Math 242-013 Winter 2026

Review (on a lot of things students get wrong)

Algebra:

$$x^{n} = \underbrace{x \cdot x \cdot \cdots \cdot x}_{\text{multiply to itself } n \text{ times}}$$

$$x^a x^b = x^{a+b}$$

$$x^{-1} = \frac{1}{x}$$

$$\rightarrow \frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$$

$$(x+y)^2 = (x+y)(x+y) = x^2 + xy + yx + y^2$$
$$= x^2 + 2xy + y^2$$

$$\frac{a}{b} + \frac{c}{d} \cdot 1 = \frac{d}{d} \cdot \frac{a}{b} + \frac{c}{d} \cdot \frac{b}{b} = \frac{da}{db} + \frac{cb}{db} = \frac{da+cb}{db}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{\frac{a}{b}\left(\frac{d}{c}\right)}{\frac{c}{d}\cancel{\left(\frac{d}{c}\right)}} = \frac{a}{b}\cdot\frac{d}{c}$$

Pre Calc

Functions have an input to output relationship (vertical line test / VLT)

$f(x) \rightarrow$ output of $f(x)$-values

↳ input of x-values

"let $y = f(x)$"

↳ we are relabelling $f(x)$ outputs as "y"

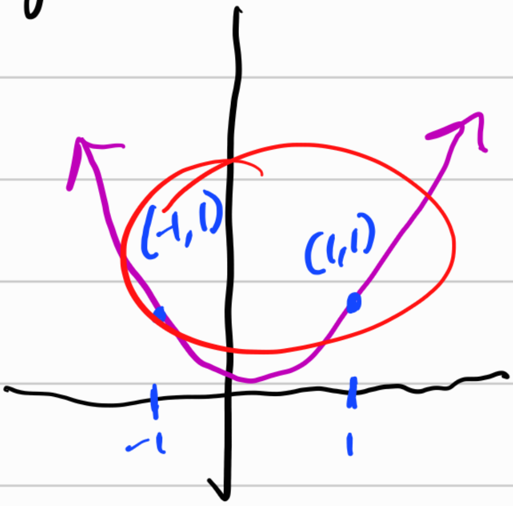
Inverses:

An inverse of a function "undo" the operation

$$f: \underset{x}{\text{inputs}} \rightarrow \underset{y}{\text{outputs}}$$

The inverse $f^{-1}: y \rightarrow x$

eg $f(x) = x^2$



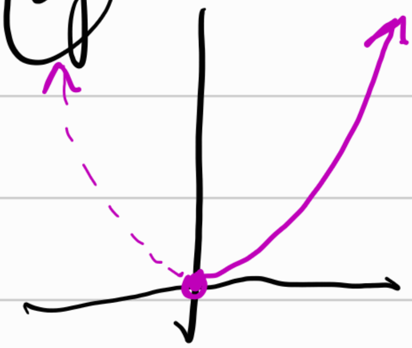
what about $f^{-1}(x)$

$f^{-1}(1)$?

↳ no clear output for the inverse.

A lot of functions they inverses unless we restrict the domain.

eg



$f(x) = x^2$ on $[0,\infty)$

then $f^{-1}(x) = \sqrt{x}$ on $[0,\infty)$

Some really important functions

Algebraic functions:

- polynomials

$x^2 + 2x + 1$

$x^3 + 32x^{65}$

0

x

- roots $\sqrt{x}$, $x^{1/3}$, $x^{3/4}$
- [name pending] $\frac{1}{x}$, $x^{-2/3}$, $\frac{1}{x^{2/3}}$

exponentials / logs: e^x, $\ln x$

Note e^x & $\ln x$ are inverses of each other

$$\begin{cases} e^x : (-\infty, \infty) \longrightarrow (0, \infty) \\ \ln x : (0, \infty) \longrightarrow (-\infty, \infty) \end{cases}$$

$\to$ $\underbrace{x = e^{\ln x}}_{\text{very useful for limit problems}}$ & $x = \ln(e^x)$

