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Radial basis functions with application to finance: American put option under jump diffusion

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ABSTRACT

In this paper, we consider a partial integro-differential equation (PIDE) problem with a free boundary, arising in an American option model when the stock price follows a diffusion process with jump components. We use a front-fixing transformation of the underlying asset variable to fix the free boundary conditions and approximate the integral term by the Laguerre polynomials. We use the Radial basis functions (RBF) method to achieve an implicit nonlinear system of first order equations and apply the Crank–Nicholson scheme. We apply the Predictor–Corrector method, to deal with the system of nonlinear equations. The proposed method is stable and the results are in agreement with those obtained by other numerical methods in literature.

1. Introduction

There is a great bulk of literature and heuristic evidence showing that the normal distribution assumption for returns is a poor fit to data, in part because the observed area in the tails of the distribution is much greater than the normal distribution permits, [1]. If the probability distribution of the stock price at any given future time is not lognormal then we underprice or overprice call and put options, depending on the distributions's tails, [2]. One heuristic evidence is the behavior of underlying assets in real options modeling such as energy prices, oil and natural gas prices. These quantities can exhibit peaks and spikes, for example, in one case the unit price of natural gas jumped from 30 euros to more than 1500 euros in one day during a period of short supply, [3]. One possible remedy is to assume an alternative distribution for the returns of the asset price that, like the normal distribution, is infinitely divisible but that has more area in the tails. For example, the inverse Gaussian (NIG) distribution is a good fit to some stock and interest rate data, [1]. Another idea to overcome the shortcomings of the inaccurate pricing of the contingent claims is using more appropriate models such as stochastic volatility models (Heston model) or models where the stock price may experience occasional jumps rather than the continuous changes. As a consequence, in contrast to the original Black–Scholes framework where for most of the option valuation problems closed-form formulas exist or standard numerical methods could be applied, in nonstandard financial models more complicated and precise techniques are required. We have chosen to extend the Black–Scholes model, by exploring the jump diffusion (Poisson) model, see [4] and radial basis function for approximation of the option prices.

In Section 2 we discuss the original Black–Scholes model and expose the mathematical model for pricing an American option under the jump-diffusion process. We postulate the option valuation problem as a partial integral–differential equation that is an extension of the famous Black–Scholes equation with an integral part that reflects the jumps of the asset price. We apply a front-fixing transformation of the PDE part and give a numerical recipe for the semi-bounded integral part.

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In Section 3 we point out the main features of the finite difference schemes as this is one of the most explored numerical methods in computational finance and origin of numerical techniques such as operator splitting and the predictor–corrector methods. In contrast to the Monte Carlo method and like finite difference schemes (FDS), the RBF method treats the option valuation problem without simulating the random movement of the asset price. We make a parallel between the FDS and the structure and application of the RBF method. We discuss the numerical accuracy of the RBF method.

In Section 4 we present the approximation of the Black–Scholes equation with the exposed radial basis functions method and the option valuation problem is reduced to solving a standard non-singular square linear system. We compare the RBF method with the method of lines for parabolic equations.

In Section 5 consists of computational results obtained by the presented RBF method which are compared with other numerical and analytical methods for pricing American options proposed in literature [5–9].

In the conclusion, we give some final remarks for the advantages of the presented radial basis function method and its possible applications.

2. Mathematical model: The modified Black–Scholes PDE

Usually in financial literature, as a mathematical model for the movement of the asset price under the risk-neutral measure is considered a standard *geometric Brownian motion* diffusion process with constant coefficients r and σ:

$$dS/S = rdt + \sigma dW_t. \tag{1}$$

The contract to be priced is a discretely monitored double barrier knock-out call option. If τ is the time to expiry T of the contract, i.e. $\tau = T - t$, $0 \le t \le T$, the price $V(S,t)$ of the option satisfies the Black–Scholes partial differential equation

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0. \tag{2}$$

In this article we explore a more general model for the random movement of the asset price. In order to make our analysis concrete, we concentrate our attention on a classical problem in financial literature, i.e. valuation of American put options under jump diffusion. The modified stochastic differential equation that models jumps is:

$$dS/S = rdt + \sigma dW_t + (\eta - 1)dq \tag{3}$$

where S is the underlying stock price, r—interest rate, σ—volatility, dW_t increments of the Gauss–Wiener process, dq—a Poisson process with arrival rate (intensity) λ defined as:

$$dq = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ 1, & \text{with probability } \lambda dt \end{cases}$$

and $\eta - 1$ impulse function producing a jump from S to $S\eta$, where $k = E(\eta - 1)$ is the expected relative jump size.

