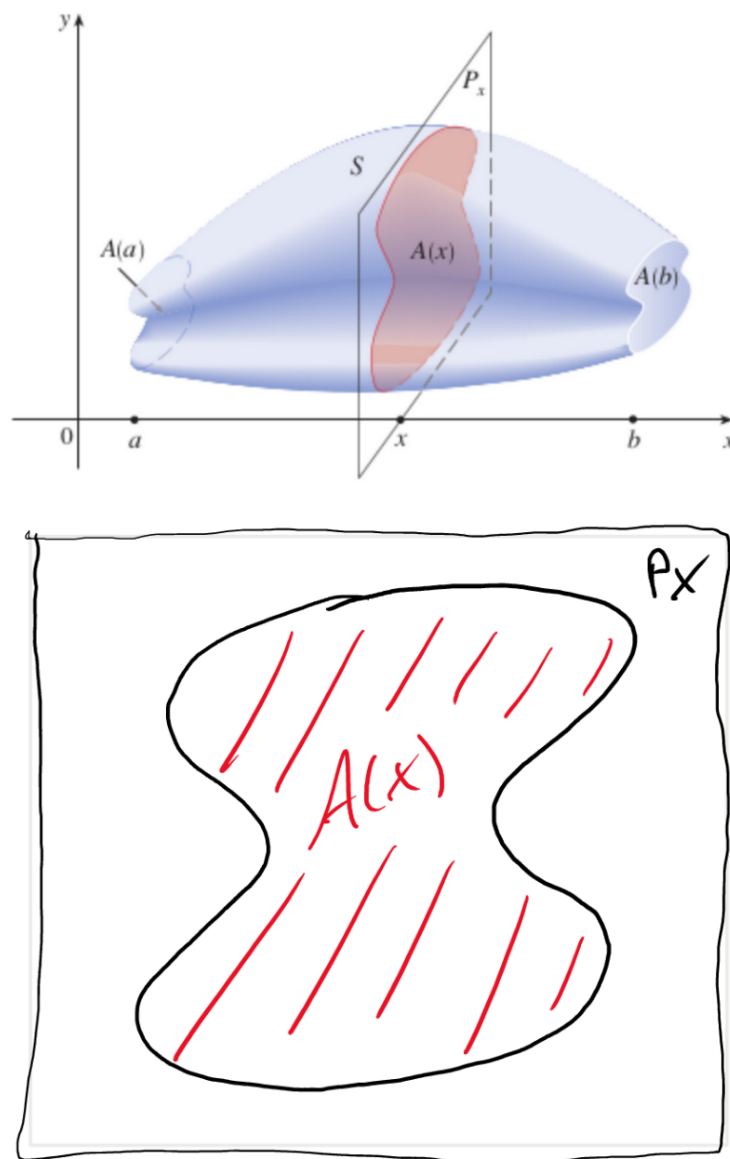# Section 6.2 Volume + Disk/Washer Method
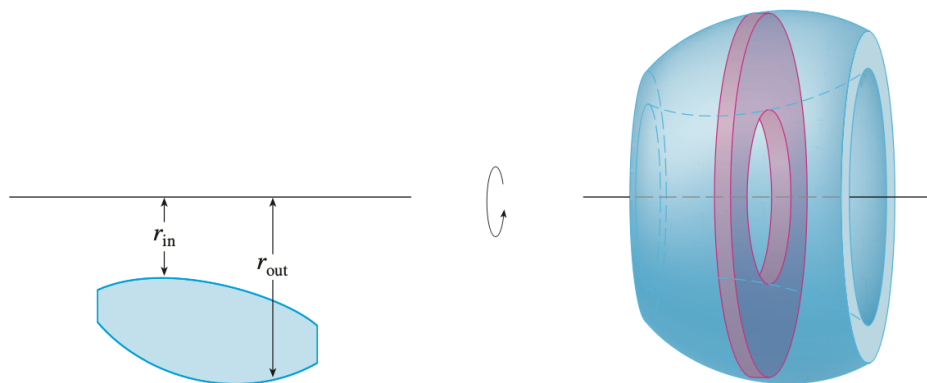

