7.1: Integration by Parts (IBP)

The product rule lets us take the derivative of a product of two functions. Now we will use it backwards to compute integrals. To see where integration by parts comes from, start with the chain rule:

$$\frac{d}{dx}\big[f(x)g(x)\big] = f'(x)g(x) + f(x)g'(x)$$

$$f(x)g(x) = \int \frac{d}{dx}\big[f(x)g(x)\big]\,dx = \int \big[f'(x)g(x) + f(x)g'(x)\big]\,dx$$

$$f(x)g(x) = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

If we move one of the integrals to the other side we get integration by parts:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx.$$

If we relabel $f(x)$ as u and $g(x)$ as v (that is, set $u = f(x)$ and $v = g(x)$), then by the chain rule, $\dfrac{du}{dx} = f'(x)$ and $\dfrac{dv}{dx} = g'(x)$ (or more familiarly: "$du = f'(x)\,dx$" and "$dv = g'(x)\,dx$") and we get Integration-By-Parts (sometimes shorten to IBP):

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx.$$

$$\text{aka} \qquad \int u\,dv = uv - \int v\,du$$

or: "ultra violet - Super (voo du)"

When integrating by parts you will want to choose u so that du makes the math easier to work with. There are many ways to remember which functions are best to choose for u. I recommend using the acronym LIATE.

LIATE which stands for "Let's Integrate A Terrible Equation" it's short hand for

- L: Logarithms (i.e. $\ln x$, $\log_b(x)$, etc.)
- I: Inverse Trig (i.e. $\arctan x$, $\arcsin x$, etc.)
- A: Algebraic (i.e. x, x^a, dx, etc.)

- T: Trig (i.e. $\sin x$, $\cos x$, etc.)
- E: Exponentials (i.e. e^x, 2^x, a^x, etc.)

Things higher on the list better candidates for u. Other acronyms that you might've seen or will see are: LIPET, ILATE, ILPTE, etc. Across all of these acronyms, the best candidates for u are Logarithmic/Inverse Trig, the middle candidate is Algebraic or Polynonial, and the worst candidates are Trig/Exponentials.

Example 1. Find $\int x \sin x \, dx$

Example 2. Find $\int \ln x \, dx$

Example 3. Find $\int e^x \sin x \, dx$

Example 4. Find $\int t^2 e^t \, dt$