Jacob Wyngaard

# MA 528 HW 4



26.3 No, it is not optimal to make the choice early as no benefit is gained while the potential for profit is decreased by limiting which option you have access to. If the choice was between an American call and an American put, early choice and exercise would be optimal if the asset value dropped to zero. In this case, the optimal strategy would be to choose the American put option and exercise it immediately.

26.5 As per put-call parity:

$$c + Ke^{-rT} = p + S_0$$

For a chooser option, we choose whether or not our option is a European call or European put at time $T_1$. Maturity at $T_2$, dividend rate $q$.

$$c + Ke^{-r(T_2-T_1)} = p + S_1 e^{-q(T_2-T_1)}$$

$$\text{Value} = \max(c,p) = \max(p, p + S_1 e^{-q(T_2-T_1)} - Ke^{-r(T_2-T_1)})$$

$$\text{Value} = p + e^{-q(T_2-T_1)} \max(0, S_1 - Ke^{-(r-q)(T_2-T_1)})$$

The Chooser Option is a package consisting of:

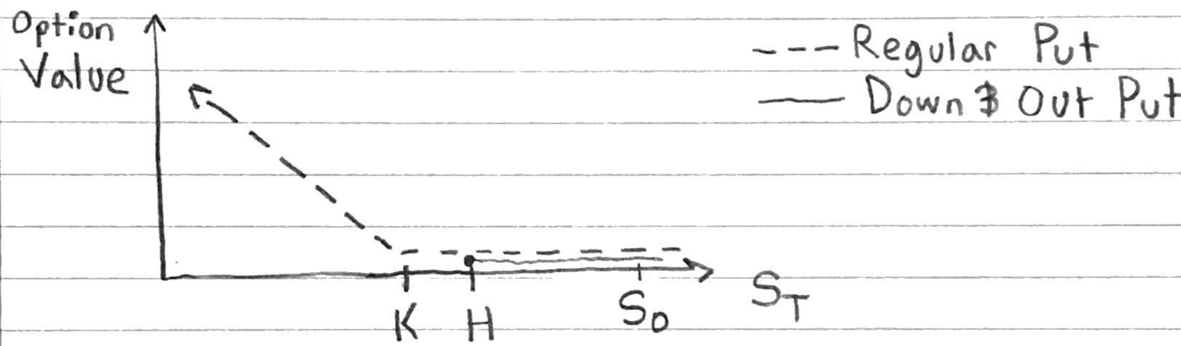
1. A put option w/ strike price $K$ and maturity $T_2$.
2. $e^{-q(T_2-T_1)}$ call options w/ strike price $Ke^{-(r-q)(T_2-T_1)}$ and maturity $T_1$.

Jacob Wyngaard

# MA 528 HW 9



26.7 A down-and-out put option ceases to exist if $S_t = H < S_0$ for some time $t$ which occurs before maturity. If $K < H$, then the option dies before it is able to be profitable, as a put option only makes money when $K > S_T$.




26.8 The payoff from the American call option is (at time T):

$$(S_T - K_0 e^{gT}) e^{-rT}$$

↑ time discounting of money

By the no-arbitrage argument w/ forwards contracts (and we could always enter into one to guarantee $S_T$) : $S_T = S_0 e^{rT}$

Thus, Payoff $= S_0 - K_0 e^{-(r-g)T}$

$$\frac{d}{dt}[\text{Payoff}] = (r-g) K_0 e^{-(r-g)T} > 0 \text{ as } r > g$$

Since the value of the payoff increases with time, it is never optimal to exercise the call early.

Jacob Wyngaard

# MA 528 HW 9



26.9 Forward Start Put Option w/ $K = 1.1 S_{T_1}$

$$\text{Payoff} = (K - S_{T_2})e^{-rT_2} = (1.1 S_{T_1} - S_{T_2})e^{-rT_2}$$

The expected value of $S_{T_i}$ is $S_0 e^{rT_i}$

$$\text{Payoff} = (1.1 e^{rT_1} - e^{rT_2}) S_0 e^{-rT_2}$$

$$\text{Payoff} = \left(1.1 S_0 - S_0 e^{r(T_2 - T_1)}\right) e^{-r(T_2 - T_1)}$$

The value of the forward start put option is the same as the value of a European put option w/ Strike price $K = 1.1 S_0$ and maturity in $T_2 - T_1$.

