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MA 528 Exam 2

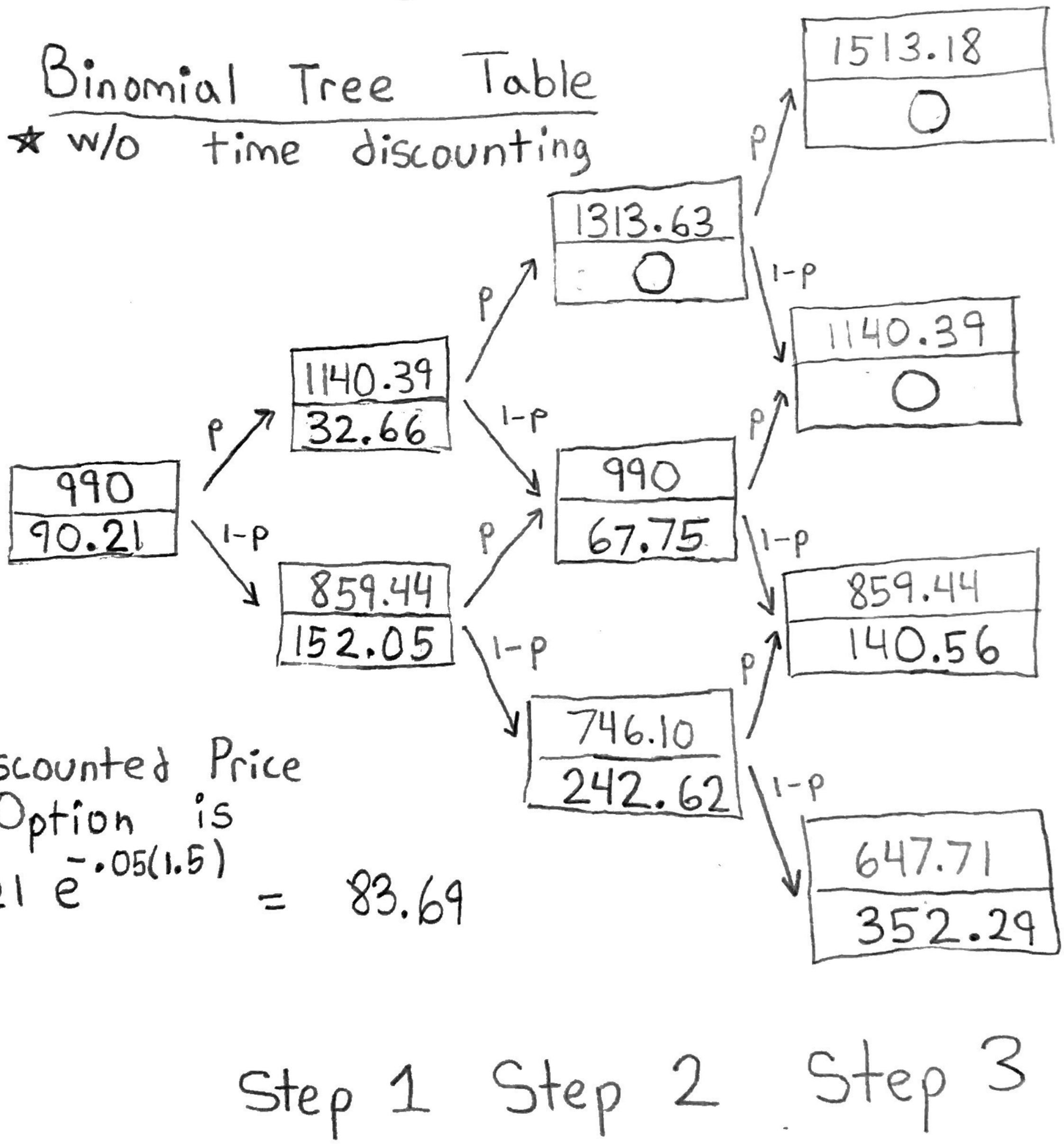
Problem #1: (20 points) A stock index is currently 990, the risk-free rate is 5%, and the dividend yield on the index is 2%. Use a three-step tree to value an 18-month European put option with a strike price of 1,000 when the volatility is 20% per annum.

$\Delta t = .5 \qquad u = e^{\sigma\sqrt{\Delta t}} = 1.1519 \qquad d = \frac{1}{u} = .8681$

$$p = \frac{e^{(r-d)\Delta t} - d}{u-d} = .5180$$

Binomial Tree Table

★ w/o time discounting



Time Discounted Price of Option is

$90.21\, e^{-.05(1.5)} = 83.69$

The 18 month European put Option has a value of \$83.69, according to this method.

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Problem #2: (20 points) A trader sells a strangle by selling a 6-month European call option with a strike price of \$50 for \$3 and selling a 6-month European put option with a strike price of \$40 for \$4. For what range of prices of the underlying asset in 6 months does the trader make a profit?

