Sheet of Formulas

1. **Cross Hedging spot with futures**: $S = spot\ price, F = futures\ price$. Optimal hedge ratio is
$$h = \rho \frac{\sigma_S}{\sigma_F}$$
with $\sigma_S = std(\Delta S)$, $\sigma_F = std(\Delta F)$, and $\rho = corr(\Delta S, \Delta F)$. Number of contracts is
$$N = h \frac{Q_A}{Q_F}$$
where $Q_A = unit\ of\ underlying\ being\ hedged$, and $Q_F =$units per future contract.

2. **Cross Hedging Stock Portfolios with an Index future:** $S = spot\ price\ of\ a\ portfolio, F = future\ price\ on\ a\ stock\ index$. Optimal hedge ratio is
$$h = \rho \frac{\sigma_S}{\sigma_F}$$
with $\sigma_S = std\left(\frac{\Delta S}{S}\right)$, $\sigma_F = std\left(\frac{\Delta F}{F}\right)$, and $\rho = corr\left(\frac{\Delta S}{S}, \frac{\Delta F}{F}\right)$. Number of contracts is
$$N = h \frac{V_A}{V_F}$$
where $V_A =$ \$- value of portfolio being hedged, and $V_F =$ \$- value of futures contract. ***If the future is near to expiry, then***
$$N = \beta \frac{V_A}{V_F}$$
where $\beta = cov\left(\frac{\Delta S}{S}, \frac{\Delta I}{I}\right) / var\left(\frac{\Delta I}{I}\right)$ with $I = level\ of\ the\ index$.

3. **Interest Rate Conversion:** Given rate r_m with compounding m times per year, the continuously compounding rate is
$$r_c = m \times ln\left(1 + \frac{r_m}{m}\right).$$

4. **Forward Rate:** let r_1 be the continuously compounded zero-rate with maturity at time T_1, and r_2 be the continuously compounded zero-rate with maturity at time $T_2 > T_1$. The forward rate for from T_1 to T_2 is
$$f = \frac{r_2 T_2 - r_1 T_1}{T_2 - T_1}.$$

5. **Forward Rate Agreement (FRA):** a FRA with fixing data $T_1$and terminal $T_2 > T_1$, will receive rate r_f equal to $(T_2 - T_1)$-year LIBOR (e.g., 6-month LIBOR with $T_2 = T_1 + .5$) for time period $[T_1, T_2]$ (this rate realized at time T_1) and will pay fixed annual rate r_K. The net payment at time T_2 is
$$L\left(r_f - r_K\right)(T_2 - T_1),$$
where L is the principal. The valuation of this FRA is
$$V = B(0, T_2)L(f - r_K)(T_2 - T_1),$$
where $B(0, T_2)$ is the discount factor and f is the forward rate for $[T_1, T_2]$ computed from LIBOR rates.

6. **Futures:** A future on a stock is
$$F = S_0 e^{(r-g)T}$$
where $S_0$is the spot price, r is the risk-free rate, g is the dividend yield, and T is the maturity. A future on a storable commodity satisfies
$$F \leq (S_0 + U)e^{rT}$$
where U is the present value of storage costs. Alternatively, $F \leq S_0 e^{(r+u)T}$ where u is the proportional rate for storage.

   Contango: $S_0 < F$ (upward sloping)
   Backwardation: $S_0 > F$ (downward sloping)

7. **Convexity Adjustment to Eurodollar Futures:**
$$forward\ rate = future\ rate - \frac{\sigma^2}{2} T_1 T_2$$
where $T_1$is the maturity date of the future, T_2 is the maturity of the underlying interest rate, and σ is the volatility of short-term interest rate.

8. **Comparative Advantage:** when a credit market treats one type of borrower better than another. E.G., a AAA-rated company can borrow fixed at 4% and floating at LIBOR – 10bps, whereas a BBB-rated company can borrow fixed at 5% and floating at LIBOR + 30bps. BBB has comparative advantage in floating because they only see an increase of 40bps rather than 1%.

