Math 242-013 Winter 2026

Review (on a lot of things students get wrong)

Algebra:

$$x^{n} = \underbrace{x \cdot x \cdot \cdots \cdot x}_{\text{multiply to itself } n \text{ times}}$$

$$x^a x^b = x^{a+b}$$

$$x^{-1} = \frac{1}{x}$$

$$\rightarrow \frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$$

$$(x+y)^2 = (x+y)(x+y) = x^2 + xy + yx + y^2$$
$$= x^2 + 2xy + y^2$$

$$\frac{a}{b} + \frac{c}{d} \cdot 1 = \frac{d}{d} \cdot \frac{a}{b} + \frac{c}{d} \cdot \frac{b}{b} = \frac{da}{db} + \frac{cb}{db} = \frac{da + cb}{db}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{\frac{a}{b}\left(\frac{d}{c}\right)}{\cancel{\frac{c}{d}\left(\frac{d}{c}\right)}} = \frac{a}{b}\cdot\frac{d}{c}$$

Pre calc

Functions have an input to output relationship (Vertical line test / VLT)

$f(x) \rightarrow$ output of $f(x)$-values

↳ input of x-values

"let $y = f(x)$"

↳ we are relabelling $f(x)$ outputs as "y"

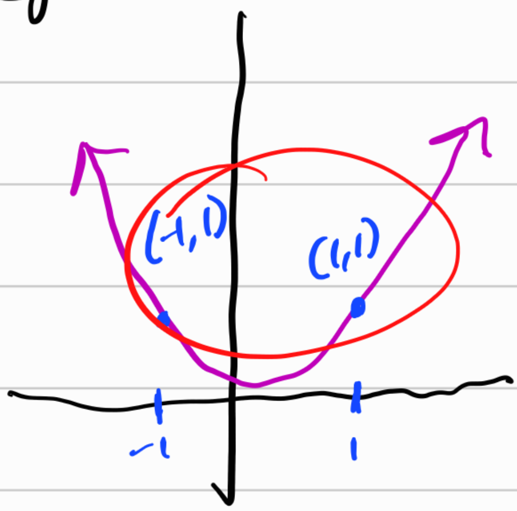
Inverses:

An inverse of a function "undo" the operation

$$f : \underset{x}{\text{inputs}} \rightarrow \underset{y}{\text{outputs}}$$

The inverse $f^{-1} : y \rightarrow x$

eg $f(x)=x^2$



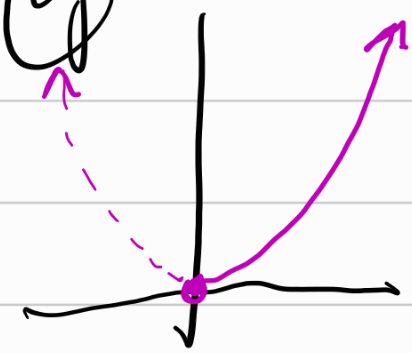
what about $f^{-1}(x)$

$f^{-1}(1)$?

↳ no clear output for the inverse.

A lot of functions they inverses unless we restrict the domain.

eg



$f(x)=x^2$ on $[0,\infty)$

then $f^{-1}(x)=\sqrt{x}$ on $[0,\infty)$

Some really important functions

Algebraic functions:

- polynomials

x^2+2x+1

x^3+32x^{65}

0

x

- roots $\sqrt{x}$, $x^{1/3}$, $x^{3/4}$
- [name pending] $\frac{1}{x}$, $x^{-2/3}$, $\frac{1}{x^{2/3}}$

exponentials / logs : e^x, $\ln x$

Note e^x & $\ln x$ are inverses of each other

$e^x : (-\infty, \infty) \longrightarrow (0, \infty)$
$\ln x : (0, \infty) \longrightarrow (-\infty, \infty)$

