DIS 2 | 11.a

$$\int_0^2 \frac{1}{x-1}\,dx$$

@ $x=1$
" $\frac{1}{0}$ "

Since we know about improper integrals we Have to check if there is an x-value that "Breaks the integral"

Since $x=1$ is between $[0,2]$ this integral is improper.

$\Rightarrow$ This "2 sub-problems"

(i) $\int_0^1 \frac{1}{x-1}\,dx$

(ii) $\int_1^2 \frac{1}{x-1}\,dx$

we have to check the conv. of both of these

(i) $\underbrace{\lim_{b\to 1^-} \int_0^b \frac{1}{x-1}\,dx}$

$$\int_0^b \frac{1}{x-1}\,dx = \Big[\ln|x-1|\Big]_0^b$$

$$= \ln|b-1| - \underbrace{\ln|-1|}_{=\ln(1) = 0}$$

$$= \ln|b-1|$$

$$\underbrace{\lim_{b\to 1^-}} \ln|\underbrace{b-1}_{\to 0}| \to \text{“}\ln(0) = -\infty\text{”}$$

So, b/c $\int_0^1 \frac{1}{x-1}\,dx = -\infty$ ← not a #

it diverges & therefore

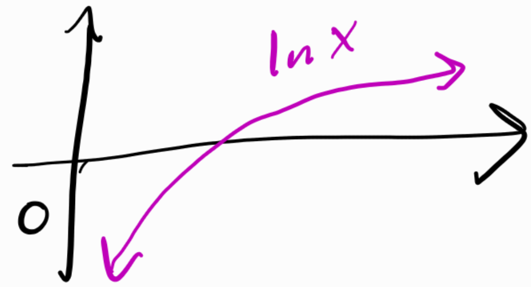
$\int_0^2 \frac{1}{x-1}\,dx$ Also diverges

(we don't have to check integral (ii))

Q: Is $\boxed{\ln(0) = -\infty}$

A: Careful what should write is

$$\lim_{x \to 0^+} \ln(x) = -\infty$$



DIS

Prob 1 (b)

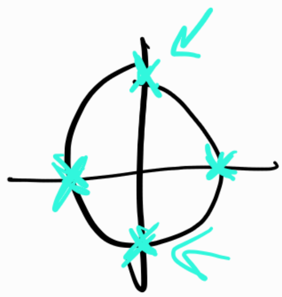
Determine the convergence of $\int_1^\infty \frac{|\sin x|}{x^2}\,dx$

The absolute values make this integral NOT fun to calculate

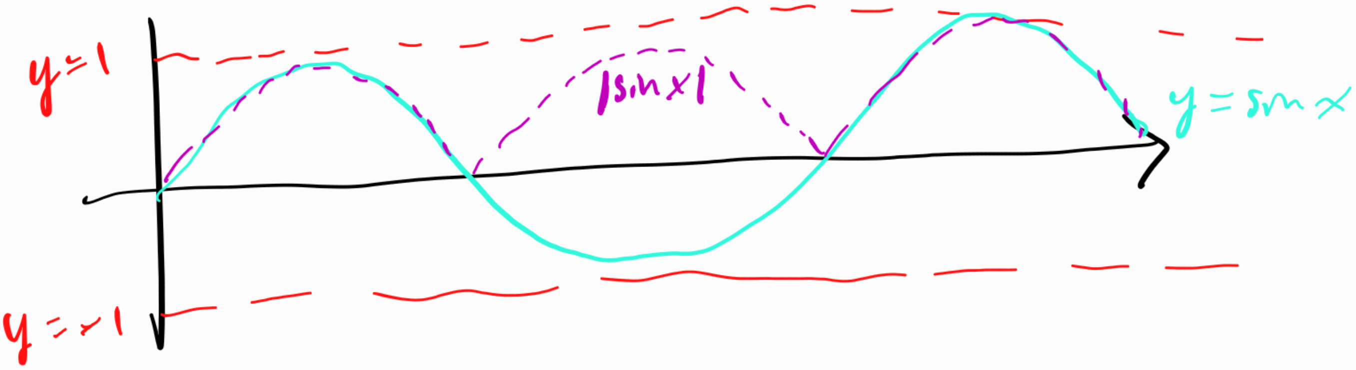
⇝ Maybe we should towards the comparison theorem

↳ This means we want to setup a chain of inequalities w/ a "nicer" function.

