FIM/ECG/MA 528 Semester Project
Due @ 11:30 PM on Tuesday November 30th, 2021

1. There are 10 Questions
2. Each question worth 20 points
3. Answer all 10 questions correctly for maximum exam score
4. Grading for each question:
   - 0 pts: work is generally incoherent
   - 10 pts: about half the work is correct
   - 15 pts: most of the work is correct but answer is not written out in full
   - 19 pts: the problem is almost entirely correct but there are 1 or more small mistakes
   - 20 pts: a completed answer with no mistakes
5. **You must show all steps to receive full credit.**
   Simply writing a number that you know is correct is incoherent.
6. **This semester project is to be done individually. This is not homework. This is a paper that is taking the place of the final examination, and therefore, it must be done by the individual.**

1. On July 1, an investor holds 50,000 shares of a certain stock. The market price is \$30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use an index futures contract. The index futures price is currently 1,500 and one contract is for delivery of \$50 times the index. The beta of the stock is 1.3. What strategy should the investor follow? Under what circumstances will it be profitable?

Solution: The investor should short contracts in the future in the amount

$$n = \beta \frac{30 \times 50000}{50 \times 1500} = 26,$$

which will be profitable if

$$50{,}000 \times (S - 30) - 26 \times 50 \times (I - 1500) > 0$$

where S and I are the finishing stock and index value, respectively. Rearrange to see that there is profit if $S > 30 + \frac{1300(I-1500)}{50{,}000}$.

2. A $50 million interest rate swap has a remaining life of 7 months. Under the terms of the swap, 3-month LIBOR is exchanged for 5.50% per annum quarterly. The average of the bid-offer rate being exchanged for 3-month LIBOR in swaps of all maturities is currently 4.25% per annum with continuous compounding. The 3-month LIBOR rate was 3.6% per annum 2 months ago. OIS rates for all maturities are 2.5% with continuous compounding. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed?

Solution: there are three remaining payments for this swap. The first occurs in 1 month, for the receiver of the fixed has a present value of

$$P_1 = \frac{1}{4}\left(e^{-\frac{.025}{12}}(5.5 - 3.6)\right) \times 50m = 23.7m.$$

For the second payment occurs in 4 months, convert LIBOR 4.25% to quarterly compounding,

$$r = 4\left(e^{.0425/4} - 1\right) = .0427$$

and for the receiver of fixed has a present value of

$$P_2 = \frac{1}{4}\left(e^{-\frac{.025\times 4}{12}}(5.5 - 4.27)\right) \times 50m = 15.25m.$$

and the third payment occurs in 7 months and for the receiver of fixed has a present value of

$$P_3 = \frac{1}{4}\left(e^{-\frac{.025\times 7}{12}}(5.5 - 4.27)\right) \times 50m = 15.15m.$$

Therefore, the value of the swap to the receiver of the fixed is $23.7 + 15.25 + 15.15 = 54.1$m. The value of the swap to someone paying the floating is -54.1m.

3. The spot price of silver is $15 per ounce. The storage costs are $0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

Solution: The present value of storage payments on an ounce of silver for the next 9 months is

$$S = \frac{.24}{4}\left(1 + e^{-\frac{.1}{4}} + e^{-\frac{.1}{2}}\right) = .1756,$$

and so the future value of an ounce is

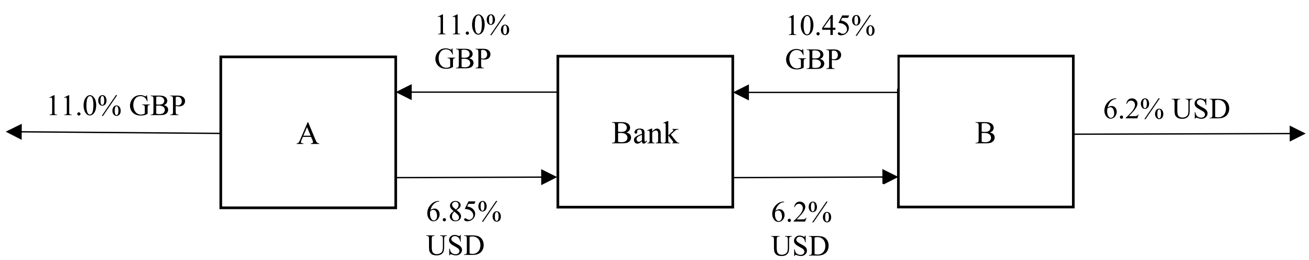
$$F = e^{\frac{.1\times 3}{4}}(15 + .1756) = 16.3576.$$

4. Company A, a British manufacturer, wishes to borrow US dollars at a fixed rate of interest. Company B, a US multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for differential tax effects):

	Sterling	US dollars
Company A	11.0%	7.0%
Company B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.

Solution:



Company A sees an improvement of 15bps for borrowing USD.

Company B sees an improvement of 15 bps for borrowing GBP

The bank nets 65bps – 55bps = 10bps.

5. A one-month European put option on a non-dividend-paying stock is currently selling for \$2.50. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur? How would you exploit this opportunity? Use a payoff table to show the payoff of your strategy at maturity.