$\underbrace{x = e^{\ln x}}$ & $x = \ln(e^x)$

very useful for limit problems

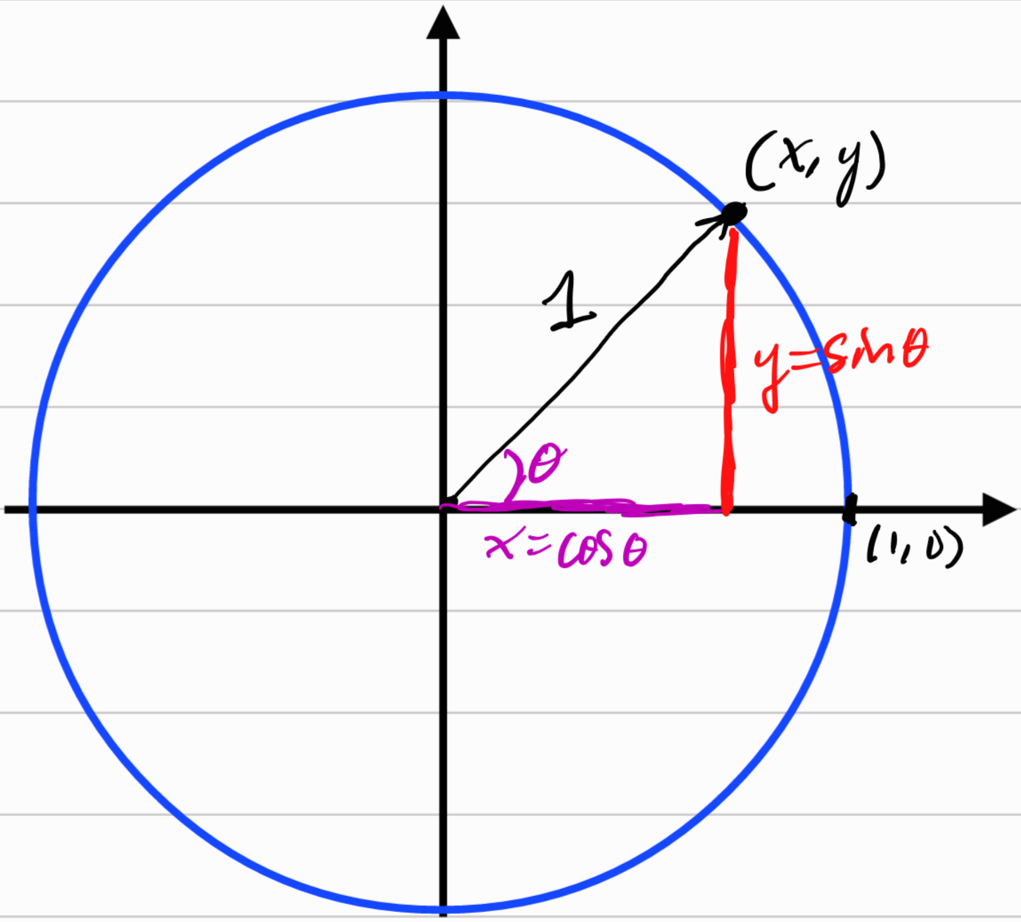
Important log properties

$$\log_b(x^a) = a \log_b(x)$$

$$\log(ab) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

Trig

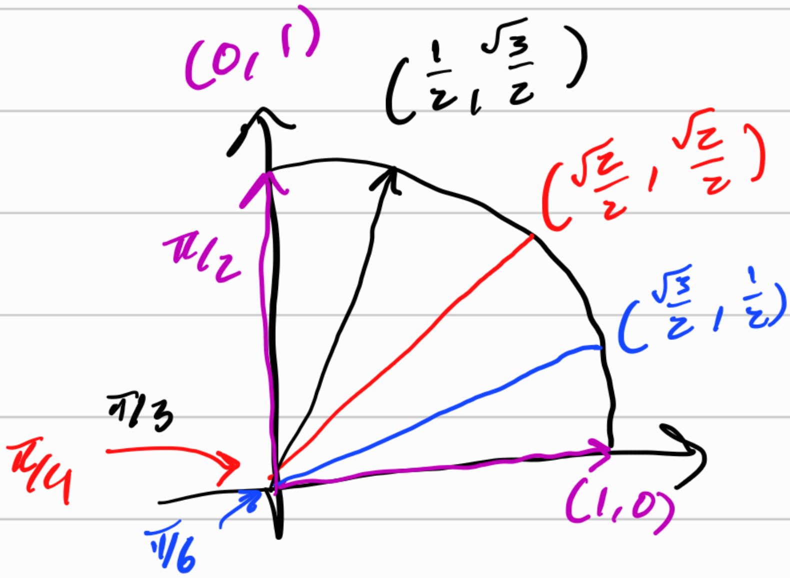


$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta},$$

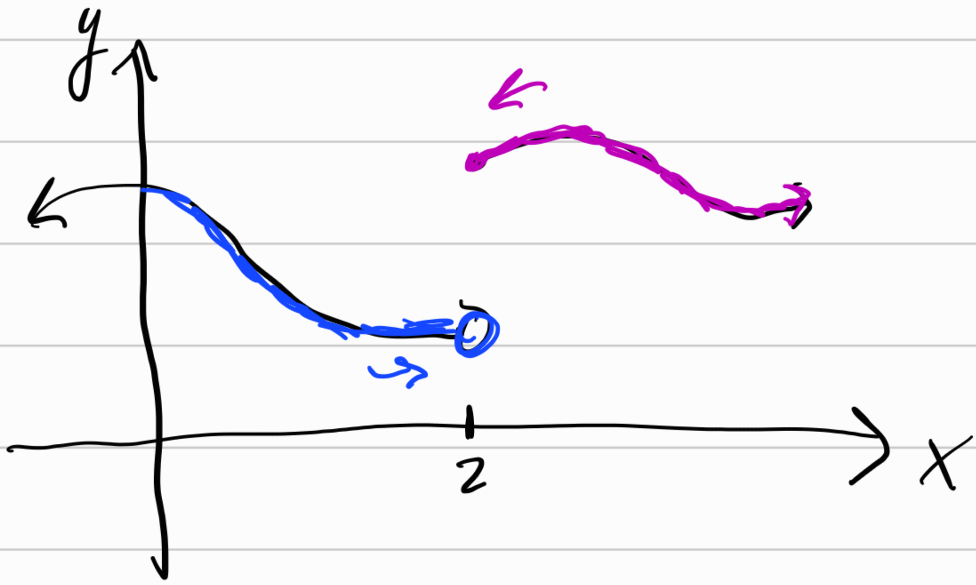
$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$



Calc 1

Limits → A limit is the "expected behaviour of a function"



Continuity → The function behaves exactly how we expect
↳ cont.

The Mathematical definition of cont. is:

$$\lim_{x \to a} f(x) = f(a)$$

↳ Another intepretation the the def.

A function is cont. if you can "push the limit inside"

↳ $\lim_{x \to a} f(x) = f\left(\lim_{x \to a} x\right) = f(a)$

Derivatives

↳ "power rule" $\frac{d}{dx}[x^a] = a x^{a-1}$

↳ chain rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

↳ product rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

↳ quotient rule

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f' \cdot g - g' \cdot f}{(g)^2}$$

$$\frac{d}{dx}\left[\frac{H}{L}\right] = \frac{L \cdot DH - H \cdot DL}{L^2}$$

You need to know how to take a derivative of ALL functions listed in the Precalc.

Whatever tricks work for tangent & secant should also work for cotangent & cosecant

L'Hôpital:

