}$$

This is equivalent to:

$$\frac{P(A \cap B)}{P(B)} > P(A) \tag{7}$$

So $P(A \cap B)$ must be positive. This can be rearranged to:

$$\frac{P(A \cap B)}{P(A)} > P(B) \tag{8}$$

Which is equivalent to:

$$\frac{P(A \cap B)}{P(B)} > P(A) \tag{9}$$

Which implies:

$$P(B \mid A) > P(B) \tag{10}$$

Since we are given that $P(B)$ is positive, and since $P(A \cap B)$, we know this must be true.

c)

A and B are disjoint

$$P(A \cup B) = P(A) + P(B) \tag{11}$$

therefore:

$$0.9 = 0.6 + P(B) \tag{12}$$

$$0.3 = P(B) \tag{13}$$

A and B are independent

$$P(A \cap B) = P(A)P(B) \tag{14}$$

And also:

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \\ &= P(A) + P(B)\,(1 - P(A)) \end{aligned} \tag{15}$$

Plugging in the values we get:

$$\begin{aligned} 0.9 &= 0.6 + P(B)\,(0.4) \\ &= 0.75 = P(B) \end{aligned} \tag{16}$$

Problem 4

a)

```
# create arithmetic sequences
logprobs <- rep(1,365)
for(k in 1:365){
  factors <- seq(from = 365,length.out= k, by = -1)
  prob <- prod(factors) / (365^k)
  if(is.nan(prob)){
    logprobs[k] <- logprobs[k-1]  + log10(tail(factors, 1) / 365)
  }
  logprobs[k] <- log10(prob)
}

logprobs[1:100]
```

0
-0.00119148080741871
-0.00357772022778088
-0.00716201415108991
-0.0119476767019067
-0.0179380403910941
-0.0251364562692497
-0.03354629408185
-0.0431709424261316
-0.054013808909731
-0.0660783203111117
-0.0793679227417985
-0.0938860818104507
-0.109636282788794
-0.126622030779445
-0.144846850885644
-0.164314288382939
-0.185027908892833
-0.206991298558434
-0.230208064222132
-0.254681833605332
-0.280416255490277
-0.307414999903981
-0.335681758304321
-0.365220243768298
-0.396034191182517
-0.42812735743591
-0.46150352161473
-0.496166485199866
-0.532120072266497
-0.569368129686126
-0.607914527331036
-0.647763158281191
-0.68891793903363
-0.731382809714386
-0.775161734292973
-0.820258700799473
-0.866677721544269
-0.914422833340457
-0.963498097728993
-1.01390760120659
-1.06565545545646

-1.11874579758183
-1.17318279034247
-1.22897062239407
-1.28611350853064
-1.34461568992994
-1.40448143440198
-1.4657150366407

b)

Problem 1

a)

Solution

The Answer is:

$$\frac{\binom{4}{1}\binom{13}{2}\binom{3}{3}13^3}{\binom{52}{5}} = 0.2637455 \tag{17}$$

Breakdown

Given that there are 5 cards and only 4 suits, we are guaranteed to have a repeat. $\binom{4}{1}$ represents the number of ways to choose the repeated suit. $\binom{13}{2}$ represents the number of ways to choose the ranks of the repeated suit cards *without* overcounting. $\binom{3}{3}$ represents the number of ways to choose the rest of the ranks (just one way, as the last three cards must each be one of the last three ranks). 13^3 represents the ways to choose the ranks for the rest of the cards. And finally we divide by the number of possible 5 card hands.

b)

The Answer is:

$$\frac{\binom{4}{1}\binom{13}{3}\binom{3}{3}13^3 + \binom{4}{2}\binom{13}{2}^2\binom{2}{2}13^2}{\binom{52}{6}} = 0.4264821 \tag{18}$$

Breakdown

The two multiplicative sequences (separated by addition) can be seen to represent two different "cases" which I will outline below.