In contrast to Eq. (1) in the new Eq. (3) the path followed by S is continuous most of the time while *finite negative or positive jumps* will appear at discrete points in time. As a result the Black–Scholes equation (2) is modified as a partial integral–differential equation by adding an integral part that reflects the jump diffusion structure of the process [10]:

$$-\frac{\partial V}{\partial t} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \lambda k)S\frac{\partial V}{\partial S} - (r + \lambda)V + \lambda \int_0^\infty V(S\eta)\rho(\eta)d\eta \tag{4}$$

endowed with initial and boundary conditions:

$$V(S,T) = \max(K - S, 0) \tag{5}$$

$$V(S,t) \to 0 \quad \text{as } S \to \infty \quad \text{and} \quad V(0,t) = K \tag{6}$$

and with the additional condition for S

$$\lim_{S \to b(t)} V(S,t) = K - b(t) \tag{7}$$

where η shows the jump amplitude and the function ρ satisfies

$$\rho(\eta) \ge 0, \quad \int_0^\infty \rho(\eta)d\eta = 1, \;\; \rho(\eta) = \frac{1}{\sqrt{2\pi}\sigma_J \eta} e^{\frac{-(\ln \eta - \mu)^2}{2\sigma_J^2}} \tag{8}$$

where μ is the mean of the lognormal jump process and $b(t)$ is the optimal exercise boundary which should be approximated. It is known that only the region $S > b(t)$ has financial meaning. The financial parameters $r, \sigma, \lambda, k, K, \sigma_J, \eta$ are constant numbers in this mathematical model. The big letter K is the strike price of the option contract.

Obviously, if the parameter $\lambda = 0$, i.e. the parameter that characterizes the jumps of the asset price is zero, then the new Eq. (3) is the standard Black–Scholes equation (1), having in mind that $\tau = T - t$.

After rescaling the variables:

$$S = K\hat{S}, \qquad P_A(S,\tau) = \hat{P_A}(\hat{S},\tau), \qquad b(t) = Kb\hat{(t)} \tag{9}$$

and using the front fixing method by applying the transformation $y = \ln \frac{\hat{S}}{b\hat{(t)}}$ in Eq. (4) so that the free boundary $b\hat{(t)}$ associated with the optimal exercise prices is converted into a fixed boundary, [11]. Thus when $\tau = T - t$, and defining $\hat{P_A}(\hat{S},\tau) = V(x,\tau)$, the problem of pricing an American put option becomes

$$\frac{\partial V}{\partial \tau} - \frac{1}{b(\tau)}\frac{db(\tau)}{d\tau}\frac{\partial V}{\partial y} = \frac{1}{2}\sigma^2\left(\frac{\partial^2 V}{\partial y^2} - \frac{\partial V}{\partial y}\right) + (r - \lambda k)\frac{\partial V}{\partial y} - (r+\lambda)V + \frac{\lambda}{b(\tau)e^y}\int_0^\infty V(u)\rho\left(\frac{u}{b(\tau)e^y}\right)du, \tag{10}$$

finally by using (8), Eq. (10) reduces to the following form:

$$\begin{aligned}\frac{\partial V}{\partial \tau} - \frac{1}{b(\tau)}\frac{db(\tau)}{d\tau}\frac{\partial V}{\partial y} = {} & \frac{1}{2}\sigma^2\left(\frac{\partial^2 V}{\partial y^2} - \frac{\partial V}{\partial y}\right) + (r - \lambda k)\frac{\partial V}{\partial y} - (r+\lambda)V \\ & + \lambda \int_0^\infty \frac{V(u)}{u}\exp\left(\frac{-\ln(u-\mu)^2}{2Kb(\tau)\exp(x)\sigma_J^2}\right)du,\end{aligned} \tag{11}$$

and

$$\left|\begin{array}{l} V(y,0) = \max(1 - b(0)e^y, 0) = 0, \quad \text{for } 0 < y < \infty, \ \text{as } b(0) = 10 \\ V(0,\tau) = 1 - b(\tau), \\ \dfrac{\partial V(0,\tau)}{\partial y} = -b(\tau) \\ \lim\limits_{y\to\infty} V(y,\tau) = 0. \end{array}\right. \tag{12}$$

Frequently $b(\tau)$ is isolated from conditions (12) and we have

$$\left|\begin{array}{l} V(y,0) = 0, \quad \text{for } 0 < y < \infty \\ V(0,\tau) - \dfrac{\partial V(0,\tau)}{\partial y} = 1 \\ \lim\limits_{y\to\infty} V(y,\tau) = 0. \end{array}\right. \tag{13}$$

The key advantage of the obtained model is that Eq. (11) permits us to apply finite difference schemes and the *radial basis functions method* without using the *linear complementarity problem* formulation for valuation of American options.