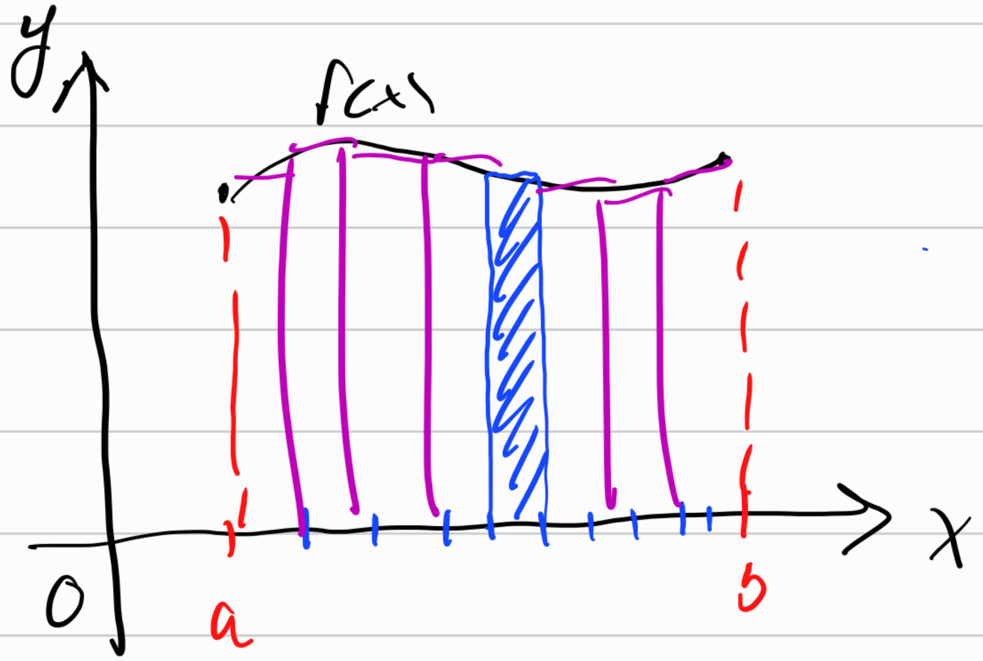
IF $\lim\limits_{\substack{x \to +\infty \\ \text{or} \\ a}} \frac{f(x)}{g(x)} = $ "$\frac{\infty}{\infty}$" or "$\frac{0}{0}$"

Then $\lim\limits_{x \to ?} \frac{f'(x)}{g'(x)} = \lim\limits_{x \to ?} \frac{f(x)}{g(x)}$

Useful $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $\boxed{\infty^0, 0^0, 1^\infty}$, etc

↳ by using $x = e^{\ln x}$

Integration (Very Important)



VERY IMPORTANT IDEA

The integral is formed by approximating the "area under curve" w/ a simple "structure" (in this case rectangles)

$$\underbrace{\int_a^b f(x)\,dx}_{\substack{\text{Area under the curve} \\ \text{(Complicated Structure)}}} = \underbrace{\lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} \underbrace{f(x_i)\Delta x}_{\text{Simple Structures}}}_{\text{approximation}}}_{\text{A refinement of the approx. until exact.}}$$

Fundemental Theorem of Calculus (FTC)

(i) $\int_a^b f'(x)\,dx = f(b) - f(a)$

(ii) $F(x) = \int_a^x f(t)\,dt$

You need to know how to integrate

Every algebraic function: $x^n + 3x^2$, $x^{1/2}$, x^{-3}, x^{-1}

Exponentials, sine & cosine, &

$\int \sec^2 x\,dx = \tan x + C$

$$\int \sec x \tan x \, dx = \sec x + C$$

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u-sub: Undos the chain rule

$$\int \frac{d}{dx}\left[f(g(x))\right] dx = \int f'(g(x)) \cdot g'(x) \, dx$$

$$f(g(x)) + C = \int f'(g(x)) \cdot g'(x) \, dx$$

$u = g(x) \rightarrow \frac{d}{dx} u = g'(x) \rightarrow du = g'(x)\, dx$

$$f(u) + C = \int f'(u) \, du$$

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§ 7.1, IBP (Theory)

↳ "Undoing a product rule"

↳ let's us push a derivative onto a "simpiler function"

$$\frac{d}{dx}\left[f(x) g(x)\right] = f'(x) \cdot g(x) + f(x) g'(x)$$

$$\int \frac{d}{dx}\left[f(x)g(x)\right]dx = \int f'(x)\cdot g(x) + f(x)g'(x)\,dx$$

$$f(x)g(x) \underbrace{+C}_{\text{drop this for notation sake}} = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$f(x)g(x) - \int f'(x)g(x)\,dx = \int f(x)g'(x)\,dx$$

If label $u = f(x)$ $\quad v = g(x)$

$$\int u\,dv = uv - \int v\,du$$

"ultra violet - super voo·du"

