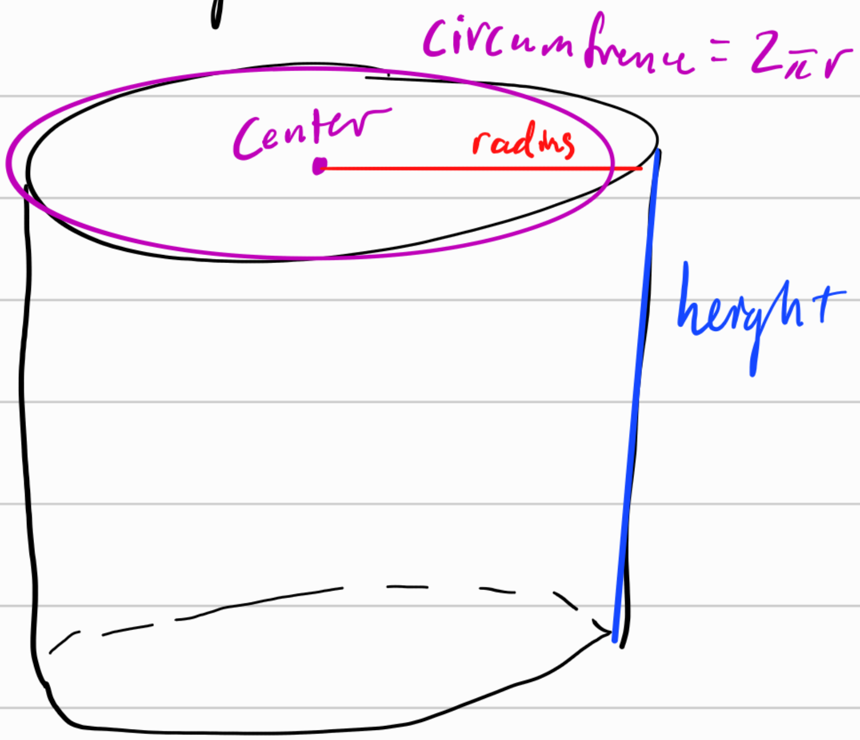
§6.3 Shell Method

Before we talk about the washer method where we form washers as our "simple area" to form a volume.

We'll now form our "simple area" as the surface area of a Cylindrical Shell

Consider a cylinder

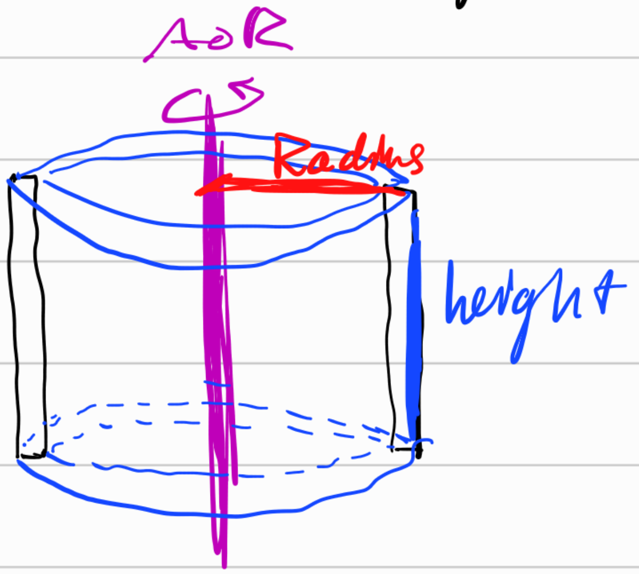


The surface area of the cylinder is given by

$$2\pi \, \text{radius} \cdot \text{height}$$

Note to talk about the surface area of the cylinder we only needed its outside Shell

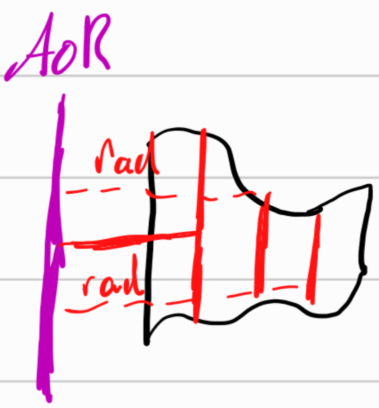
Consider a region rotated as such



This looks like a cylinder (w/ a hole)

