

7.1: Integration by Parts (IBP)

The chain rule let us ask “*is this integral a chain rule?*” and now we will learn Integration by Parts which will let us ask “*is this integral a **product rule?***” To see how it comes up, let’s look at the product rule and try to integrate it:

$$\begin{aligned}\frac{d}{dx}[f(x)g(x)] &= f'(x)g(x) + f(x)g'(x) \\ f(x)g(x) &= \int \frac{d}{dx}[f(x)g(x)] dx = \int [f'(x)g(x) + f(x)g'(x)] dx \\ f(x)g(x) &= dx = \int f'(x)g(x) dx + \int f(x)g'(x) dx\end{aligned}$$

If we move one of the integrals to the other side we get integration by parts:

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

If we relabel $f(x)$ as u and $g(x)$ as v (that is, set $u = f(x)$ and $v = g(x)$), then by the chain rule, $\frac{du}{dx} = f'(x)$ and $\frac{dv}{dx} = g'(x)$ (or more familiarly: “ $du = f'(x) dx$ ” and “ $dv = g'(x) dx$ ”) and we get Integration-By-Parts (sometimes shorten to IBP):

$$\begin{aligned}\int f(x)g'(x)dx &= f(x)g(x) - \int g(x)f'(x)dx. \\ &\text{aka} \\ \int u dv &= uv - \int v du\end{aligned}$$

or: “ultra*violet - SUPER (voo*du)”

When integrating by parts you will want to choose dv so that you can actually integrate the function. This means, your priority will be setting dv first in many cases. As there are many ways to try and remember the order, my suggestion is:

LIATE which stands for “**L**et’s **I**ntegrate **A** **T**errible **E**quation” it’s short hand for

- **L**: Logarithms (i.e. $\ln x$, $\log_b(x)$, etc.)
- **I**: Inverse Trig (i.e. $\arctan x$, $\arcsin x$, etc.)
- **A**: Algebraic (i.e. x , x^a , dx , etc.)
- **T**: Trig (i.e. $\sin x$, $\cos x$, etc.)
- **E**: Exponentials (i.e. e^x , 2^x , a^x , etc.)

Things higher on the list are harder to integrate so you most likely set that as u and set dv to be whatever appear below it. Other acronyms that you might’ve seen or will see are: LIPET, ILATE, ILPTE, etc. When it comes to inverse trig and logs, you can

Example 1. Find $\int x \sin x \, dx$

Example 2. Find $\int \ln x \, dx$

Example 3. Find $e^x \sin x \, dx$

Example 4. Find $t^2 e^t dt$