7.1: Integration by Parts (IBP)

The chain rule let us ask "*is this integral a chain rule*?" and now we will learn Integration by Parts which will let us ask "*is this integral a **product rule***?" To see how it comes up, let's look at the product rule and try to integrate it:

$$\frac{d}{dx}\Big[f(x)g(x)\Big] = f'(x)g(x) + f(x)g'(x)$$
$$f(x)g(x) = \int \frac{d}{dx}\Big[f(x)g(x)\Big]dx = \int \Big[f'(x)g(x) + f(x)g'(x)\Big]\,dx$$
$$f(x)g(x) = dx = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

If we move one of the integrals to the other side we get integration by parts:

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

If we relabel $f(x)$ as u and $g(x)$ as v (that is, set $u = f(x)$ and $v = g(x)$), then by the chain rule, $\frac{du}{dx} = f'(x)$ and $\frac{dv}{dx} = g'(x)$ (or more familiarly: "$du = f'(x)\,dx$" and "$dv = g'(x)\,dx$") and we get Integration-By-Parts (sometimes shorten to IBP):

$$\int {\color{red}f(x)}{\color{blue}g'(x)}dx = {\color{red}f(x)}{\color{blue}g(x)} - \int {\color{blue}g(x)}{\color{red}f'(x)}dx.$$

aka

$$\int {\color{red}u}{\color{blue}dv} = {\color{red}u}{\color{blue}v} - \int {\color{blue}v}{\color{red}du}$$

or: "ultra*violet - SUPER (voo*du)"

When integrating by parts you will want to choose dv so that you can actually integrate the function. This means, your priority will be setting dv first in many cases. A there are many ways to try and remember the order, my suggestion is:

LIATE which stands for "**L**et's **I**ntegrate **A** **T**errible **E**quation" it's short hand for

- **L**: Logarithms (i.e. $\ln x$, $\log_b(x)$, etc.)
- **I**: Inverse Trig (i.e. $\arctan x$, $\arcsin x$, etc.)
- **A**: Algebraic (i.e. x, x^a, dx, etc.)
- **T**: Trig (i.e. $\sin x$, $\cos x$, etc.)
- **E**: Exponentials (i.e. e^x, 2^x, a^x, etc.)

Things higher on the list are harder to integrate so you most likely set that as u and set dv to be whatever appear below it. Other acronyms that you might've seen or will see are: LIPET, ILATE, ILPTE, etc. When it comes to inverse trig and logs, you can

§ 7.1 - IBP

Integrating is very hard we can't just do the integral

So we have a bunch of tricks & techniques

↳ u-sub.

eg $\int \sin x \, dx = -\cos x + C$ ✓

we can do this b/c we know the antiderivative

eg $\int \sin(7x) \, dx = -\frac{1}{7}\cos(7x) + C$

At this point this is already a u-sub integral. But in Calc 2 you don't need to do this u-sub.

In list of integral techniques to try

(i) "Do it"

(ii) u-sub → "undos a chain rule"

(iii) IBP → "undos a product rule" (in some sense)

↳

We have an integral of the form

$$\int (\text{function } 1) \cdot (\text{function } 2)\, dx$$

we make one function "u" & take derivative

we make one function "dv" & integrate

$$\int u\, dv = uv - \int v\, du$$

ultraviolet - Super voo du

A way to identify a u & dv is

Let's Integrate A Terrible Equation

make u

L → logs

I → inverse trig (arcsinx, arccosx, etc.)

A → algebraic (x^2+x+1, $x^{1/2}$, 1, $\frac{1}{x}$, ...etc.)