We also note that Eq. (11) is non-linear having the non-linear term $\frac{1}{b(\tau)}\frac{db(\tau)}{d\tau}\frac{\partial V}{\partial y}$.

As the Eq. (11) is a partial integral–differential equation, i.e. it consists of a PDE part and an integral part, the solution of the this problem for valuation of American put options will be divided into two steps:

1. Approximation of the integral part of Eq. (11).
2. Implementation of the radial basis function for the PDE part.

And Duffy in [3] has propose a modified form of Eq. (4) in which the integrand is defined on a bounded interval:

$$\frac{\partial u}{\partial t} = Lu + \lambda \int_A^B u(x+y,t)\Gamma_\delta(y)dy.$$

Another similar question is that the PDE part of Eq. (11) is also defined on a semi-infinite interval $[0,\infty]$, but this problem is easily resolved having in mind the variable S is the underlying asset value. The S-domain is truncated at the value $S_{\max}$, sufficiently large such that computed values are not appreciably affected by the upper boundary. As in financial practice it is well known that $S \geq 3K$, (K is the strike price), has no financial interest, then $S_{\max} = 3K$ is enough to be chosen. In finite difference practice usually, $S_{\max} = 2K$. More information for finding precise values of the $S_{\max}$ are explored in [12].

3. Radial basis functions approach

3.1. Comparison with the finite difference method

As usual, in the finite difference approximation the S-domain is truncated at the value $S_{\max}$, sufficiently large such that computed values are not appreciably affected by the upper boundary. The computational domain $[0, S_{\max}] \times [0, T]$ is discretized by a uniform mesh with steps ΔS, Δt. Therefore we obtain the nodes S_j and t_n, where $(S_j = j\Delta S,\ t_n = n\Delta t)$, $j = 0,\dots,N$, $n = 0,\dots,N_1$ so that $S_{\max} = N\Delta S$, $T = N_1\Delta t$, N_1 and N are integers.

1. The choice of a specific numerical scheme is based on its property of convergence. The requirement rests on the Lax equivalence theorem.
2. The parabolic nature of the Black–Scholes equation ensures that the initial condition $V(S,0) = (S-K)^{+}1_{[L,U]}(S)$ being square-integrable the solution is smooth in the sense that $V(\cdot,t) \in C^{\infty}(\mathbb{R}^{+}),\ \forall t \in (t_{i-1}, t_i^{-}],\ i = 1,\ldots,F$. Thus rough initial data give rise to smooth solutions in infinitesimal time.

As there is still no closed-form solution of Eq. (11), we will point most of the frequently used finite difference methods in numerical analysis that are listed in the book of Duffy [3]:

1. Explicit and implicit numerical schemes—the so called θ-method.
2. Implicit–explicit Runge–Kutta methods.
3. Operator splitting methods.
4. Predictor–Corrector methods.

We will not enter into the details of the advantages and disadvantages of the above listed numerical methods, but we would like to point out some practical problems that we would like to avoid such as conditional stability [13], non-smooth solutions and undesired spurious oscillations [14], lower-order of accuracy [15], unreasonable computational time [16], artificial numerical diffusion [17], errors induced by splitting [3] and the impossibility of multi-dimensional implementation [1].

3.2. Advantages of the radial basis functions approach

We listed in the introduction the following three main advantages of the radial basis functions (RBF) method that could be observed from the structure and the way of the application of this approach:

1. The RBF is an extremely flexible interpolation method because it does not depend on the locations of the approximation nodes and applies immediately for scattered data. Thus, unlike finite difference schemes, application and proving convergence do not differ in case of uniform and non-uniform grids. Often the RBF method is referred to meshless methods, [18].
2. The RBF generalization in any number of dimensions is immediate and analogous to the one-dimensional case. Thus pricing multi-asset option problems is significantly simplified and does not seem too arduous a task as it is generally concerned with computational finance, [19,20].
3. The RBF method depends on the shape parameter as explained in Section 4 and shown in Section 5, and by adjusting it, we can get a higher accuracy [21].

