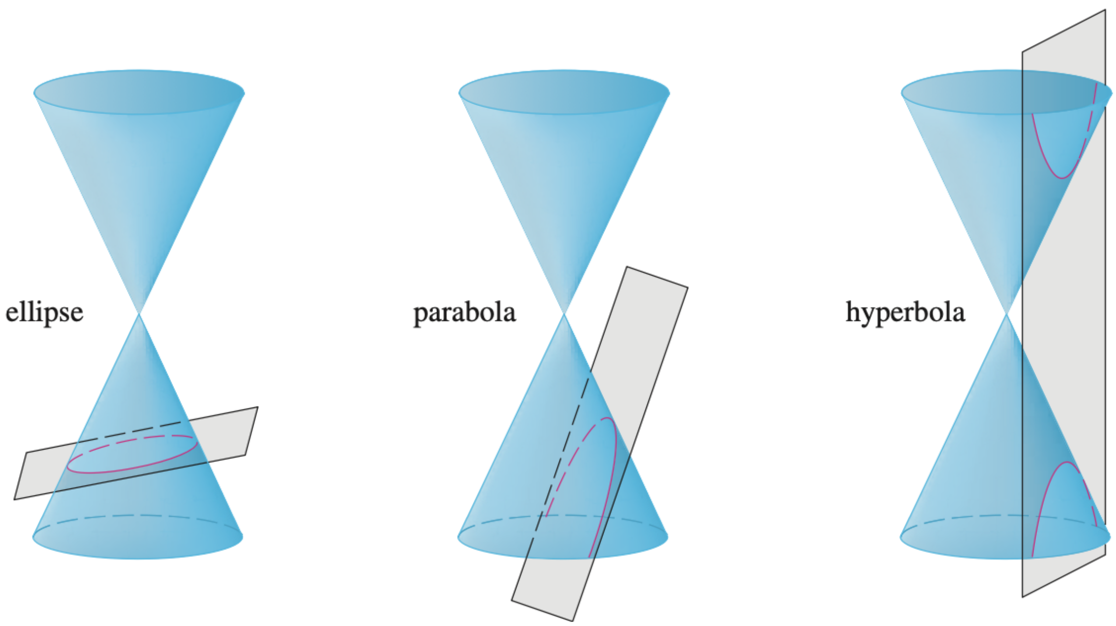
MATH 242 SPRING 2025
Chap 10 Parametric Equations and Polar Coordinates

UD ID: ______________________ Name: ______________________

10.5 Conic Sections

In this section, we give geometric definitions of parabolas, ellipses, and hyperbolas and derive their standard equations. They are called **conic sections**, or **conics** because they result from intersecting a cone with a plane as shown below:



1. Parabolas

A parabola is a set of points in a plane that are equidistant from a fixed point F (called the focus) and a fixed line (called the **directrix**). The point halfway between the focus and the directrix lies on the parabola is called the **vertex**. The line through the focus perpendicular to the directrix is called the axis of the parabola.

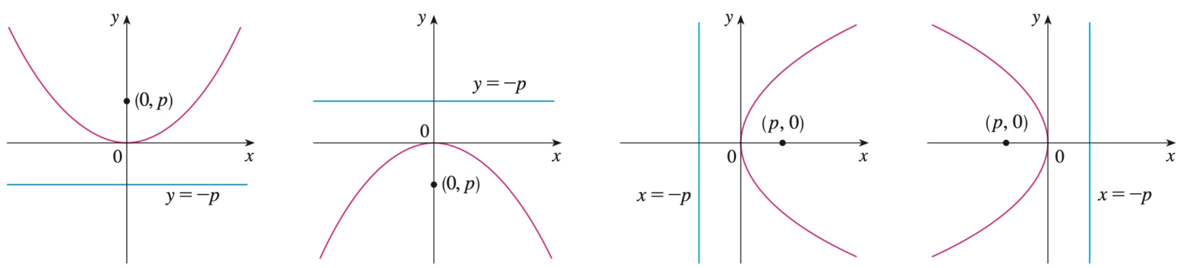
1 An equation of the parabola with focus $(0, p)$ and directrix $y = -p$ is

$$x^2 = 4py$$

If we interchange x and y in (1), we obtain

2

$$y^2 = 4px$$



(a) $x^2 = 4py,\ p > 0$ (b) $x^2 = 4py,\ p < 0$ (c) $y^2 = 4px,\ p > 0$ (d) $y^2 = 4px,\ p < 0$

