EENG20005 Electrical Energy Conversion and Supply 24-25

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Part A. AC power system

Case 1A Single-phase AC system

Task 1.1

Fig1.11

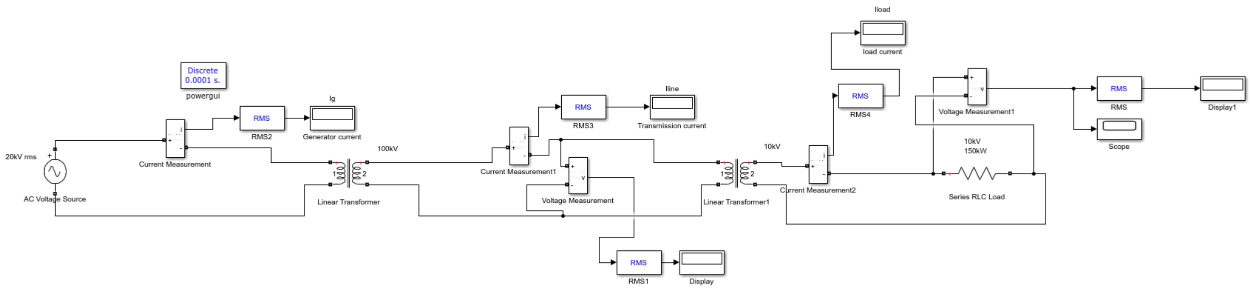


Fig1.11 shows the complete circuit of a single phase AC power system consisting of generatior, transmission, line and a load.

The active (real) power of the load is set as 150kw and the nominal (rms) voltage of the load is set to 10kV. Line frequency is set to 50Hz on AC source and RLC load)

Power generation (AC voltage source) is valued at 20kV rms due to my student number – converted into a Pk value in Simulink:

$V_{rms} = \frac{V_{pk}}{\sqrt{2}}$

$V_{pk} = 20000 * \sqrt{2} = 28284$ V

The turn ratios of the transformers are calculated using the relationship $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$ [**eq1.11**] as power is equal at both sides of the transformer.

The turn ratio of transformer 1:

$T_1 = \frac{V_1}{V_2} = \frac{N_1}{N_2}$: $T_1 = \frac{100k}{20k} = 5$

The turn ratio of transformer 2:

$T_2 = \frac{V_1}{V_2} = \frac{N_1}{N_2}$: $T_2 = \frac{10k}{100k} = 0.1$

Task 1.2

To work out the Generator current (I_G), Transmission line current (I_{line}) and consumer load current (I_{load}).
Equation [**eq1.11**] is once again used.
Starting with Consumer load power, we can first use the relationship P = IV to calculate the I_{load}:

$I_{load} = \frac{P}{V} = \frac{150k}{10k} = 15A$

Next, we can use the relationship $\frac{V_1}{V_2} = \frac{I_2}{I_1}$ to work out the value of transmission line current I_{line}:

$\frac{V_{load}}{V_{line}} = \frac{I_{line}}{I_{load}}$ [**eq1.21**]:

$I_{line} = (\frac{10k}{100k}) \times 15 = 1.5A$

Finally, we can use [**eq1.11**] again to calculate the generator current I_G:

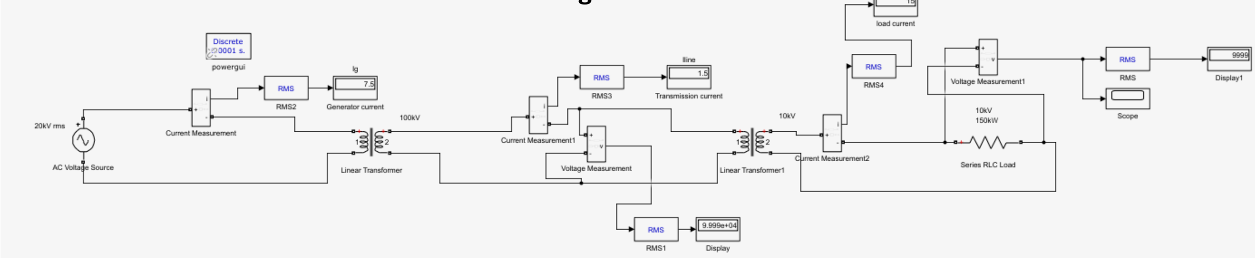
$\frac{V_G}{V_{line}} = \frac{I_{line}}{I_G}$ [**eq1.22**]:

$I_G = \frac{I_{line} * V_{line}}{V_G}$

$I_G = \frac{1.5 * 100k}{20k} = 7.5A$

Running the simulation verifies our calculations as shown in **Fig1.21**

Fig1.21



Waveform for load current (**Fig1.22**), transmission line current (**Fig1.23**) and generator current (**Fig1.24**) are shown below:

Fig1.22

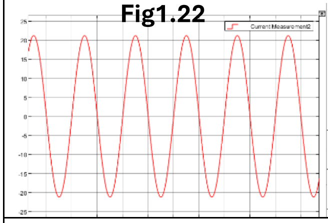


Fig1.22

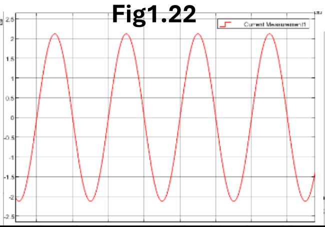
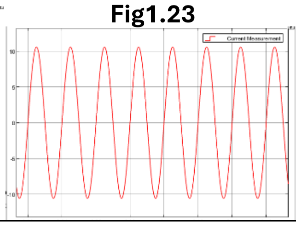


Fig1.23



Task 1.3

The benefit of transmitting power at a higher voltage is that due to the conservation of power, the current at the transmitter line will be lower, this will lead to less heat due to resistance, leading to a lower loss in energy during transmission.

If the generator and transmission voltages are aligned to the consumer voltage, there will be no change in current between generator and transmission line. This will lead to a greater loss in energy during the transmission due to a higher current in the transmission line.

Case 1B

Task 1.4

Now if the consumer 1 continues to draw active power in case 1 but with a 0.75 power factor due to load being an inductive electric machine.

We know the real power (P_G) of the generator will be equal to the active power as set before (150kW). To find the generator reactive power Q_G we can use the fact that S(VA), P(W) and Q(Var) form a right angled triangle with angle θ in between which can be calculated using power factor = cos(θ).

$\text{Tan}(\theta) = \frac{Q_G}{P_G}$
$\text{Cos}(\theta) = 0.75$ (power factor)
$\theta = 0.723$ (rad)

$P_G * \tan(\theta) = Q_G$
$150{,}000 * \tan(0.723) = Q_G$
$Q_G = 132358\text{W}$

We can then enter this reactive power into the RLC load (into the RLC load inductive reactive power)

To calculate Transmission line current (I_{line}), we can first work out generator current I_G, which is given by equation $P(W) = V_{rms} * I_{rms} * \cos(\theta)$. As we know both V_{rms} and P(W)

$P(W) = V_{rms} * I_{rms} * \cos(\theta)$
$\frac{\text{P(W)}}{V_{rms} * \cos(\theta)} = I_{rms} = I_G$
$I_G = \frac{150k}{20k * \cos(0.723)}$
$I_G = 10\text{A}$

Now using [**eq1.22**] we can calculate the transmission line current (I_{line})

$I_{line} = \frac{I_G * V_G}{V_{line}} = \frac{10 * 20k}{100k} = 2\text{A}$

We can then use [**eq1.21**] to calculate the load current (I_{load})

$I_{load} = \frac{I_{line} * V_{line}}{V_{load}} = \frac{2 * 100k}{10k} = 20\text{A}$

These results are verified in the simulation model as shown in **Fig 1.31**

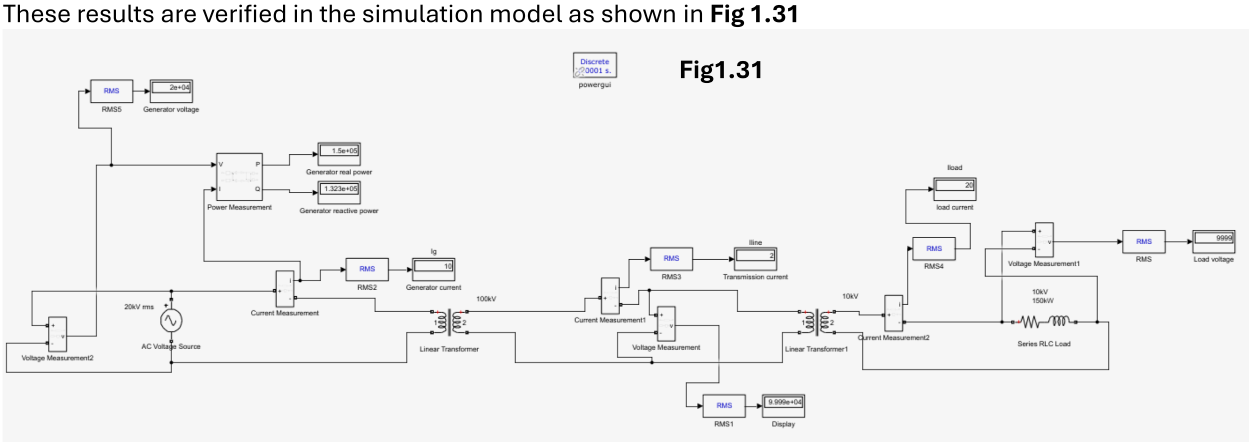


Fig1.31

Compared to the currents in case 1.1 and 1.2, the currents observed in this case are higher, which shows that as reactive power increases, the power factor of the system decreases and increases the currents at the generator, line, and load. This would lead to higher resistive loss along the system, especially for long transmission lines, and therefore increase the power loss when transmitting over long distances.

Task 1.5

We can measure the apparent using only voltage and current probes.

Real power (P(W)) is 150kW

Reactive power Q(Var) is calculated using Q(Var) = $V_{rms}I_{rms}\sin(\theta)$ = (20,000)(10)sin(0.723)

Q(Var) = 132327W $\sqrt{150000^2 + 132327^2}$

Apparent power S(Va) = $\sqrt{P(W)^2 + Q(Var)^2} = \sqrt{150000^2 + 132327^2}$

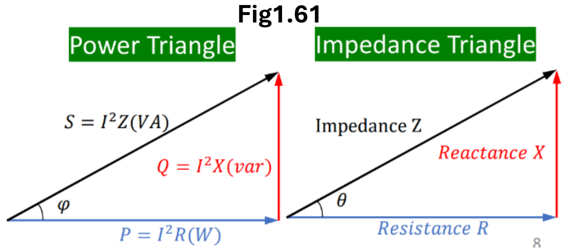
S(Va) = 200026

Which is equal to the measured values in the simulation shown in **Fig1.31**.

Task 1.6

Now we have replaced the constant load block with a passive load formed by a resistor and inductor in series To make the new circuit satisfy the same power and voltage specifications, we need to find the resistance and inductance values such that V_{load} and I_{load} remain the same. This can be done using the power triangle shown in **Fig1.61**

Fig1.61



We know that real, apparent and reactive power remain the same.

First we can find the inductance of the inductor using Q(Var) = I^2X(Var)

X(Var) = $\frac{Q(Var)}{I^2} = \frac{132358}{20^2}$ = 330.895 Ω

Reactance is related to inductance by the equation $X_L = 2\pi fL$ where L is the inductance

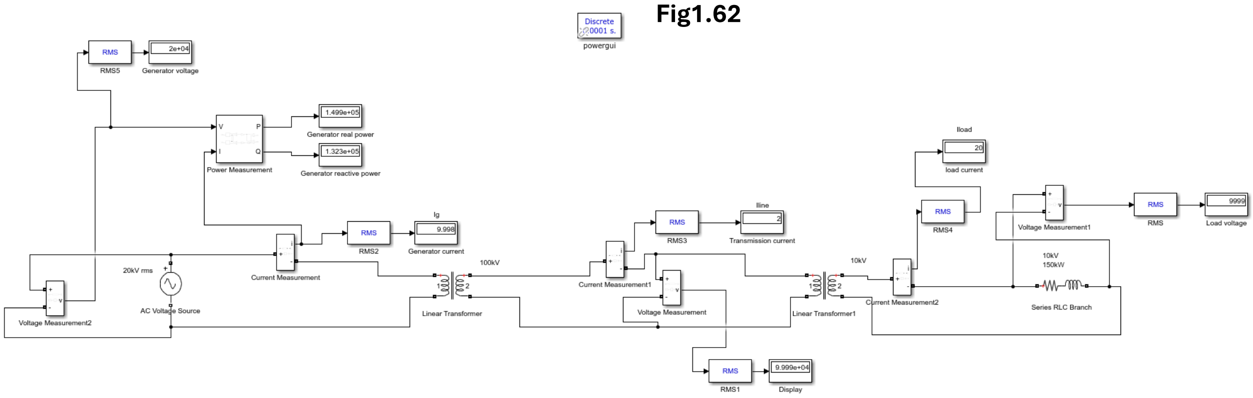
L = $\frac{X_L}{2\pi * f} = \frac{330.895}{2\pi * 50}$ = 1.053H

We can then calculate the resistance of the resistor using the relationship between real power P(W) and resistance R(W) P(W) = I^2R(W)

R(W) = $\frac{\mathrm{P(W)}}{I^2} = \frac{150\mathrm{k}}{20^2}$ = 375 Ω

After entering these 2 values into our series resistor and inductor branch and running the simulation we can confirm that the power and voltage specifications stated earlier are still satisfied shown in **Fig1.62**

Fig1.62

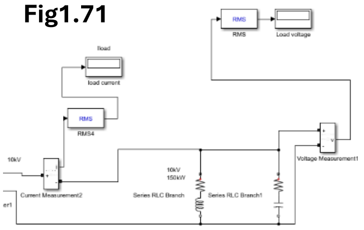


Case 1C

Task 1.7

The circuit diagram of just the load with the added capacitor is shown in **Fig1.71**

Fig1.71



To calculate the required capacitance value C to bring the power factor of the load from 0.75 to 0.95, we first find the impedance Z of the load

$Z = R + j\omega L = 375 + j(2\pi(50)(1.053)$

$Z = 375 + j330.81$

We can then calculate the magnitude $|Z|$ and phase θ of this impedance.

