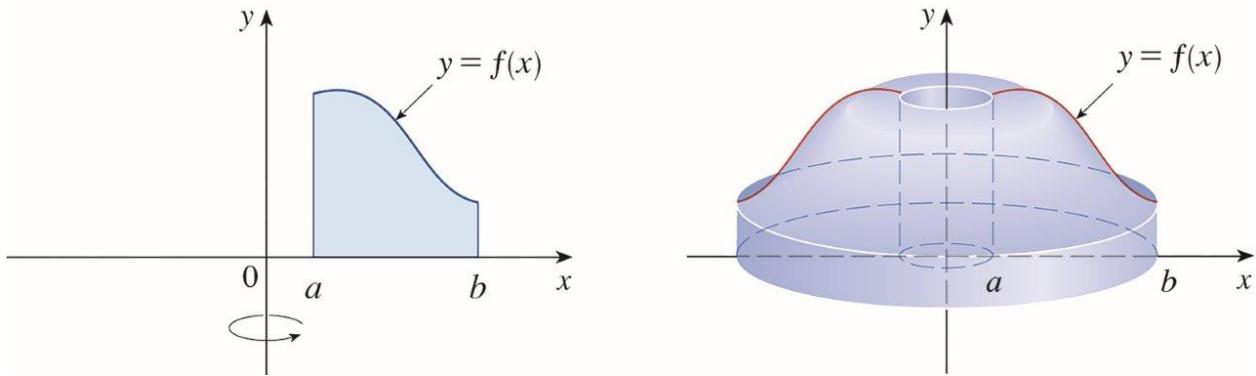
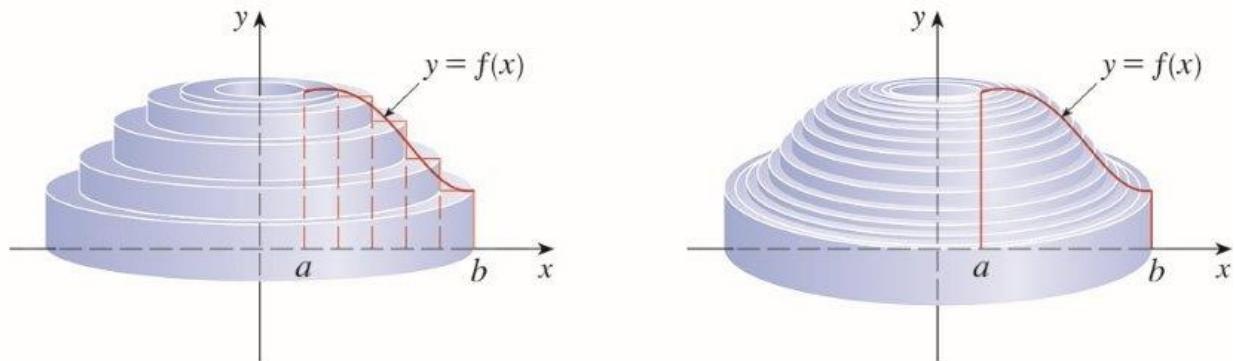


6.3: Shell Method

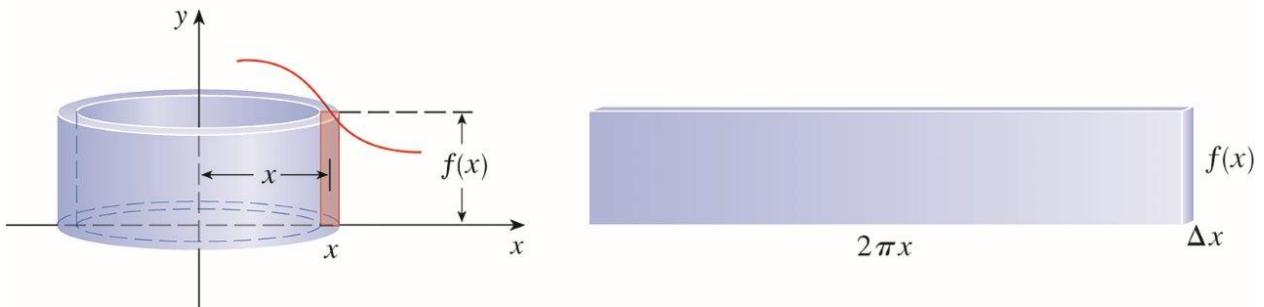
Consider the following region bounded by $y = f(x)$, the x -axis, $x = a$, and $x = b$. We will then rotate the region around the y -axis.



