

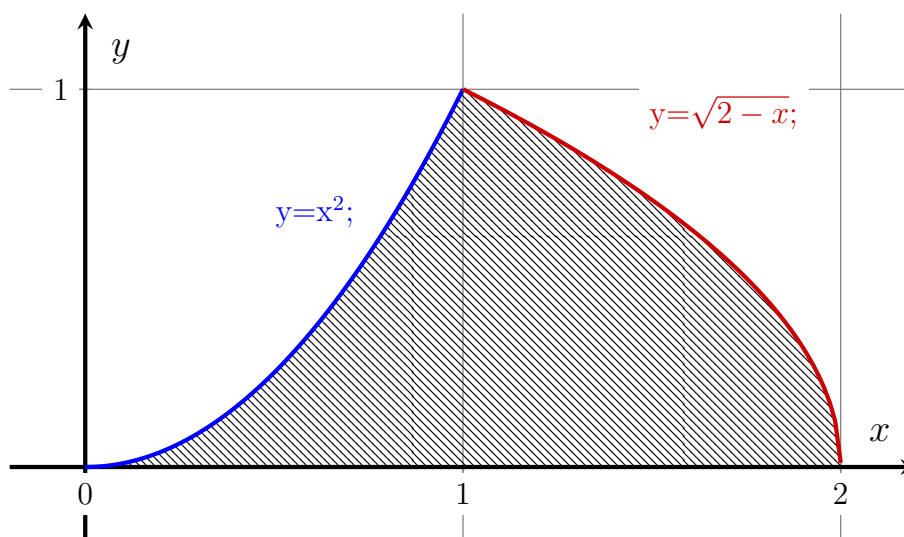
Midterm 1 Questions

Show all work clearly. Submit to Gradescope.
No calculator usage or software / program assistance.

THERE ARE 3 PAGES IN TOTAL:

- 2 pages of problems (6* Problems in total // *problem 1 has 6 parts)
- 1 page of given information

Problem 1. Consider the region R bounded by $y = x^2$ and $y = \sqrt{2-x}$ in the first quadrant. An accurate graph is provided below:
(ignore the semi-colons the graph doesn't render w/o them for some reason)



The following parts are all based on this region R .

- (a : 4 points) Rewrite the region's edges given by $y = x^2$ and $y = \sqrt{2-x}$ as functions as y .
- (b : 5 points) Calculate the area of the region R .
- (c : 10 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line $y = 1$ using the **Shell method**.
- (d : 10 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line $x = 1$ using the **Washer Method**.
- (e : 12 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line $x = -1$.
- (f : 12 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line x -axis.

Problem 2. (5 points) Set up BUT DO NOT SOLVE the Partial Fraction Decomposition for $\frac{3x^2 + 4x - 5}{x^2(x-1)(2x^2+3)^2}$

Problem 3. (10 points) Evaluate $\int \frac{10}{(x-1)(x^2+9)} dx$

Problem 4. (10 points) Evaluate $\int_1^\infty \frac{e^{-1/x}}{x^2} dx$

Problem 5. (10 points) Evaluate the integral $\int \sec(x) \tan(x) \ln(\sec(x)) dx$.

Problem 6. (12 points) Set up BUT DO NOT EVALUATE the integral that represents the arc length of the function $y = x^2$ on the interval $[1, \infty)$.

IF you had to evaluate this integral, what integral techniques or special considerations (if any) would you have to do/deal with? If you are not sure what to write, do the first few steps associated with solving the resulting arc length integral.

List of Given Information

1. Pythagorean Identities

(a) $\sin^2(x) + \cos^2(x) = 1$

(b) $\tan^2(x) + 1 = \sec^2(x)$

(c) $1 + \cot^2(x) = \csc^2(x)$

2. Double Angle Identities

(a) $\sin(2x) = 2 \sin x \cos x$

(b) $\cos(2x) = \cos^2(x) - \sin^2(x)$

3. Half-Angle Identities

(a) $\sin^2(x) = \frac{1 - \cos(2x)}{2}$

(b) $\cos^2(x) = \frac{1 + \cos(2x)}{2}$

4. Trig-Integrals

(a) $\int \sec x \, dx = \ln |\sec x + \tan x| + C$

(b) $\int \tan x \, dx = \ln |\sec x| + C$

(c) $\int \csc x \, dx = \ln |\csc x - \cot x| + C$

(d) $\int \cot x \, dx = \ln |\csc x| + C$