$$-1 \le \sin x \le 1$$



$\Rightarrow \underbrace{0 \leq |\sin x| \leq 1}$



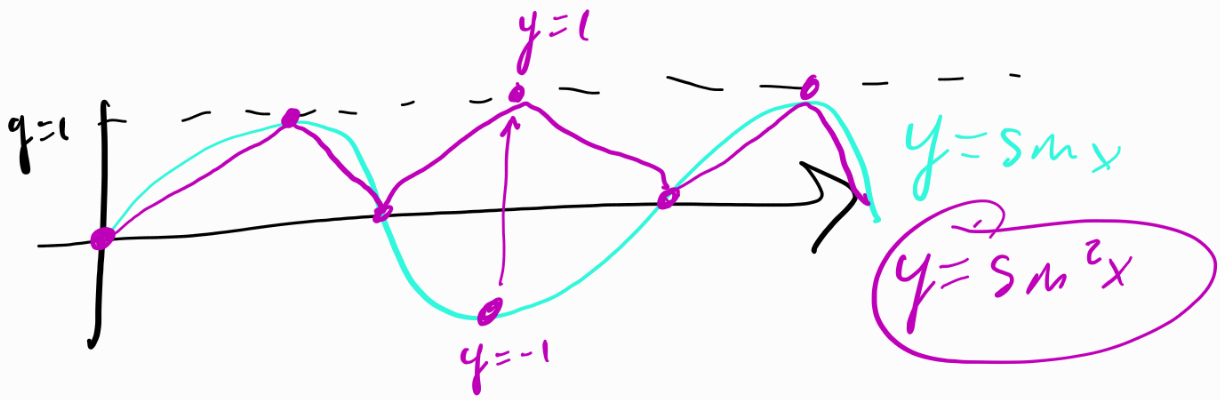
$$\frac{0}{x^2} \leq \frac{|\sin x|}{x^2} \leq \frac{1}{x^2}$$

$$\Rightarrow \underbrace{\int_1^\infty \frac{|\sin x|}{x^2}\,dx}_{\text{conv. by comp. w/ this}} \leq \underbrace{\int_1^\infty \frac{1}{x^2}\,dx}_{\text{conv. p-test}}$$

Using the comparison theorem

Q: Is $\sin^2 x$ also restricted to $[0, 1]$?

A: Yes.



$-1 \le \sin x \le 1 \implies 0 \le \sin^2 x \le 1$

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Prob 2 | a

Given unknown const. a, b, d

we're going to do PFD of

$$\int \frac{d}{(x+a)(x+b)}\,dx$$

PFD

$$\frac{d}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

$$d = A(x+b) + B(x+a)$$

method (i)

@ $x = -b$

$$d = A(\underbrace{-b+b}_{0}) + B(-b+a)$$

$$d = \boxed{B}(-b+a) \quad \text{Solving } B$$

$$B = \frac{d}{a-b}$$

@ $x = -a$

$$d = \underbrace{A(-a+b)} + B(\cancel{-a+a})^{0}$$

$$d = A(b-a)$$

$$A = \frac{d}{b-a}$$

$$\frac{d}{a-b}$$

unknown

$$\frac{d}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

$$= \left(\frac{d}{b-a} \cdot \frac{1}{x+a}\right) + \left(\frac{d}{a-b} \cdot \frac{1}{x+b}\right)$$

$$\int (\dots)\,dx = \int \underbrace{\frac{d}{b-a}} \cdot \underbrace{\frac{1}{x+a}} + \underbrace{\frac{d}{a-b}} \cdot \underbrace{\frac{1}{x+b}}\,dx$$