- Case 1: Three cards share the same rank

  $\binom{4}{1}$ represents the number of ways of choosing the suit for the three cards. $\binom{13}{3}$ represents the number of ways of choosing the ranks for these 3 cards of the same suit. $\binom{3}{3}$ represents the number of ways to choose the rest of the ranks (just one way, as the last three cards must each be one of the last three ranks). 13^3 represents the ways to choose the ranks for the rest of the cards. And finally we divide by the number of possible 5 card hands.

- Case 2: Two sets of two cards each share the same rank

  $\binom{4}{2}$ represents the number of ways of choosing the two suits for the four cards. $\binom{13}{2}^2$ represents the number of ways of choosing the ranks for these 2 sets of cards of the same suit. $\binom{2}{2}$ represents the number of ways to choose the rest of the ranks (just one way, as the last two cards must each be one of the last three ranks). 13^2 represents the ways to choose the ranks for the rest of the cards. And finally we divide by the number of possible 6 card hands.

Problem 2: simulations

```
probAllSuits <- function(reps, handsize){

# Initialize the deck
  H <- rep("h", 13)
  S <- rep("s", 13)
  C <- rep("c", 13)
  D <- rep("d", 13)
  deck <- c(H,S,C,D)



  count <- 0 # Tracks number of hands w/ all ranks
  for (i in 1:reps){
  # samples handsize number of cards from the deck
    hand <- sample(deck, handsize, replace=FALSE)
    # if all ranks are present, increment counter
    if(length(table(hand)) == 4){
      count = count + 1
    }
  }

  sprintf("Probability of all suits appearing in %s-card
             hand after %s reps: %s" ,
             handsize,
             reps,
             count / reps)

}
```

```
probAllSuits(10000,5)
```

Probability of all suits appearing in 5-card hand after 10000 reps: 0.2747

```
probAllSuits(10000,6)
```

Probability of all suits appearing in 6-card hand after 10000 reps: 0.4203

Problem 3

a)

To me this sounds like the following:

$$P(A\Delta B) = P(A \cup B) - P(A \cap B) \tag{19}$$

From the notes we have:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{20}$$

Therefore:

$$P(A\Delta B) = P(A) + P(B) - 2P(A \cap B) \tag{21}$$

b)

From the problem we have:

$$P(A \mid B) > P(A) \tag{22}$$

This is equivalent to:

$$\frac{P(A \cap B)}{P(B)} > P(A) \tag{23}$$

So $P(A \cap B)$ must be positive. This can be rearranged to:

$$\frac{P(A \cap B)}{P(A)} > P(B) \tag{24}$$

Which is equivalent to:

$$\frac{P(A \cap B)}{P(B)} > P(A) \tag{25}$$

Which implies:

$$P(B \mid A) > P(B) \tag{26}$$

Since we are given that $P(B)$ is positive, and since $P(A \cap B)$, we know this must be true.

c)

A and B are disjoint

$$P(A \cup B) = P(A) + P(B) \tag{27}$$

therefore:

$$0.9 = 0.6 + P(B) \tag{28}$$

$$0.3 = P(B) \tag{29}$$

A and B are independent

$$P(A \cap B) = P(A)P(B) \tag{30}$$

And also:

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \\ &= P(A) + P(B)\,(1 - P(A)) \end{aligned} \tag{31}$$

Plugging in the values we get:

$$\begin{aligned} 0.9 &= 0.6 + P(B)\,(0.4) \\ &= 0.75 = P(B) \end{aligned} \tag{32}$$

Problem 4

a)

```
# create arithmetic sequences
logprobs <- rep(1,365)
for(k in 1:365){
  factors <- seq(from = 365,length.out= k, by = -1)
  prob <- prod(factors) / (365^k)
  if(is.nan(prob)){
    logprobs[k] <- logprobs[k-1]  + log10(tail(factors, 1) / 365)
  }
  logprobs[k] <- log10(prob)
}

logprobs[1:100]
```

