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MA 528 HW 7

14.3 The distribution of the company's cash position at the end of one year has a mean of $2+c_0$ and a standard deviation of 4. The probability that this cash position is negative after one year is:

$$\int_{-\infty}^{-2-c_0} \frac{e^{-x^2/32}}{\sqrt{2\pi}\,4}\,dx$$

[As a weiner process has a Gaussian distribution]

Setting this equal to .05 and solving for c_0, we get:

$\boxed{\text{Initial Cash Position: \$4.58 Million}}$

14.4 a) $\boxed{\text{A Weiner Process with drift rate } \mu_1+\mu_2 \text{ and variance } \sigma_1^2+\sigma_2^2}$

b) $\boxed{\text{A Weiner Process with drift rate } \mu_1+\mu_2 \text{ and variance } \sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2}$

14.5 $\boxed{\text{A Normal Distribution with mean 20 and standard deviation } 5\sqrt{3} \approx 8.66}$

$$\sigma_{total} = \sqrt{3\cdot 3^2 + 3\cdot 4^2} = 5\sqrt{3}$$

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14.14 a) The probability distributions are all Normal distributions with parameters given below:

Time	Mean	Standard Deviation
1 Month	2.1	.4
6 Monthes	2.6	$.4\sqrt{6} \approx 1.00$
1 Year	3.2	$.4\sqrt{12} \approx 1.39$

☆ Parameters given in millions of dollars

b) The probability of a negative cash position after time t, with mean μ and standard deviation σ, is:

$$\int_{-\infty}^{0} \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}\,dx$$

$\boxed{\text{The probability of a negative cash position after 6 months is .004 and after 1 year is .011.}}$

c)

$$\text{Prob}(c \leq 0) = \int_{-\infty}^{0} \frac{e^{-(x-2-.1t)^2/.32t}}{\sqrt{.32\pi t}}$$

as $\mu = 2+.1t$ and $\sigma = .4\sqrt{t}$
(t in months)

$\frac{d\,\text{Prob}(c\leq 0)}{dt} = 0$ when $\text{Prob}(c \leq 0)$ is maximized.

Using Mathematica

This happens at $t=20$.

$\boxed{\text{Therefore, the probability of a negative position is maximized at 20 monthes.}}$

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14.16 $dS = (\mu dt + \sigma dz)S$

a) $y = 2S \rightsquigarrow S = \frac{y}{2}$

$\frac{dy}{dS} = 2 \rightsquigarrow dS = \frac{dy}{2}$

$$\boxed{dy = (\mu dt + \sigma dz)y}$$

b) $y = S^2 \rightsquigarrow S = \sqrt{y}$

$\frac{dy}{dS} = 2S$; $\frac{d^2y}{dS^2} = 2$, $\frac{\partial y}{\partial t} = 0$

Ito's Lemma:

If $dx = a(x,t)dt + b(x,t)dz$ and $G = G(x,t)$

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial G^2}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}b\,dz$$

Here, $x = S$, $a = \mu S$, and $b = \sigma S$

$$\boxed{dy = (2\mu + \sigma^2)y\,dt + 2\sigma y\,dz}$$

c) $y = e^S$; $\frac{dy}{dS} = e^S$; $\frac{d^2y}{dS^2} = e^S$; $\frac{\partial y}{\partial t} = 0$

$$\boxed{dy = \left(\mu\ln(y) + \frac{\sigma^2}{2}\ln(y)^2\right)y\,dt + \sigma y\ln(y)\,dz}$$

d) $y = \frac{e^{r(T-t)}}{S}$; $\frac{dy}{dS} = \frac{-e^{r(T-t)}}{S^2}$; $\frac{d^2y}{dS^2} = \frac{2e^{r(T-t)}}{S^3}$; $\frac{\partial y}{\partial t} = \frac{-re^{r(T-t)}}{S}$

$\frac{dy}{dS}a = -\mu$; $\frac{\partial y}{\partial t} = -r$; $\frac{d^2y}{dS^2}b^2 = \sigma^2$

$$\boxed{dy = -(r + \mu - \sigma^2)y\,dt - \sigma y\,dz}$$