x

└ just #'s ┘

$$= \frac{d}{b-a} \ln|x+a| + \frac{d}{a-b} \ln|x+b| + C$$

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Problem 3 | a

Find c so that

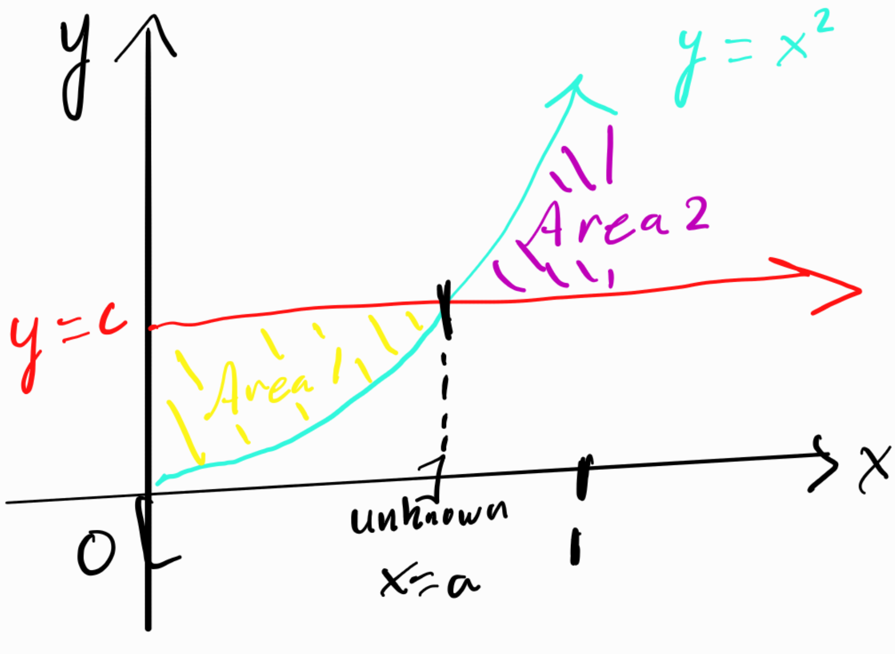
the area above $y=x^2$ & below $y=c$

is Equal to

the area below $y=x^2$ & above $y=c$

on the interval $[0, 1]$.

$y = \#$ is always a horiz. line



Area 2 = Area 1

I don't know where the line $y=c$ intersects $y=x^2$ (because I don't know what c is)

So I @ least label it

let a be the x-value where $y=c$ & $y=x^2$ intersect.

$$\text{Area } 1 = \int_0^a c - x^2 \, dx$$

$$\text{Area } 2 = \int_a^1 x^2 - c \, dx$$

$$\int_0^a c - x^2 \, dx = \int_a^1 x^2 - c \, dx$$

$$\left[cx - \frac{1}{3}x^3\right]_0^a = \left[\frac{1}{3}x^3 - cx\right]_a^1$$

$$ca - \frac{1}{3}a^3 - \underbrace{\left(c \cdot 0 - \frac{1}{3}0^3\right)}_{=0} = \left(\frac{1}{3}(1)^3 - c\right)$$