Solution: first, notice that the option is mis-priced because $P < e^{-rT}K - S$, that is

$$2.5 < e^{-\frac{.6}{12}}50 - 47 = 2.75.$$

Therefore, an arbitrage is to long the put, long the stock, and finance this position at rate 6%. This position costs \$$(2.5 + 47)$ to set up. Its payoff table is as follows:

outcome	put	stock	bank	net
$S_T < 50$	$50 - S_T$	S_T	$-e^{\frac{.06}{12}}(2.5 + 47) = -49.75$	.25
$S_T > 50$	0	S_T	$-e^{\frac{.06}{12}}(2.5 + 47) = -49.75$	$S_T - 49.75 > .25$

Thus, the position always nets at least \$.25.

6. A stock price is currently $50. Over each of the next two 3-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. Use a binomial tree to compute the value of a 6-month European call option with a strike price of $51. What is the delta hedge at time 0?

Solution: First, notice that $50 \times 1.06 \times .95 = 50.35 < 51$, and so a call option only finishes in-the-money if there are two up moves. The two up-move outcome has stock price $50(1.06)^2 = 56.18$. The risk-neutral probability of an upward move is

$$q = \frac{e^{\frac{.05}{4}} - .95}{1.06 - .95} = .5689.$$

Therefore, the call option value is

$$C = e^{-\frac{.05}{2}}.5689^2(56.18 - 51) = 1.6351$$

The delta hedge is $\Delta = \frac{C^u - C^d}{50(u-d)}$, and obviously $C^d = 0$ because there is no change of finishing in-the-money, and

$$C^u = e^{-\frac{.05}{4}}.5689(56.18 - 51) = 2.9103.$$

Finally, the initial delta hedge is

$$\Delta = \frac{2.9103}{50 \times .11} = .5291.$$

7. Suppose that x is the yield on a perpetual government bond that pays interest at the rate of \$1 per annum. Assume that x is expressed with continuous compounding, that interest is paid continuously on the bond, and that x follows the process

$$dX_t = a(X_0 - X_t)dt + \sigma X_t dW_t$$

where a, X_0, and σ are positive constants, and W_t is a Wiener process. What is the process followed by the bond price? What is the expected instantaneous return (including interest and capital gains) to the holder of the bond? Note: the yield on a perp is equal to 100 times the annual coupon amount over the price.

Solution: The price of the perp equals to 100 times \$1 over the yield. The price is

$$P_t = \frac{100}{X_t},$$

And using Ito's lemma we find that the bond price following differential,

$$dP_t = -\frac{100}{{X_t}^2} dX_t + \frac{100}{4{X_t}^3}(dX_t)^2 = 100\left(\frac{\sigma^2}{4X_t} - \frac{a(X_0 - X_t)d}{{X_t}^2}\right) dt - 100\frac{\sigma}{X_t} dW_t.$$

The expected instantaneous return to the hold of the bond is

$$dR_t = 100\left(\frac{\sigma^2}{4X_t} - \frac{a(X_0 - X_t)d}{{X_t}^2}\right) dt + dt,$$

where the 2nd dt is the instantaneous coupon payment.

8. The price of a European call that expires in six months with a strike price of €30 is €2. The underlying stock price is €29, and a dividend of €0.50 is expected in two months and again in five months. The term structure is flat, withall continuously compounded risk-free interest rates being 10%. What is the price of a European put option that expires in six months and has a strike priceof €30?

Solution: The present value of dividends is

$$D = .5\left(e^{-\frac{.1}{6}} + e^{-\frac{.1\times 5}{12}}\right) = .9713.$$

By put-call parity, for European calls and puts we have

$$p + S = c + e^{-rT}K + D$$

And therefore, the European put option's price is

$$p = 2 + 28.5369 + .9713 - 29 = €2.5082.$$

9. What is the price of a European call option on a stock with stock price \$50, a dividend rate of 1% per annum, a strike price of \$50, a risk-free interest rate of 3% per annum, a volatility of 25% per annum, and time to maturity of 6 years? What is the option's delta?

Solution: Use the Black-Scholes formula,

$$c = 50e^{-.01\times 6}N(d_1) - 50e^{-.03\times 6}N(d_2) = \$13.55,$$

where $d_1 = \frac{\log(50/50)+(.02+.5\times.25^2)6}{\sqrt{.25\times.5}}$ and $d_2 = d_1 - \sqrt{.25\times 6}$. The delta of the option is

$$\Delta = e^{-.01\times 6}N(d_1) = .65.$$

10. The futures price of an asset is currently 78 and the risk-free rate is 3%. A six-month put on the futures with a strike price of 80 is currently worth 6.5. What is the value of a six month call on the futures with a strike price of 80 if both the put and call are European? What is the range of possible values of the six-month call with a strike price of 80 if both put and call are American?

Solution: the put-call parity for European options on futures is

$$p + e^{-rT}F = c + e^{-rT}K,$$

and so the put call price is

$$c = 6.5 + e^{-.03/2}(78 - 80) = 6.5 - 1.9702 = 4.5298.$$

Let C and P denote the American call and put prices, respectively. The range of prices for the American call are

$$e^{-rT}F - K + P \leq C \leq F - e^{-rT}K + P,$$

which in this case means

$$P - 3.1613 \leq C \leq P - .8090.$$