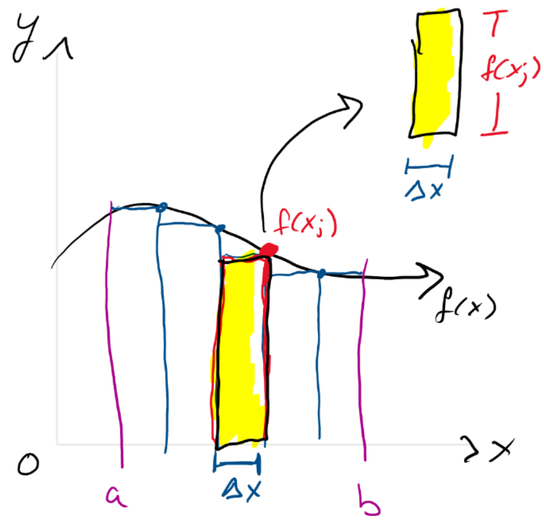
Section 6.1 Area Between Curves

When finding the area of an awkward shaped where functions can be used. We approximate it using Riemann sums. That is



$$\text{Area of } R \approx \sum_{j=1}^{n} f(x_j)\Delta x$$

where each $f(x_j)$ represents a height of our small rectangle and Δx is our width. By refining the size of our rectangles we get the integral:

$$\text{Area of } R = \lim_{n\to\infty} \sum_{j=1}^{n} f(x_j)\Delta x = \int_a^b f(x)\, dx$$

If $f(x)$ and $g(x)$ are two curves such that $f(x) \geq g(x)$ on $[a, b]$, then the area bounded by $f(x)$ and $g(x)$ on the interval $[a, b]$ is

$$A = \underline{\hspace{6cm}}$$

If we were to draw a picture then

- $f(x)$ would be the ______________________ function (i.e. closer to $+\infty$)
- $g(x)$ would be the ______________________ function (i.e. closer to $-\infty$)
- Visually, $f(x)$ is (**above** / **below**) $g(x)$.

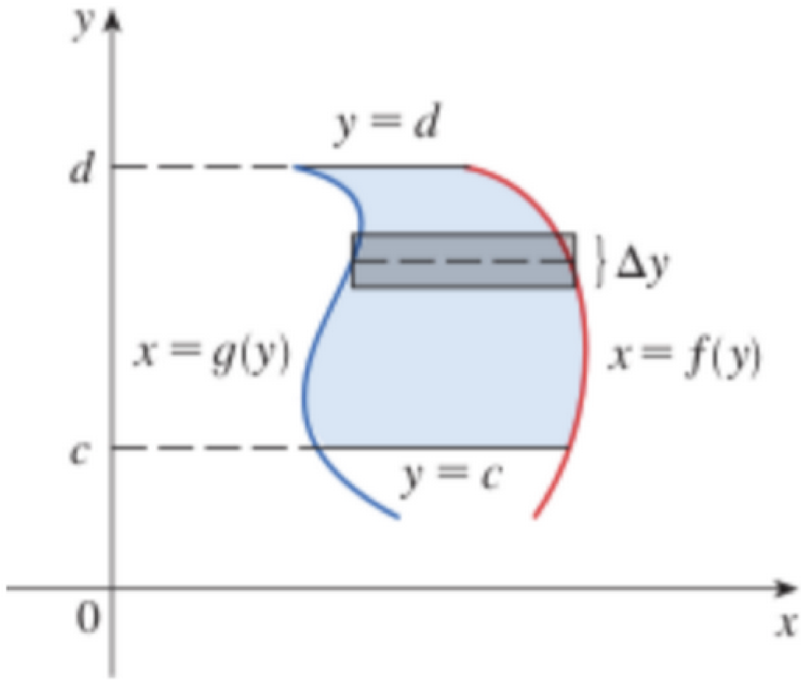
A sketch *could look like*

Example 1. Find the area bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \dfrac{\pi}{2}$.

In general, **the Area between two curves** $f(x)$ and $g(x)$ on $[a, b]$ is

$$A = \underline{\hspace{6cm}}$$

Consider a region as below. Here, the left and right edges are NOT functions of x, but instead can be described as functions of y. ie. The curves are described by some function $f(y)$ and some function $g(y)$.



If an area R (as above) is bounded by the curves $x = f(y)$ and $x = g(y)$ curves such that $f(y) \geq g(y)$ and the horizontal lines $y = c$ and $y = d$, then

- the **right** function $f(y)$ would be the closer to ($+\infty$ / $-\infty$)
- the **left** function $g(y)$ would be the closer to ($+\infty$ / $-\infty$)
- This means that the **right function** / "more positive" function $f(y)$ plays the role of the (top / bottom)
- This means that the **left function** / "more negative" function $f(y)$ plays the role of the (top / bottom)

If an area R (as above) is bounded by the curves $x = f(y)$ and $x = g(y)$ curves such that $f(y) \geq g(y)$ and the horizontal lines $y = c$ and $y = d$, then

Area of A = ______________________

Example 2. Find the area bounded by $y = x - 1$ and $y^2 = 2x + 6$.

Example 3. Find the area bounded by $y = \ln x$ and $y = 1$, in the first quadrant.