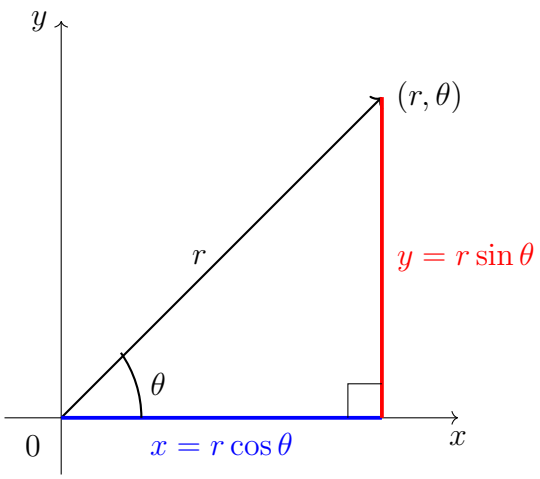
10.3: Polar Coordinates

Up until now, we've only ever worked with 1 way of describing where a point is located. That is, as an (x, y) point on the *cartesian coordinate system* which tells us how far away a point is from both the x-axis and y-axis. This is great is life resolved around boxes (*cough* Manhattan *uncough*), but we live on a sphere, which doesn't handle rectangles very well, as such we can determine a point on the planet using *Longitude and Latitude* which refer to your angle relative towards the Earth's equator and prime meridian. That will be handled in Calc 3, and we will focus on a more simpler coordinate system, and will be your first example of a *transformation of coordinates*:

Polar Coordinate System:



In an (x, y)-cartesian coordinate system:

- x is the distance from the y-axis
- y is the distance from the x-axis

In an (r, θ)-**polar coordinate** system

- r is the **distance** from the **origin (i.e.** $(0, 0)$**)** (however, it does make sense to talk about negative r values) and is given by $r^2 = x^2 + y^2$
- θ is the **angle** the point makes with the x-axis **measured counter-clockwise** and is given by $\tan\theta = \dfrac{y}{x}$

Some additional things: A "standard polar coordinate" requires $r \geq 0$ and $0 \leq \theta < 2\pi$. However, we will often see that it is convenient to allow for negative r and θ to be any angle.

Converting Between Systems:

To convert from cartesian (x, y) to polar (r, θ) we can use:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

To convert from polar (r, θ) to cartesian (x, y) we can use:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Example 1. Plot the *polar coordinates*:

(a) $\left(1, \frac{5\pi}{4}\right)$

(b) $\left(2, -\frac{2\pi}{3}\right)$

(c) $\left(-3, \frac{3\pi}{4}\right)$

Example 2. Convert the cartesian coordinate $(1, -1)$ to polar.

1 Graphing

Example 1.

1. Graph the polar equation $r = 2$

2. Graph the polar equation $\theta = 1$

Example 2. Graph the polar equation $r = 2\cos\theta$. Can this be written as a cartesian equation?

Example 3. Graph the $r = \sin(6\theta) + 2$ first on an (r, θ)-cartesian coordinate system then on an (r, θ)-polar coordinate system.