$|Z| = (375^2 + 330.81^2)^{\frac{1}{2}} = 500.1$

$\theta = \tan^{-1}(330.81/375)$ – using the argand diagram

$\theta = 41.42°$ (lagging) = $-41.42°$

Using equation $P(W) = V_{rms}I_{rms}\cos(\theta)$ and $Z = \frac{V}{I}$

$P(W) = \frac{V^2}{Z}*\cos(\theta)$ **[Eq1.71]**

Using the power triangle we can deduce $\tan(\theta) = \frac{Q(Var)}{P(W)}$, $Q(Var) = P(W)*\tan(\theta)$ **[Eq 1.72]**

Subbing **[Eq1.71]** into **[Eq1.72]** we get $Q(Var) = \frac{V^2}{Z}*\cos(\theta)*\tan(\theta)$

$Q_1 = \frac{V^2}{Z}*\cos(-41.42)*\tan(-41.42)$ – where Q_1 = original reactive power

After power correction the real power remains the same $P(W) = \frac{V^2}{Z}*\cos(-41.42)$, but reactive power is lower

$Pf = \cos(\theta) = 0.95$, $\theta = \cos^{-1}(0.95) = 18.2°$ (lagging) = $-18.2°$

New reactive power $Q_2 = P(W)*\tan(\theta) = \frac{V^2}{Z}*\cos(-41.42)*\tan(-18.2)$

Change in reactive power Q_{comp} required to increase power factor from 0.75 to 0.95 = $Q_2 - Q_1$

$Q_{comp} = \frac{V^2}{Z}*\cos(-41.42°)*[\tan(-41.42°) - \tan(-18.2°)] = \frac{V^2}{X_C}$

$X_C = \frac{Z}{\cos(-41.42°)*[\tan(-41.42°) - \tan(-18.2°)])} = \frac{500.1}{[(0.75)(0.553)]} = 1204.8$

$C = \frac{1}{2\pi*f*X_C} = \frac{1}{2\pi*50*1204.8} = 2.64\mu F$

For the power factor correction case we can calculate the reactive power $Q_2 = |P(W)\tan(\theta)|$

$Q_2 = \left|\frac{10000^2}{500.1} * \cos(-41.42°)\tan(-18.2°)\right| = 49300W$

$P(W) = \frac{10000^2}{500.1}*\cos(-41.42°) = 149946W$, $S(Va) = \sqrt{P(W)^2 + Q(Var)^2} = 157842W$

The power triangle is shown in **Fig1.72**: $\theta = \tan^{-1}(\frac{49300}{149946}) = 18.2°$, $\cos(18.2°) = 0.95$ (power factor)

The real and reactive power is validated in simulation using the capacitance calculated in **Fig1.73**

Fig1.72

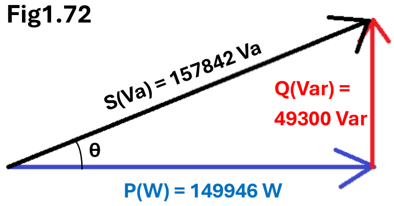
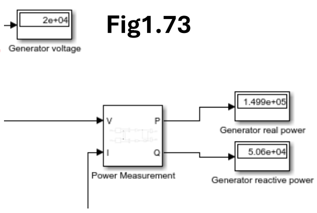


Fig1.73



Task 1.8

A phasor diagram can be drawn using the generator voltage as reference to show the current and voltage phases on each of the RLC components of the circuit.

We know the resistor has a 0 phase difference between voltage and current, $\theta_R = 0$

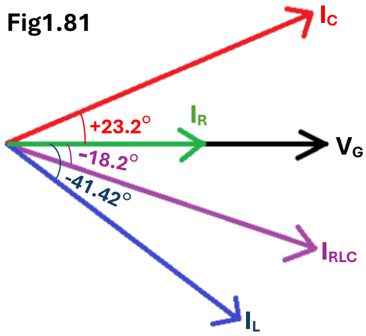
The inductor phase was calculated as cos(θ) = p.f. originally
$\cos(\theta_L) = 0.75$
$\theta_L = 41.42°$ (lagging) = -41.42° – current lags voltage by 41.42°

The total phase of the system after power factor correction is calculated as -18.2° (lagging)

The phase of the capacitor is the angle needed to increase the reduce the phase of the overall system (inductor and resistor) to the final system phase of -18.2°

$\theta_C = -18.2° - -41.42° = 23.2°$
This is drawn on the phasor diagram in **Fig1.81**

Fig1.81



Task 1.9

A higher power factor correction reduces the current flowing through the system as it reduces the reactive power that needs to be supplied, as reactive power decreases the apparent power also decreases which means less current is needed to supply the same amount of real power.

With a lower current, resistive losses (I^2R) in the system decrease this is especially useful in the transmission line as transmission of current along a long length increases the overall resistance and therefore improves the efficiency of the transmission line.

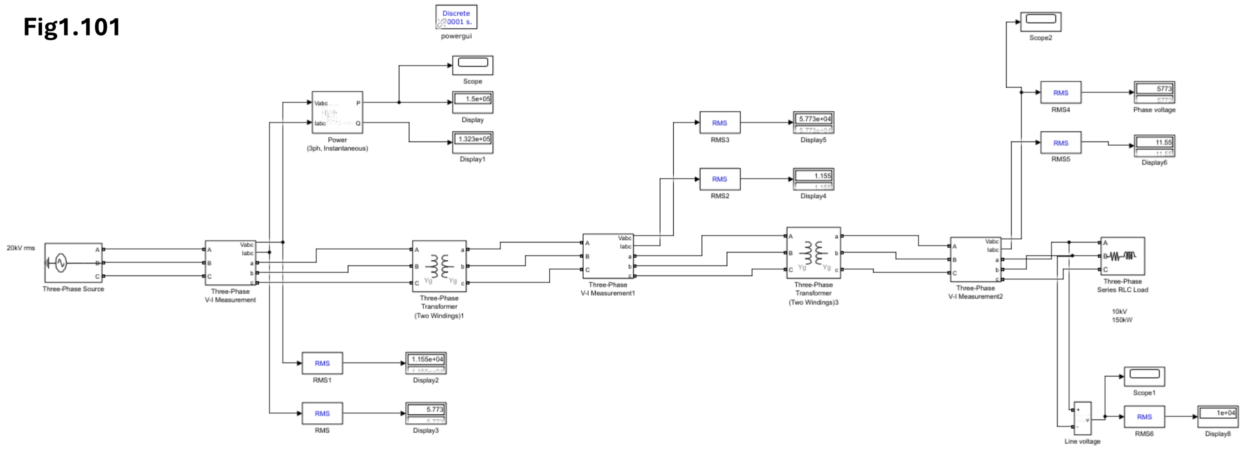
Since loss in a line is given by $P = I^2*R$, for a conductor resistance R will increase with length, a lower current will reduce the losses experienced.

Case 1D

Task 1.10

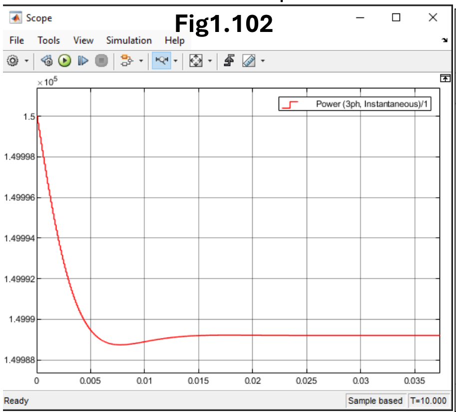
The circuit constructed in section 1B converted into an equivalent 3 phase circuit with the same load specifications with a power factor of 0.75 is shown in **Fig1.101**

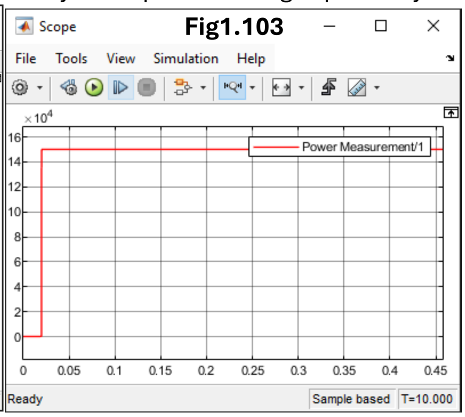
Fig1.101



The circuit features a three phase AC voltage source connected in star connection with it's neutral point connected to ground and connected to a series of transformers, the first transformer steps up the voltage from 20,000V to 100,000V and so reduces current (P = I*V where power P is constant) and the second transformer reduces the voltage from 100,000V to 10,000V therefore increasing the current. This is then connected to a three phase RL load connected in star connection with a floating neutral point.
The equivalence of this circuit can be validated by running simulation and we can see that the generator real power P (150,000W) and reactive power Q (133,000W) are equivalent to the measured values in case 1B

The total instantaneous power transmitted by the three-phase system is shown in **Fig1.102**
The total instantaneous power transmitted by the equivalent single-phase system is shown in **Fig1.103**

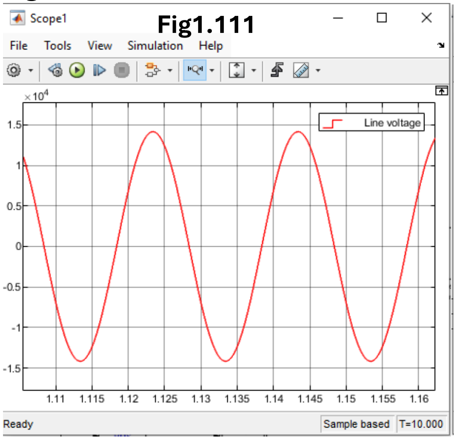


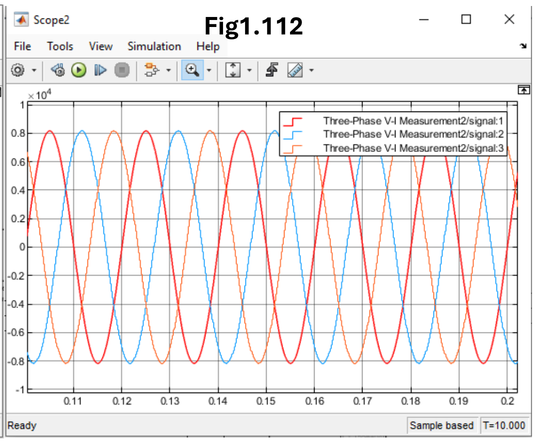


For single phase voltage supply, voltage waveform is initially zero in the beginning (time = 0) and so there is a small period where the power will be equal to zero, but for three phase since there are 3 phases of current and voltage (spaced 120 degrees apart since in a balanced system), there is always power supplied, so after a small transient period the power will remain constant but never be at zero.

Task 1.11

The line voltage on the load side is shown in **Fig1.111** and the phase voltage on the load side is shown in **Fig1.112**





The waveform in **Fig1.111** agrees with our expected value for line voltage on the load side which is RMS value 10,000, since RMS value is calculated using $V_{rms} = \frac{V_{pk}}{\sqrt{2}}$ we can see that the peak value of $\frac{14,100}{\sqrt{2}}$= 9970V ≈ 10,000V which is the expected line voltage after the transmission line.

Fig1.112 agrees with the expected phase voltage, if we use the equation $V_{line} = V_{phase}*\sqrt{3}$ we can find V_{phase} = 5773V, calculating the RMS value of these phase-to-phase voltages we find V_{rms} = 5773 which aligns with our expected value.

We can express the V_{phase}, V_{line}, I_{phase} and I_{line} mathematically with the following expressions
$P_{phase} = V_{phase}*I_{phase}$*p.f. where p.f. is the power factor and P is the real power per phase
$V_{line} = V_{phase}*\sqrt{3}$
For a balanced three phase system in star connection: $I_{line} = I_{phase}$

Task 1.12

If the grid frequency is increased from 50Hz to 60Hz we can use the impedance triangle (shown in **Fig1.61**) to see how the passive RL load would need to be changed to satisfy the same power specifications.

The impedance triangle gives the relationship between overall impedance, reactance and resistance, where the angle θ determines the power factor (pf) of the system pf = cos(θ).

By increasing the frequency of the system the reactance (X) of the RL load would increase as reactance of the load (X_L) is given by $X_L = 2\pi*f*L$ where L is inductance of load and f is the frequency of the grid.

If the reactance of the load increases the impedance (Z) will increase $Z = \sqrt{R^2 + X_L^2}$, as impedance and reactance increases the angle θ in the impedance triangle (**Fig1.61**) will increase and so the power factor of the system will decrease as pf = cos(θ).

To keep the power factor the same, we would need to decrease the angle θ, which could be done by lowering the inductance of the load or increasing the resistance of the load.

Case 1E

Task 1.13

If we have a second load drawing in an apparent power (S) of 100kVA with a power factor of 0.9, we can work out the composite power factor of the 2 loads.

First our original load has a power factor of 0.75, a real power (P) of 150,000W and a reactive power (Q) of 132,288 Var

Our new load has a power factor of 0.9 and an apparent power (S) of 100,000 VA.

To find composite power factor, we can use the expression p.f. composite = $\frac{P_{tot}}{S_{tot}}$ where P_{tot} is the total real power of the system and S_{tot} is the total apparent power of the system.

