

7.1: Integration by Parts (IBP)

The chain rule let us ask “*is this integral a chain rule?*” and now we will learn Integration by Parts which will let us ask “*is this integral a product rule?*” To see how it comes up, let’s look at the product rule and try to integrate it:

$$\begin{aligned}\frac{d}{dx} [f(x)g(x)] &= f'(x)g(x) + f(x)g'(x) \\ f(x)g(x) &= \int \frac{d}{dx} [f(x)g(x)] dx = \int [f'(x)g(x) + f(x)g'(x)] dx \\ f(x)g(x) &= dx = \int f'(x)g(x) dx + \int f(x)g'(x) dx\end{aligned}$$

If we move one of the integrals to the other side we get integration by parts:

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

If we relabel $f(x)$ as u and $g(x)$ as v (that is, set $u = f(x)$ and $v = g(x)$), then by the chain rule, $\frac{du}{dx} = f'(x)$ and $\frac{dv}{dx} = g'(x)$ (or more familiarly: “ $du = f'(x) dx$ ” and “ $dv = g'(x) dx$ ”) and we get Integration-By-Parts (sometimes shorten to IBP):

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

aka

$$\int u dv = uv - \int v du$$

or: “ultra*violet - SUPER (voo*du)”

When integrating by parts you will want to choose dv so that you can actually integrate the function. This means, your priority will be setting dv first in many cases. A there are many ways to try and remember the order, my suggestion is:

LIATE which stands for “Let’s Integrate A Terrible Equation” it’s short hand for

- **L**: Logarithms (i.e. $\ln x$, $\log_b(x)$, etc.)
- **I**: Inverse Trig (i.e. $\arctan x$, $\arcsin x$, etc.)
- **A**: Algebraic (i.e. x , x^a , dx , etc.)
- **T**: Trig (i.e. $\sin x$, $\cos x$, etc.)
- **E**: Exponentials (i.e. e^x , 2^x , a^x , etc.)

Things higher on the list are harder to integrate so you most likely set that as u and set dv to be whatever appear below it. Other acronyms that you might’ve seen or will see are: LIPET, ILATE, ILPTE, etc. When it comes to inverse trig and logs, you can

Example 1. Find $x \sin x \, dx$

Example 2. Find $\ln x \, dx$

Example 3. Find $e^x \sin x \, dx$

Example 4. Find $t^2 e^t dt$