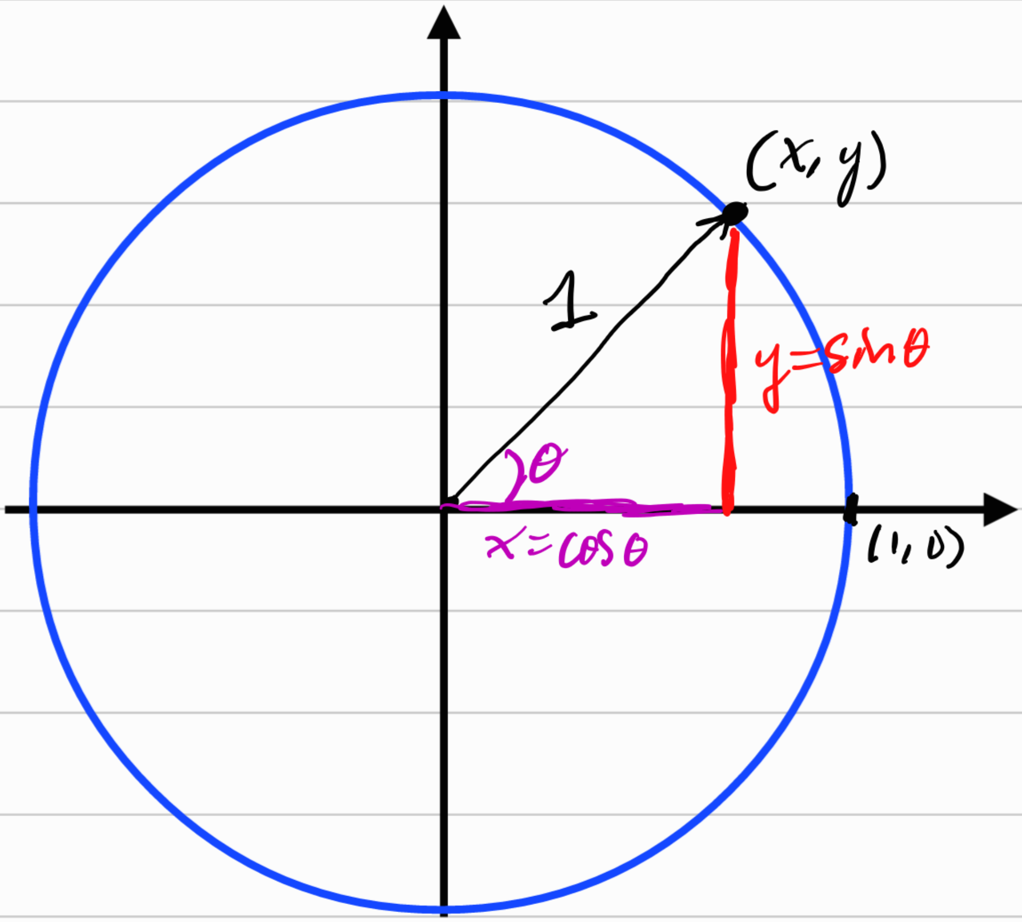
Important log properties

$$\log_b(x^a) = a \log_b(x)$$

$$\log(ab) = \log(a) + \log(b)$$

$\log(\frac{a}{b}) = \log(a) - \log(b)$

Trig

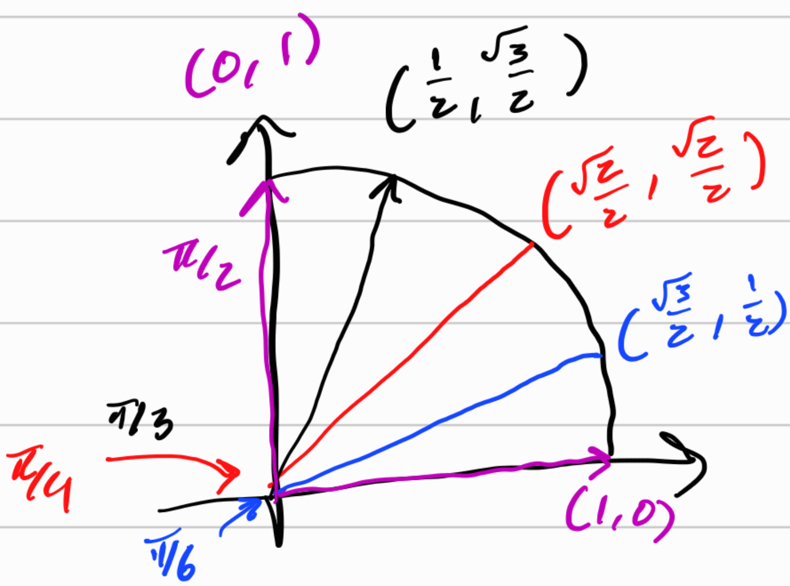


$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$,

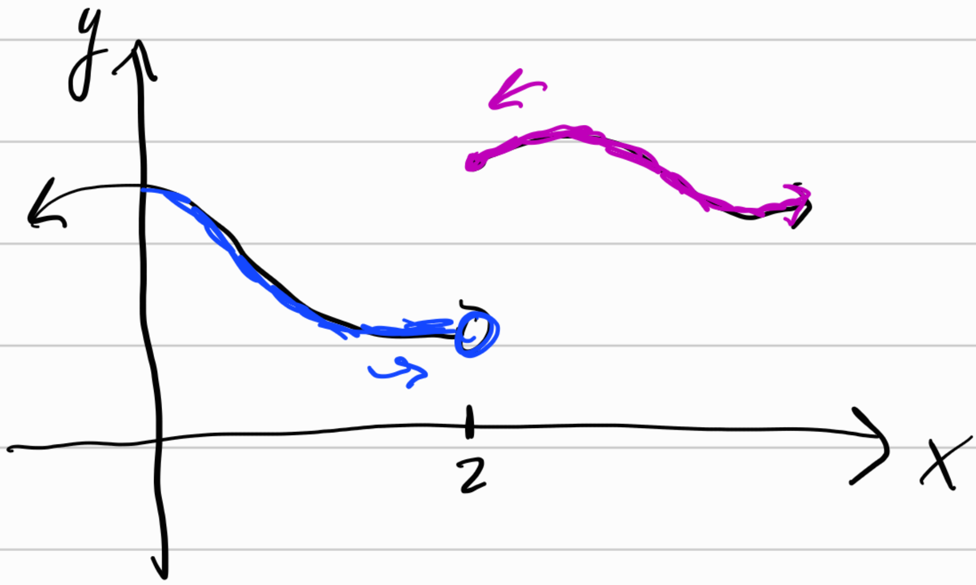
$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$



Calc 1

Limits → A limit is the "expected behaviour of a function"



Continuity → The function behaves exactly how we expect

↳ cont.

The Mathematical definition of cont. is:

$$\lim_{x \to a} f(x) = f(a)$$

Another interpretation the the def.

A function is cont. if you can "push the limit inside"

$$\lim_{x \to a} f(x) = f\left(\lim_{x \to a} x\right) = f(a)$$

Derivatives

"power rule" $\frac{d}{dx}[x^a] = a x^{a-1}$

