

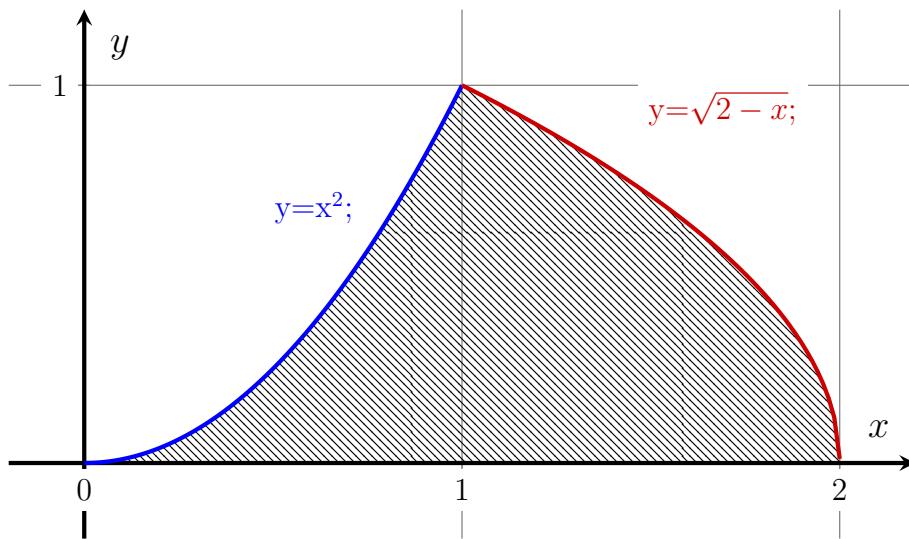
Midterm 1 Questions

**Show all work clearly. Submit to Gradescope.
No calculator usage or software / program assistance.**

THERE ARE 3 PAGES IN TOTAL:

- 2 pages of problems (6* Problems in total // *problem 1 has 6 parts)
- 1 page of given information

Problem 1. Consider the region R bounded by $y = x^2$ and $y = \sqrt{2-x}$ in the first quadrant. An accurate graph is provided below:
(ignore the semi-colons the graph doesn't render w/o them for some reason)



The following parts are all based on this region R .

- (a : 4 points) Rewrite the region's edges given by $y = x^2$ and $y = \sqrt{2-x}$ as functions as y .
- (b : 5 points) Calculate the area of the region R .
- (c : 10 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line $y = 1$ using the **Shell method**.
- (d : 10 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line $x = 1$ using the **Washer Method**.
- (e : 12 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line $x = -1$.
- (f : 12 points) Set up BUT DO NOT EVALUATE an integral(s) for the volume generated by rotating the region R about the line x -axis.

Problem 2. (5 points) Set up BUT DO NOT SOLVE the Partial Fraction Decomposition for $\frac{3x^2 + 4x - 5}{x^2(x - 1)(2x^2 + 3)^2}$

Problem 3. (10 points) Evaluate $\int \frac{10}{(x - 1)(x^2 + 9)} dx$

Problem 4. (10 points) Evaluate $\int_1^\infty \frac{e^{-1/x}}{x^2} dx$

Problem 5. (10 points) Evaluate the integral $\int \sec(x) \tan(x) \ln(\sec(x)) dx$.

Problem 6. (12 points) Set up BUT DO NOT EVALUATE the integral that represents the arc length of the function $y = x^2$ on the interval $[1, \infty)$.

IF you had to evaluate this integral, what integral techniques or special considerations (if any) would you have to do/deal with? If you are not sure what to write, do the first few steps associated with solving the resulting arc length integral.

List of Given Information

1. Pythagorean Identities

- (a) $\sin^2(x) + \cos^2(x) = 1$
- (b) $\tan^2(x) + 1 = \sec^2(x)$
- (c) $1 + \cot^2(x) = \csc^2(x)$

2. Double Angle Identities

- (a) $\sin(2x) = 2 \sin x \cos x$
- (b) $\cos(2x) = \cos^2(x) - \sin^2(x)$

3. Half-Angle Identities

- (a) $\sin^2(x) = \frac{1 - \cos(2x)}{2}$
- (b) $\cos^2(x) = \frac{1 + \cos(2x)}{2}$

4. Trig-Integrals

- (a) $\int \sec x \, dx = \ln |\sec x + \tan x| + C$
- (b) $\int \tan x \, dx = \ln |\sec x| + C$
- (c) $\int \csc x \, dx = \ln |\csc x - \cot x| + C$
- (d) $\int \cot x \, dx = \ln |\csc x| + C$