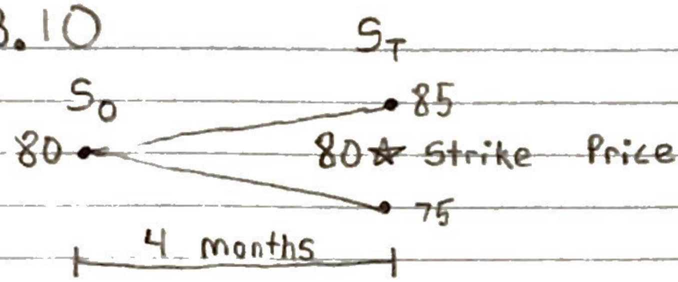
Jacob Wyngaard

MA 528 HW 6

13.10



$r = .05$

Gain from Put Option is either 0 or $5e^{-.05\frac{1}{3}}$

Gain from Stock is either $5e^{-.05\frac{1}{3}}$ or $-5e^{-.05\frac{1}{3}}$

Let the cost of the put option be f
Consider the purchase of 1 stock and n put options

	$S_T = 75$	$S_T = 85$
Put Option Value	$5ne^{-.05/3}$	0
Stock Value	$75e^{-.05/3}$	$85e^{-.05/3}$
Σ	$(75+5n)e^{-.05/3}$	$85e^{-.05/3}$

No Arbitrage means both sums equal $80 + nf$

Setting the sums equal means $n = 2$

$$nf = 85e^{-.05/3} - 80 \; ; \; f \approx 1.80$$

$\boxed{\text{The cost of the put option is \$ 1.80}}$

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13.15 $\Delta t = 1/12;\ r = .05;\ r_f = .08;\ \sigma = .12$

$u = e^{\sigma\sqrt{\Delta t}} \rightsquigarrow \boxed{u = 1.0352}$

$d = e^{-\sigma\sqrt{\Delta t}} \rightsquigarrow \boxed{d = .9660}$

$$p = \frac{e^{(r-r_f)\Delta t} - d}{u - d} \rightsquigarrow \boxed{p = .4553}$$

13.16 $S_0 = 78;\ \sigma = .3;\ r = .03;\ \Delta t = 1/6$

$u = e^{\sigma\sqrt{\Delta t}} \rightsquigarrow \boxed{u = 1.1303}$

$d = e^{-\sigma\sqrt{\Delta t}} \rightsquigarrow \boxed{d = .8847}$

$$p = \frac{e^{r\Delta t} - d}{u-d} \rightsquigarrow \boxed{p = .4898}$$

Value of 4 month European Call ($t = 1/3$) Option w/ $K = 80$ using two step binomial tree (so $\Delta t = 1/6$)

$$\begin{tikzcd}[column sep=small, row sep=small]
 & & 78u^2 & \text{Makes } (78u^2-80)e^{-rt} \\
 & 78u \arrow[ur, "p"] \arrow[dr, "1-p"] & & \\
78 \arrow[ur, "p"] \arrow[dr, "1-p"'] & & 78 & \text{Makes } 0 \\
 & 78d \arrow[ur, "p"] \arrow[dr, "1-p"'] & & \\
 & & 78d^2 & \text{Makes } 0
\end{tikzcd}$$

$F = p^2(78u^2 - 80)e^{-rt}$

$\boxed{f = \$4.67}$

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If a trader sells 1000 options, they hedge their position with purchasing Δ stock.

Initially, $\Delta = \dfrac{f_u - f_d}{uS_0 - dS_0}$

$f_u = e^{-r\Delta t}(S_0 u^2 - K)p$ (option will be worth the expected value)

$f_d = 0$

$f_u = 9.58 \rightsquigarrow \Delta = .500$

$\boxed{\text{The trader should also purchase 500 shares of stock.}}$

13.17 $\Delta t = \frac{1}{2}$; $\sigma = .18$; $r = .04$; $\underset{y}{\text{dividend yield}} = .025$

Index Current at 1500

$u = e^{\sigma\sqrt{\Delta t}} \rightsquigarrow \boxed{u = 1.1357}$

$d = e^{-\sigma\sqrt{\Delta t}} \rightsquigarrow \boxed{d = .8805}$

$p = \dfrac{e^{(r-y)\Delta t} - d}{u - d} \rightsquigarrow \boxed{p = .4977}$

Value of 12 month put option w/ $K = 1480$:

American Option: Option can be exercised after 1st time step.