26.18 $V(S, \frac{1}{2}) = \begin{cases} 0 & \text{Index} \leq 1000 \\ 100 & \text{Index} > 1000 \end{cases}$

$$dS = (r-q)S\,dt + \sigma S\,dz = .085\,dt + .25\,dz$$

By Ito's Lemma: $\ln\left(\frac{S}{S_0}\right)$ normally distributed w/ mean $(.05 - \frac{.2^2}{2})T = .03T$ and standard deviation $.2\sqrt{T}$ ; $T = \frac{1}{2}$

$$V(S,0) = 1000 e^{-.08\frac{1}{2}} Pr\left[ V(S, \tfrac{1}{2}) > 100 \mid S = 960 \text{ at } t = 0 \right]$$

$$V(S,0) = 1000 e^{-.04}(1 - N(z)) ; \quad z = \frac{\ln\left(\frac{1000}{960}\right) - .03T}{.2\sqrt{T}} = .1826$$

$V(S,0) = 41.08$, the value of the derivative is \$41.08

Jacob Wyngaard

# MA 528 HW 9



29.1 $\$20 \text{ million } (.04-.02)\frac{1}{4} = \$100{,}000$

$\boxed{\text{\$100,000 will be paid in 3 months.}}$

29.3 $p = Ke^{-rT}N(-d_-) - F_0e^{-rT}N(-d_+)$

$F_0 = (125-10)e^{rT} = 127.09$ present value of coupons

$r = .1,\ \sigma = .08,\ T = 1,\ K = 110$

$$d\pm = \frac{\ln\left(\frac{F_0}{K}\right) \pm \sigma^2/2\, T}{\sigma\sqrt{T}}$$

$d_+ = 1.845$ ; $d_- = 1.765$

$p = \$.12$ $\boxed{\text{The value of the put option is \$.12}}$

29.5 Value of Caplet is:

$L\delta_K P(0, t_{K+1})[F_K N(d_+) - R_K N(d_-)]$

$L = 1000,\ \delta_K = \frac{1}{4},\ t_{K+1} = \frac{3}{2},\ F_K = .12,\ \sigma_K = .12$

$$d\pm = \frac{\ln(F_K/R_K) \pm \sigma_K^2 t_K/2}{\sigma_K\sqrt{t_K}} \quad ;\ R_K = .13$$

$$\text{Value} = 1000\tfrac{1}{4}e^{-.115\cdot\frac{3}{2}}[.12N(d_+) - .13N(d_-)]$$

where $d\pm = \dfrac{\ln(.12/.13) \pm .12^2\,\frac{5}{4}/2}{.12\sqrt{5/4}} = \underset{d_+}{-.5295},\ \underset{d_-}{-.6637}$

$\boxed{\text{The value of the option is \$.54}}$

Jacob Wyngaard

# MA 528 HW 9



29.6 The implied volatility is the standard deviation of $\ln(P)$, where $P$ is the bond price, divided by the square root of $T$, the time to maturity. For a 9 year option on a ten year bond, the time to maturity is 1 year. Thus, the real volatility would be less than that of a 5 year option as $\frac{1}{\sqrt{9}} < \frac{1}{\sqrt{5}}$. Therefore, the resultant price would overestimate the price.

29.18 $V_1$ - value of swaption to pay fixed rate $S_K$ and receive LIBOR for $t \in [T_1, T_2]$

$V_2$ - Value of swaption to receive fixed rate $S_K$ and pay LIBOR for $t \in [T_1, T_2]$

$f$ - value of foward swap to to receive $S_K$ and pay LIBOR for $t \in [T_1, T_2]$

Consider the two portfolios:

$\Omega_1$: Swaption from $V_2$

$\Omega_2$: Swaption from $V_1$ and forward swap from $f$

At maturity:

If swap rate $> S_K$, $\Omega_1$ doesn't exercise swaption and $\Omega_2$ makes net zero.

If swap rate $< S_K$, $\Omega_1$ exercises swaption and makes the same $\Omega_2$, which doesn't exercise its swaption.

Therefore, since both portfolios return the payoff, their values are the same and so $V_1 + f = V_2$.

Jacob Wyngaard

# MA 528 HW 9



If $s_K$ = current forward swap rate, then $f = 0$ so $V_1 = V_2$.

29.14 As per DerivaGem, the value of the swap option is \$3.75.