$$-\left(\frac{1}{3}a^3 - c\cdot a\right)$$

$$ca - \frac{1}{3}a^3 = \frac{1}{3} - c - \frac{1}{3}a^3 + ca$$

$$-ca \quad + \frac{1}{3}a^3 \qquad\qquad + \frac{1}{3}a^3 \quad - ca$$

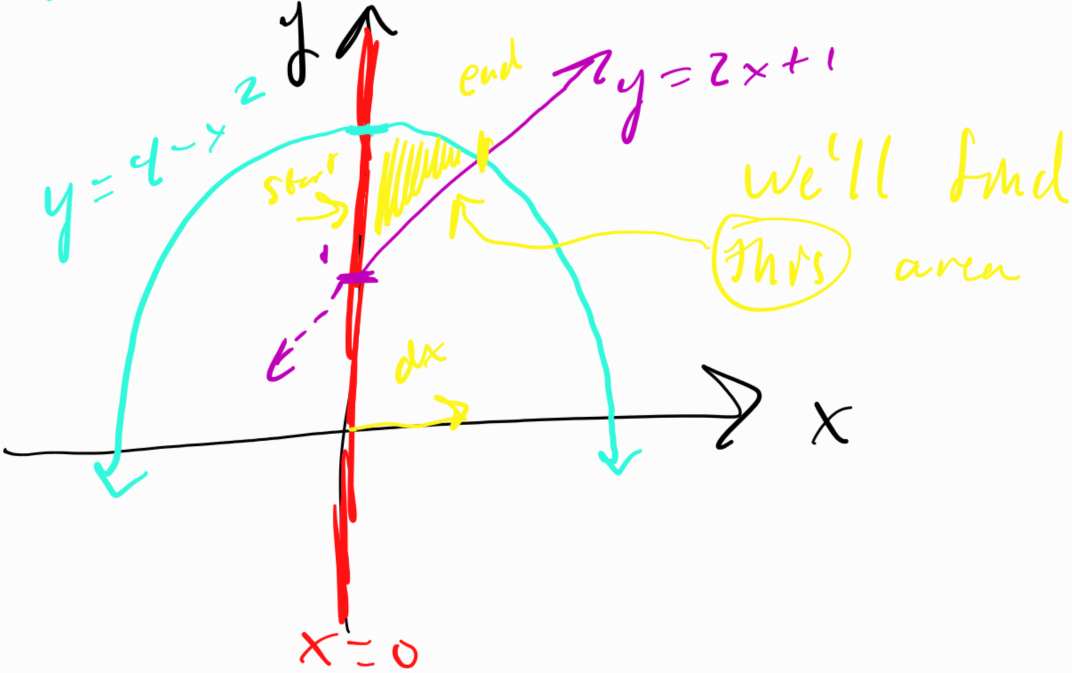
$$0 \qquad 0$$

$$0 = \frac{1}{3} - c$$

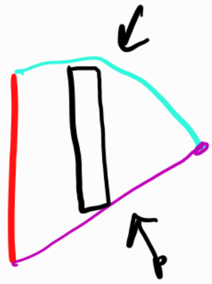
$$c = \frac{1}{3}$$

DIS | Prob 3 | b(

$y = 4 - x^2$, $y = 2x+1$, $x = 0$



$$\int_{?}^{?} \text{top} - \text{bot.} \; d?$$



$\Rightarrow dx$ because it's less work than the dy (in this case)

$$\int_{\text{start}}^{\text{end}} (4-x^2) - (2x+1)\,dx$$

Need to solve for the bounds

I know [start] by the picture @ $x=0$

For the [end] we "set the eq. equal & solve"

$$4-x^2 = 2x+1$$

$$0 = x^2+2x-3 = (x+3)(x-1)$$

$$\Rightarrow x = 1 \quad \& \quad x=-3$$

we choose $x=1$ (again by the picture)

$$\int_0^1 (4-x^2) - (2x+1)\,dx$$

$$= \int_0^1 -x^2-2x+3\,dx$$

$$= \left[-\frac{1}{3}x^3 - x^2 + 3x \right]_0^1$$

$$= -\frac{1}{3} - 1 + 3 - 0$$

$$= -\frac{1}{3} - \frac{3}{3} + \frac{9}{3} = \frac{5}{3}$$

$$\int x e^{x^2} \, dx$$

function · function

=> can't do

Integration Techniques

(i) Do it

(ii) u-sub

(iii) IBP

(iv) PFD for Rational

(v) trig-sub

So b/c function · function is this a u-sub?

Possible u-sub values are

$u = x^2$ or $u = e^{x^2}$

$du = 2x \, dx$

$$\int x e^{x^2} dx = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{x^2} + C$$

teleport

$u = e^{x^2}$

$$\boxed{du = e^{x^2} \cdot \underbrace{2x}_{\text{chain rule}}\, dx}$$

$$\int \underbrace{x e^{x^2} dx}_{\frac{1}{2} du} = \frac{1}{2} \int du = \frac{1}{2} u + C$$

$$= \frac{1}{2} e^{x^2} + C$$

DIS | Prob 1 | b)

$$\boxed{\int_0^{\infty} \frac{1}{x^2} dx}$$

we always need to check is there an x-value that breaks the function in the integral

@ $x = 0$ the function breaks

" $\frac{1}{0}$ "

We have to break up the integral as

$$\boxed{\int_0^{\#} \frac{1}{x^2}\,dx} \quad \& \quad \boxed{\int_{\#}^{\infty} \frac{1}{x^2}\,dx}$$

This # can be anything between $(0, \infty)$

why not choose $\# = 1$

$$\Rightarrow \int_1^{\infty} \frac{1}{x^2}\,dx \Rightarrow \text{conv. by p-test}$$

$$\Rightarrow \int_0^1 \frac{1}{x^2}\,dx = \lim_{a \to 0^+} \boxed{\int_a^1 \frac{1}{x^2}\,dx}$$

$$\int_a^1 \frac{1}{x^2}\,dx = -\left[\frac{1}{x}\right]_a^1 = \frac{1}{a} - 1$$

$$\lim_{a \to 0^+} \left(\frac{1}{a} - 1\right) = \infty$$

So $\int_0^{\infty} \frac{1}{x^2}\,dx$ div. b/c a part of the integral $\left(\int_0^1 \frac{1}{x^2}\,dx\right)$ diverged