We can find the apparent power of our original load using the equation p.f. = $\frac{P}{S}$ [**Eq1.131**], we can find the apparent power S_1 to be $\frac{150{,}000}{0.75}$= 200,000 VA

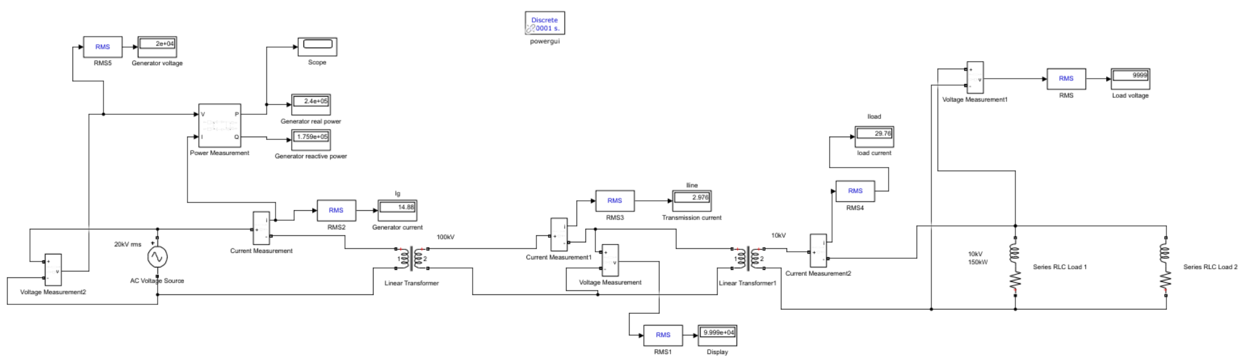
Next, we can find the real power (P) of the second load using equation [**Eq1.131**] and solving for P, P = p.f. * S and so P_2 = 0.9 * 100,000 = 90,000W

We can then sum both the real powers to find total real power P_{tot} and sum the apparent powers to find total apparent power S_{tot},
P_{tot} = 150,000 + 90,000 = 240,000W
S_{tot} = 200,000 + 100,000 = 300,000Va

Now we can find the composite power factor of the system (p.f. composite)
p.f. composite = $\frac{P_{tot}}{S_{tot}} = \frac{240{,}000}{300{,}000}$= 0.8

We can validate this in simulation by adding the load in as shown below and looking at the real power and reactive power readings.
The reactive power (Q) can be calculated for our composite system using the power triangle shown in **Fig1.61** relating real, reactive and apparent power to find the following expression $Q_{tot} = \sqrt{S_{tot}^2 - P_{tot}^2}$= 180,000 Var
The circuit used is shown in **Fig1.131**

Fig1.31



Measuring the generator real and reactive power in simulation we can see that for P(W) = 240,000W and Q(Var) = 176,000 Var which are consistent with our analytical results therefore verifying the simulation.

Task 1.14

For a three-phase system with a neutral line and unbalanced load, we can work out the neutral line current by finding the phase currents and their phases and adding them together: $I_n = I_A + I_B + I_C$.

Since apparent power (S) is given by $S = V_{phase} * I_{phase}$ we can use this to find out current in each phase path by finding their respective apparent power as well as the V_{phase}

V_{phase} is given by the equation $V_{line} = V_{phase} * \sqrt{3}$, since we know line voltage is 10,000, we can find $V_{phase} = \frac{10{,}000}{\sqrt{3}}$ = 5773V

Next we can use the ratio of the load power as well as the total apparent power calculated in previous sections (200,000 VA) to find the respective phase apparent power (S_{phase})

Since the load power between phase A,B,C is 3:2:1 we know that $S_A = S_{tot}*\frac{1}{2}$, $S_B = S_{tot}*\frac{1}{3}$ and $S_C = S_{tot}*\frac{1}{6}$

We obtain the values: $S_A = 200{,}000*\frac{1}{2}$ = 100,000Va, $S_B = 200{,}000*\frac{1}{3}$ = 67,000Va and $S_C = 200{,}000*\frac{1}{6}$ = 33,000Va

We can then calculate the respective phase current magnitudes using $I_{phase} = \frac{S_{phase}}{V_{phase}}$

$I_A = \frac{100{,}000}{5773}$= 17.3A, $I_B = \frac{66{,}000}{5773}$ = 11.43A and $I_C = \frac{33{,}000}{5773}$= 5.72A

In order to find the phase angle of the currents we can find the phase angle between the current and voltage waveforms and add it to the phases of the voltages which are (0°,-120°,-240°) in a three phase system.

$\theta = \cos^{-1}(\text{p.f.}) = \cos^{-1}(0.75) = 41.4°$

So the currents lag voltage by 41.4 degrees and we can find the corresponding phases of the currents.

I_A = 0° +41.4° = 41.4°, I_B = -120° +41.4° = -78.6°, I_C = -240° +41.4° = -198.6°

Now that we have the magnitude and phases of the currents we can convert them into cartesian form and add them to find the magnitude of the neutral line current.

$I_A = 13.13 + 11.57j$, $I_B = 2.26 – 11.2j$, $I_C = -5.42 + 1.82j$

Adding them together we get $I_N = 9.97 + 1.89j$

Which we can use to find the magnitude of neutral line current: $I_N = \sqrt{9.97^2 + 1.89^2}$ = 10.14A

The simulation circuit is shown in **Fig1.141**

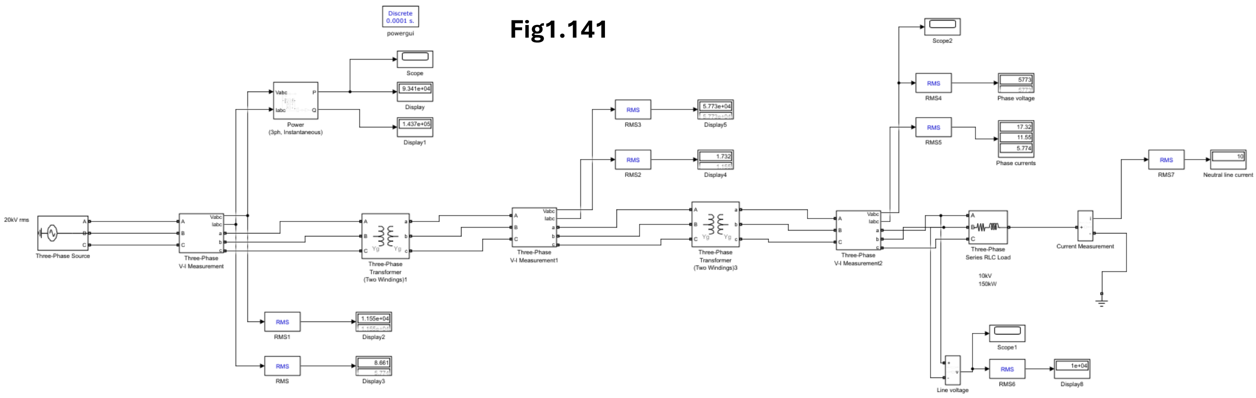


Fig1.141

Running the simulation we can see I_N (RMS) = 10A which is the expected value we worked out analytically above validating the simulation.

Part B. Power Converters

Case 2A

Task 2.1

A single phase rectifier uses an AC voltage generator connected in parallel to 4 ideal diodes shown in **Fig2.11**.

When the AC signal is in the positive half, diodes D1 and D2 are forward biased allowing a current to flow through them and outputting a positive voltage across the load, when the AC signal is in the negative half, diodes D3 and D4 conduct allowing the current to flow through them and outputting a positive voltage across the load again, the diodes are labelled in **Fig2.11** as **D1,D2,D3,D4**.

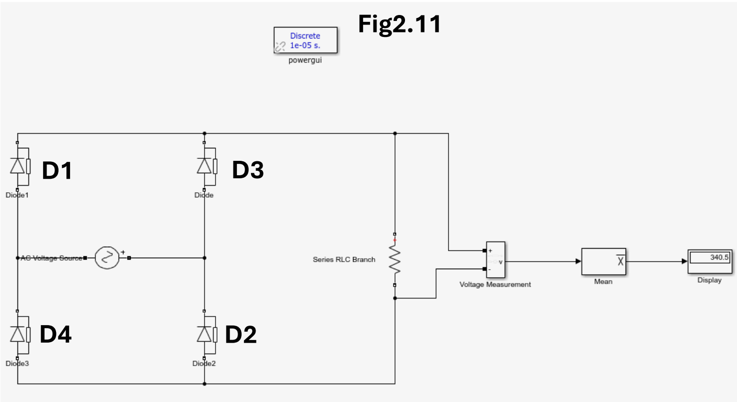
With the specifications provided (V_{mains} = 380 Vrms, 50Hz, Load resistance = 10Ω) we can calculate the average value of the rectified voltage using the equations: $V_{AVE} = 2V_{pk}/\pi$ and $V_{pk} = V_{rms} \times \sqrt{2}$

$V_{AVE} = 2V_{pk}/\pi = 2(380 * \sqrt{2})/\pi$

$V_{AVE} = 342.1V$

This is validated through running the simulation with the specifications obtaining a mean voltage value of 340.5V.

Fig2.11



Task 2.2

At the start with a capacitance value of 1μF, we obtain a voltage waveform as shown in **Fig2.21**

By increasing the capacitance value to 0.02F, the peak-to-peak voltage ripple is reduced to 22.5V, the mean value of voltage has changed to 524.2V, 4% of that is 21V, so the peak to peak voltage ripple has been successfully reduced to 4% of the average voltage value, the waveform is shown in **Fig2.22**.

Fig2.21

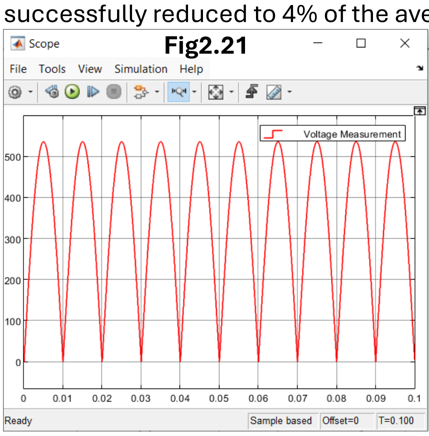
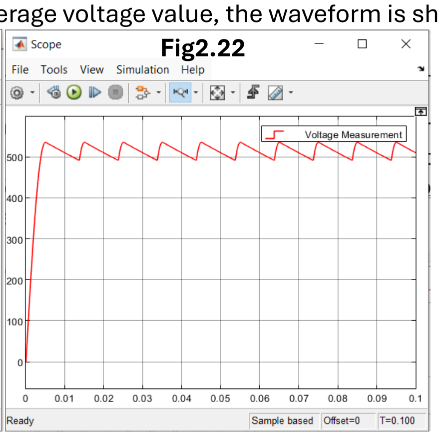


Fig2.22



As the capacitance is increased, the ripple is smoothed (smaller peak to peak ripple), bringing the average DC link voltage closer to the peak rectified voltage. This is because the capacitor smooths out the ripples by charging up when voltage is increasing and discharging when voltage falls. A larger capacitor stores more charge and discharges slower leading to a smoother output.

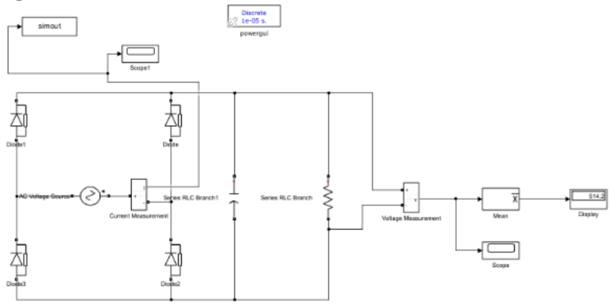
Changing the load R to 5Ω increases the peak to peak voltage ripple which decreases the average voltage value. As the resistance is lower the current passing through both the resistor and capacitor increases. This increased current with a fixed capacitor leads to an increased discharge rate of the capacitor leading to an increased voltage ripple. As the voltage ripple is greater it has a lower minimum peak value.

This lower minimum peak value reduces the average voltage value throughout the waveform.

Task 2.3

We can measure input AC current using the setup in **Fig2.31**

Fig2.31



This leads to a FFT shown in **Fig2.32**

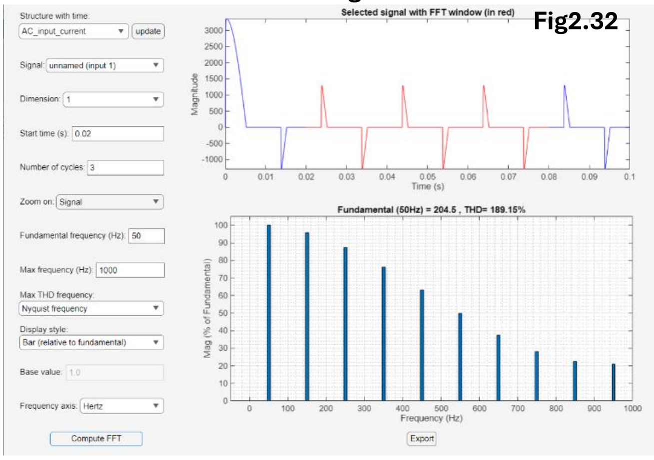


Fig2.32

We can see that there are components of frequency at odd multiples of the fundamental frequency (50Hz)

Case 2B

Task 2.4

The single phase two switch inverter is shown in **Fig 2.41**, each DC voltage source is $V_{DC}/2$ = 200V

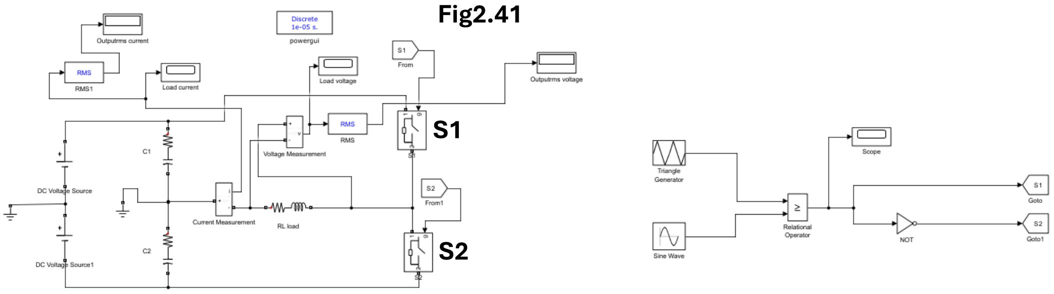


Fig2.41

A PWM box is created using a triangle wave which is compared with a sine wave. This is used to control the two switches in complementary fashion (one switch is open while the other is closed).

