# Section 7.2 Trigonometric Integrals

In this section, we use trigonometric identities to integrate certain combinations of trigonometric functions. We start with powers of sine and cosine.

**Strategy for Evaluating** $\int \cos^n x \sin^m x\, dx$

1. If the power of cosine is odd ($n = 2k+1$), save one cosine factor and use

$$\cos^2 x = 1 - \sin^2 x$$

   to express the remaining factors in terms of sine:

$$\int \cos^{2k+1} x \sin^m x\, dx = \int (\cos^2 x)^k \cos x \sin^m x\, dx = \int (1 - \sin^2 x)^k \sin^m x \cos x\, dx.$$

   Then substitute $u = \sin x$.

2. If the power of sine is odd ($m = 2k+1$), save one sine factor and use

$$\sin^2 x = 1 - \cos^2 x$$

   to express the remaining factors in terms of cosine:

$$\int \cos^n x \sin^{2k+1} x\, dx = \int \cos^n x (\sin^2 x)^k \sin x\, dx = \int \cos^n x (1 - \cos^2 x)^k \sin x\, dx.$$

   Then substitute $u = \cos x$.

3. If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x), \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x).$$

   It is sometimes helpful to use the identity

$$\sin x \cos x = \tfrac{1}{2} \sin 2x.$$

**Example 1:** $\int \sin^2 x \, \cos^3 x \, dx$

**Example 2:** $\int \sin^5 x \, \cos^2 x \, dx$

**Strategy for Evaluating** $\int \tan^n x \sec^m x \, dx$

1. If the power of tangent is odd ($n = 2k+1,\ k \geq 0$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\int \tan^{2k+1} x \sec^m x \, dx = \int (\tan^2 x)^k \sec^m x \ \tan x \, dx$$

$$\int (\tan^2 x)^k \sec^m x \ \tan x \, dx = \int (\sec^2 x - 1)^k \sec^{m-1} x \, (\sec x \tan x) \, dx.$$

   Then substitute $u = \sec x$.

2. If the power of secant is even ($m = 2k,\ k \geq 1$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\int \tan^n x \sec^{2k} x \, dx = \int \tan^n x \, (\sec^2 x)^{k-1} \sec^2 x \, dx$$

$$\int \tan^n x \, (\sec^2 x)^{k-1} \sec^2 x \, dx = \int \tan^n x \, (1 + \tan^2 x)^{k-1} \sec^2 x \, dx.$$

   Then substitute $u = \tan x$.

**Example 3:** $\int \tan^6 x \sec^4 x \, dx$

**Example 4:** $\int_0^{\pi/2} (1 - \sin x)^2 \, dx$

**Example 5:** $\int \sin x \, \sec^5 x \, dx$

**Example 6:** $\int \sqrt{1 + \cos(2x)}\, dx$

**Strategy for Evaluating**
$\int \sin(mx)\cos(nx)\,dx$**,** $\int \sin(mx)\sin(nx)\,dx$**, or** $\int \cos(mx)\cos(nx)\,dx$

Use the corresponding identities:

$$\begin{aligned}
&\text{(a)} \quad \sin A \cos B = \tfrac{1}{2}\big[\sin(A-B) + \sin(A+B)\big],\\
&\text{(b)} \quad \sin A \sin B = \tfrac{1}{2}\big[\cos(A-B) - \cos(A+B)\big],\\
&\text{(c)} \quad \cos A \cos B = \tfrac{1}{2}\big[\cos(A-B) + \cos(A+B)\big].
\end{aligned}$$

**Example 7:** $\displaystyle\int \sin(5x)\cos(4x)\,dx$