chain rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

product rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

quotient rule

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f' \cdot g - g' \cdot f}{(g)^2}$$

$$\frac{d}{dx}\left[\frac{H}{L}\right] = \frac{L \cdot DH - H \cdot DL}{L^2}$$

You need to know how to take a derivative of ALL functions listed in the Precalc.

Whatever tricks work for tangent & secant should also work for cotangent & cosecant

L'Hôpital:

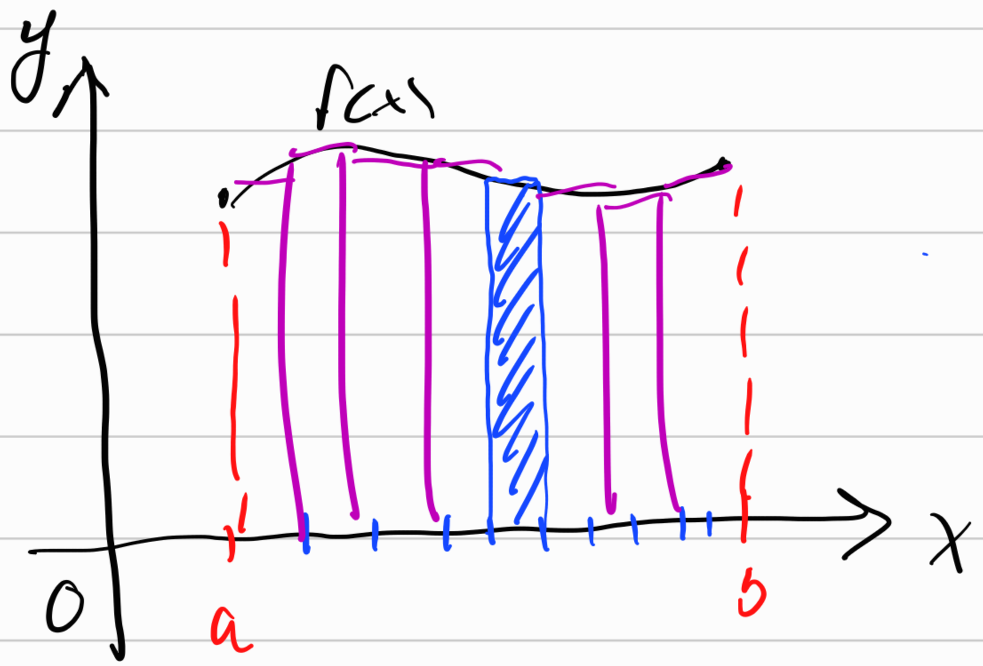
IF $\lim_{x \to \pm\infty \text{ or } a} \frac{f(x)}{g(x)} = $ " $\frac{\infty}{\infty}$ " or " $\frac{0}{0}$ "

Then $\lim_{x \to ?} \frac{f'(x)}{g'(x)} = \lim_{x \to ?} \frac{f(x)}{g(x)}$

Useful $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $\boxed{\infty^0,\ 0^0,\ 1^\infty}$, etc