When **S1** is closed and **S2** is open, current flows through the top half of circuit and voltage across load is $+V_{DC}/2$, when **S1** is open and **S2** is closed, current flows through bottom half of circuit and voltage across the load is $-V_{DC}/2$. **S1** and **S2** are switches labelled on **Fig2.41**

Fig2.42 shows the load voltage and **Fig2.43** shows the output current.

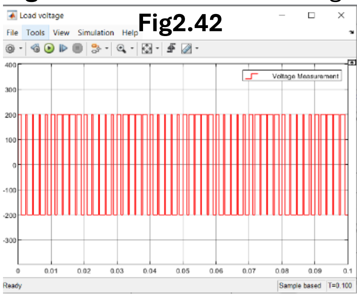


Fig2.42

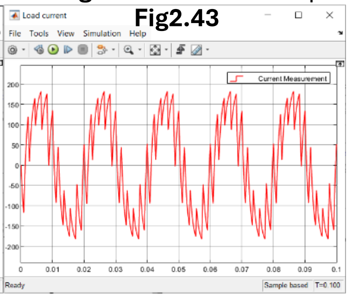


Fig2.43

True RMS and RMS values for output voltage and load current are different in this case, we get (V_{load} RMS = 113V, I_{load} RMS = 107.8A, V_{load} True RMS = 199.9V, I_{load} True RMS = 114.9A)

Task 2.5

Originally with an inductance value of 1mH, we obtain a peak to peak current ripple of 94.4A, the fft of the current waveform is shown in **Fig2.51**. After increasing the inductance value to 0.03H, we get a peak to peak voltage ripple of 8.2A (less than 10A), the fft of the waveform is shown below **Fig2.52**.

Fig2.51

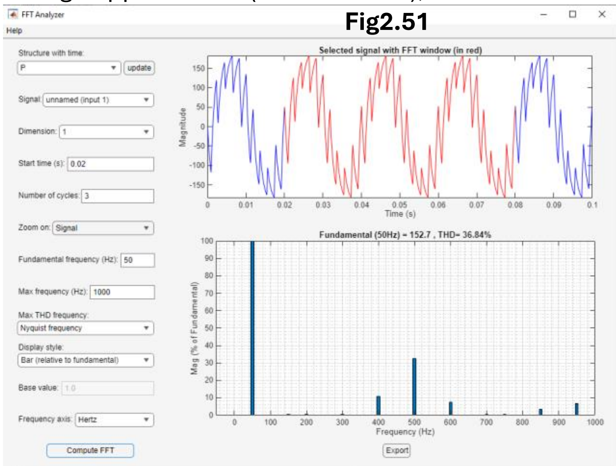
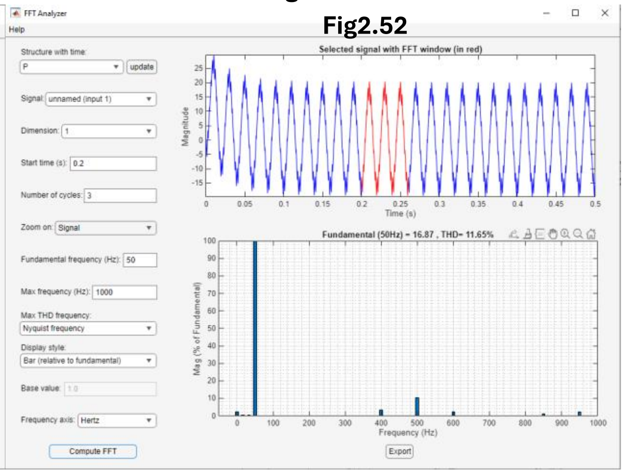


Fig2.52



The spectrum of the smoother current waveform is mostly similar to the original waveform (before changing inductance) however it has lower peak value (and RMS value) as well as less overall ripples. The downwards trend of the peaks at the beginning suggest that the load current takes longer to reach steady state.

The FFT of the new load current has lower magnitude in the higher frequency components which is due to the higher inductance reducing the rate of change of current. ($V= L\frac{dI}{dt}$) so for an increased L, $\frac{dI}{dt}$ reduces for a fixed voltage (V). Since the magnitude of frequency components is related to the number of sharp changes (ripples) in the waveform, a smoother waveform will have smaller high frequency magnitudes

The additional lower frequency components also come from this reduced rate of change of current.

Task 2.6

We can reduce the output voltage by changing the modulation index.

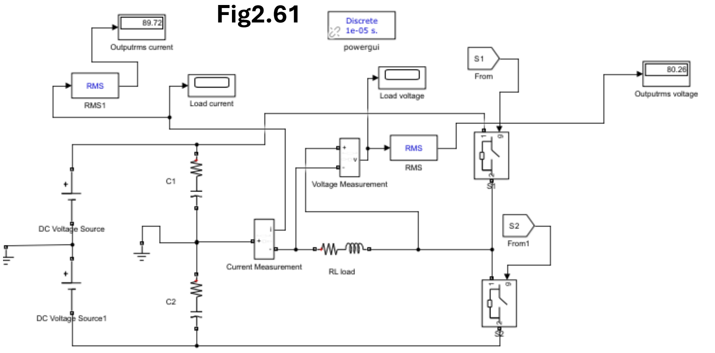
V_{AC} RMS = $M.V_{DC}/2\sqrt{2}$ as V_{out} pk = $M.V_{DC}/2$ and $V_{RMS} = V_{pk}/\sqrt{2}$

If we want the reduce the output voltage to 80V RMS

$80 = M.V_{DC}/2\sqrt{2}$, $M = (80.\ 2\sqrt{2}) / 400$

M = 0.566 (we know that M is amplitude of the sine wave $M = V_M$ as triangle wave has fixed amplitude of 1) so we can change the amplitude of the sine wave, this is confirmed by running the simulation. Which shows the output voltage is reduced to 80.26V

Fig2.61



We can compare the performance against an equivalent AC source shown in by taking the FFT of the load current (**Fig2.62**) and comparing it to the load current of the circuit in **Fig2.61** (shown in **Fig2.63**)

Fig2.62

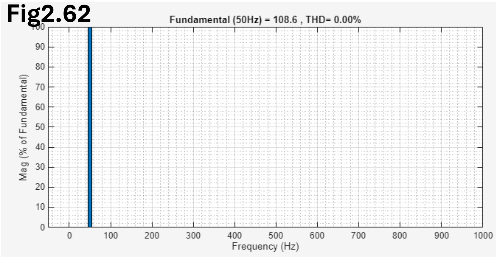
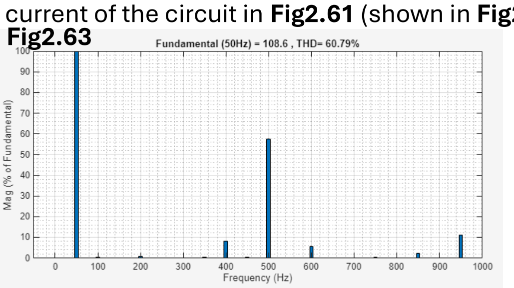


Fig2.63



Task 2.7

We can change the fundamental frequency of the converter output voltage to 100hz by simply changing the frequency of the sine wave from 50hz to 100hz, changing the modulating wave frequency will increase the output voltage frequency.

This is validated by using FFT on the voltage waveform to find the fundamental frequency shown in **Fig2.71**

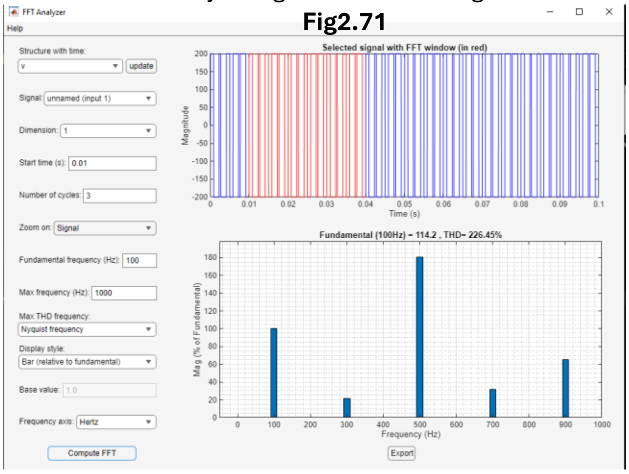


Fig2.71

This fft spectrum shows that the voltage waveform has fundamental frequency at 100hz and other components at the 3rd, 5th and 9th harmonics

One reason to change fundamental frequency would be to reduce energy losses in reactive components such as inductors (Z = jωL) and capacitors ($Z = \frac{1}{j\omega C}$) since their impedance is related to frequency.

Another reason to change fundamental frequency is to comply with regional standards, in Europe the standard frequency is 50Hz while in America the standard frequency is 60Hz.

Task 2.8

By keeping all system specifications the same and increasing the switching frequency to 5kHz (frequency of the triangle wave), the frequency of load voltage is significantly increased, the RMS value of load voltage is decreased (from 113V with f_{sw} = 500Hz to 107.8V when f_{sw} = 5000Hz), however the output load voltage frequency is significantly higher.

The load current RMS values are decreased slightly (from 107.8A when f_{sw} = 500Hz to 102.9A when f_{sw} = 5000Hz), the output current waveform is significantly smoother with a significantly lower current ripple (from 94.4A when f_{sw} = 500Hz to 8.5A when f_{sw} = 5000Hz) shown in **Fig2.81**

This means that the inductance required for a lower current ripple is reduced for a higher switching frequency (f_{sw}), When we're using a switching frequency of 500Hz we require a 30mH inductance to reduce current ripple to below 10A, however for a switching frequency of 5000Hz we only require a 1mH inductance to reduce current ripple to below 10A.

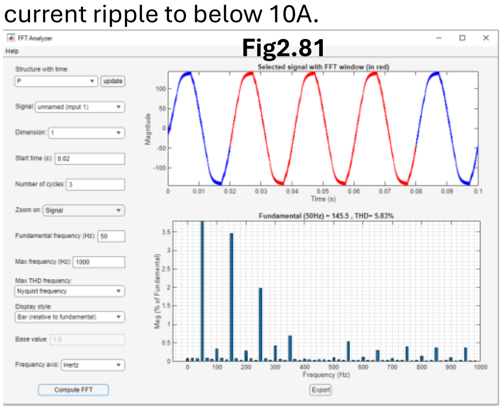


Fig2.81

Computing the FFT of the load current compared to task 2.5 where current ripple is less than 10A due to a higher inductance value. We see that for the new load current there are more frequency components.
This is because the new current waveform is smoother but contains more ripples per cycle which would require more frequency components to represent in the frequency domain.

Task 2.9

Replacing the switches with IGBT and antiparallel diodes, the waveforms are the same as with the ideal switches (task 2.8). However, when running the simulation with the antiparallel diodes removed the RMS voltage is decreased (from 107V to 9.2V) and RMS current is decreased (from 102.1A to 9.2A)

The load current waveform is shown in **Fig2.91**

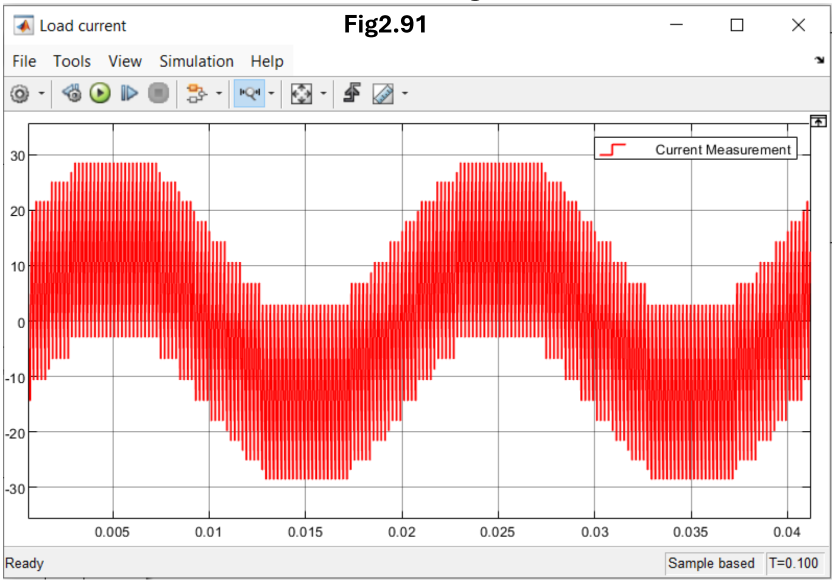


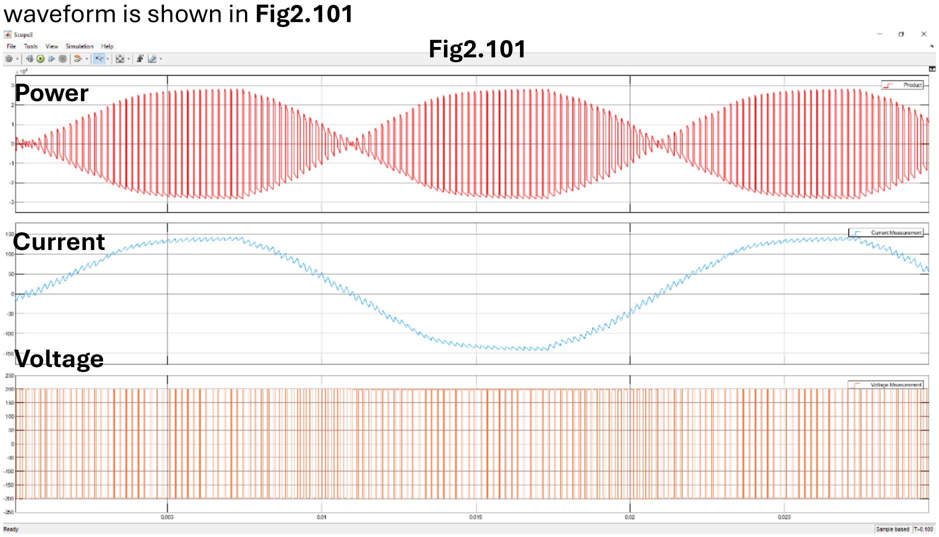
Fig2.91

The current ripple is much higher than with diodes, where the original ripple was 8.8A, now it is increased to 31A ripple when the diodes are removed.

