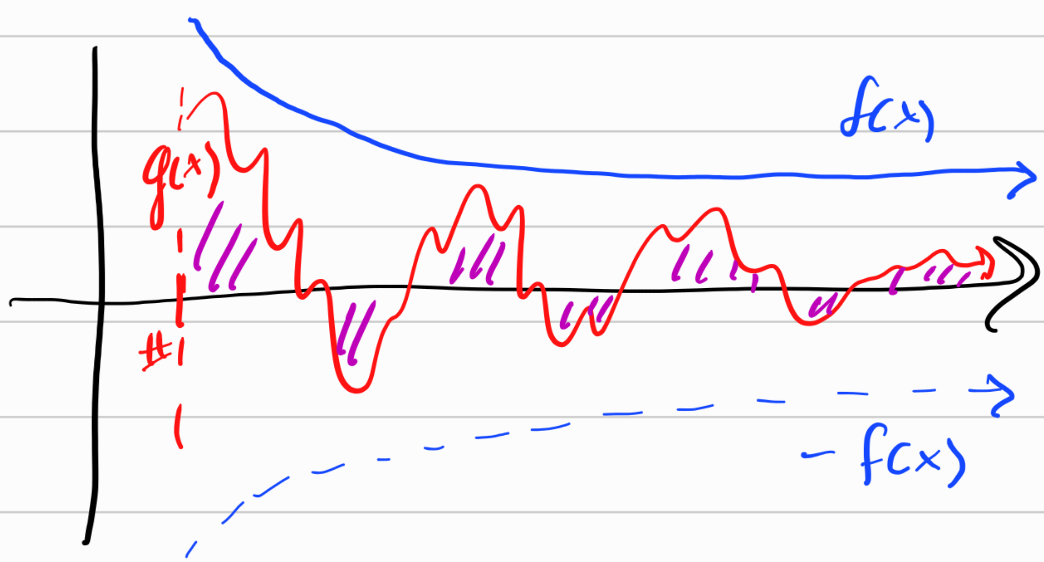
Finishing §7.8: Comparison Theorem

$\underbrace{\S 7.8}$ → Improper integrals

i.e. $\int_{\#}^{\infty} f(x)\,dx$ or $\int_{\#}^{\#} \underbrace{(\text{prob pt. in here})}_{}dx$

$\frac{1}{x}$ @ $x=0$

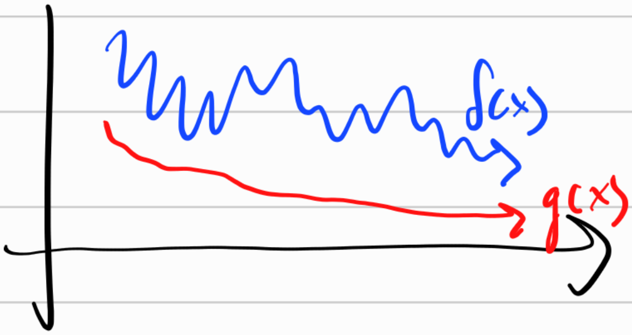
The thing of concern is whether the area under the curve is finite



$\int_{\#}^{\infty} g(x)\,dx$

If we have $0 \leq |g(x)| \leq f(x)$

If $\underbrace{\int_1^{\infty} f(x)\,dx}_{\text{conv.}}$ so does $\underbrace{\int_1^{\infty} |g(x)|\,dx}_{\text{conv.}}$



$0 \le g(x) \le f(x)$

$\int_1^\infty g(x)\,dx$ dN.

Then so does $\int_1^\infty f(x)\,dx$

Sometimes, evaluating an integral is hard, but if all we care about is a question about convergence, then we have a handy theorem that takes care of things for us:

The Comparison Theorem:

Suppose that f and g are continuous functions where $\boxed{f(x) \geq g(x) \geq 0}$ for all $x \geq a$ where a is some constant number.

- **IF** $\displaystyle\int_a^\infty f(x)\,dx$ is **convergent**, **then** this implies $\displaystyle\int_a^\infty g(x)\,dx$ is **convergent**.
- **IF** $\displaystyle\int_a^\infty g(x)\,dx$ is **divergent**, **then** this implies $\displaystyle\int_a^\infty f(x)\,dx$ is **divergent**.

This theorem can be thought of as:

- If the **BIGGER thing** converges then so does the smaller one.
- If the **SMALLER thing** diverges then so does the bigger one.

Example 7. Show that $\displaystyle\int_1^\infty \frac{1+e^{-x}}{x}\,dx$ is divergent.

Note $e^{-x} > 0$ for all x

$1 + e^{-x} > 1$

$\dfrac{1+e^{-x}}{x} > \dfrac{1}{x} \quad (x>0)$

$$\underbrace{\int_1^\infty \frac{1+e^{-x}}{x}\,dx}_{\text{this div. by comp. w/ } \int_1^\infty \frac{1}{x}dx.} \geq \underbrace{\int_1^\infty \frac{1}{x}\,dx}_{\text{div. by p-test}}$$