11/03/2017 **Quiz02 Math331** Section B **Student** Matt Sirota

- **Write out each formula, procedure, and then present the desired numbers.**
- **Any answer without the desired formula and details gets no credit.**
- **α=0. 05, pf(100,1,12)=1−0.000001, pf(21.81, 2,12)=1−0.0001, pf(1.34,2,12)=1−0.298.**

An evaluation of a new coating applied to three different materials was conducted at two different laboratories. Each laboratory tested 3 samples from each of the treated materials. The recorded results are listed as below. We want to check the significance of materials, laboratories and interaction.

	Materials (B)		
LABS (A)	1	2	3
1	4.1	3.1	3.5
	3.9	2.8	3.2
	4.3	3.3	3.6
2	2.7	1.9	2.7
	3.1	2.2	2.3
	2.6	2.3	2.5

1. Evaluate overall mean, column means, row means and cell means. ----------20pts

overall mean = $4.1+3.1+3.5+3.9+2.8+3.2+4.3+3.3+3.6+2.7+1.9+2.7+3.1+2.2+2.3+2.6+2.3+2.5$

$= 54.1 / 18 =$ **3.006** $\cdot \frac{1}{n}\sum_{i=1}^{n} x_i$

Lab 1 means

$r=1 \quad \bar{x} = 3.567$

$r=2 \quad \bar{x} = 3.3$

$r=3 \quad \bar{x} = 3.733$

$c=1 \quad \bar{x} = 4.1$

$c=2 \quad \bar{x} = 3.067$

$c=3 \quad \bar{x} = 3.433$

Lab 2 means

$r=1 \quad \bar{x} = 2.433$

$r=2 \quad \bar{x} = 2.533$

$r=3 \quad \bar{x} = 2.467$

$c=1 \quad \bar{x} = 2.8$

$c=2 \quad \bar{x} = 2.133$

$c=3 \quad \bar{x} = 2.50$

cell means

	1	2	3	average
1	4.1	3.067	3.433	3.533
2	2.8	2.133	2.50	2.478
average	3.45	2.60	2.965	3.006

2 Evaluate Sum of Squares due to two factors, interaction and error. ---------- 40pts

$$SSE = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n_{ij}} \left(x_{ijk} - \bar{x}_{ij}\right)^2$$

$= (4.1-4.1)^2 + (3.9-4.1)^2 + (4.3-4.1)^2 + (3.1-3.067)^2 + (2.8-3.067)^2 + (3.3-3.067)^2$

$+ (3.5-3.433)^2 + (3.2-3.433)^2 + (3.6-3.433)^2 + (2.7-2.8)^2 + (3.1-2.8)^2 + (2.6-2.8)^2$

$+ (1.9-2.133)^2 + (2.2-2.133)^2 + (2.3-2.133)^2 + (2.7-2.5)^2 + (2.3-2.5)^2 + (2.5-2.5)^2$

SSE = 0.60

$$SSA = c\sum_{i=1}^{r} n_{ij}\left(\bar{x}_i - \bar{\bar{x}}\right)^2 = 3\cdot 3\left[(3.533-3.006)^2 + (2.478-3.006)^2\right]$$

SSA = 5.01

$$SSB = r \sum_{j=1}^{c} n_{i,j} (\bar{x}_j - \bar{x})^2$$

$$= 2 \cdot 3 \left[(3.45 - 3.006)^2 + (2.6 - 3.006)^2 + (2.9665 - 3.006)^2 \right] = 2.18 = SSB$$

$$SSAB = \sum_{i=1}^{r} \sum_{j=1}^{c} n_{i,j} (\bar{x}_{i,j} - \bar{x}_i - \bar{x}_j + \bar{x})^2$$

$$= (4.1 - 3.533 - 3.45 + 3.006)^2 + (3.067 - 3.533 - 2.6 + 3.006)^2 + (3.433 - 3.533 - 2.9665 + 3.006)^2$$
$$+ (2.8 - 2.478 - 3.45 + 3.006)^2 + (2.133 - 2.478 - 2.6 + 3.006)^2 + (2.5 - 2.478 - 2.9665 + 3.006)^2$$

$$= 0.0447765 \cdot 3 = 0.1343 = SSAB$$

98/100

3. Determine the degree of freedom for all Sum of Squares. ---------- 10pts

$df_A = r - 1 = (1)$

$df_B = c - 1 = (2)$

$df_{AB} = (r-1)(c-1) = (2)$

$df_E = n - cr = 18 - (2)(3) = (12)$

$df_{total} = n - 1 = (17)$

4. Produce the two ANOVA table based on the results obtained in the above. ----------15pts

p-value: $1 - pf(f, df, n-rc)$

Source	df	SS	MS	F	p-value
A	1	5.01	5.01	100.2	$1-pf(100,1,12) = 0.000001$
B	2	2.18	1.09	21.8	$1-pf(21.81,2,12) = 0.0001$
A×B	2	0.1343	0.06715	1.34	$1-pf(1.34,2,12) = 0.298$
Error	12	0.60	0.05		
Total	17	7.9743			

$MSA = \frac{SSA}{r-1}$ $\quad MSB = \frac{SSB}{c-1}$ $\quad MSAB = \frac{SSAB}{(r-1)(c-1)}$ $\quad MSE = \frac{SSE}{n-cr}$

$F_1 = \frac{MSA}{MSE}$ $\quad F_2 = \frac{MSB}{MSE}$ $\quad F_3 = \frac{MSAB}{MSE}$

SST = $\sum SS$

5. Conclude your study. ----------15pts

$\alpha = 0.05$

H_{01}: all rows have a common population mean

$0.000001 < 0.05$ so Reject

H_{02}: all columns have a common population mean

$0.0001 < 0.05$ so reject

H_{03}: all cells have a common population mean

$0.298 > 0.05$ so accept