Task 2.10

We can multiply the voltage and current waveforms to obtain a graph of apparent power against time. The waveform is shown in **Fig2.101**

Fig2.101



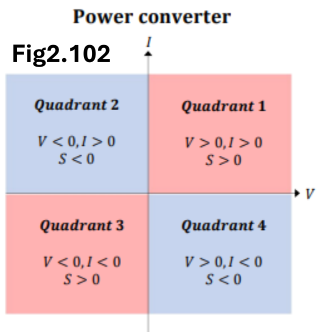


Fig2.102

We can see that the first quadrant is when apparent power (S) is positive, Voltage and current are both positive shown in **Fig2.102.**

Looking at the following waveforms we can see that the converter load system will operate beyond the first quadrant when the current and voltage are not both positive, so the load system operates in the 1st and 2nd quadrant when the current value is positive (every half cycle of the load current) and operates in the 3rd and 4th quadrant when the current value negative (every half cycle). So it will operate beyond the first quadrant every cycle of the load current.

When the current is positive, the operating mode will vary between quadrant 1 and quadrant 2 depending on whether the voltage is positive or negative, this is determined by the frequency of the load voltage.

Case 2C

Task 2.11

Single phase full H bridge inverter using ideal switches (**Fig2.111**), verified by running simulation and plotting load current(**Fig2.112**) and load voltage(**Fig2.113**) which oscillates between 400V and -400V which is expected for a V_{DC} = 400V in this system. Table of output for switch combinations shown in **Fig2.114**.

Fig2.111

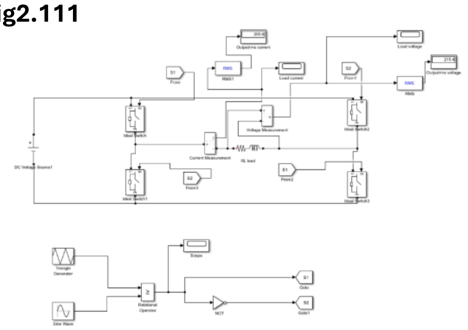


Fig2.112

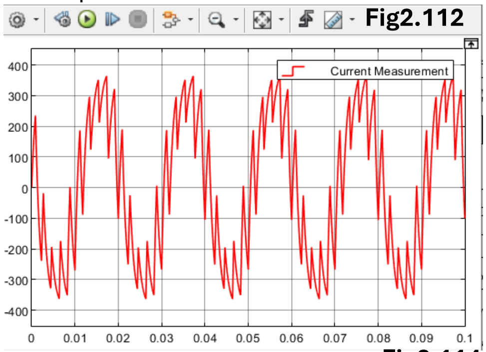


Fig2.113

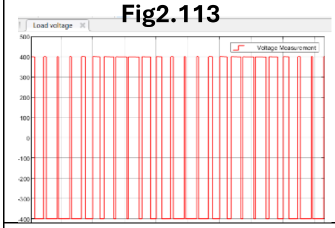


Fig2.114

S1 (top left)	S2 (top right)	S3 (bottom left)	S4 (bottom right)	V_{load}
on	off	off	on	$+V_{DC}$
off	on	on	off	$-V_{DC}$

Task 2.12

A H bridge inverter is more complex compared to a half bridge inverter requiring 4 switches compared to half bridge inverter which requires only 2 switches, this increases complexity of the circuit as well as cost to produce.

However, the H bridge inverter produces an output load voltage varying between $+V_{DC}$ and $-V_{DC}$ compared to half bridge inverter producing voltage waveform between $+V_{DC}/2$ and $-V_{DC}/2$ which means it has a better DC link utilization resulting in an extended range of output voltage amplitude.

Comparing the output voltage waveform of the Full H bridge inverter (shown in **Fig2.121**) to a half bridge inverter (shown in **Fig2.122**) (both have same 400V V_{DC} voltage supply).

Fig2.121

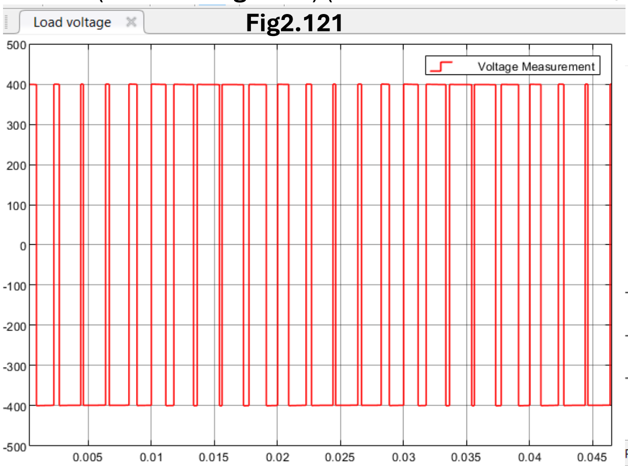
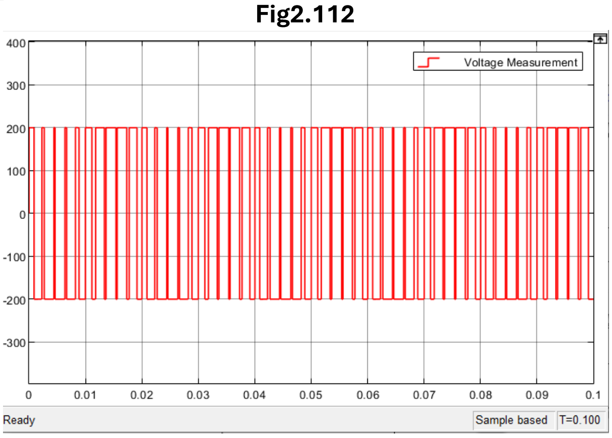


Fig2.112

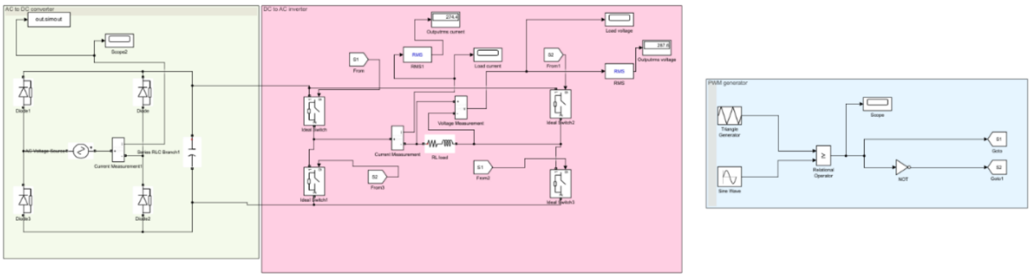


The full H bridge inverter has output waveform between $+V_{DC}$ and $-V_{DC}$ while the half bridge inverter has output waveform between $+V_{DC}/2$ and $-V_{DC}/2$ showing that full H bridge inverter has better DC link utilisation.

Task 2.13

Joining the diode rectifier in case 2A and full H bridge inverter to mimic a variable frequency drive we get the circuit shown in **Fig2.131**:

Fig2.131



The load current is shown in **Fig2.132** and voltage waveforms is in **Fig2.133**

Fig2.132

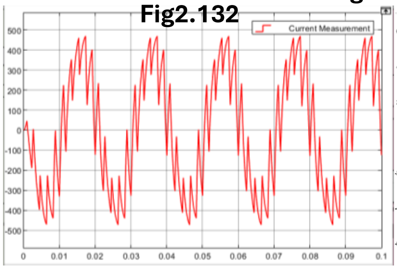
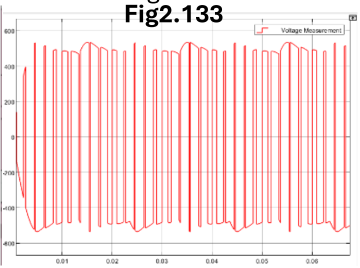


Fig2.133



These waveforms are similar to the results obtained using an ideal DC voltage source where voltage waveform varies from $+V_{DC}$ to $-V_{DC}$ which in this case has a maximum value of +530V and -530V.

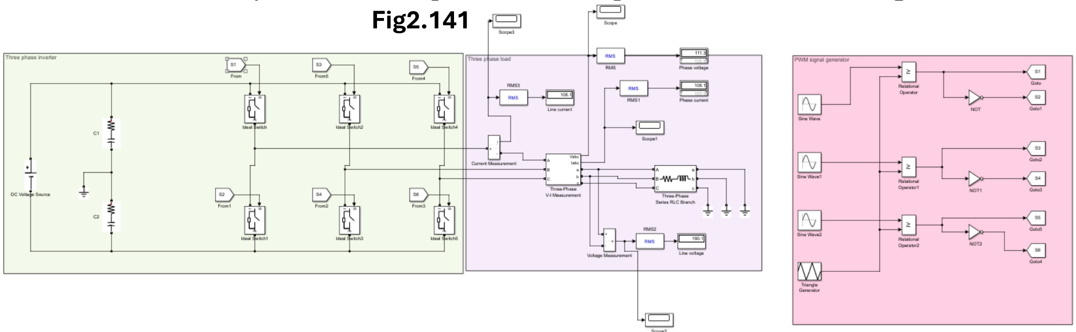
The differences are between case when ideal DC source is used is that that the voltage and current waveforms experience a short transient period in the beginning and since the voltage supplied to the inverter is not constant (DC voltage ripples between 510V and 530V so the maximum absolute value of the converted voltage waveform will vary between 530V and 510V, rippling effects of the “constant” DC voltage supplied to inverter can be seen). If a DC source is used the maximum value of output voltage will be constant.

Case 2D

Task 2.14

We can create a three phase half bridge inverter using the circuit shown in **Fig2.141**

Fig2.141



We can work out the expected RMS value for V_{phase} (phase to neutral voltage) using the equation: V_{phase} RMS = $M.V_{DC}/2\sqrt{2}$ where M is the modulation index (0.8) and V_{DC} is the DC link voltage (400V) in this case. Substituting the values in we obtain the following result: $V_{phase} = 0.8*400/2\sqrt{2}$ = 113V which is approximately equal to the RMS value calculated in simulation (111.3V). Using the relationship V_{line} RMS = $\sqrt{3}*V_{phase}$ RMS = 195.7V which is approximately equal to the RMS value calculated in simulation (190.1V).

Below are the various simulation waveforms: Load line voltage (**Fig2.142**), Load phase voltage (**Fig2.143**) and load line current (**Fig2.144**).

Fig2.142

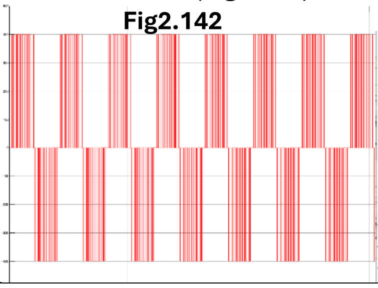


Fig2.143

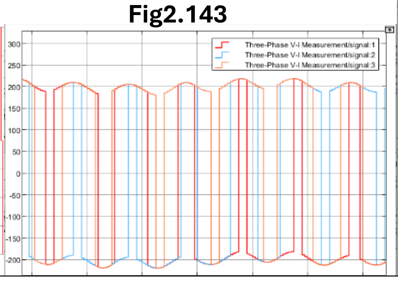
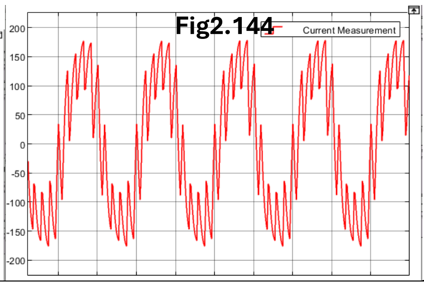


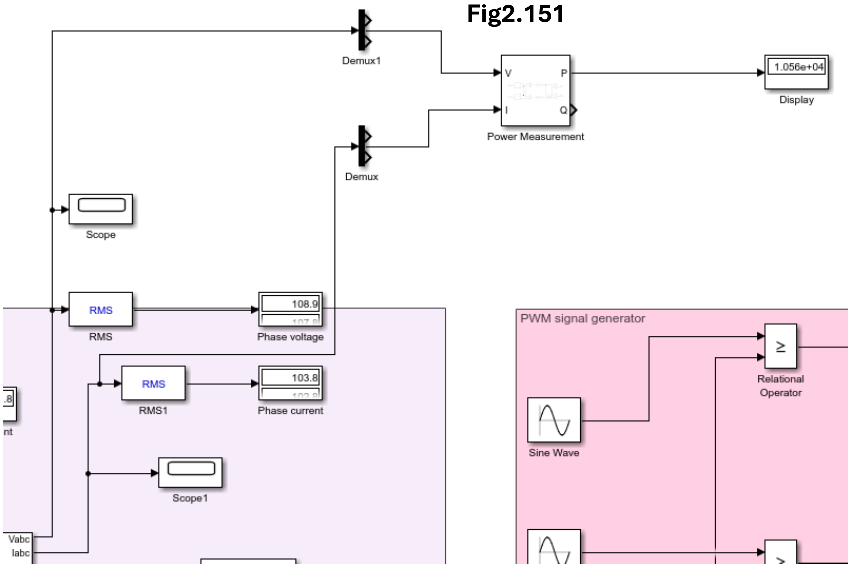
Fig2.144



Task 2.15

The output phase power can be measured using the power measurement as shown in **Fig2.151.**
Output power can be changed by changing the modulation index (amplitude of the sine wave)
In this case we do not need to adjust modulation index to achieve an output load phase real power of 10kW.

