Perpetual Notes Document

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1 Number Systems

Suppose A is a 2-dim unital algebra over $\mathbb{R}$.
Consider: $B = \{1, \alpha\}$, $\boxed{x + y\alpha \in A}$
Note: $\alpha \in A$ implies that $\exists\ a, b \in \mathbb{R}$ such that $\alpha^2 = a + b\alpha$

$$\begin{aligned}
&\Rightarrow \alpha^2 - bd = a \\
&\Rightarrow 4\alpha^2 - 4b\alpha + b^2 = 4a + b^2 \\
&\Rightarrow (2\alpha - b)^2 = 4a + b^2 \in \mathbb{R}
\end{aligned}$$

This motivates another definition:
$B' = \{1, \beta\}$ where $\beta :- 2\alpha - b$, therefore $\beta^2 \in \mathbb{R}$

Case 1:

$$\begin{aligned}
\beta^2 = 0 \rightsquigarrow \text{set } \beta = \epsilon,\ \therefore \epsilon^2 = 0 \\
(a + b\epsilon)(c + d\epsilon) = ac + (ad + bc)\epsilon \\
A = \mathbb{R}(\epsilon)
\end{aligned}$$

Case 2:

$$\begin{aligned}
\beta^2 < 0 \rightsquigarrow \text{set } i = \frac{\beta}{|\beta^2|},\ \therefore i^2 = -1 \\
(a + bi)(c + di) = (ac - bd) + (ad + bc)i \\
A = \mathbb{C}
\end{aligned}$$

Case 3:

$$\begin{aligned}
\beta^2 > 0 \rightsquigarrow \text{set } j = \frac{\beta}{\beta^2},\ \therefore j^2 = 1 \\
(a + bj)(c + dj) = (ac + bd) + (ad + bc)j \\
A = \mathbb{R}(j) \\
\text{also} \cong \mathbb{R} \times \mathbb{R}
\end{aligned}$$

$$\begin{aligned}
e^{\pi i} = -1\ ??? \\
{}^{\mathbb{R}[x]}/_{x^2+1} \\
\hat{f}(w) = \int_{-\infty}^{\infty} e^{-iw} f(t) dt \\
re^{i\theta} = r\cos(\theta) + ir\sin(\theta)
\end{aligned}$$

'Complex' Numbers?? $a + bi \cong \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

$$\rho = \frac{3.332\ g\ cm^{-1}}{\pi(\frac{1}{2} * 1.25\ cm)^2}$$
$$\rho = 2.715\ g\ cm^{-3}$$

kj

$$\%u_{radius} = \frac{\frac{1}{2} * 0.05}{1.25}$$
$$\%u_{radius} = 2\%$$

$$\%u_{total} = 2 * \%u_{radius} + \%u_{slope}$$
$$\%u_{total} = 2 * (2\%) + 6.18\%$$
$$\%u_{total} = 10.18\%$$

$$\rho = 2.72 \pm 0.28\ g\ cm^{-3}$$