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15.4 Find price of a 3 month $(T = \frac{1}{4})$ European put option given $S_0 = 50$, $K = 50$, $r = .1$, and $\sigma = .3$

cumulative Normal distribution

$$p = Ke^{-rT}N(-d_-) - S_0 N(-d_+)$$

where $d_\pm = \dfrac{\ln(S_0/K) + (r \pm \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$

$d_+ = .2417$
$d_- = .0917$

$\boxed{\text{Thus, } p = \$2.37}$

15.5 Solve 15.4 if a dividend of \$1.50 is expected in two months.

$$S_0 \rightarrow S_0 - 1.5e^{-.1(\frac{1}{6})} = 48.52$$

$d_+ \rightarrow .0414$
$d_- \rightarrow -.1086$

$\boxed{\text{Thus, } p = \$3.03}$

15.8 $\ln(S)$ is normally distributed
$\ln(S) = .09875dt + .35dz$

a) Find $Pr[S_T \geq 40 \mid T = 1/2]$

$= Pr[\ln(40) \leq \ln(38) + .0494 + .35\sqrt{1/2}\, dz$

$= Pr[$A normal distribution has a z score $\geq \frac{\ln(40/38) - .0494}{.35\sqrt{1/2}}]$

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This equals $1 - N\left(\frac{\ln(40/38) - .0494}{.35/\sqrt{2}}\right) = .4968$

$\boxed{\text{Thus, the probability that this option is exercised is } .4968}$

b) Find $Pr[\, S_T < 40 \mid T = 1/2\,]$

This is just $1 - Pr[S_T \geq 40 \mid T = 1/2]$

$\boxed{\text{Thus, the probability that this option is exercised is } .5032}$

15.16 $c = 2.5$, $S_0 = 15$, $K = 13$, $T = 1/4$, $r = .05$

Find σ

$$2.5 = 15\, N(d_+) - 13e^{-.05/4} N(d_-)$$

where $d_\pm = \dfrac{\ln(15/13) + (.05 \pm \frac{\sigma^2}{2})/4}{\sigma/2}$

Using DerivaGem, we find

$\boxed{\sigma = .3964}$

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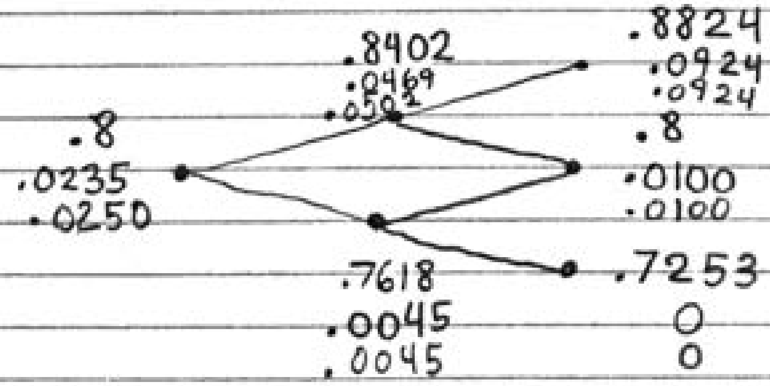
17.1 If the index goes down to I, the portfolio loses $10(1 - I/800)$ million dollars. Buying put options on 10 million/800 = 12,500 times the index ensures the losses are capped at $10(1 - 700/800)$ or 1.25 million dollars. Thus, the portfolio is "insured" on losses over \$1.25 million.

17.4 $S = .8$, $\sigma = .12$, $r_d = .06$, $r_f = .08$, $K = .79$
Two Step Binomial Tree w/ $\Delta t = 1/6$

$$u = e^{\sigma\sqrt{\Delta t}} = 1.0502; \; d = u^{-1} = .9522$$

$$p = \frac{e^{(r_d - r_f)\Delta t} - d}{u - d} = .4538$$



$\boxed{\text{European call option worth \$.0235}}$

$\boxed{\text{American call option worth \$.0250}}$

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17.7 $T=\frac{2}{3}$, $K=.5$, $S_0=.52$, $\sigma=.12$, $r_d=.04$, $r_f=.08$
Find Value of European put option.

$$p=\left(Ke^{-(r_d-r_f)T}N(-d_-)-S_0N(-d_+)\right)e^{-r_fT}$$

where $d_\pm=\dfrac{\ln(S_0/K)+(r_d-r_f\pm\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$

$d_-=.0791$
$d_+=.1771$

$\boxed{\text{Thus, the price of this European put option is \$.0162.}}$

18.4 $c=\underset{\substack{\uparrow\\ Pr[S_T=56|T=1/2]}}{P}\underbrace{6e^{-.06\frac{1}{2}}}_{\text{time discounted profit}}$

$Pr[S_T=56|T=1/2]=\dfrac{1-d}{u-d}$

$d=\frac{46}{50}=.92$ $u=\frac{56}{50}=1.12$

Thus, $Pr[S_T=56|T=1/2]=.4$

$\boxed{\text{Therefore, the call option is worth \$2.33.}}$

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18.7 $p = Ke^{-rT}N(-d_-) - F_0e^{-rT}N(-d_+)$

where $d_{\pm} = \dfrac{\ln(F_0/K) \pm \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$