Fig2.151



Part 3. Ferromagnetic Actuator

Case 3A

Task 3.1 & Task 3.2

After completing the code for the FEMM model and running the simulation, we obtain the model shown in **Fig3.11**

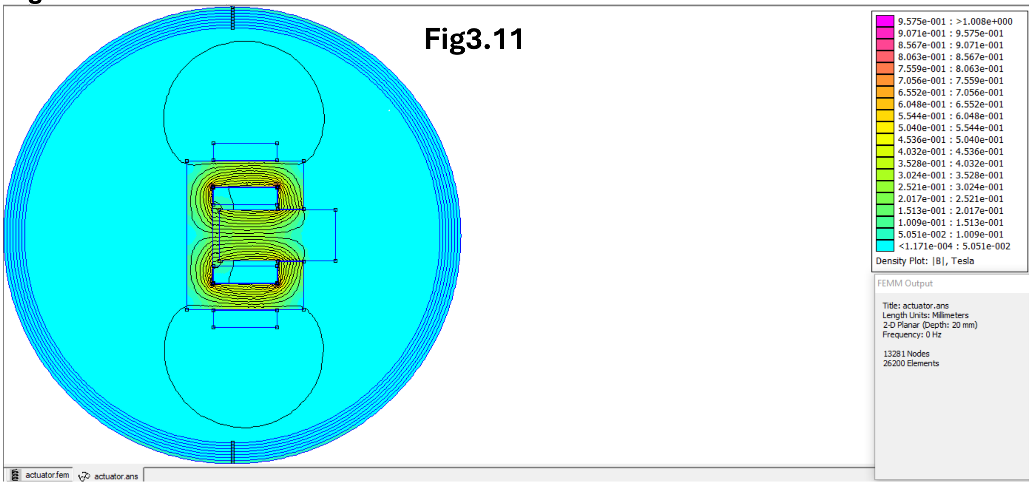


Fig3.11

We can compare the smartmesh feature when it is off (shown in **Fig3.12**) and when it is on (shown in **Fig3.13**) as well as introducing refinement along the variable air gap.

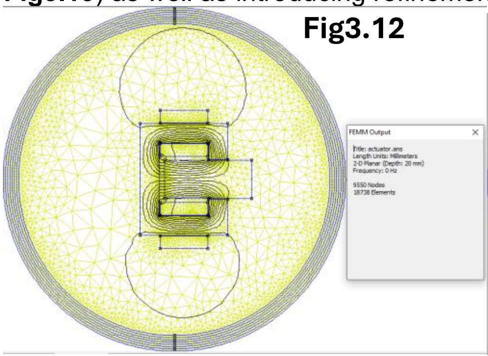


Fig3.12

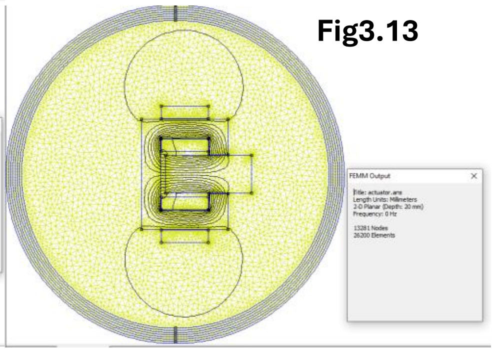


Fig3.13

Smartmesh off model has a total of 18738 elements
Smartmesh on model as well as further mesh refinement along the variable air gap has an increase in the number of elements (26200 elements):

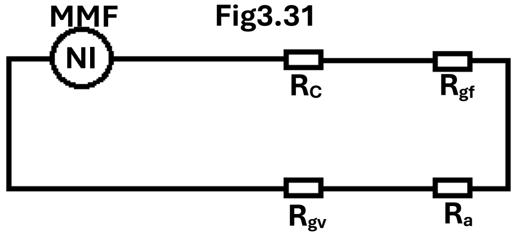
A higher mesh density achieved through using smartmeshing and further refinement along the variable air gap will increase the accuracy of the numerical solution particularly in the variable air gap region where the magnetic field potential varies the most.

Case 3B

Task 3.3

Since the Ferromagnetic actuator consists of 2 copper windings which are symmetrical, we can model the actuator as 2 separate magnetic circuits and the total reluctance of the system is the reluctance of one circuit multiplied by 2.
Fig3.31 shows the magnetic equivalent circuit of one copper winding:

Fig3.31



Where $\mathbf{R_c}$ = Reluctance of the core, $\mathbf{R_{gf}}$ = Reluctance of fixed air gap, $\mathbf{R_a}$ = Reluctance of armature and $\mathbf{R_{gv}}$ = Reluctance of the variable air gap.

By calculating the mean length of the flux through the respective components we obtain the following results:
L_c = 145mm, L_a = 74.9mm, L_{gf} = 0.5mm and L_{gv} = 5.1mm

We can then use the reluctance equation R = $\frac{L}{\mu_0 * \mu_r * A}$ where L = mean length travelled by flux in component, μ_0 is permeability of free space ($4\pi*10^{-7}$ H/m), since we are going to use mm in our calculations we divide by 1000 ($4\pi*10^{-10}$). μ_r = relative permeability of the material we are using and A = cross sectional area of the material.

We then consider fringing effects. For fringing effects (when flux is passing through the air gaps) the effective area A_{eff} is given by A_{eff} = (T +2g) * (W + 2g) where T is thickness of material, W is width of material and g is the gap length.

Calculating the areas and mean lengths the following reluctances can be worked out:
R_c = 288468 $H^{-1}mm^{-1}$ R_a = 74504 $H^{-1}mm^{-1}$
R_{gf} = 994718 $H^{-1}mm^{-1}$ R_{gv} = 5073063 $H^{-1}mm^{-1}$
$R_{gf(eff)}$ = 902239 $H^{-1}mm^{-1}$ $R_{gv(eff)}$ = 2677008 $H^{-1}mm^{-1}$

The total Reluctance of the system is the sum of reluctance of every component in one path x 2
R_T (No fringing) = 12861506 $H^{-1}mm^{-1}$
R_T (Fringing) = 7884438 $H^{-1}mm^{-1}$

Task 3.4

Using the Reluctances calculated in the previous section, we can then use the equation L = N^2/R to calculate the Inductance of one winding, where N is the number of turns of the winding and R is the reluctance of one path (R_{path}). Then we can multiply by 2 to find the total inductance (L_T) of the actuator since the design is symmetrical.

We have the following values:
N = 100
R_{path} (No fringing) = 6430753 $H^{-1}mm^{-1}$
R_{path} (Fringing) = 3942216 $H^{-1}mm^{-1}$
Using the equation for inductance of one path and multiplying by 2 we obtain
L_T (Non fringing) = 0.0031 H.mm
L_T (Fringing) = 0.0051 H.mm

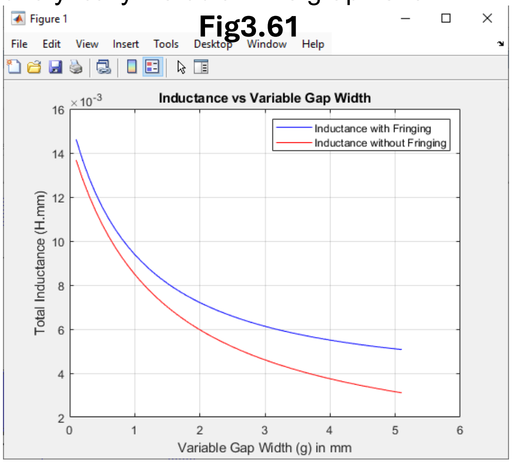
Task 3.5

I have completed the code for analytical modelling

Case 3C

Task 3.6

After writing the relevant code to find total inductance with and without fringing effects of the system analytically we obtain the graph shown in **Fig3.61**



We can see that as the variable gap width increases, the total inductance of the system decreases. This is consistent as Inductance is given by L = N^2/R, as the variable gap increases, the length of the gap increases which increases R and therefore decreases L.
We can also see that when considering fringing effects the total inductance is higher, this is consistent as effective area of gap increases which decreases R of variable gap and so increases L.

Task 3.7

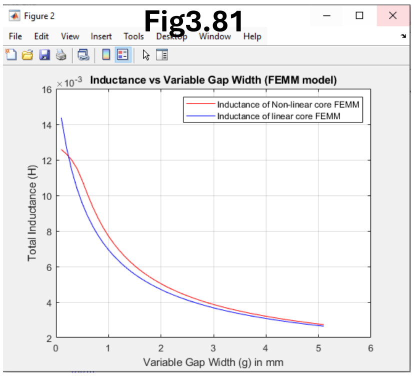
The equation relating flux linkage (ψ) and winding current (I): ψ = LI [**eq3.71**]

Task 3.8

We can use FEA modelling technique to find the Flux linkage (ψ) as well as the winding current for the windings (I) which can then be used to calculate the inductance at various variable gap lengths.

After writing the relevant code which performs a for loop where each iteration translates the armature towards the core (decreasing variable air gap 5.1mm to 0.1mm in steps of 0.1mm) and calculating the associated induction value at each step for one winding, we can multiply the calculated inductance by 2 as we have 2 symmetrical windings and we can store these results and plot them on a graph.

The graph showing how the inductance varies with air gap distance using FEA technique for both Linear and Non Linear core model is shown in **Fig3.81**



The analytical method of calculating inductance with fringing effects (**Fig3.61**) is similar to the inductance calculated with linear FEMM model as both models take into account fringing effects.
However for the non linear model, the core is saturated when the airgap is small leading to a decrease in the rate of increase of inductance as air gap length decreases and so inductance curve is different to analytical method.

Task 3.9

One assumption in the analytical method is that there is no flux leakage and that the core is not saturated, this means the flux passing through each component is assumed to be uniform.

One assumption in the FEMM method is that the system assumes DC static conditions, so the effects of eddy currents are ignored leading to energy dissipation in the system which overestimates stored magnetic energy. Since in later sections we use Femm method to calculate the force based on this overestimated stored energy, we will calculate higher force values than in practice.

Case 3D

Task 3.10

The energy balance law states that energy in an isolated system must be balanced and is as $E_{elec} = E_{mech} + E_{mag} + E_{loss}$

Where E_{elec} is the energy supplied to the system, E_{mech} is the energy transferred to mechanical energy, E_{mag} is the energy supplied to the magnetic field and E_{loss} is the energy loss in the system (Resistive losses in windings, AC winding loss, Hysteresis and Eddy currents).

In this system we assume that it is linear (The core does not enter the saturation region) and that we neglect Energy loss $E_{loss} = 0$.

Finding ΔE_{mech} allows us to calculate electromagnetic force $\Delta E_{mech} = F*\Delta X$ where F is the average electromagnetic force exerted on the armature.

First consider the armature static in open configuration (gap = 5.1mm):

Since it is static, there is no movement $\Delta E_{mech} = 0$, so $\Delta E_{elec} = \Delta E_{mag}$

When a winding current I is applied at time t= 0 such that the current ramps up from 0A – 10A, we can calculate ΔE_{elec} and therefore ΔE_{mag}

ΔE_{elec} can be calculated by integrating current (I) and induced voltage (V) over time, using Faraday's law E $= \int \frac{d\psi}{dt}$, we obtain $\Delta E_{elec} = \int i(t).\frac{d\psi}{dt}dt$ and since ψ = LI we can rearrange the equation to find $\Delta E_{elec} = \Delta E_{mag1} = \frac{1}{2}L_1I_1^2$ where L is inductance and I is current

Secondly when armature is in closed position (gap = 0.1mm), since the core is linear we obtain the same equation $\Delta E_{mag2} = \frac{1}{2}L_2I_2^2$

Considering the movement from open position to closed position, assuming movement is slow (induced voltage from motion is 0)

$\Delta E_{elec} = \int_{t1}^{t2} i(t)v(t)$ using faraday's law again $\Delta E_{elec} = \int_{t1}^{t2} I(t).\frac{d\psi}{dt}.dt = \int_{\psi1}^{\psi2} I.d\psi$, now we can use $\Delta E_{mech} = \Delta E_{elec} - (\Delta E_{mag2} - \Delta E_{mag1})$ to find the electromagnetic force. $\Delta E_{mech} = \int_{\psi1}^{\psi2} I.d\psi - (\frac{1}{2}L_2I_2^2 - \frac{1}{2}L_1I_1^2) = F*\Delta x$.

