

Instructions:

- Write your name and email address. If you are using more than one sheet of paper, make sure that you staple all the sheets together.
- Remember that collaboration of any kind is not allowed.

Total points = 10

1. The linear system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x} = [x_1 \ x_2]^T$, $\mathbf{b} = [3 \ 3.0001]^T$ and $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix}$ has the solution $[1 \ 1]^T$ (no need to show this here). Now, perturb \mathbf{A} slightly to $\tilde{\mathbf{A}} = \begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix}$ and consider the linear system $\tilde{\mathbf{A}}\mathbf{x} = \mathbf{b}$. Compute the new solution using 5-digit rounding arithmetic. Compute the condition number of \mathbf{A} .

3 points

2. Given the following data of 10 points (x_i, y_i) where $i = 1, 2, \dots, 10$

x_i	4.00	4.20	4.50	4.70	5.10	5.50	5.90	6.30	6.80	7.10
y_i	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.50	326.72

- (a) Construct the least squares approximation of degree 1 and compute the error. 2 points
- (b) Construct the least squares approximation of degree 2 and compute the error. 2 points
3. Apply the **Jacobi's iterative method** to solve the system of linear equations

$$\mathbf{Ax} = \mathbf{b}$$

where $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$, $\mathbf{b} = [6 \ 6 \ -6 \ 6 \ 6]^T$ and

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 1 & 0 & 0 \\ -1 & -3 & 1 & 1 & 0 \\ 2 & 1 & 5 & -1 & -1 \\ -1 & -1 & -1 & 4 & 0 \\ 0 & 2 & -1 & 1 & 4 \end{bmatrix}$$

Use error tolerance $\text{TOL} = 10^{-3}$ and the maximum number of iterations $N = 25$. (Hint: You may need to write a computer program or MATLAB code but show only the output.) 3 points