Problem 1

ⓐ $\int_1^e x^2 \ln x \, dx \longrightarrow \int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2 \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{3}x^3$

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \frac{1}{x} \, dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 \, dx$$

$$= \frac{1}{3}x^3 \ln x - \overbrace{\left(\frac{1}{3}\cdot\frac{1}{3}\right)}^{=1/9}x^3 + C$$

$$\int_1^e x^2 \ln x \, dx = \left[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right]_1^e$$

$$= \frac{1}{3}e^3 \overbrace{\cancel{\ln(e)}}^{1} - \frac{1}{9}e^3$$

$$- \left(\frac{1}{3}(1)^3 \overbrace{\cancel{\ln(1)}}^{0} - \frac{1}{9}(1)^3\right)$$

$$= \frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9} = \boxed{\frac{2e^3+1}{9}}$$

(b) $\int \ln\sqrt{x}\,dx$

$u = \ln\sqrt{x}$ $\qquad$ $dv = dx$

$du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\,dx$ $\qquad$ $v = x$

Chain rule

$= \frac{1}{2x}\,dx$

$$\int \ln\sqrt{x}\,dx = x\ln\sqrt{x} - \int x\,\frac{1}{2x}\,dx$$

$$= x\ln\sqrt{x} - \frac{1}{2}\int dx$$

$$= x\ln\sqrt{x} - \frac{1}{2}x + C$$

Problem 2

ⓐ $\int_0^{\pi/2} \cos^3 x \sin^{2026} x \, dx \to \int \cos^3 x \sin^{2026} x \, dx$

$u = \sin x$ & $\cos^2 x = 1 - \sin^2 x$

$du = \cos x \, dx$

$$\int \cos^2 x \sin^{2026} x \cdot \underbrace{\cos x \, dx}_{du}$$

$\cos^2 x \to 1 - \sin^2 x \to 1 - u^2$, $\sin^{2026} x \to u^{2026}$

$$= \int (1-u^2) u^{2026} \, du = \int u^{2026} - u^{2028} \, du$$

$$= \frac{1}{2027} u^{2027} - \frac{1}{2029} u^{2029} + C$$

$$= \frac{\sin^{2027}(x)}{2027} - \frac{\sin^{2029}(x)}{2029} + C$$

$$\int_0^{\pi/2} \cos^3 x \sin^{2026} x \, dx = \left[\frac{\sin^{2027}(x)}{2027} - \frac{\sin^{2029}(x)}{2029} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2027} - \frac{1}{2029} - 0$$

(b) $\int [1 + \sin(2x)]^2 \, dx$

$\sin^2(☺) = \frac{1-\cos(2☺)}{2}$

$☺ = 2x$

$$= \int 1 + 2\sin(2x) + \sin^2(2x) \, dx$$

$$= \int 1 + 2\sin 2x + \frac{1}{2} - \frac{1}{2}\cos(2 \cdot 2x) \, dx$$

$$= \int \frac{3}{2} + 2\sin 2x - \frac{1}{2}\cos(4x) \, dx$$

$$= \frac{3}{2}x - 2 \cdot \frac{1}{2}\cos 2x - \frac{1}{2} \cdot \frac{1}{4}\sin(4x) + C$$

$$= \frac{3}{2}x - \cos(2x) - \frac{1}{8}\sin(4x) + C$$

Problem 3

$$\int \frac{dx}{1-x^2}$$

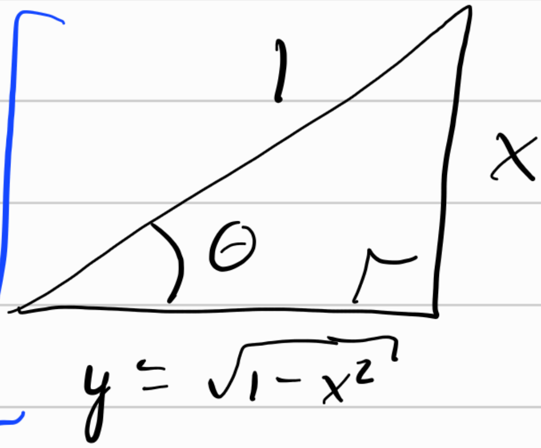
$1 - x^2 = y^2$

$1 = x^2 + y^2$

$x = \sin\theta$

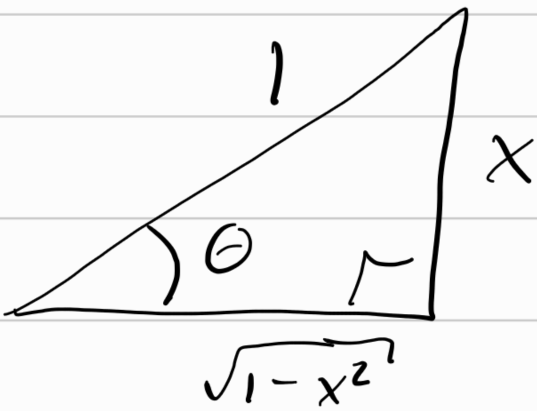
$dx = \cos\theta \, d\theta$

$\Leftarrow$



$$\int \frac{dx}{1-x^2} = \int \frac{\cos\theta \, d\theta}{1-\sin^2\theta} = \int \frac{\cos\theta \, d\theta}{\cos^2\theta} = \int \frac{d\theta}{\cos\theta}$$

$$= \int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + C$$



$\Rightarrow \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\sqrt{1-x^2}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{x}{\sqrt{1-x^2}}$

$$\Rightarrow \boxed{\ln\left|\frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}\right| + C}$$

Problem 4

u-sub, IBP, & Trig-sub all work for this one.

Reasons:

Trig-sub: $1-x^2$ looks "close enough" to $a^2+b^2=c^2$

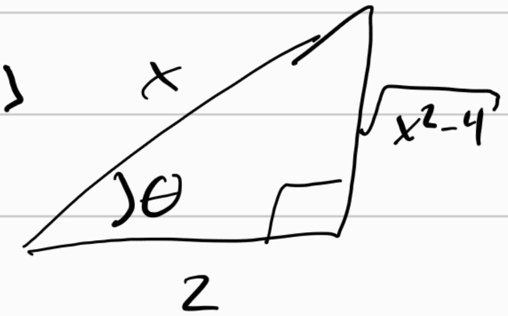
IBP: $dv = \frac{x}{\sqrt{1-x^2}}\,dx$ is doable so I would do this & set $u = x^2$

u-sub: If I set $u=1-x^2$ then $du = -2x\,dx$ & absorbs one x & the remaining x^2 can be replaced w/ $1-u$

Problem 5

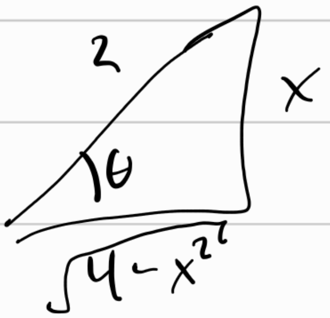
ⓐ False. IBP is based on the product rule

ⓑ False $2\sec\theta = x$

$\Rightarrow$ 

$\Rightarrow \sin\theta = \dfrac{\sqrt{x^2-4}}{x}$

ⓒ False $\sin\theta = \dfrac{x}{2}$ & $\sin(2\theta) = 2\sin\theta\cos\theta$

$\Rightarrow$ 

$\Rightarrow \cos\theta = \dfrac{\sqrt{4-x^2}}{2}$

$$\sin 2\theta = 2 \cdot \frac{x}{2} \cdot \frac{\sqrt{4-x^2}}{2}$$