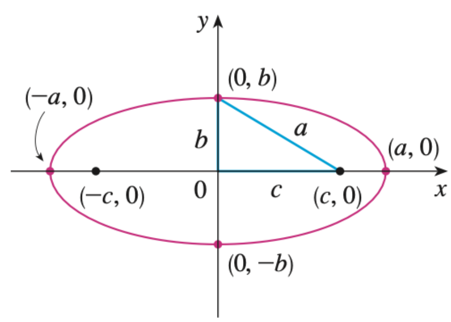
2. Ellipses

An ellipse is the set of points in a plane the **sum** of whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are called the foci (plural of focus).

4 The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a \geqslant b > 0$$

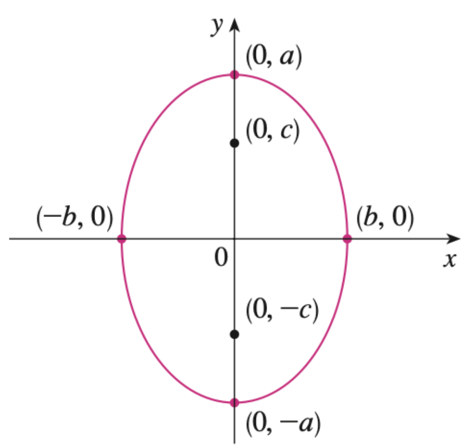
has foci $(\pm c, 0)$, where $c^2 = a^2 - b^2$, and vertices $(\pm a, 0)$.



5 The ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad a \geqslant b > 0$$

has foci $(0, \pm c)$, where $c^2 = a^2 - b^2$, and vertices $(0, \pm a)$.



3. Circles

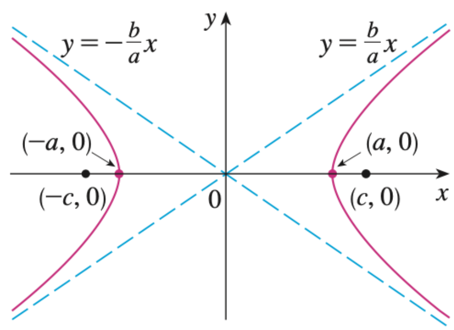
4. Hyperbolas

A hyperbola is the set of all points in a plane the **difference** of whose distances from two fixed points F_1 and F_2 (the foci) is a constant.

7 The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

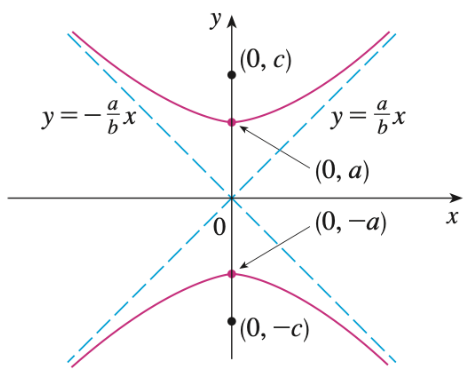
has foci $(\pm c, 0)$, where $c^2 = a^2 + b^2$, vertices $(\pm a, 0)$, and asymptotes $y = \pm(b/a)x$.



8 The hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

has foci $(0, \pm c)$, where $c^2 = a^2 + b^2$, vertices $(0, \pm a)$, and asymptotes $y = \pm(a/b)x$.



Shifted Conics

We shift conics by taking the standard equations 1, 2, 3, and 4 and replacing x and y by $x - h$ and $y - k$.

Example 1: Identify the type of conic section whose equation is given.

(a) $4x^2 = y^2 + 4$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these

(b) $x^2 = 4y - 2y^2$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these

(c) $4x^2 = y + 4$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these

(d) $x^2 - 2x + 2y^2 - 8y + 7 = 0$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these

(e) $x^7 - 2x + y^5 = 0$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these

(f) $2(x - 2)^2 - 2y = -2(y - 1)^2$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these

(g) $3y - \frac{1}{2}x^2 = 5$

○ parabola ○ ellipse ○ hyperbola ○ circle ○ none of these