Using the Washer method for this region would be a pain. Instead we can use instead cylindrical shells to find the volume.



Since each volume chunk is given by a cylinder we can find the “length” of each chunk using the circumference of the associated circle ($2\pi \cdot \text{radius}$) and the height of each chunk. For this specific example, the radius is x (the distance to the y -axis) and the height is the value of the top function $y = f(x)$ minus the bottom function $y = 0$.

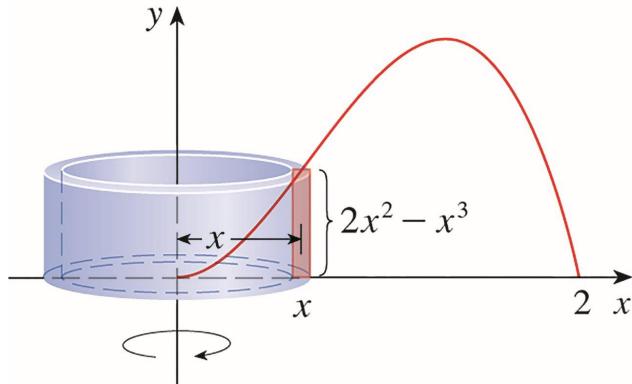


In general the shell method is given by (either dx or dy)

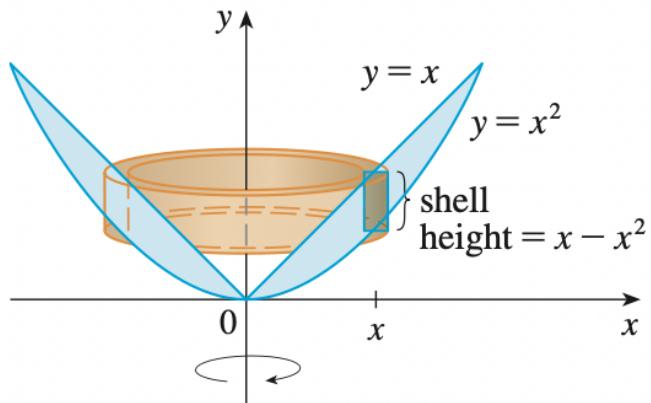
$$\int_a^b 2\pi \ r(x) \ h(x) \ dx$$

- $r(x)$ is the radius of the shell (i.e. “current” distance to axis of rotation)
- $h(x)$ is the height of the shell (i.e. top-bottom or right-left)

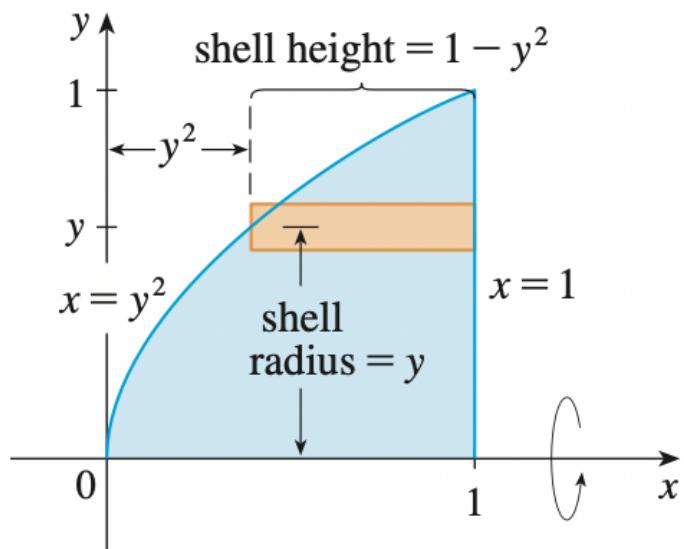
Example 1. Find the volume of the solid obtain by rotating about the y -axis with the region bounded by $y = 2x^2 - x^3$ and $y = 0$.



Example 2: Find the volume of the solid obtained by rotating about the y -axis with the region between $y = x$ and $y = x^2$.



Example 3: Use cylindrical shells to find the volume of the solid obtained by rotating about the x -axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



Example 4: Set up the volume V of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