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Early Exercise Expected Value: $(1-p)(1480-1500d)e^{-r\Delta t}$
\$78.41

Holding on Expected Value: $(1-p)^2(1480-1500d^2)e^{-r2\Delta t}$
\$76.87

Early Exercise is the more profitable strategy

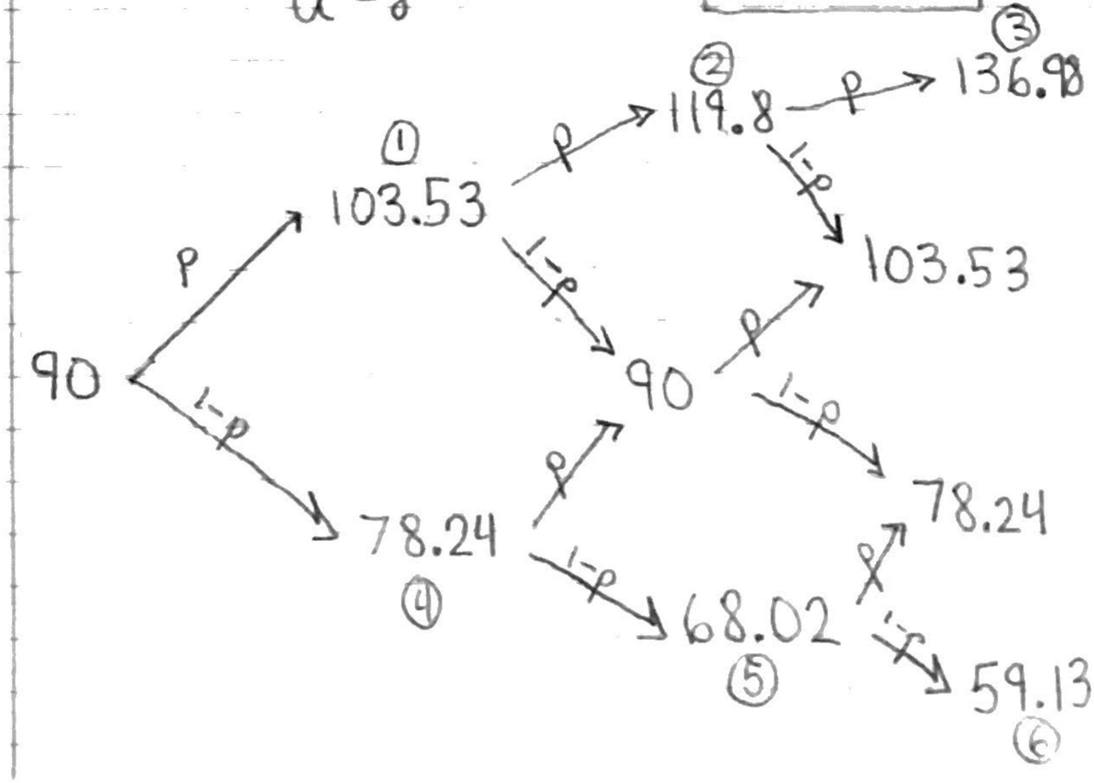
$$\boxed{f = \$78.41}$$

13.18 Commodity Futures Price = \$90
Three Step Tree w/ $\Delta t = 1/4$
$\sigma = .28$, $r = .03$

$u = e^{\sigma\sqrt{\Delta t}}$ $\rightsquigarrow$ $\boxed{u = 1.1503}$

$d = 1/u$ $\rightsquigarrow$ $\boxed{d = .8694}$

$p = \dfrac{1-d}{u-d}$ $\rightsquigarrow$ $\boxed{p = .4651}$



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a) Value American Call Option w/ $K=93$ and maturity in 9 months

Exercise Time	Time Discounted Expected Profit
1/4	\$ 4.86
2/4	\$ 5.56
3/4	\$ 7.90

$\boxed{f = \$7.90}$

b) Same as a) but now it's a put option

Exercise Time	Time Discounted Expected Profit
1/4	\$ 7.84
2/4	\$ 8.51
3/4	\$ 10.83

$\boxed{F = \$10.83}$