The surface area of the cylindrical Shell is 2π Radius $\cdot$ Height

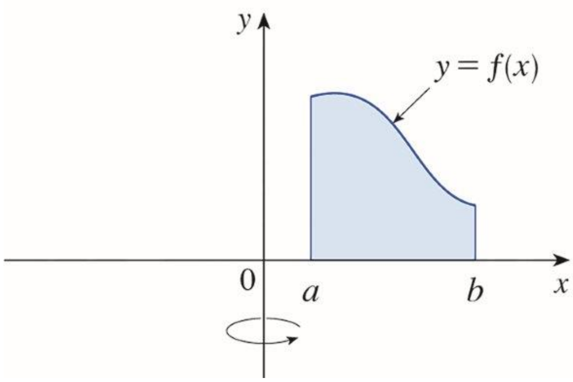
Well now we can take this the extreme.

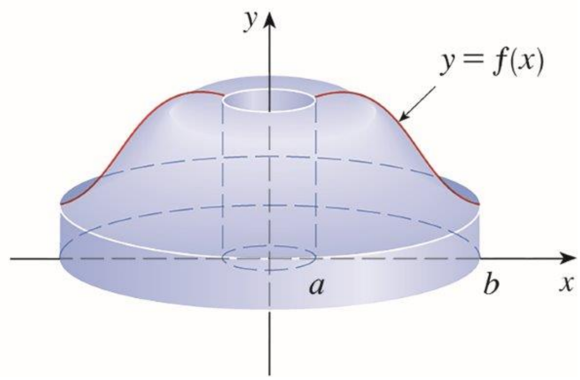


We can approximate the volume generated using cylindrical shells (Note the radius depends on our "current location" in the integral)

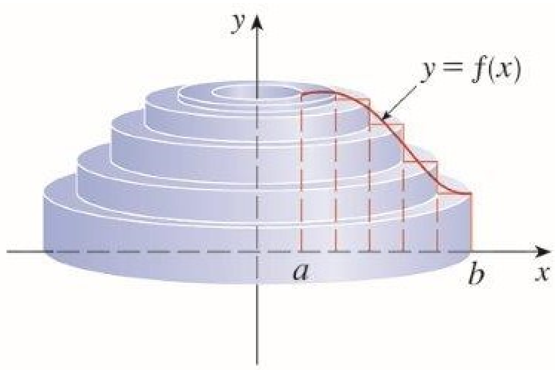
6.3: Shell Method

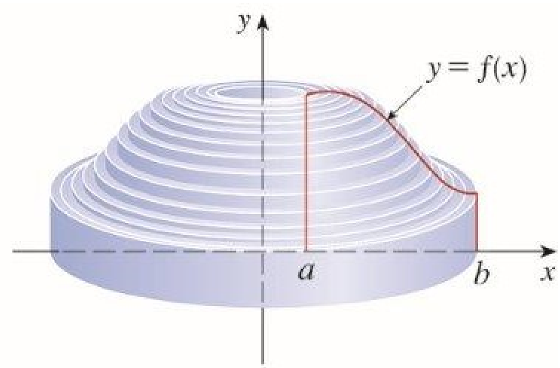
Consider the following region bounded by $y = f(x)$, the x-axis, $x = a$, and $x = b$. We will then rotate the region around the y-axis.



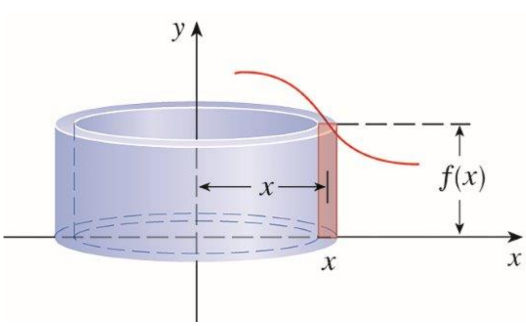


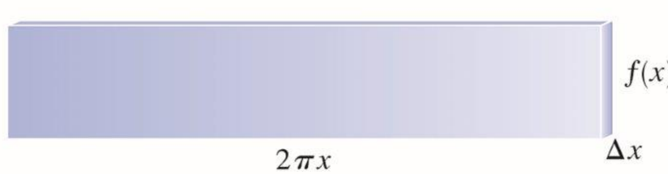
Using the Washer method for this region would be a pain. Instead we can use instead cylindrical shells to find the volume.





Since each volume chunk is given by a cylinder we can find the "length" of each chunk using the circumference of the associated circle (2π·radius) and the height of each chunk. For this specific example, the radius is x (the distance to the y-axis) and the height is the value of the top function $y = f(x)$ minus the bottom function $y = 0$.



