Jacob Wyngaard

FIM/ECG/MA 528 Midterm #1 (Thursday September 29th, 2022)

1. There are 5 Questions
2. Each question worth 20 points
3. Answer all 5 questions correctly for maximum exam score
4. Grading for each question:
   - 0 pts: work is generally incoherent
   - 10 pts: about half the work is correct
   - 15 pts: most of the work is correct but answer is not written out in full
   - 19 pts: the problem is almost entirely correct but there is small mistakes
   - 20 pts: a completed answer with no mistakes

5. **You must show all steps to receive full credit.**
6. Simply writing a number that you know is correct is incoherent.

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Problem #1: On July 1, an investor holds 50,000 shares of a certain stock. The market price is $30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use an index futures contract. The index futures price is currently 1,500 and one contract is for delivery of $50 times the index. The beta of the stock is 1.3. What strategy should the investor follow? Under what circumstances will it be profitable?

The investor should short $1.3 \frac{50000 \cdot 30}{1500 \cdot 50}$ $\left(N^* = \beta \frac{V_A}{V_F}\right)$ or 26 index futures contracts.

This hedge will be profitable if the index specified in the contract drops points. Assuming we have a risk free rate of zero, for each point drop in the index, the investor would gain $1,300.

How to get $N^* = \beta \frac{V_A}{V_F}$:

$$\text{Risk} = Var(\Delta V_A - N^* \Delta V_F)$$

coefficient of correlation

$$\text{Risk} = \sigma_P^2 V_A^2 + \sigma_F^2 N^{*2} V_F^2 - 2N^* \rho\, \sigma_P \sigma_F V_A V_F$$

$$\frac{\partial \text{Risk}}{\partial N^*} = 2\sigma_F^2 N^* V_F^2 - 2\rho\, \sigma_P \sigma_F V_A V_F$$

$$N^* = \rho \frac{\sigma_P}{\sigma_F} \frac{V_A}{V_F} \quad ; \quad \beta = \rho \frac{\sigma_P}{\sigma_F}$$

$$N^* = \beta \frac{V_A}{V_F}$$

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Problem #2: Suppose that S is spot and F is a futures prices for Brent Crude Oil with times to maturity T. Give a cash-and-carry arbitrage argument to prove

$$F \leq Se^{(r+c)T}$$

where $r > 0$ is the interest rate (assumed constant) and $c > 0$ is the rate of storage cost (assumed constant). For the purposes of this problem, assume that a futures contract is the same as a forward contract.

With a Cash and Carry strategy, we would borrow S dollars at rate r, and purchase some Brent Crude Oil. To complete arbitrage, we enter into a forward contract to sell the Brent Crude Oil for F.

The cost of borrowing S dollars and storing the Brent Crude Oil from purchase $(t=0)$ until the delivery date $(t=T)$ is $Se^{(r+c)T}$.

Thus, the profit from this arbitrage scheme is $F - Se^{(r+c)T}$.

For abitrage to be impossible, profit should be less than or equal to zero.

$\boxed{\text{Therefore, } F - Se^{(r+c)T} \leq 0 \text{ or } F \leq Se^{(r+c)T}}$

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Problem #3: Suppose that risk-free zero interest rates with continuous compounding are as follows:

Maturity (months)	Rate (% per annum)
3	3.1
6	3.3
9	3.4
12	3.6
15	3.7
18	3.8

Assuming that risk-free zero rates are as in this table, what is the value of an FRA where the holder will pay LIBOR and receive 4.75% (quarterly compounded) for a three-month period starting in one year on a principal of $1,000,000? The forward LIBOR rate for the three-month period is 5.5% (quarterly compounded).

Holder pays $\frac{1}{4}\cdot(.055-.0475)\cdot 1000000 = \$1,875$ at the 15 month mark

This payment is currently worth $1875e^{-.037\cdot\frac{15}{12}}$ or $1,790.26

$$\boxed{\text{Thus, the value of this FRA for the holder is } -\$1,790.26.}$$

if the FRA is settled at the end of the three month period.
If it's settled at the beginning of the period, $e^{-.037\cdot\frac{15}{12}}$ is replaced with $e^{-.036\frac{12}{12}}$

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Problem #4:: The spot price of silver is $15 per ounce. The storage costs are $0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

Assuming no arbitrage, $F_T = Se^{RT} + \sum_i p_i e^{r_i(T-t_i)}$ where S is the spot price, T is the time from now to maturity, R is the interest rate from now to time T, and $\{p_i, t_i, r_i\}$ are the sets of payments, payment times, and the interest rates for those times to maturity.

$$F_{.75} = 15e^{.1 \cdot .75} + .24e^{.1 \cdot .75} + .24e^{.1 \cdot .5} + .24e^{.1 \cdot .25}$$

$$\boxed{F_{.75} = \$16.93 \text{ per ounce}}$$

where $F_{.75}$ is the futures price of silver for delivery in 9 months.

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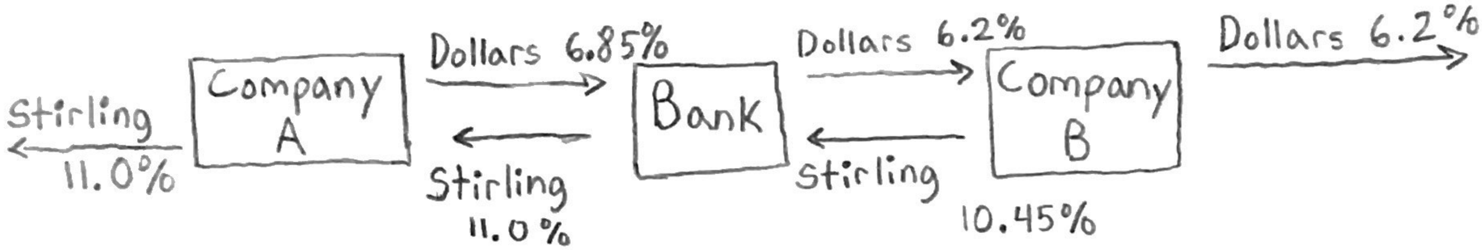
Problem #5: Company A, a British manufacturer, wishes to borrow US dollars at a fixed rate of interest. Company B, a US multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for differential tax effects):

	Sterling	US dollars
Company A	11.0%	7.0%
Company B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.



Company A

- Borrows Sterling at 11.0%
- Lends Sterling to Bank at 11.0%
- Borrows Dollars from Bank at 6.85%

Net: Borrows Dollars at 6.85%

Company B

- Borrows Dollars at 6.2%
- Lends Dollars to Bank at 6.2%
- Borrows Stirling from Bank at 10.45%

Net: Borrows Stirling at 10.45%