In the Section 3.1, we have described the main features of the finite difference schemes (FDS). Let us expose the main idea of the RBF method in order to make a parallel with the FDS, see [22,2]. Let us at each data location $x_i,\ i = 1,\ldots,n$, place a translate of the function $\phi(x) = |x|^3$, i.e. at x_i the function $\phi(x - x_i) = \|x - x_i\|^3$. We then try, if it is possible, to form a linear combination of all these functions.

$$V(x,\tau) = \sum_{j=0}^{\infty} \lambda_j(\tau)\phi(u - u_j). \tag{14}$$

In d dimensions, we use the rotated version of the same radial function and we could write them as $\phi(\|x - x_i\|)$, where $\|\cdot\|$ denotes the standard Euclidean norm. In particular, we note that the algebraic complexity of the interpolation problem has not increased with the number of dimensions. We will always end up with a square symmetric system of the same size as the number of data points. Cubic splines have thus been generalized to apply also to scattered data in any number of dimensions, [22]. For different types of radial basis functions with their corresponding names and respectively, the properties of each type could be found in [23].

4. Discretization by radial basis functions

Let us first make a summary of our efficient and accurate numerical scheme pro-pricing American options under the jump diffusion process. We have used a *front fixing transformation* to reach a partial-integral differential equation (PIDE) problem with a *fixed boundary*. In this section we will do the following procedures to develop an *efficient and accurate numerical scheme* for pricing American options:

1. We will interpolate the integral term by the Laguerre polynomials.
2. We use the radial basis functions (RBF) as a flexible interpolation method and we achieve a *system of nonlinear equations*.
3. To overcome the nonlinearity of the system, we apply the Cranck–Nicholson and the Predictor–Corrector methods as well to reach stable and correct results.
4. In the next section of numerical results we compare our results with other numerical schemes to show the validity of the presented algorithm of radial basis functions.

We take the general form of the RBF method as represented in (14) for the problem (11), by considering $\phi(u)$ as the multi quadratic polynomials:

$$\phi(u) = \sqrt{u^2 + c^2} \tag{15}$$

where c is the shape parameter which we can adjust to come close to the exact solution as explained in Section 3.2.

If L is a linear partial differential operator, B is the Dirichlet boundary operator and Ω is a bounded region in $\mathbb{R}^n$ with boundary $\partial\Omega$, we seek the option price $V(S,t) \in C(\overline{\Omega})$ satisfying

$$\begin{cases} LV = A(V, V_x, V_{xx}) & \text{in } \Omega, \\ B(V, V_x) = 0 & \text{on } \partial\Omega \end{cases} \tag{16}$$

where A and B are the functions of option price and its first and second derivatives with respect to x.

Now by approximating $V(S,t)$ by its interpolating function $\widetilde{V}(S,t)$ in (16) we have:

$$\begin{cases} L\widetilde{V} = A(\widetilde{V}, \widetilde{V}_x, \widetilde{V_{xx}}) & \text{in } \Omega, \\ B(\widetilde{V}, \widetilde{V}_x) = 0 & \text{on } \partial\Omega. \end{cases} \tag{17}$$

To determine the unknown coefficients $\lambda_j(t),\ j = 1, 2, \ldots, N$, we can use the collocation method. For simplicity, we use centers as collocation points and we get:

$$\begin{cases} L\widetilde{V}(\mathbf{x}_i) = A(\widetilde{V}, \widetilde{V}_x, \widetilde{V_{xx}})(\mathbf{x}_i), & 1 \le i \le N_\Omega, \\ B(\widetilde{V}, \widetilde{V}_x)(\mathbf{x}_i) = 0, & N_\Omega + 1 \le i \le N. \end{cases} \tag{18}$$

where N_Ω is the number of interior collocation points, N is the total number of collocation points and V is defined by (14).

First we approximate the integral term by the Laguerre polynomials with the weight function of e^x and using (14):

$$\begin{aligned} \int_0^\infty \frac{V(u)}{u} \exp\left(\frac{-\ln(u-\mu)^2}{2Kb(\tau)\exp(x)\sigma_J^2}\right) du &= \sum_{j=0}^{N} \lambda_j(\tau) \int_0^\infty \frac{\phi(u-u_j)}{u} \exp\left(\frac{-\ln(u-\mu)^2}{2Kb(\tau)\exp(x)\sigma_J^2}\right) du \\ &= \sum_{j=0}^{N} \lambda_j(\tau) \left(\sum_{k=0}^{N_2} w_k \frac{\phi(r_k-u_j)}{r_k} \exp\left(\frac{\ln(u-\mu)^2}{2Kb(\tau)\exp(x)\sigma_J^2} + r_k\right)\right) \end{aligned} \tag{19}$$

where r_k and w_k, $k = 1(1)N_2$, are the roots and weights of the Laguerre polynomials respectively, by getting $N_2 = 15$ fixed.

