

## 10.1: Parametric Curves

Up until now we have only talked about the situation in which 1 variable is described as a function of another variable  $y = f(x)$ ,  $x = g(y)$ ,  $r(\theta)$ , etc. However, in a lot of cases, given 2 variables, it's quite unlikely we will be able to do this. The simplest example is  $x^2 + y^2 = 1$ , the unit circle. However, there is a way to describe the equation of a circle using only 1 variable. This idea gives us parametric equations and paves the way for calc 3 / vector calculus.

### Parametric Equations:

The variables  $x$  and  $y$  are given by a **parametric system of equations** if there is a 3rd variable  $t$  such that

$$x = f(t) \quad \text{and} \quad y = g(t).$$

This equivalent to saying:

- A **parametric description of  $x$  and  $y$**
- A **parametrization of  $x$  and  $y$  by  $t$**
- **$x$  and  $y$  are given parametrically by  $t$**

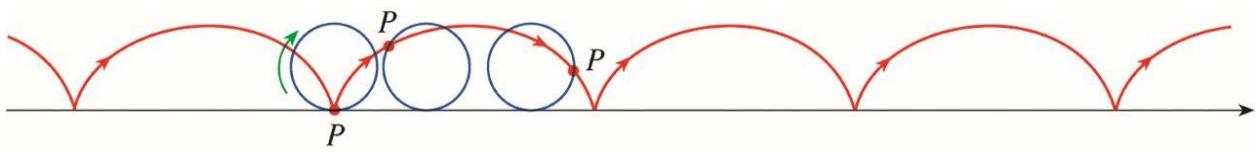
In some sense, a parametric description of the variables  $x$  and  $y$  gives a sense of following the “*path over time*” that  $x$  and  $y$  sketch out.

**Example 1.** What is the function that the parametric equations  $x = t$  and  $y = t$  describe?

**Example 2.** What is the graph of the parametric curve  $x = t^2 - 1$  and  $y = t - 1$ ? Can you rewrite this system as 1 equation with only 2 variables instead of 2 equations with 3 variables?

**Example 3.**

- (a) What is the shape that the parametric equations  $x = \cos(t)$  and  $y = \sin(t)$  describe on the interval  $0 \leq t \leq 2\pi$ ?
- (b) What do the parametric equations  $x = \cos(2t)$  and  $y = \sin(2t)$  describe on the interval  $0 \leq t \leq 2\pi$ ?



**Example 4.** Consider a pebble  $P$  stuck in a tire. If we assume the pebble starts at the bottom of the tire, how do we model the motion of the pebble if the tire moves at unit speed?