Revenue from Option Sales = \$7

Loss from 6 month European call option

$= \text{Max}(S_T - 50, 0)$

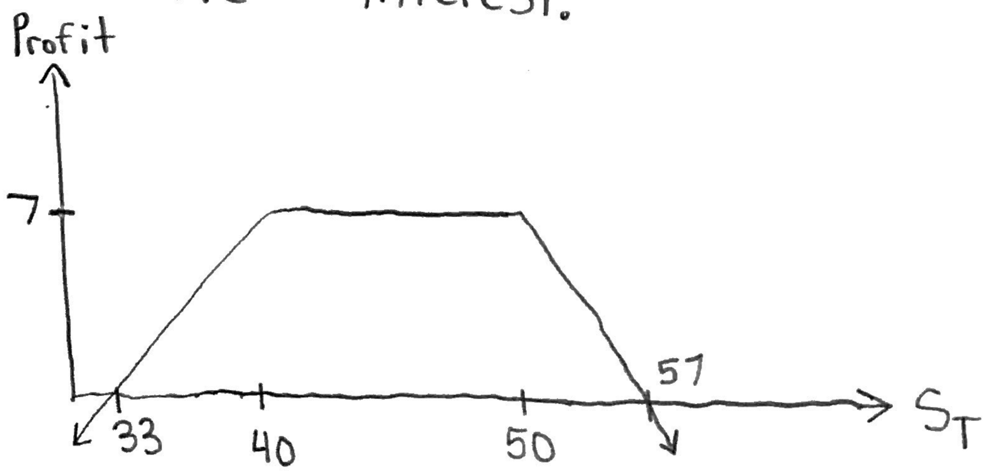
Loss from 6 month European put option

$= \text{Max}(40 - S_T, 0)$

Thus, the trader makes a profit if $7 - \text{Max}(S_T - 50, 0) - \text{Max}(40 - S_T, 0) \geq 0$

$\boxed{\text{The trader makes a profit if the prices of the underlying asset in 6 months is in the range of \$33 to \$57.}}$

Assuming no time discounting of cash or that the \$7 sales revenue doesn't receive interest.



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Problem #3: (20 points) A stock is trading at \$105, you're long a 3-month American call with strike K = \$100, and the risk-free rate is 5%. No dividends will be paid in the coming 3 months, and you have a strong feeling that the asset price will go down leaving the option OTM. Compare the following two strategies:

a) Exercise the option and invest the proceeds.
b) Short the asset, hold the option, and invest the proceeds.

Which strategy is better? Show your computations.

Note that all future profits are time discounted to reflect current value.

Option a)

Make $S_0 - K = \$5$ and invest for three months at $r = .05$.

$\boxed{\text{Current Value of Profit} = \$5}$

Option b)

Profit from Short (current value) $= S_0 - e^{-rT} S_T = 105 - e^{-.05(\frac{1}{4})} S_{1/4}$

Profit from Option (current value, if exercised) $= e^{-.05(\frac{1}{4})}(S_{1/4} - 100)$

$\boxed{\text{Current Value at the minimum of Profit} = \$105 - 100e^{-.05(\frac{1}{4})}}$

Option b) is better as it allows the trader to invest more money at the risk free rate while having the call option to hedge against any losses from the short.

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Problem #4: Consider the following model for a non-dividend-paying stock price,

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

where W(t) is a standard Wiener process. Let $S(0) = 100, \mu = .05$, and $\sigma = .2$.

1. (10 points) What are the 2-sided 95% confidence limits for the stock price at the end of 1 year? That is, find interval $[a, b]$ such that $P(S(1) > b) = .025$ and $P(S(1) < a) = .025$.
2. (10 points) Suppose the stock is sold short at time t=0 and the proceeds of the sale invested in a risk-free account at 4% annual (continuously compounded). How much additional cash is needed to cover losses at the end of 1 year with 95% confidence?

1. The 95% Confidence Interval goes from 1.96 standard deviations below the mean to 1.96 standard deviations above the mean. $dS = \mu S dt + \sigma S dz$ implies $\ln(S_T)$ is normally distributed with mean $\ln(S_0) + (\mu - \frac{\sigma^2}{2})T$ and standard deviation $\sigma^2 T$.
$\ln(S_T)$ has the confidence interval $[4.59517, 4.67517]$

$\boxed{\text{The two sided confidence limits for the stock price after 1 year is } \$[99.01, 107.25]}$

2. Value of Invested Sale Proceeds after 1 year: $100e^{.04} = \$104.08$

Additional Cash Needed: \$3.17

$\boxed{\text{\$3.17 Additional Cash is needed to cover losses at the end of 1 year with 95\% confidence.}}$

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Problem #5: Consider the following model for the short-term interest rate

$$dr(t) = \lambda(\bar{r} - r(t))dt + \sigma dW(t),$$

where W(t) is a standard Wiener process. Short-term loans made at this rate earn $r(t)$, such that an initial principal P_0 at end of 1 year will be

$$P_1 = P_0 e^{\int_0^1 r(t)dt}.$$

1. (15 points) Show that $\int_0^1 r(t)dt$ is normally distributed.
2. (5 points) What is expected value of P_1 for $\lambda = 5, \bar{r} = .04, \sigma = .2$, and given $r(0) = .05$?