By taking $x = x_j$ and $t = t_j$, $j = 1(1)N$, we set the left hand of (19) as the following matrix:

$$[G(b(\tau))]_{i,j} = \sum_{k=0}^{N_2} w_k \frac{\phi(r_k-u_j)}{r_k} \exp\left(\frac{-\ln(u-\mu)^2}{2Kb(\tau)\exp(x_i)\sigma_J^2} + r_k\right). \tag{20}$$

And more to circumvent the nonlinearity of the PDE part of Eq. (11), we deal with it by the predictor–corrector schemes:

Predictor:

$$\begin{aligned} \frac{V(x_j,\tau_{i+1}) - V(x_j,\tau_i)}{h_1} &= \frac{b(\tau_{i+1}) - b(\tau_i)}{h_1 b(\tau_i)} V_x(x_j,\tau_i) \\ &+ \frac{1}{2}\left[\left(\frac{1}{2}\sigma^2 V_{xx}(x_j,\tau_{i+1}) + \left(r - \lambda\kappa - \frac{1}{2}\sigma^2\right) V_x(x_j,\tau_{i+1}\right) - (r+\lambda)V(x_j,\tau_{i+1}))\right. \\ &+ \left.\left(\frac{1}{2}\sigma^2 V_{xx}(x_j,\tau_i) + \left(r - \lambda\kappa - \frac{1}{2}\sigma^2\right) V_x(x_j,\tau_i) - (r-\lambda)V(x_j,\tau_i)\right)\right] \\ &+ \frac{1}{2}(G_1(b(\tau_i))\lambda(\tau_{i+1}) + G_2(b(\tau_{i+1}))\lambda(\tau_{i+1})) + q, \quad i = 1(1)N_1,\ h_1 = \frac{N_1}{T} \end{aligned} \tag{21}$$

where

$$\lambda(\tau_i) = \begin{pmatrix} \lambda_1(\tau_i) \\ \lambda_1(\tau_i) \\ \vdots \\ \lambda_1(\tau_i) \end{pmatrix}_{N\times 1}, \qquad q = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{N\times 1} \tag{22}$$

and

$$b(\tau_i) = 1 - V(0,\tau_i) \tag{23}$$

is written based on (12), as well setting $G = G_1$ and $G = G_2$ in (21) as the coefficients of $\lambda(\tau_i)$ and $\lambda(\tau_{i+1})$ respectively which is represented in the last term of (21).

Subsequently the corrector scheme we construct is:

$$\begin{aligned}\frac{V^{\mu+1}(x_i,\tau_{j+1}) - V(x_i,\tau_j)}{h_1} &= \frac{1}{2}\frac{b^{\mu+1}(\tau_{j+1}) - b(\tau_j)}{h_1 b(\tau_j)}(V_x(x_j,\tau_j) + V_x^{\mu}(x_i,\tau_{j+1}))\\ &\quad + \left(\frac{1}{2}\sigma^2 V_{xx}^{\mu}(x_i,\tau_{j+1}) + \left(r - \lambda\kappa - \frac{1}{2}\sigma^2\right)V_x^{\mu}(x_i,\tau_{j+1}\right) - (r+\lambda)V^{\mu}(x_i,\tau_{j+1}))\\ &\quad + G_2(b(\tau_{j+1}))(\lambda(\tau_{j+1})) + q, \quad \mu = 1(1)M \end{aligned} \tag{24}$$

for a fixed number M i.e. $(V^{M+1}(x_i,t_j) \simeq V(x_i,t_j))$.

By setting

$$V(u_i,\tau_j) = \sum_{j=0}^{\infty}\lambda_j(\tau_j)\phi(u_i - u_j) \tag{25}$$

in relation (21) and (24), we reach to the system of nonlinear equations of the form:

$$B_1\lambda(t_{j+1}) = A(B_1\lambda(t_{j+1}), B_2\lambda(t_{j+1}), B_3\lambda(t_{j+1}), C_1\lambda(t_j), C_2\lambda(t_j), C_3\lambda(t_j)) \tag{26}$$

where

$$b_{1,ij} = \begin{cases}\phi_{ij}, & 1 \le i \le N_\Omega,\ 1 \le j \le N,\\ B\phi_{ij}, & N_\Omega + 1 \le i \le N,\ 1 \le j \le N,\end{cases} \tag{27}$$