↳ by using $x = e^{\ln x}$

Integration (Very Important)



VERY IMPORTANT IDEA

The integral is formed by approximating the "area under curve" w/ a simple "structure" (in this case rectangles)

$$\underbrace{\int_a^b f(x)\,dx}_{\text{Area under the curve}} = \underbrace{\lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} \underbrace{f(x_i)\,\Delta x}_{\text{Simple Structures}}}_{\text{approximation}}}_{\text{A refinement of the approx. until exact.}}$$

(Complicated Structure)

Fundemental Theorem of Calculus (FTC)

(i) $\int_a^b f'(x)\,dx = f(b) - f(a)$

(ii) $F(x) = \int_a^x f(t)\,dt$

You need to know how to integrate

Every algebraic function: $x^n + 3x^2$, $x^{1/2}$, x^{-3}, x^{-1}

Exponentials, sine & cosine, &

$\int \sec^2 x\,dx = \tan x + C$

$$\int \sec x \tan x \, dx = \sec x + C$$

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u-sub: Undos the chain rule

$$\int \frac{d}{dx}\left[f(g(x))\right] dx = \int f'(g(x)) \cdot g'(x) \, dx$$

$$f(g(x)) + C = \int f'(g(x)) \cdot g'(x) \, dx$$

$u = g(x) \rightarrow \frac{d}{dx} u = g'(x) \rightarrow du = g'(x) \, dx$

$$f(u) + C = \int f'(u) \, du$$

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§ 7.1, IBP (Theory)

↳ "Undoing a product rule"

↳ let's us push a derivative onto a "simpiler function"

$$\frac{d}{dx}\left[f(x) g(x)\right] = f'(x) \cdot g(x) + f(x) g'(x)$$

$$\int \frac{d}{dx}\left[f(x)g(x)\right]dx = \int f'(x)\cdot g(x) + f(x)g'(x)\,dx$$

$$f(x)g(x)\underbrace{+C}_{\text{drop this for notation sake}} = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$f(x)g(x) - \int f'(x)g(x)\,dx = \int f(x)g'(x)\,dx$$

If label $u = f(x)$ $v = g(x)$

$$\int u\,dv = uv - \int v\,du$$

"ultra violet – super voo · du"

7.1: Integration by Parts (IBP)

The chain rule let us ask "*is this integral a chain rule*?" and now we will learn Integration by Parts which will let us ask "*is this integral a* ***product rule***?" To see how it comes up, let's look at the product rule and try to integrate it:

$$\frac{d}{dx}\Big[f(x)g(x)\Big] = f'(x)g(x) + f(x)g'(x)$$

$$f(x)g(x) = \int \frac{d}{dx}\Big[f(x)g(x)\Big]dx = \int \Big[f'(x)g(x) + f(x)g'(x)\Big]\, dx$$

$$f(x)g(x) = dx = \int f'(x)g(x)\, dx + \int f(x)g'(x)\, dx$$

If we move one of the integrals to the other side we get integration by parts:

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

If we relabel $f(x)$ as u and $g(x)$ as v (that is, set $u = f(x)$ and $v = g(x)$), then by the chain rule, $\frac{du}{dx} = f'(x)$ and $\frac{dv}{dx} = g'(x)$ (or more familiarly: "$du = f'(x)\ dx$" and "$dv = g'(x)\ dx$") and we get Integration-By-Parts (sometimes shorten to IBP):

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

aka

$$\int udv = uv - \int vdu$$

or: "ultra*violet - SUPER (voo*du)"

When integrating by parts you will want to choose dv so that you can actually integrate the function. This means, your priority will be setting dv first in many cases. A there are many ways to try and remember the order, my suggestion is:

LIATE which stands for "**L**et's **I**ntegrate **A** **T**errible **E**quation" it's short hand for

- **L**: Logarithms (i.e. $\ln x$, $\log_b(x)$, etc.)
- **I**: Inverse Trig (i.e. $\arctan x$, $\arcsin x$, etc.)
- **A**: Algebraic (i.e. x, x^a, dx, etc.)
- **T**: Trig (i.e. $\sin x$, $\cos x$, etc.)
- **E**: Exponentials (i.e. e^x, 2^x, a^x, etc.)

Things higher on the list are harder to integrate so you most likely set that as u and set dv to be whatever appear below it. Other acronyms that you might've seen or will see are: LIPET, ILATE, ILPTE, etc. When it comes to inverse trig and logs, you can

Example 1. Find $x \sin x \, dx$

Example 2. Find $\ln x \, dx$

Example 3. Find $e^x \sin x \; dx$

Example 4. Find $t^2e^t\ dt$