0
-0.00119148080741871
-0.00357772022778088
-0.00716201415108991
-0.0119476767019067
-0.0179380403910941
-0.0251364562692497
-0.03354629408185
-0.0431709424261316
-0.054013808909731
-0.0660783203111117
-0.0793679227417985
-0.0938860818104507
-0.109636282788794
-0.126622030779445
-0.144846850885644
-0.164314288382939
-0.185027908892833
-0.206991298558434
-0.230208064222132
-0.254681833605332
-0.280416255490277
-0.307414999903981
-0.335681758304321
-0.365220243768298
-0.396034191182517
-0.42812735743591
-0.46150352161473
-0.496166485199866
-0.532120072266497
-0.569368129686126
-0.607914527331036
-0.647763158281191
-0.68891793903363
-0.731382809714386
-0.775161734292973
-0.820258700799473
-0.866677721544269
-0.914422833340457
-0.963498097728993
-1.01390760120659
-1.06565545545646
-1.11874579758183
-1.17318279034247
-1.22897062239407
-1.28611350853064
-1.34461568992994
-1.40448143440198
-1.4657150366407

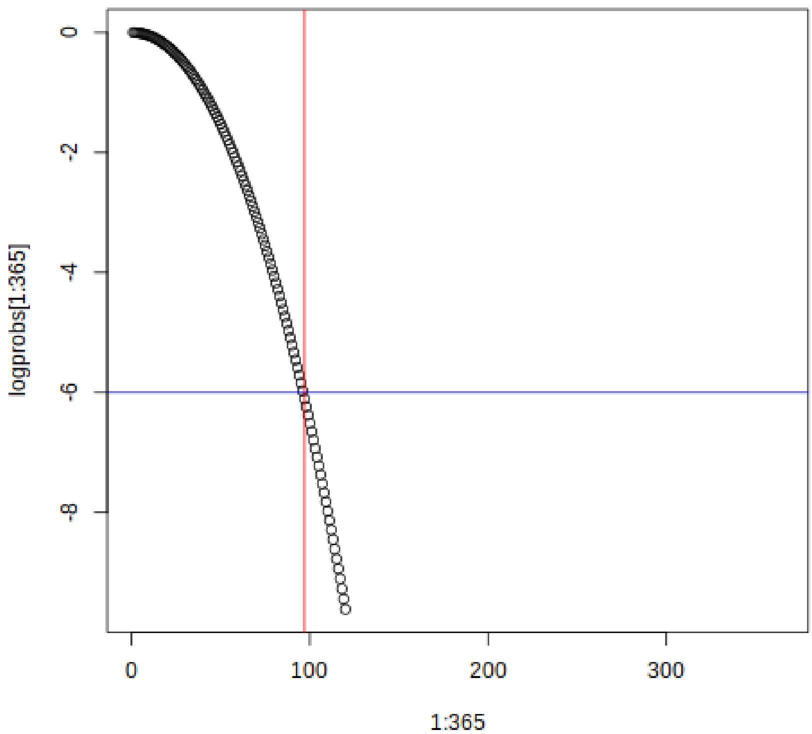
b)

```
lloydsFunc <- function(){
  for(k in 1:365){
    factors <- seq(from = 365,length.out= k, by = -1)
    prob <- prod(factors) / (365^k)
    if(!is.nan(prob)){
      if(prob <= 1e-6){
        return(k)
    }
  }
  }
}
sprintf("Hey Lloyd...yadda yadda yadda... ANSWER: %s",lloydsFunc())
```

```
Hey Lloyd...yadda yadda yadda... ANSWER: 97
```

c)

```
plot(x=1:365,y=logprobs[1:365])
abline(h=-6, col="blue")
abline(v=lloydsFunc(), col="red")
```


logprobs[1:365]
0
-2
-4
-6
-8
0
100
200
300
1:365