$$b_{2,ij} = \begin{cases}L\phi_{ij}, & 1 \le i \le N_\Omega,\ 1 \le j \le N,\\ B\phi_{ij}, & N_\Omega + 1 \le i \le N,\ 1 \le j \le N,\end{cases} \tag{28}$$

$$b_{3,ij} = \begin{cases}L^2\phi_{ij}, & 1 \le i \le N_\Omega,\ 1 \le j \le N,\\ B\phi_{ij}, & N_\Omega + 1 \le i \le N,\ 1 \le j \le N,\end{cases} \tag{29}$$

$$c_{1,ij} = \begin{cases}\phi_{ij}, & 1 \le i \le N_\Omega,\ 1 \le j \le N,\\ 0, & N_\Omega + 1 \le i \le N,\ 1 \le j \le N,\end{cases} \tag{30}$$

$$c_{2,ij} = \begin{cases}L\phi_{ij}, & 1 \le i \le N_\Omega,\ 1 \le j \le N,\\ 0, & N_\Omega + 1 \le i \le N,\ 1 \le j \le N,\end{cases} \tag{31}$$

and

$$c_{3,ij} = \begin{cases}L^2\phi_{ij}, & 1 \le i \le N_\Omega,\ 1 \le j \le N,\\ 0, & N_\Omega + 1 \le i \le N,\ 1 \le j \le N,\end{cases} \tag{32}$$

where $\phi_{ij} = \phi(\|\mathbf{x}_i - \mathbf{x}_j\|)$.

By achieving the option price $V(x_i,\tau_{j+1})$ at each time τ_{j+1}, we should update it as follows to avoid arbitrage opportunities:

$$V(x_i,\tau_j) = \max\Big(V(x_i,\tau_j),\ 1 - b(\tau_j)\exp(x_i)\Big). \tag{33}$$

5. Numerical results

The presented procedure of radial basis functions in this section has a simple computer implementation similar to the approach of the finite difference schemes, but in contrast to them it is much faster [20] where the RBF method turns out to be 20–40 times faster in one and two space dimensions and has approximately the same memory requirements compared with an adaptive finite difference method. This is confirmed by the numerical results in Table 2 of our experiment in Table 1 as in [9], we note also the small number of time steps and size S-grid of the RBF method. We observe a very fast convergence of the RBF method and we obtain a third-order accuracy with only 40 time steps and 15 space nodes.

In this section we present radial basis function method results exploring the numerical examples for pricing American put options in [9]. We show the error convergence in Figs. 1–3 according to the variation of the shape parameter, time and stock mesh size when one of these three variables is fixed considering the parameters in Table 1 for all experiments. We obtain the value of the American put option prices for three different stock prices at $S_0 = 90, 100, 110$ and compute the error value for each of them. For comparison, we use the reference values in [9] that are respectively 10.003822 at $S_0 = 90$, 3.241251 at $S_0 = 100$ and 1.419803 at $S_0 = 110$. We see from Table 2, that our results are in agreement till the 4-th to 6-th decimal point. We point in Fig. 1 the optimal values of the shape parameter c when the solution error reaches its minimal value.

Table 1
Input data used to value American options under the lognormal jump diffusion process.

Underlying asset price and jump diffusion parameter values						
Asset price parameters		Jump parameters			Fix contract parameters	
Volatility (σ)	Interest rate (r)	Jump volatility (σ_J)	Mean (μ)	Jump intensity (λ)	Maturity (T)	Strike price (K)
0.15	0.05	0.45	−0.90	0.10	0.25	100

Table 2
American put option values using a predictor–corrector scheme and the Crank–Nicolson method.

Number of time steps	Size of the space S-grid	$S_0 = 90$		$S_0 = 100$		$S_0 = 110$	
		Value	Error	Value	Error	Value	Error
40	15	10.007723	3×10^{-3}	3.23832	2×10^{-3}	1.424355	4×10^{-3}
50	20	10.003251	5×10^{-4}	3.239984	1×10^{-3}	1.410948	8×10^{-3}
60	25	10.003547	3×10^{-4}	3.2406385	6×10^{-4}	1.4198438	4×10^{-5}
70	30	10.003968	1×10^{-4}	3.2402608	9×10^{-4}	1.4200659	2×10^{-4}
80	35	10.0038211	1×10^{-6}	3.2413965	1×10^{-4}	1.418803	3×10^{-4}