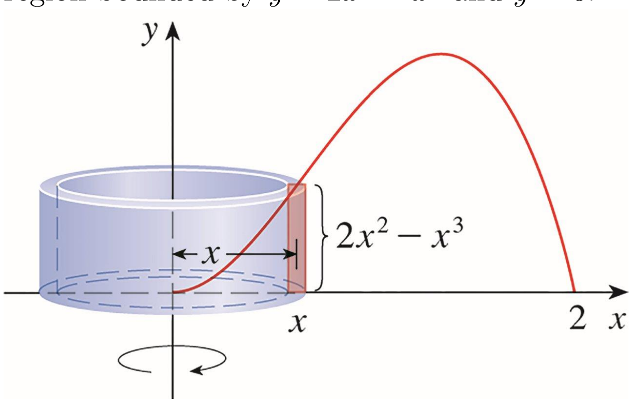


In general the shell method is given by (either dx or dy)

$$\int_a^b 2\pi\, r(x)\, h(x)\, dx$$

- $r(x)$ is the radius of the shell (i.e. "current" distance to axis of rotation)
- $h(x)$ is the height of the shell (i.e. top-bottom or right-left)

Example 1. Find the volume of the solid obtain by rotating about the y-axis with the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

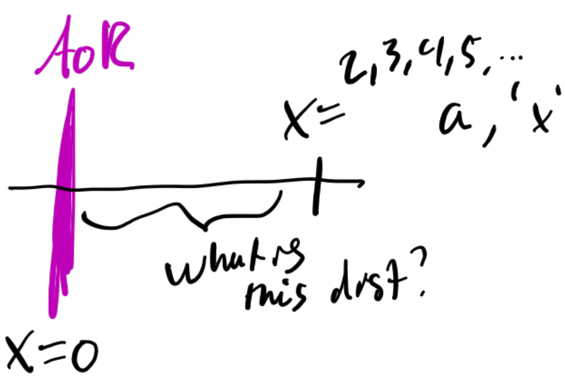


For shell method
- we want parrellel slices to AoR so vert. slices.
- ie. if AoR is $X = \#$ line then we do same vert. slice (dx).

$$2\pi \int_{\text{start}}^{\text{end}} \text{radius} \cdot \text{height}\; d\left(x \text{ or } y\right)$$

$$\text{height} = \underbrace{\text{top of region} - \text{bot of region}}_{\text{for } dx} = \underbrace{2x^2 - x^3}_{\text{top}} - \underbrace{0}_{\text{bot}}$$

radius = "current" dist. to axis of rotation



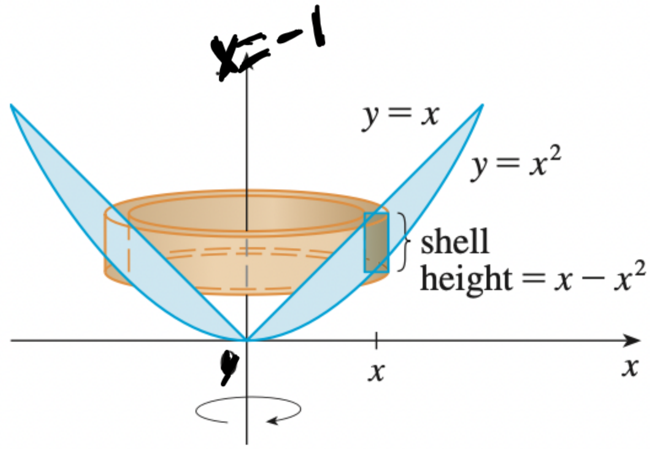
$\hookrightarrow = x - 0$ ← location of AoR

