9/22/2017 Quiz01 Math331 Section B **Name** Matthew Sirota

According to the historical data, the average body length of male graduates in New Jersey is 174cm. Recently, we got the following sample of male graduates in Stevens Institute of Technology: ~~170, 172, 177, 168, 169, 184, 180, 175, 185, 176, 178, 181, 174, 179, 166, 185.~~

165, 166, 168, 169, 170, 172, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185

1. Provide 5-number summary of the sample and construct the box plot. --15pts.

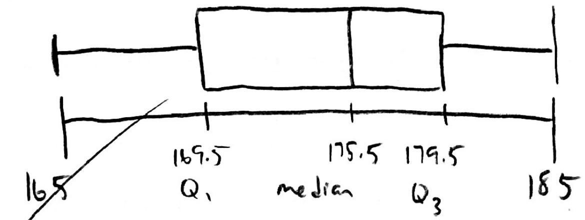
Minimum: 165

Q_1 = 169.5

median: 175.5

Q_3: 179.5

maximum: 185



2. Evaluate sample mean, sample variance and sample standard deviation, and then identify the tail skewness sample. --15pts

n = 16

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{16}\sum_{i=1}^{16} x_i = 174.94$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{15}\sum_{i=1}^{16}(x_i - 174.94)^2 = 38.20$$

$$s = \sqrt{s^2} = \sqrt{38.20} = 6.18$$

-5

3. Assume a simple and random sample $X_1, \ldots, X_n$ from the population $X \sim U(0, 2\theta+1)$, the uniform distribution with probability density $f(x) = \frac{1}{2\theta+1}$ for $x \in [0, 2\theta+1]$. Find the moment estimator $\hat{\theta}$. ------------------20pts

$$E(x) = \int_0^{\infty} x f(x)\,dx = \int_0^{2\theta+1} \frac{x}{2\theta+1}\,dx = \frac{x^2}{2(2\theta+1)}\Big|_0^{2\theta+1}$$

$$\frac{(2\theta+1)(2\theta+1)}{2(2\theta+1)} = \frac{2\theta+1}{2} = E(x)$$

$$\bar{X} = \frac{2\theta+1}{2} \qquad \hat{\theta} = \frac{2\bar{X}-1}{2}$$

4. Assume $X_1, \ldots, X_n$ is a simple and random sample from the population $X \sim G(p)$ with probability mass function $f(x) = p(1-p)^{x-1}$, for $x = 1, 2, \ldots,$ and $p \in (0,1)$. Derive the maximum likelihood estimator $\hat{p}$.-------20pts

$$L(p, x) = \prod_{i=1}^{n} f(x) = \prod_{i=1}^{n} p(1-p)^{x_i - 1} = p^n (1-p)^{\sum_{i=1}^{n} x_i - n}$$

$$\log L(p,x) = n \log p + \left(\sum_{i=1}^{n} x_i - n\right)\log(1-p)$$

$$\frac{d \log L(p,x)}{dp} = \frac{n}{p} - \frac{\sum_{i=1}^{n} x_i - n}{1-p}$$

$$\hat{p} = \frac{1}{\bar{X}} \qquad \bar{X}$$

-2

93/100

Next two questions are based on the sample 170, 172, 177, 168, 169, 184, 180, 175, 165, 176, 178, 181, 174, 179, 166, 185.

5. If $X_1,\dots,X_n$ is a simple and random sample from the population $X\sim N(\mu,\sigma^2)$, $\bar{X}$ is the average of the sample, then $T=\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{\sigma^2}\sim\chi^2_{n-1}$. For $\sigma^2=38$, Compute $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\sigma^2}$ based on the above data, and then compute $P\left(T>\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\sigma^2}\right)$.(Hint: $pchisq(x,y)=P(\chi^2_y\le x)$, pchisq(15,15)=0.5486, pchisq(15,16)=0.4754, pchisq(10,15)=0.1803, pchisq(10,16)=0.1334) ---10pts

$\bar{X} = 174.94$ $\quad \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{38} = \frac{572.94}{38} = 15.0774$

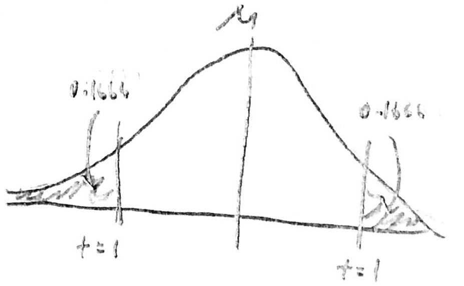
$P(T>15) = 1-P(T\le 15) = 1-0.5486 = 0.4514$

6. If $X_1,\dots,X_n$ is a simple and random sample from the population $X\sim N(\mu,\sigma^2)$, $\bar{X}$ defines the sample average and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$ gives the sample variance, then $T=\frac{\bar{X}-\mu}{\sqrt{S^2/n}}\sim t_{n-1}$. Suppose $\mu=173.39$, based on the above data, (i) compute $\frac{\bar{x}-\mu}{\sqrt{s^2/n}}$ and then $P\left(T>\frac{\bar{x}-\mu}{\sqrt{s^2/n}}\right)$. (ii) Construct the level 1-0.3332 Student's t CI for μ. (Hint: pt(x,y)=P($t_y\le x$), pt(0.96,15)=0.8239, pt(0.96,16)=0.8243, pt(1,15)=0.8334, pt(1,16)=0.8339). ---------- 20pts

i) $\frac{\bar{X}-\mu}{\sqrt{s^2/n}} = \frac{174.94-173.39}{\sqrt{38.20/16}} = 1.00313$

$P(T>1) = 1-P(T\le 1) = 1-0.8334 = 0.1666$

ii) $\alpha/2 = 0.1666$



$-1 = \frac{\bar{X}-\mu}{\sqrt{s^2/n}} = 1$

$-1.545 = 174.94-\mu = 1.545$

$173.39 \le \mu \le 176.485$

I am 66.68% confident that the true values for μ are between 173.39 and 176.485