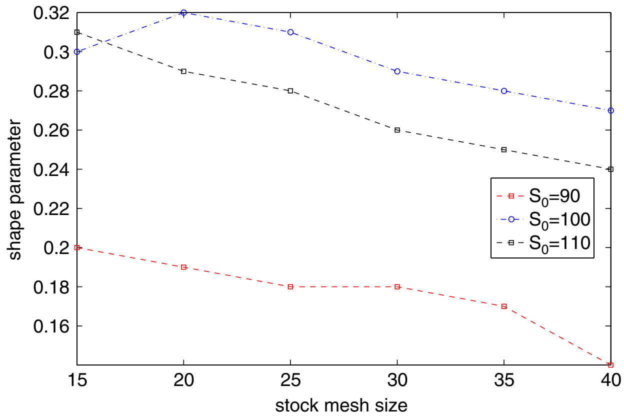

Fig. 1. The variation of shape parameter with respect to the stock mesh size by fixing the time mesh size ($N_1 = 40$). The values of the shape parameter c are generated by a selecting procedure so that the numerical error is minimal.

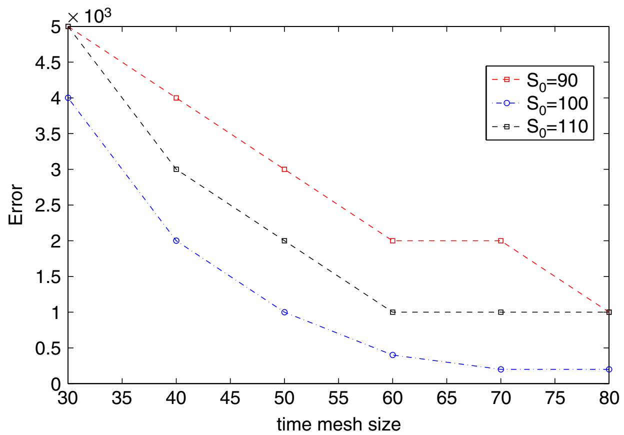

Fig. 2. The variation of the error with respect to the variation of time mesh size by fixing the stock mesh size ($N = 15$).

We would like to note that the curve of $S_0 = 100$ in Fig. 1 increases when the stock mesh size increases from 15 to 20, although the other curves gradually decrease according to the increase of the stock mesh size. We would like to explain why this curve of $S_0 = 100$ turns out to be different from the others, i.e. it is *not decreasing* as stock mesh size increases. To explain this 'phenomenon' of '*non decreasing*' $S_0 = 100$, it is important to explain how Fig. 1 has been drawn. As the number of timesteps is fixed in Fig. 1 and three different parameters, i.e. the underlying asset price S_0, the stock mesh size

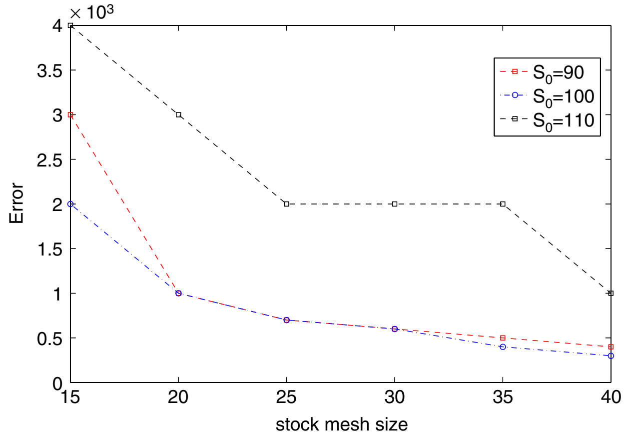

Fig. 3. The variation of the error with respect to the variation of stock mesh size by fixing the time mesh size ($N_1 = 40$).

(number of space steps in the S-grid shown in Table 2) and the optimal shape parameter c remain, we apply the following procedure to define the *optimal shape parameter* c:

1. First, we choose a *fixed* underlying *asset value* S_0.
   (a) Then, we *fix the value* of the *stock mesh size*, for example $j = 15$ as in Table 2.
      i. We apply the MQ RBF method for *different values* $c_1, c_2, \ldots, c_n$ of the parameter c;
      ii. Among the $c_1, c_2, \ldots, c_n$, we choose this value c, when the numerical error is *minimal*. This explains the sense of the '*optimal*' value of the shape parameter c_{optimal};
      iii. Then, we point the value c_{optimal} on Fig. 1.
   (b) We fix a *subsequent* value of the *stock mesh size j*, for example $j = 20$ as in Table 2 and we follow the same procedure (i)–(iii) of the previous point (a).
2. For a different value of the underlying valued S_0 we follow the same procedure (a)–(b) of point 1.