Task 3.11

We can plot the flux linkage against current graphs of the actuator at different variable gap lengths using analytical (shown in **Fig3.111** and **Fig3.112**) and FEMM methods (shown in **Fig3.113** and **Fig3.114**).
First using the analytical method, we can plot the graph for the fringing and non fringing case by using the equation $\psi = LI$ as this is the actuator is linear in both these cases. Where ψ is the flux linkage, L is the inductance and I is the current. Using inductance values calculated in task 3.5 for both fringing and non fringing cases we can calculate flux linkage against current for various air gap lengths.
First we extract the inductance value for the gap we want (5.1mm,4.1mm,3.1mm,2.1mm,1.1mm and 0.1mm), then we use a for loop to iterate through a range of current values (0A to 10A with a step of 2.5A) to plot the curve for that specific gap. Then we take the inductance of the next selected gap length and repeat.

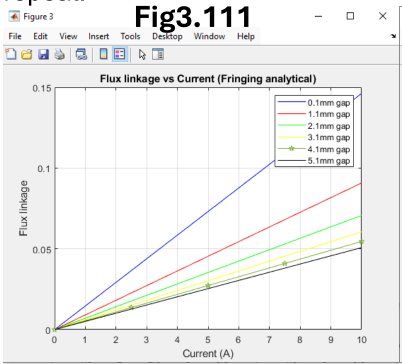


Fig3.111

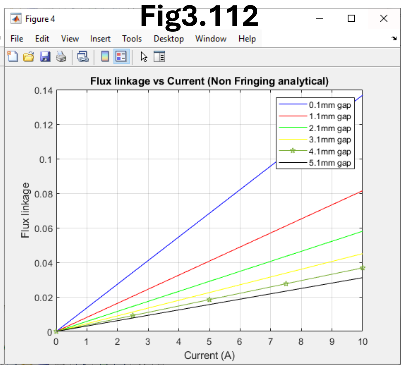


Fig3.112

We can then calculate the co energy for the open (5.1mm gap) and closed (0.1mm) armature positions. Since co energy (W_{co}) is the area under the curve, we can find the co energy by using the equation $W_{co} = \frac{1}{2} * \psi_{max} * I_{max}$ where ψ_{max} is the maximum flux linkage for the gap position and I_{max} is the maximum current value (10A).
Fringing: $W_{co\text{-}open}$ = 0.2537J, $W_{co\text{-}closed}$ = 0.7309J
Non fringing: $W_{co\text{-}open}$ = 0.1555J, $W_{co\text{-}closed}$ = 0.6839J

Next, using FEA method we can plot the graph for linear and non linear cases.
For this method, we first set the armature position on simulation to be 5.1mm and iterate through the range of current values (0A to 10A with a step of 1A) using a for loop and each iteration we set the current of the winding, we then use the mi_analyse() function and extract the flux linkage using the mo_getcircuitproperties() function, we repeat this for all current values in the loop then move the armature 1mm to the left using the mi_movetranslate(x,y) function and repeat the above steps.

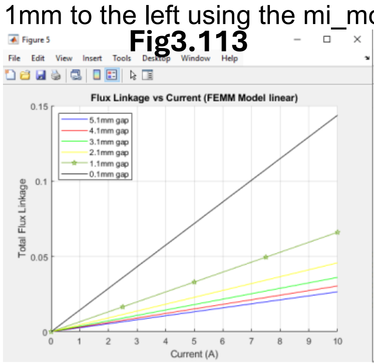


Fig3.113

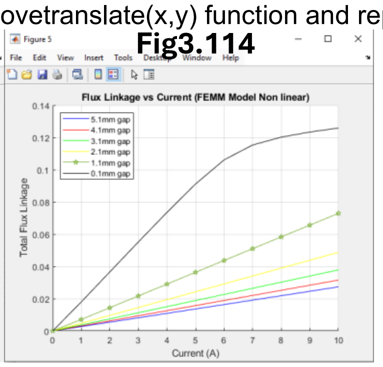


Fig3.114

For the linear model we can calculate the co energy by integrating under the curve using the trapz() function and non linear we set the armature position and can calculate the co energy at that position using the mo_blockintegral(17) function
Linear: $W_{co\text{-}open}$ = 0.1322J, $W_{co\text{-}closed}$ 0.7186J
Non linear: $W_{co\text{-}open}$ = 0.1363, $W_{co\text{-}closed}$ = 0.8034J

We can see that analytical methods produce a flux linkage versus current curve similar to the FEMM model linear curve showing that our equations used to approximate the circuit is accurate under linear conditions.
For the non linear curve for the 5.1mm gap the flux curve is not linear for the 5.1mm gap showing that as current increases the core becomes saturated and flux linkage no longer increases.

Task 3.12

Using the flux linkage curves plotted in **task 3.11** we can calculate electromagnetic force and plot a force airgap curve.

First using analytical method we can perform the co energy calculation using the same method as above for fringing and non fringing cases ($W_{co} = \frac{1}{2} * \psi_{max} * I_{max}$).

Next using the equation $\Delta E_{mech} = \frac{F}{\Delta x}$ [**eq3.121**] where ΔE_{mech} is the change in co energy (co energy of given gap length – co energy of closed position), F is the average force and Δx is the distance the armature moves (given gap position – closed position (0.1mm))

Performing these calculations for the fringing and non fringing cases we obtain the graph shown in **Fig3.121.**

Second, using the FEA method, for the linear case we can use the trapz() function for each gap length flux linkage vs current curve to calculate the co energy values. Then using [**eq3.121**] we can calculate average force against air gap position.

For the non linear case, we use the mo_blockintegral(17) function for the air gap lengths to find all of the co energies at those points and carry out the same calculation using [**eq3.121**] to find the force.

The Force airgap curves for FEA method are shown in **Fig3.122.**

Fig3.121

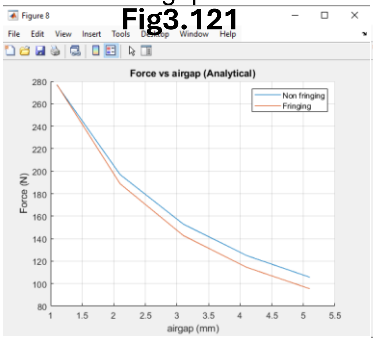
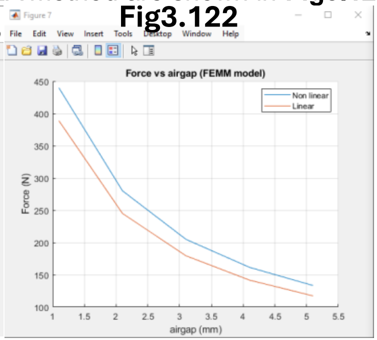


Fig3.122



The FEMM model graph shows that the average force is higher than the analytical model graph.

The difference in these values can be because the FEMM model calculates flux linkage by setting the winding current and setting armature position then analysing the circuit to find the flux linkage, the co energy is also calculated using the trapz() function for linear and mo_blockintegral(17) function giving more accurate co energy values to calculate the force with. While the analytical method uses equations approximating the behaviour of the circuit to calculate the flux linkage which can lead to the lower force values seen in the above graphs.

DEENG20005 Electrical Energy Conversion and Supply 24-25

Jake Evans (student no. [redacted]

Part 4. DC Machines

Case 4A Simulink Modelling of Brushed Permanent Magnet DC machines

Task 4.1

The equivalent circuit diagram for a steady state brushed PM DC machine is shown in **Fig4.11**, and for PM DC machine in dynamic state it is shown in **Fig4.12**

Fig4.11

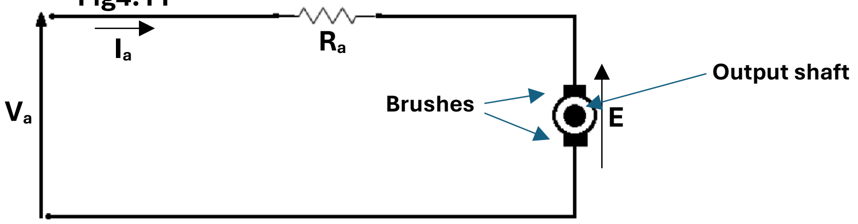
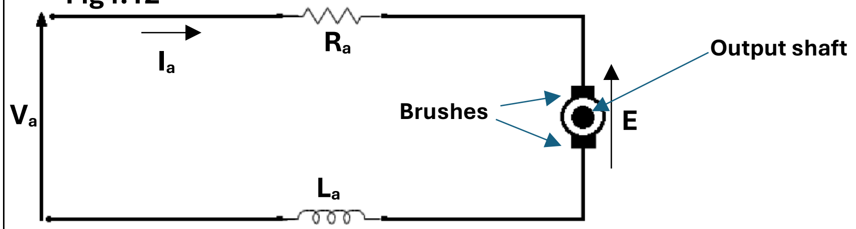


Fig4.12



Voltage equation for the steady state circuit (**Fig4.11**): $V_a = I_aR_a + E$ [**eq4.11**]and can be written in terms of the electro – mechanical conversion constant as $V_a = I_aR_a + Ke\omega$.
Where V_a is applied DC voltage to the armature, I_a is current through armature, R_a is resistance of armature, Ke is electro – mechanical conversion constant and ω is angular velocity of the output shaft.

Voltage equation for dynamic state circuit (**Fig4.12**):$V_a = I_aR_a + Ke\omega + L_a\frac{dI_a}{dt}$ [**eq4.12**], The new term $L_a\frac{dI_a}{dt}$ is the inductance of the winding multiplied by the rate of change of current in the armature.

The steady state describes the system when all variables have settled to a constant value (no changing current or ω). In this case the back emf E = Ke*ω is constant as angular velocity ω is constant and the inductance of the armature winding does not affect the system as I_a is constant ($\frac{dI_a}{dt}$ = 0).

Dynamic state describes the system when the motor is changing speed ω, in this case the back emf is not constant E = Ke*ω (ω is varying) and the armature winding experiences inductance effects ($\frac{dI_a}{dt}$ is not equal to 0) the term $L_a\frac{dI_a}{dt}$ represents the induced voltage in the winding due to the changing current.

Task 4.2

The steady state circuit operating in Motor mode has the following torque equation:
$T_{em} = T_{load} + T_f$
Where T_{em} is total torque produced by the motor, T_{load} is the torque of the load and T_f is the torque generated by the friction opposing motion.
Since T_{em} is given by: $T_{em} = Ke*I_a$ and T_f is given by: $T_f = B*\omega$ where B is the Rotor Viscous Friction constant.
We can rewrite the equation as $Ke*I_a = T_{load} + B*\omega$

The dynamic state circuit operating in motor mode has the following torque equation:
$T_{em} = T_{load} + T_f + T_{inertia}$
Where $T_{inertia}$ is the generated rotary inertia.

Task 4.3

For a PMDC machine, we can calculate the no load speed using the steady state torque equation $T_{em} = T_{load} + T_f$
We can rewrite it as $Ke*I_a = T_{load} + B*\omega$ and as $T_{load} = 0$, we can rearrange it to obtain $T_{em} = T_f$ and so
$\omega = \frac{Ke*I_a}{B}$ [**eq4.31**].

Since we do not have the steady state current I_a we can use relationship $V_a = I_aR_a + Ke*\omega$ and rearrange to make I_a the subject to substitute into [**eq4.31**] to express ω in terms of the values we know.

Substituting and rearranging we obtain the expression $\omega = \frac{Ke*V_a}{Ke^2+B*R_a}$

Since we have both $V_a = 50V$, Ke = 0.025, B = 0.02 and $R_a = 1$.
$\omega = \frac{50*0.025}{0.025^2+0.02*1}$= 60.6rad/s

which we can express in terms of revolution per minute (RPM)

rotational speed (RPM) = $\frac{60.6}{2\pi}*60$ = 579RPM

Stall torque is given by $T_{stall} = \frac{V_a*Ke}{R_a}$
$T_{stall} = \frac{50*0.025}{1}$= 1.25N.m

Task 4.4

We can use Simulink to model the PM DC machine, using 3 levels:
The top level (shown in **Fig4.41**) describes the whole circuit, using input voltage, armature current output and rotor angular velocity output to produce a simulation waveform (shown in **Fig4.41**)

The middle level (shown in **Fig 4.42**) describes the Torque speed control loop and is modelled based on the torque equation: $T_{em} = T_{load} + T_f + T_{inertia}$, as T_{load} is constant and set to 0, we set it to 0 in this circuit. We can use the fact that $T_{em} = Ke*I_a$, $T_f = B\omega$ and $T_{inertia} = \mathbf{J}\frac{d\omega}{dt}$ and rearrange the T_{em} equation to find the following expression $\omega = \int \frac{K_e * I_a - B\omega - 0}{J}$ [**eq4.41**] as well as $E = Ke*\omega$. Which allows us to create the circuit above (Where J is the rotor mass moment of inertia).

The bottom level (shown in **Fig4.43**) describes the voltage current loop and is modelled based on the voltage equation $V_a = E + I_aR_a + L_a\frac{dI}{dt}$ which can be rearranged into $I_a = \int \frac{V_a - E - I_a * R_a}{J}$.