Definition of Volume

Consider a solid S whose "ends" we can place at $x = a$ and $x = b$. Let P_x be a plane that is perpendicular to the x-axis and goes through a point x(as pictured above). Let $A(x)$ be the cross-sectional area of S that lies on P_x. Then the volume of S is given by
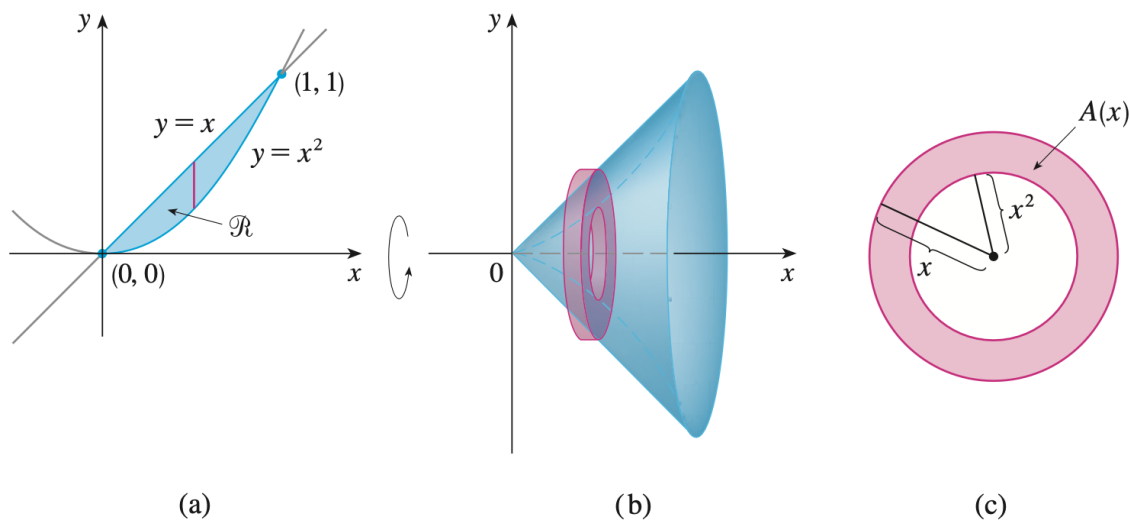
$$\text{Volume of } S \text{ on } [a, b] = \int_a^b A(x) \, dx$$

- **Washer Method**: If the cross-section is a washer, we find the inner radius r_{in} and outer radius r_{out} from a sketch and compute the area of the washer by subtracting the area of the inner disk from the area of the outer disk:

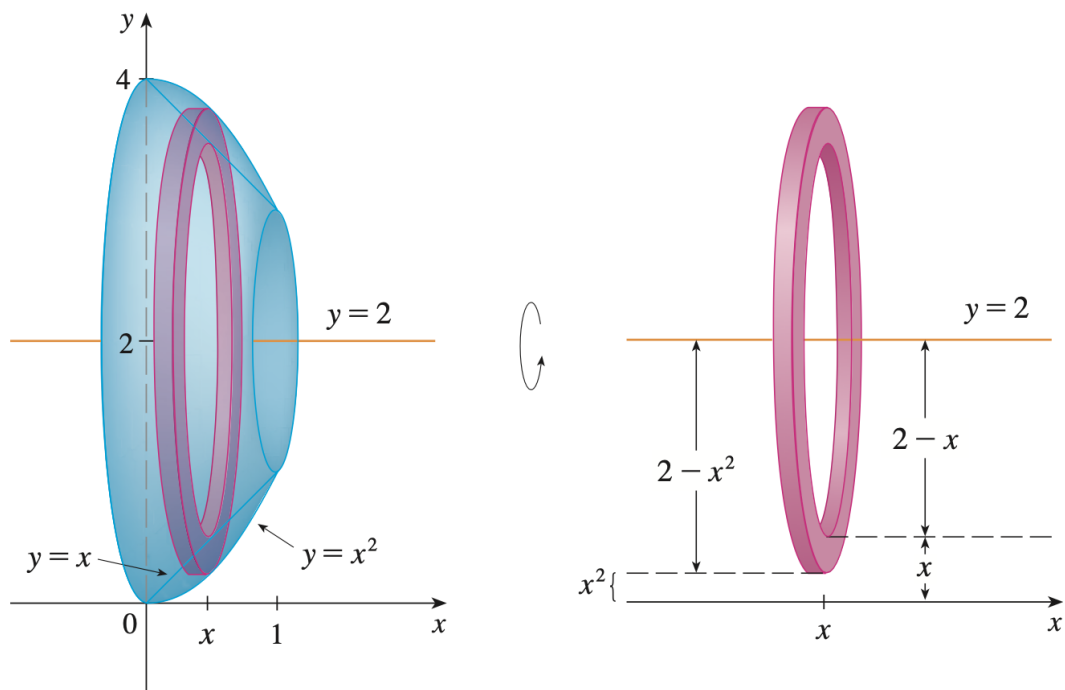$$A = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$$

Example 3: The region $\Re$ enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Set up the volume V of the resulting solid.

(a)   (b)   (c)

Example 4: Set up the volume formula V of the solid obtained by rotating the region in Example 3 about the line $y = 2$.

y

4

$y = 2$

2

$y = x$ $y = x^2$

0   x 1 x

$2 - x^2$

$2 - x$

$x^2\{$ x

x x

$y = 2$

Problem 6.2.13. (a) Find the area bounded between the curves $y = \sqrt{x-1}$, $y = 0$, and $x = 5$.

(b) Find the volume using (via washer method) generated by the area from part (a) about the y-axis.

Problem 6.2.19. (a) Find the area bounded between the curves $y = x^3$ and $y = \sqrt{x}$

(b) Find the volume using (via washer method) generated by rotating the area in part (a) about the x-axis.

Problem 6.2.19(cont.). (c) Set up the integral for the volume using (via washer method) generated by rotating the area in part (a) about the line $y = -1$.

(d) Set up the integral for the volume using (via washer method) generated by rotating the area in part (a) about the line $y = 1$.

(e) Set up the integral for the volume using (via washer method) generated by rotating the area in part (a) about the line $x = -1$.