So, the *structure* of the procedure described above explains that *we choose the optimal value* c_{optimal} of the shape parameter c for a *fixed* value of the space parameters *j so that the numerical error is minimal* and Fig. 1 illustrates *only* this value c_{optimal}. We note that the numerical error defines the optimal value c_{optimal} in the procedure (ii) of (a), and thus *the values of c do not depend proportionally on the size of the stock mesh*. An analogous example is the red curve $S_0 = 90$ on Fig. 1, i.e. the values of c in the region from $j = 25$ to $j = 30$. We see in Fig. 1 that the values of the shape parameter c do not decrease for increasing values of the stock mesh size from $j = 25$ to $j = 30$.

To the authors knowledge there is no evidence in literature that *the shape parameter c depends proportionally on the size of the grid*, i.e. the shape parameter c could be expressed as a deterministic function $c(n, j)$ where n and j are the number of time and space steps, respectively. It is sure that the curves for S_0 *should gradually decrease* according to the *increase* of the stock mesh size *but when the shape parameter c is fixed*, i.e. there is convergence of the numerical solution. And in Figs. 2 *and* 3 we demonstrate also that the *solution error* decreases by increasing independently the time and stock mesh size.

The problem of making a *suitable choice* of a proper RBF method is general and often difficult to be done in a determinant way. There is often a doubt whether the *multiquadratic* (MQ) or *inverse multiquadratic* (IMQ), as well as the *quadratic* or *inverse quadratic* radial basis functions (RBF) method should be used and usually these couples are related and both applied [24,18].

One of the reasons is that the application of different kinds of radial basis functions methods depends on the shape parameter c, i.e. for example, it happens for some values of c the MQ RBF method gives higher order accurate results than those obtained by the IMQ RBF method, but these accurate numerical results could be obtained by the IMQ RBF method for a *different shape parameter c*, [24]. And, in a lot of cases it is only *experimentally* defined which type of the RBF methods is preferable.

We have compared both the MQ and IMQ RBF methods at the same conditions, i.e. *for the same values of the shape parameter c* shown in Fig. 1. We apply the IMQ RBF method for $S_0 = 90$ and we reach an agreement of the numerical results till the 3-rd and 4-th decimal point, while from Table 2 we observe that the MQ RBF results are in agreement till the 4-th to 6-th decimal point. Thus, in the case $S_0 = 90$, for the IMQ RBF method we have obtained a *lower accuracy* than that obtained by the MQ RBF method applied. In the other two cases, $S_0 = 100$ and $S_0 = 110$, the IMQ RBF method does not match well with the exact solution in Table 2, i.e. with the reference values in [9], but accurate values could be obtained for a different value of the shape parameter c. (As this process requires an analogous implementation code and trivial calculations as presented in the figures and tables for the MQ RBF method we leave this task to the reader.) As the results of the IMQ RBF method for $S_0 = 100$ and $S_0 = 110$ using the values of the shape parameter c in Fig. 1 are not only inaccurate and but also have no practical sense in option pricing (the error is about one unit) we omit their presentation.

The computational time of both IMQ and MQ RBF methods is relatively the same as the implementation code is slightly changed (and as a result we do not observe substantial influence on the CPU time for both of the RBF methods).

Most articles in literature describe and confirm the validity of the RBF method for pricing European, barrier and Asian options of single asset in the original Black–Scholes framework or under the so called pure lognormal process [24,19,20,25]. In case of pricing the classical American put options under a jump diffusion process we have demonstrated high accuracy and computational speed efficiency of the RBF algorithm presented in Section 4. In some cases the advantage of the RBF method is demonstrated even over fundamental heuristic methods used in computational finance [2] such as the traditional binomial trees [25].

6. Conclusions

In this article we explore jump diffusion option pricing models where the stock price may experience occasional jumps rather than only continuous changes. Such models capture the important leptokurtic features of the market better than the standard Black–Scholes model and in contrast to stochastic volatility models are not too complicated so that practical algorithms such as the presented radial basis functions method are easy to implement. The radial basis functions turns out to be an extremely flexible interpolation method avoiding most of the frequently met problems in computational finance such as the unstable or slow convergent numerical solutions, multi-asset valuation, complexity of computer implementation, unreasonable time and memory requirements. We price numerically the American put option and obtain highly accurate results that are in good agreement with those obtained by other numerical and analytical methods in literature. The RBF method is applied to standard European options as well as path-dependent options in [24]. Thus the RBF is a very competitive and challenging method compared to the standard numerical methods in finance, i.e. the Monte Carlo simulation, binomial trees and finite difference schemes.

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