T → trig

E → exponentials

make dv

Example 1. Find $\int \overset{u}{x} \overset{dv}{\sin x \, dx} = uv - \int v \, du$

$u = x$ $\xrightarrow{\frac{d}{dx}}$ $du = dx$

$dv = \sin x \, dx$ $\xrightarrow{\text{integrate}}$ $v = -\cos x$

u ↑ L I A $= x = u$

T $= \sin x \, dx = dv$ E ↓ dv

$$\int x \sin x \, dx = x(-\cos x) - \int (-\cos x) \, dx$$
$$= -x\cos x + \int \cos x \, dx$$
$$= -x\cos x + \sin x + C$$

Example 2. Find $\int \ln x \, dx$

There's no "easy" antiderivative, so we can't just "do the integral"

There's nothing to u-sub here. → Try IBP.

u L → $\ln x = u$ I A → $dx = dv$ T E dv

$$\int \ln x \cdot 1 dx$$

$u = \ln x$ $\xrightarrow{\frac{d}{dx}}$ $du = \frac{1}{x} dx$

$dv = dx$ $\xrightarrow{\int}$ $v = x$

$$\int \ln x \, dx = \overset{u}{\ln x} \cdot \overset{v}{x} - \int \overset{v}{x} \cdot \overset{du}{\frac{1}{x} dx}$$

$$= x \ln x - \int dx$$

$$\int \ln x \, dx = x \ln x - x + C$$

"If dv isn't clear, but u is.

Then dv is probably going to be dx"

↳ You want to make sure in any case you can actually integrate dv

Example 3. Find $\int e^x \sin x \, dx$

L I A T E (u: $\sin x$ at T; dv: e^x at E)

$u = \sin x$ $\quad$ $dv = e^x dx$

$du = \cos x \, dx$ $\quad$ $v = e^x$

$$= e^x \sin x - \int e^x \cos x \, dx$$

new u L I A T E new dv ($\cos x$ at T; e^x at E)

$$\int e^x \cos x \, dx$$

new $u = \cos x$ $\quad$ new $dv = e^x dx$

new $du = -\sin x \, dx$ $\quad$ new $v = e^x$

$$\int e^x \cos x \, dx = e^x \cos x - \int e^x (-\sin x) \, dx$$

$$= e^x \cos x + \int e^x \sin x \, dx$$

$\boxed{\text{Back to OG Prob.}}$

$$\int e^x \sin x \, dx = e^x \sin x - \left[e^x \cos x + \int e^x \sin x \, dx \right]$$

$$(- - - - -) = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$+ \int e^x \sin x \, dx$ $\qquad$ $\underbrace{+ \int e^x \sin x \, dx}_{0}$

$$\int e^x \sin x \, dx + \int e^x \sin x \, dx = e^x \sin x - e^x \cos x + 0$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x = \frac{e^x}{2}\left[\sin x - \cos x\right] + C$$

We need IBP for $\int f(x)\cdot g(x)\,dx$ is b/c we

Can't Do $\int f\cdot g\,dx \to \int f\,dx \cdot \int g\,dx$

This is super wrong

So for this we need IBP.

Example 4. Find $\int_0^1 t^2 e^t \, dt$ $\quad u\ \text{L I } \overset{t^2}{\text{A}} \text{ T } \overset{e^t}{\text{E}}\ dv$

Definite integrals can be annoying.

↳ Do the indefinite integral 1st

$$\int t^2 e^t \, dt = t^2 e^t - 2\boxed{\int e^t \, t \, dt}$$

$u = t^2$
$du = 2t\,dt$
$dv = e^t\,dt$
$v = e^t$

$$\int e^t \cdot t \, dt = t e^t - \int e^t \, dt$$
$$= t e^t - e^t + C$$

$u = t$
$du = dt$
$dv = e^t\,dt$ ← Star of IBP
$v = e^t$

$$\int t^2 e^t \, dt = t^2 e^t - 2\left[t e^t - e^t + C\right]$$

arb. const.

$$= t^2 e^t - 2te^t + 2e^t + C$$

$$\int_0^1 t^2 e^t \, dt \overset{\text{FTC}}{=} \left[t^2 e^t - 2te^t + 2e^t\right]_0^1$$

$$= 1^2 e^1 - 2\cdot 1 \cdot e^1 + 2\cdot e^1$$
$$- (0 \cdot \text{idc} - 2\cdot 0 \cdot \text{idc} + 2e^0)$$

$$= e \underbrace{- 2e + 2e} - \underbrace{0 + 0} - 2$$

$$= e - 2$$