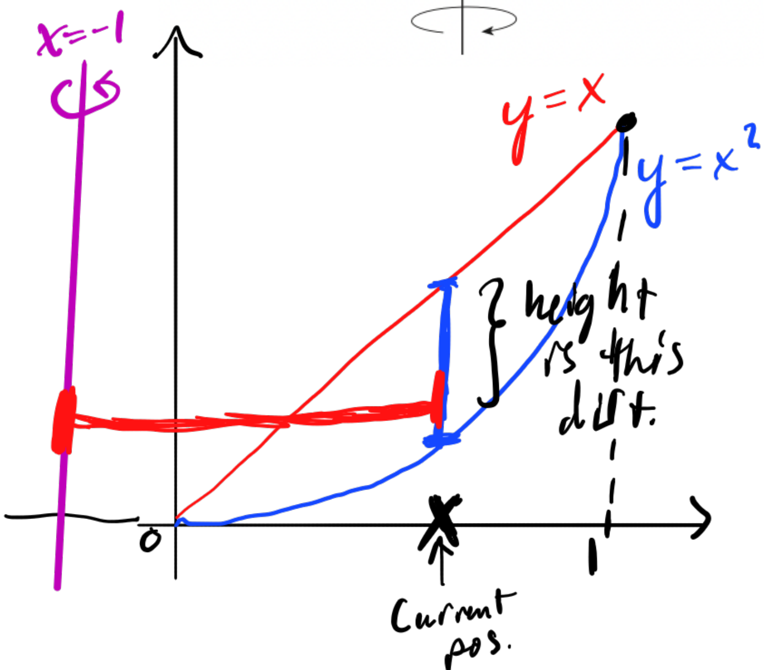
"current pos." in integral.

So

$$2\pi \int_0^2 x\left(2x^2 - x^3\right) dx$$

Example 2: Find the volume of the solid obtained by rotating about ~~the y-axis~~ $x=-1$ with the region between $y = x$ and $y = x^2$.





$$2\pi \int_0^1 \text{rad}\ \text{height}\ dx$$

$\text{height} = \underbrace{\text{top} - \text{bot}}_{dx}$

$= x - x^2$

rad = current dist. to AoR

In the integral our "current position" is always x (dx)

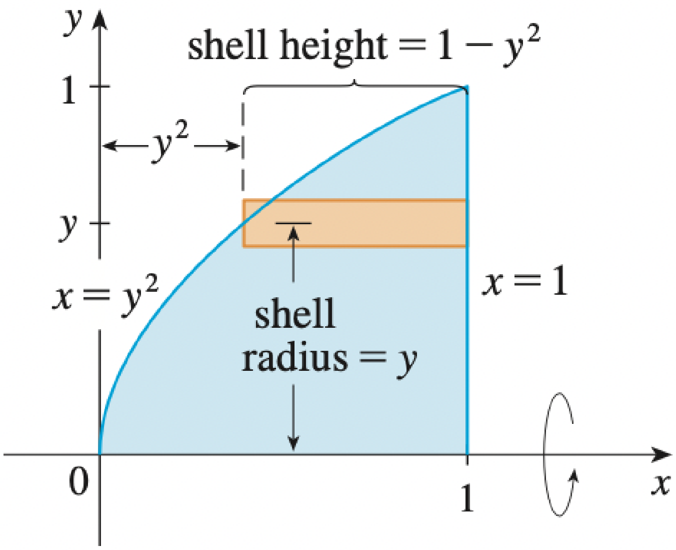
How far are we from the AoR?

$\text{current dist. to AoR} = \underbrace{\text{right} - \text{left}}_{dx} = x - (-1)$

current pos. (x), AoR (-1)

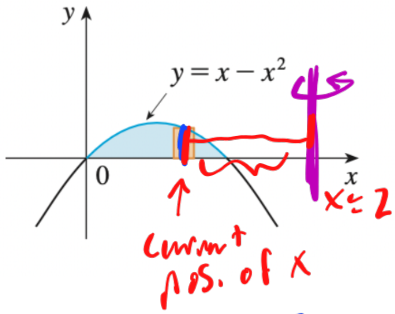
So: $$2\pi \int_0^1 \underbrace{(x+1)}_{\text{rad}}\underbrace{(x-x^2)}_{\text{height}}\, dx$$

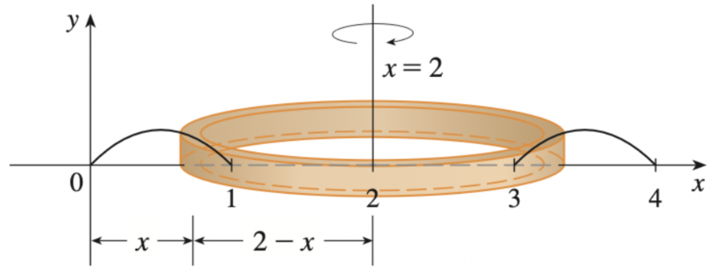
Example 3: Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



$$2\pi \int_0^1 y\,(1-y^2)\,dy$$

Example 4: Set up the volume V of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.





height $= x - x^2 - 0$

$$\Rightarrow 2\pi \int_0^1 (2-x)(x-x^2)\,dx$$

rad: $2 - x$

$x - x^2 = 0$ @ $x = 1$ & $x = 0$