

10.1: Parametric Curves

Up until now we have only talked about the situation in which 1 variable is described as a function of another variable $y = f(x)$, $x = g(y)$, $r(\theta)$, etc. However, in a lot of cases, given 2 variables, its quite unlikely we will be able to do this. The simplest example is $x^2 + y^2 = 1$, the unit circle. However, there is a way to describe the equation of a circle using only *1 variable*. This idea gives us parametric equations and paves the way for calc 3 / vector calculus.

Parametric Equations:

The variables x and y are given by a **parametric system of equations** if there is a 3rd variable t such that

$$x = f(t) \quad \text{and} \quad y = g(t).$$

This equivalent to saying:

- A **parametric description of x and y**
- A **parametrization of x and y by t**
- **x and y are given parametrically by t**

In some sense, a parametric description of the variables x and y gives a sense of following the “*path over time*” that x and y sketch out.

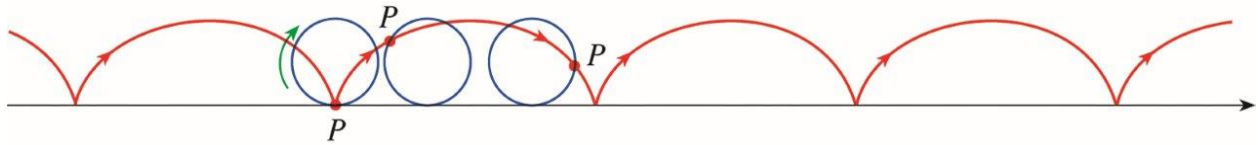
Example 1. What is the function that the parametric equations $x = t$ and $y = t$ describe?

Example 2. What is the graph of the parametric curve $x = t^2 - 1$ and $y = t - 1$? Can you rewrite this system as 1 equation with only 2 variables instead of 2 equations with 3 variables?

Example 3.

- (a) What is the shape that the parametric equations $x = \cos(t)$ and $y = \sin(t)$ describe on the interval $0 \leq t \leq 2\pi$?

- (b) What do the parametric equations $x = \cos(2t)$ and $y = \sin(2t)$ describe on the interval $0 \leq t \leq 2\pi$?



Example 4. Consider a pebble P stuck in a tire. If we assume the pebble starts at the bottom of the tire, how do we model the motion of the pebble if the tire moves at unit speed?