9. **European Put-Call parity:** $p + S_0 e^{-gT} = c + Ke^{-rT}$.

10. **American put-call parity:** $S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT}$, where D is present value of dividends.

11. **Binomial Trees:** risk-neutral up probability,

$$p = \frac{e^{(r-g)\Delta t} - d}{u - d}.$$

For Cox-Ross-Rubenstein Model,

$$u = e^{\sigma\sqrt{\Delta t}} \;\; and \;\; d = e^{-\sigma\sqrt{\Delta t}}.$$

The value V for a derivative contract is

$$V = e^{-r\Delta t}(pV^u + (1-p)V^d)$$

and the Delta is

$$\Delta = \frac{V^u - V^d}{S(u-d)}.$$

12. **Wiener Process (Brownian Motion):** a standard Wiener process is W_t such that

   a. $W_0 = 0$
   b. $W_t - W_s$ is normally distribution with mean zero and variance $|t-s|$
   c. W_t has independent increments.

13. **Ito's Lemma:** for $dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t$ with W_t a standard Wiener process,

$$df(t, S_t) = \left(f_t(t, S_t) + \frac{\sigma^2(t, S_t)}{2} f_{ss}(t, S_t) + \mu(t, S_t) f_s(t, S_t)\right) dt + \sigma(t, S_t) f_s(t, S_t) dW_t.$$

14. **Black-Scholes Formula:** the Black-Scholes price of a European call option is

$$c(t, s) = se^{-g(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2)$$

where $d_1 = \frac{\ln\left(\frac{s}{K}\right) + \left(r - g + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$. The European put option price is

$$p(t, s) = Ke^{-r(T-t)}N(-d_2) - se^{-g(T-t)}N(-d_1).$$

15. **Black's Formula:** for an option a European call option on a futures contract,

$$c(t, f) = e^{-r(T-t)}\big(fN(d_1) - KN(d_2)\big),$$

where f is the current future price, $d_1 = \frac{\ln\left(\frac{f}{K}\right) + \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$.

16. **Black-Scholes Greeks:** for European call and put options on non-dividend paying stock,

a. $\Delta = c_s = N(d_1)$ for call, $\Delta = p_s = -N(-d_1)$ for put,
b. $\Gamma = c_{ss} = p_{ss} = N'(d_1)/\left(s\sigma\sqrt{T-t}\right)$
c. $\theta = c_t = \frac{-sN'(d_1)\sigma}{(2\sqrt{T-t})} - rKe^{-r(T-t)}N(d_2)$ for call, and $\theta = p_t = \frac{-sN'(d_1)\sigma}{(2\sqrt{T-t})} + rKe^{-r(T-t)}N(-d_2)$
d. $vega = c_\sigma = p_\sigma = N'(d_1)s\sqrt{T-t}$
e. $\rho = c_r = N(d_2)K(T-t)e^{-r(T-t)}$ for call, and $\rho = p_r = -N(-d_2)K(T-t)e^{-r(T-t)}$ for put.

17. **Bond Option Price:** the price of a call option on a bond is

$$c(t, F_B) = P(t,T)\left(F_B N(d_1) - KN(d_2)\right)$$

where $P(t,T)$is the discount factor, the forward bond price is $F_B = \frac{B(t)-I}{P(t,T)}$ where $B(t)$ is the current bond price and I is the present value of coupons paid during the life of the option, $d_1 = \frac{\ln\left(\frac{F_B}{K}\right)+\left(\frac{\sigma_B{}^2}{2}\right)(T-t)}{\sigma_B\sqrt{T-t}}$ and $d_2 = d_1 - \sigma_B\sqrt{T-t}$. The forward bond price volatility is

$$\sigma_B = Dy_t\sigma_y$$

where D is the underlying bond's modified duration, y_t is the forward yield, and σ_y is the volatility of y_t.