Fig4.41

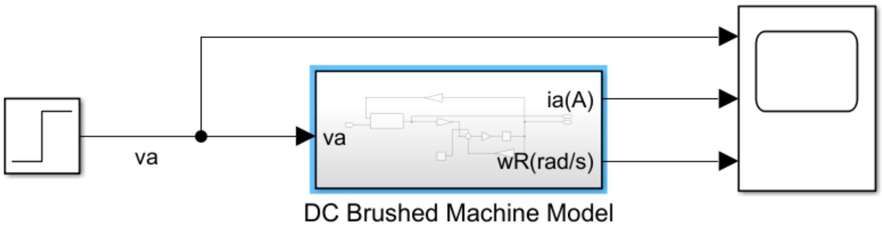


Fig4.42

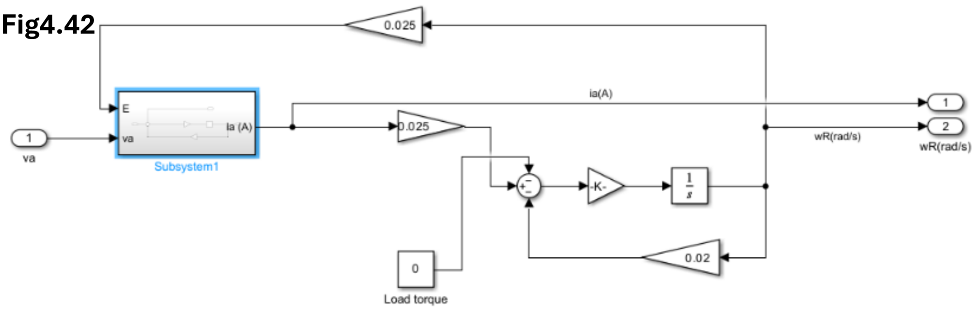
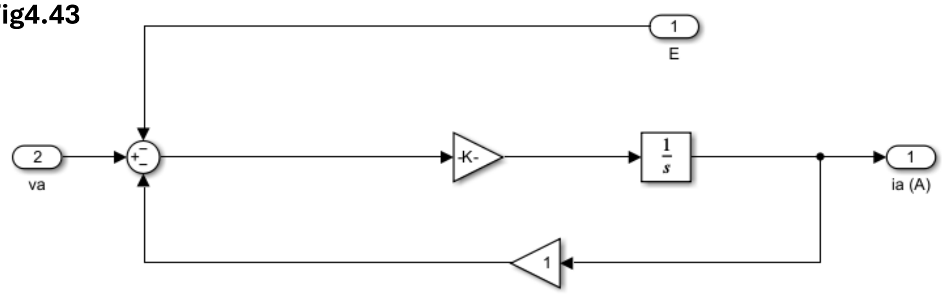


Fig4.43



Task 4.5

We can use the completed circuit above to simulate the no load speed with a run time of 20 seconds to ensure it reaches steady state. The waveform we receive is shown in **Fig4.51**.

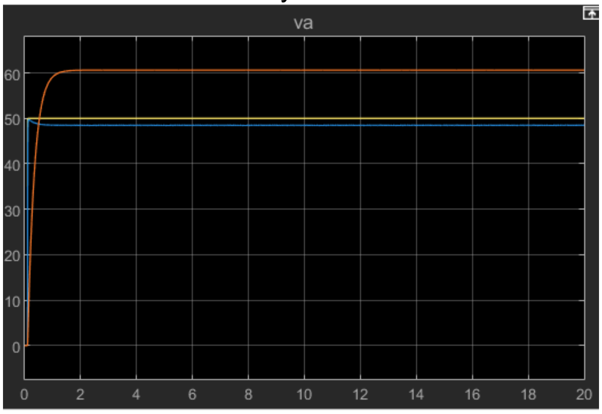


Fig4.51

The no load speed shown on the simulation is 60 rad/s which roughly equal to the calculated no load speed calculated in task 4.3, validating the design.

Task 4.6

Fig4.61 shows the modified torque-speed level with the varying load and **Fig4.62** shows the waveform

Fig4.62

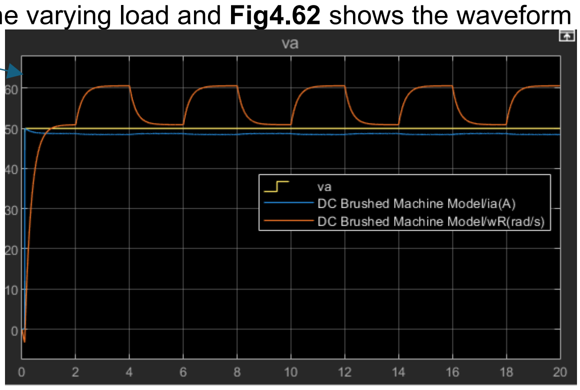
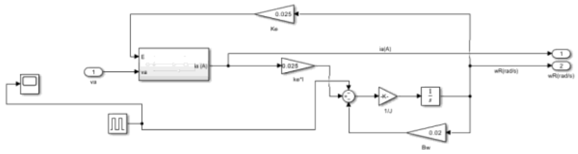


Fig4.61



The varying current can be explained by the torque equation Ke*I_a = T_{load} + B*ω, rearranging for I_a = (T_{load} +B*ω)/Ke where T_{load} is torque of load. Since the load is varying and ω is changing and all other terms in the equation are constant I_a will vary synchronised with the period of T_{load}.

In startup state ω = $\int \frac{K_e * I_a - B\omega - T_{load}}{J}$ using torque equation. Initially B*ω is 0 and I_a is very small, so ω is determined by T_{load}, since it is negative the angular speed of rotor will initially be negative and become positive as I_a increases.

In steady state, ω = $\frac{Ke * I_a - T_{load}}{B}$, since I_a, Ke and B are constant, ω is determined solely by T_{load} and so when T_{load} = 0, ω is at maximum and when T_{load} =0.2, ω is at minimum value.

Case 4B Simulink Modelling of Wound Field DC Machines

Task 4.7

The dynamic equation for a field wound DC machine is given by $V_a = I_aR_a + E + L_a\frac{dIa}{dt}$ [**eq4.12**]

Since we know that $E = Ke*\omega$ and $T_{em} = Ke*I_a$ and for a field wound DC machine the electro-mechanical conversion constant $Ke = M*I_f$, we can substitute these terms into [**eq4.12**] to get $V_a = I_aR_a + M*I_f*\omega + L_a\frac{dIa}{dt}$.

Where I_f is field current and M is the mutual coupling parameter.

We also obtain the following equation for torque:

$T_{em} = T_{load} + T_f + T_{inertia}$

Where T_{em} is total torque of system, T_{load} is torque on the load, T_f is torque due to friction and $T_{inertia}$ is generated rotary inertia.

Since $T_{em} = Ke*I_a = M*I_f*I_a$, $T_f = B*\omega$ and $T_{inertia} = J\frac{d\omega}{dt}$, we can substitute into the above torque equation to find:

$M*I_f*I_a = T_{load} + B*\omega + J\frac{d\omega}{dt}$.

Assuming zero load torque the equation can be simplified to:

$M*I_f*I_a = B*\omega + J\frac{d\omega}{dt}$.

Task 4.8

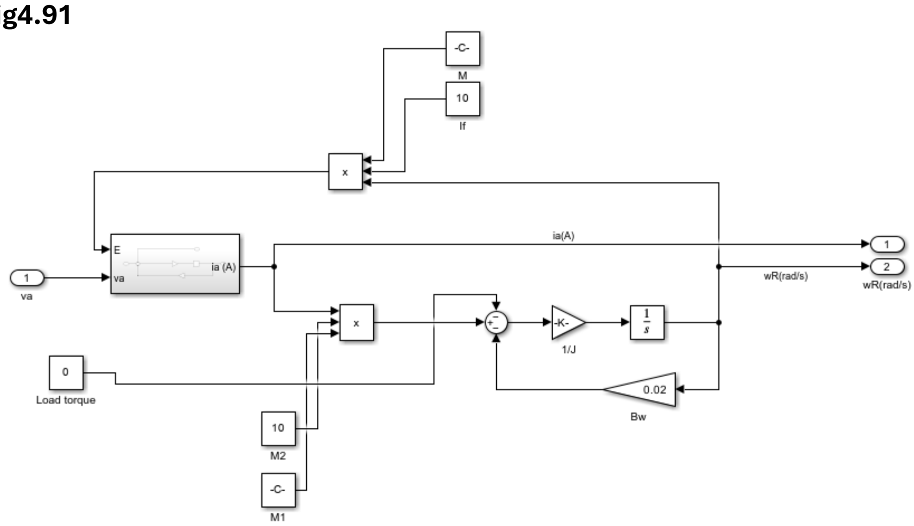
Since $Ke = M*I_f$, we can rearrange to calculate M using the given Ke and I_f values:

$M = Ke/I_f = 0.025/10 = 0.0025H$

Task 4.9

We can use the voltage and torque equations to model the field wound DC machine in Simulink
The top level and inner level are kept the same.
The middle level (torque speed control) is shown in **Fig4.91**

Fig4.91



This is designed using the torque equation rearranged: $\omega = \int \frac{M*I_f*I_a - B\omega}{J}$ and $E = M*I_f*I_a$.

Task 4.10

Stall torque can be calculated using $T = \frac{V_a*M*I_f}{R_a} = \frac{50*0.0025*5}{1} = 0.625$Nm
Using the torque equation, we can predict how the no load speed will change if the field current I_f is reduced from 10A to 5A in the steady state (the following equation accounts for friction torque $B*\omega$)

$T_{em} = T_f + T_{inertia}$, $M*I_f*I_a = B*\omega + J\frac{d\omega}{dt}$

Steady state so $\frac{d\omega}{dt} = 0$

$\omega = \frac{M*I_f*I_a}{B}$

So, for a decreased field current (I_f), no load speed should decrease.
We can validate this using the simulation and reducing the field current to 5A, the resulting waveform (**Fig4.101**) shows a decreased no load speed value than before (60rad/s decreases to 30 rad/s).

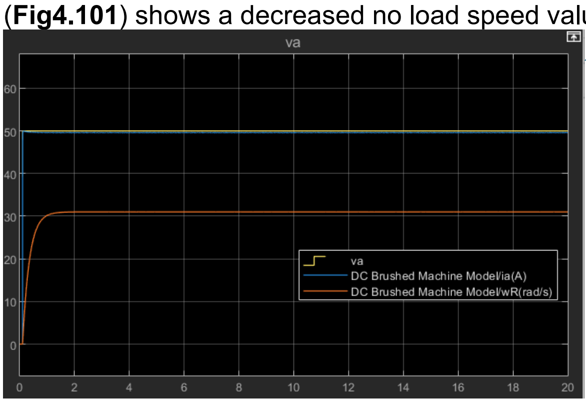


Fig4.101

Case 4C Case Study of a Wound Field DC Machine

Task 4.11

Using the rated conditions of the machine we can find the torque constant (Ke) and mutual inductance (M)
Using $P_{mech} = T_{em}*\omega$ [**eq4.111**], $T_{em} = Ke*I_a$ [**eq4.112**] and $Ke = M*I_f$ [**eq4.113**]
First convert rotational speed in RPM to rad/s $\omega = \frac{600*2\pi}{60} = 157$

Using [**eq4.111**]: $T_{em} = \frac{P_{mech}}{\omega} = \frac{1{,}500{,}000}{157} = 23{,}873$ N.m

Using [**eq4.112**]: $Ke = \frac{Tem}{I_a} = \frac{23{,}873}{2650} = 9.01$N.m.A^{-1}

Using [**eq4.113**]: $M = \frac{Ke}{I_a} = 9.01/83 = 0.1087$H

Task 4.12

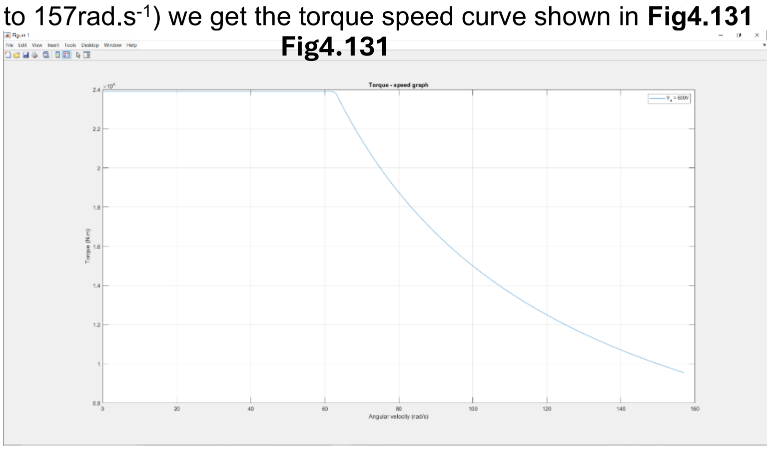
We can calculate efficiency of the machine by finding input power (P_{in}) of the machine

$P_{in} = V_a*I_a + V_f*I_f = 600*2650 + 600*83 = 1{,}639{,}800$W
Efficiency = $\frac{P_{out}}{P_{in}}*100 = \frac{1{,}500{,}000}{1{,}639{,}800}*100 = 91.5\%$

Task 4.13

Initially for low speeds (ω), since the output power does not exceed the rated output power (1500kW) which is calculated by P = T*ω, the machine will operate at it's maximum rated torque (calculated earlier – 23873N.m).
As speed increases, the output power will increase until it reaches the rated power where after that in order for speed to increase the torque will need to decrease.
We can calculate the maximum torque after this point using the equation $T_{max} = P_{max}/\omega$.
Calculating these maximum torque values at each speed (0 to 1500RPM) converted to radians/second (0 to 157rad.s^{-1}) we get the torque speed curve shown in **Fig4.131**

Fig4.131



This is consistent with the rated conditions of the machine as using the equation P = T*ω, we can see that the speed where the maximum torque starts to decrease will be 62.8 rad/s which is validated by the graph where the maximum torque begins to decrease after 62rad/s

Task 4.14

Maximum torque can be calculated using the equation $T_{max} = M*I_a*I_f$, in order to reduce torque the field current (I_f) needs to be decreased

This will decrease V_a as well as $V_a = \frac{T*R_a}{M*I_f} + M*I_f*\omega$ which will affect the input power.

Output power will be affected by Torque friction ($B*\omega$)

Fig4.141 shows the efficiency speed curve of the motor.

Fig4.141