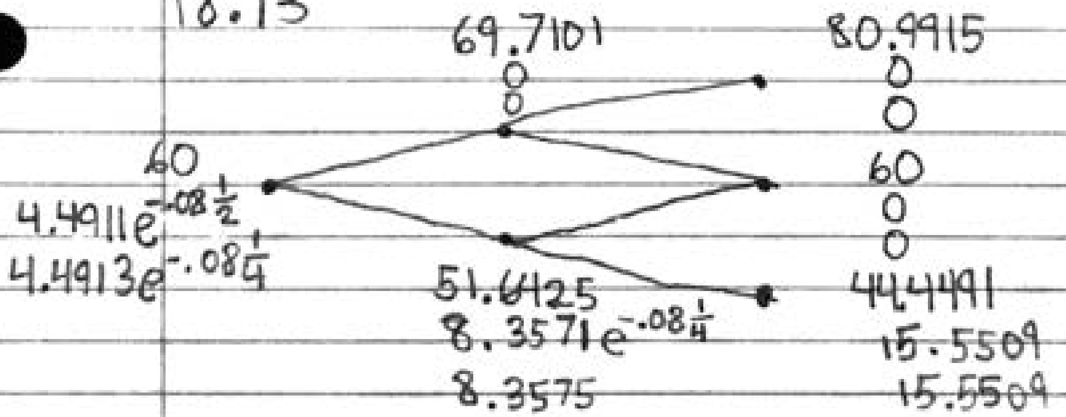
$F_0 = 19$, $K = 20$, $r = .12$, $\sigma = .2$

$d_+ = -.3327$ $\qquad d_- = -.4618$

$N(.4618) = .6778$ $\qquad N(.3327) = .6303$

Therefore, the put option is worth \$1.50.

18.13



The American option is worth exercising early, after 3 months if $S_T = 51.6425$

The value of the six month European put option is $4.4911e^{-.08\frac{1}{2}} = \$4.3155$

The value of the six month American put option is $4.4913e^{-.08\frac{1}{4}} = \$4.4026$

Since $F_0 = 60 = K$ and $c = p$, the put-call parity $c + Ke^{-rT} = p + F_0e^{-rT}$ holds

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19.3 $\Delta = \frac{\partial C}{\partial S} = N(d_+)$

$$d_+ = \frac{\ln(S_0/K) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

At the money $\rightsquigarrow$ $S_0 = K$ so $\ln(S_0/K) = 0$

$r = .1$, $\sigma = .25$

$d_+ = .3712$ $\rightsquigarrow$ $N(d_+) = .64$

$$\boxed{\Delta = .64}$$

19.8 The trader loses \$.1 with each transaction. The total cost of the transactions must be \$4, the price of the option. Thus, the stock is expected to be bought around 20 times and sold around 20 times.

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19.14 $S_0 = .8$, $K = .81$, $r_d = .08$, $r_f = .05$, $\sigma = .15$
$T = \frac{7}{12}$

Delta $\Delta = \frac{\partial V}{\partial S} = N(d_+)e^{-r_f T}$ where $d_\pm = \frac{\ln(S_0/K) + (r_d - r_f \pm \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$

$d_+ = .1016 \rightsquigarrow N(d_+) = .5405$

$\boxed{\text{Thus, } \Delta = .5250}$

Gamma $\Gamma = \frac{\partial^2 V}{\partial S^2} = N'(d_+)\underbrace{\frac{1}{S_0\sigma\sqrt{T}}}_{\frac{\partial d_+}{\partial S}} e^{-r_f T} = \frac{e^{-d_+^2/2}}{\sqrt{2\pi}} \quad \frac{e^{-r_f T}}{S_0\sigma\sqrt{T}}$

$\boxed{\Gamma = 4.206}$

Vega $\mathcal{V} = \frac{\partial V}{\partial \sigma} = S_0\sqrt{T}\,N(d_+)$ for a Black-Scholes call option

$\boxed{\mathcal{V} = .2355}$

Theta $\Theta = \frac{\partial V}{\partial T} = N'(d_+)\left(\frac{-S_0\sigma}{2\sqrt{T}}\right)e^{-r_f T} + \underbrace{r_f N(d_+)e^{-r_f T}}_{\text{Extra term added for currency exchange}} - r_d K e^{-r_d T} N(d_-)$

$\Theta =$

$\boxed{\Theta = -.0399}$

Rho $= KTe^{-r_d T}N(d_-)$ for call option

$\boxed{\text{Rho} = .2231}$

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Delta is the instantaneous change in the option value as the yen price increases. It is the local ratio of the increase in option price to increase in yen price. Gamma is the instantaneous change in Delta when the yen price increases, and is the local ratio of Delta change per yen price change. Vega is the local ratio of the change in option value per change in Volatility. Theta is the instantaneous loss of option value as time progresses. Finally, Rho is the ratio of the change in option value to the change in the U.S. risk free rate. In this case, since Rho is .2231, if the U.S. risk free rate rises from .08 to .09, then the value of the option is expected to rise $ .002231.