1. If $r(t) = \bar{r}(1-e^{-\lambda t}) + \sigma \int_0^t e^{-\lambda(t-s)} dW(s) + r_0 e^{-\lambda t}$

then $Y(t) = e^{\lambda t} r(t) = \bar{r}(e^{\lambda t}-1) + \sigma \int_0^t e^{\lambda s} dW(s)$

$dY(t) = \lambda \bar{r} e^{\lambda t} dt + \sigma e^{\lambda t} dW(t)$

$d(e^{\lambda t} r(t)) = \lambda e^{\lambda t} r(t) dt + e^{\lambda t} dr(t)$

$d(e^{\lambda t} r(t)) = dY(t) \rightsquigarrow dr(t) = \lambda(\bar{r} - r(t))dt + \sigma dW(t)$

Therefore, $r(t) = \bar{r}(1-e^{-\lambda t}) + \sigma \int_0^t e^{-\lambda(t-s)} dW(s) + r_0 e^{-\lambda t}$

$\int_0^1 r(t)\,dt = \bar{r}\left(1 - \frac{1}{\lambda} + \frac{e^{-\lambda}}{\lambda}\right) + \sigma \int_0^1\int_0^t e^{-\lambda(t-s)} dW(s)dt + r_0\left(\frac{1}{\lambda} - \frac{e^{-\lambda}}{\lambda}\right)$

$\int_0^1 r(t)dt = \bar{r} + (r_0 - \bar{r})\left(\frac{1}{\lambda} - \frac{e^{-\lambda}}{\lambda}\right) + \sigma \int_0^1\int_0^t e^{-\lambda(t-s)} dW(s)\,dt$

$\sigma \int_0^1\int_0^t e^{-\lambda(t-s)} dW(s)\,dt = \underset{\text{normal distribution}}{\phi}\left(0, \sigma^2\left(\frac{1}{\lambda} - \frac{e^{-\lambda}}{\lambda}\right)^2\right)$

$\boxed{\text{Therefore, } \int_0^1 r(t)dt \text{ is normally distributed with mean } \bar{r} + (r_0-\bar{r})\left(\frac{1}{\lambda}-\frac{e^{-\lambda}}{\lambda}\right) \text{ and standard deviation } \sigma\left(\frac{1}{\lambda}-\frac{e^{-\lambda}}{\lambda}\right)}$

2. $\langle P_1 \rangle$ or Expectation Value of P_1 is equal to $P_0 \int_{-\infty}^{\infty} e^x \underbrace{\frac{e^{-(x-\mu)/2\sigma^{*2}}}{\sqrt{2\pi}\,\sigma^*}}_{\text{distribution of } \int_0^1 r(t)dt} dx = \boxed{1.0437\ P_0}$

Using $\mu = \bar{r} + (r_0 - \bar{r})\left(\frac{1}{\lambda} - \frac{e^{-\lambda}}{\lambda}\right) = .042$ and $\sigma^* = \sigma\left(\frac{1}{\lambda} - \frac{e^{-\lambda}}{\lambda}\right)$ from 1.

$\sigma^* = .04$

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```
"""
Checking Midterm 2 Problem 5 Answer
"""
import numpy as np
import matplotlib.pyplot as plt
import scipy.integrate

def rsArray(n):
    # n - size of r Array
    rsArray = np.zeros(n)

    dt = 1/n

    for k in range(n):
        if k == 0:
            rsArray[k] = .05

        if k != 0:
            dr = 5*(.04-rsArray[k-1])*dt + .2*np.random.normal(0,1)*np.sqrt(dt)
            rsArray[k] = rsArray[k-1] + dr

    return rsArray

#numerical integration
def Intrt2(rsArray):
    intVal = scipy.integrate.simps(rsArray, x=None, dx=1/len(rsArray))
    return intVal

def MonteCarloAns(n,numTrials):
    IntVals = np.zeros(numTrials)
    AnsSum = 0
    for m in range(numTrials):
        myArray = rsArray(n)
        myIntVal = Intrt2(myArray)
        IntVals[m] += myIntVal
        AnsSum += np.exp(myIntVal)

    IntAvg = np.mean(IntVals)
    IntStDev = np.std(IntVals)
    Answer = round(AnsSum/numTrials,4)
    return Answer

print("The Answer to Problem is " +str(MonteCarloAns(50,10000)) + " P0")
```

Python code returns Values around 1.043 P_0