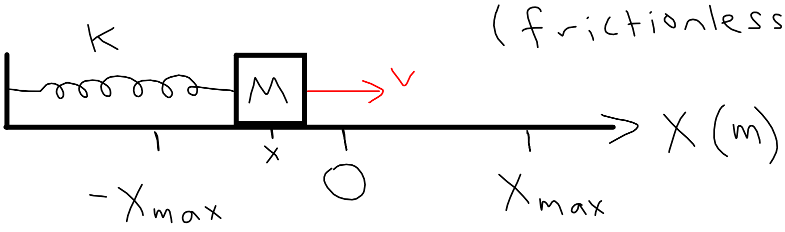


(frictionless)

Reminder:
$KE = \frac{1}{2} M V^2$
$F_s = -k(\Delta x)$
$PE_s = \frac{1}{2} k (\Delta x)^2$

Example

Given: $X_{max} = 5$ cm
$M = 2$ kg
$K = 23$ N/m
$X = -2$ cm

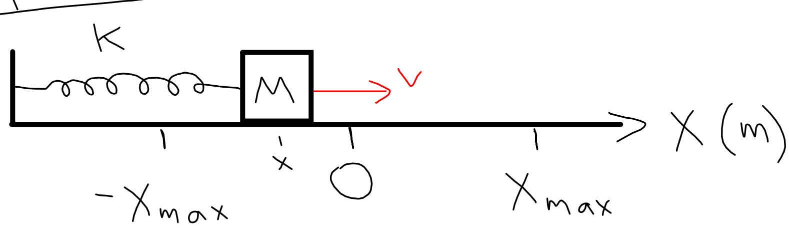
$\boxed{\text{find } V}$

Total Energy:
when $X = X_{max} \rightarrow V = 0$
$\Rightarrow E = \frac{1}{2}(23)(0.05)^2 + C$
$= 0.02875$ J

$x = -0.02$ m

$0.02875 = \frac{1}{2}(2)V^2 + \frac{1}{2}(23)(-0.02)^2$
$\Rightarrow \boxed{V = 0.155 \text{ m/s}}$

Example 2

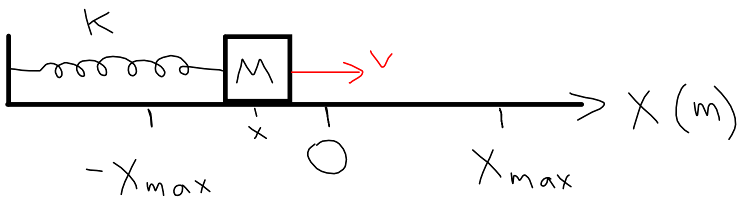


Given: $k = 35$ N/m
$M = 7$ kg
$X = -3$ cm
$V = 0.4$ m/s

$\boxed{\text{find } X_{max}}$

$\boxed{X_{max} = 18.1 \text{ cm}}$

Example 3



Given: $k = 45\ N/m$
$M = 10\ kg$
$x = -4\ cm$
$v = 8\ m/s \rightarrow$

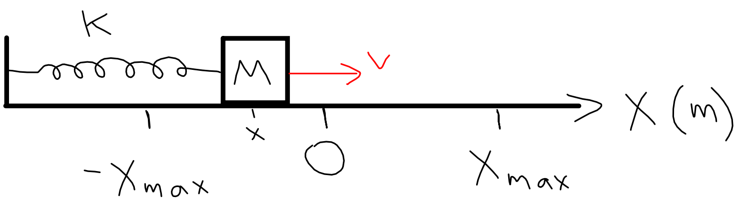
How much time elapses before x = x_{max}?

$$\cancel{X_f = X_i + V_i t + \tfrac{1}{2} a t^2}$$
$$\cancel{V_f = V_i + a t}$$
$\vdots$
etc.

invalid

$(a \neq \text{const.})$

Simple Harmonic Motion



Goal: Find x(t)

$$F_s = -kX$$

$$Ma = -kX$$

$$M\frac{d^2x}{dt^2} = -kX$$

Differential Equation

$\Downarrow$ solution

$$\boxed{x(t) = X_{max} \sin\left(\sqrt{\frac{k}{m}}\, t + \phi\right)}$$

Check:

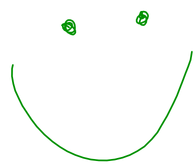
$$m\frac{d^2x(t)}{dt^2} \stackrel{?}{=} -kX(t) \qquad x(t) = X_{max}\sin\left(\sqrt{\frac{k}{m}}\,t+\phi\right)$$

$$\frac{dx}{dt} = X_{max}\sqrt{\frac{k}{m}}\cos\left(\sqrt{\frac{k}{m}}\,t+\phi\right)$$

$$\frac{d^2x}{dt^2} = -X_{max}\left(\frac{k}{m}\right)\sin\left(\sqrt{\frac{k}{m}}\,t+\phi\right)$$

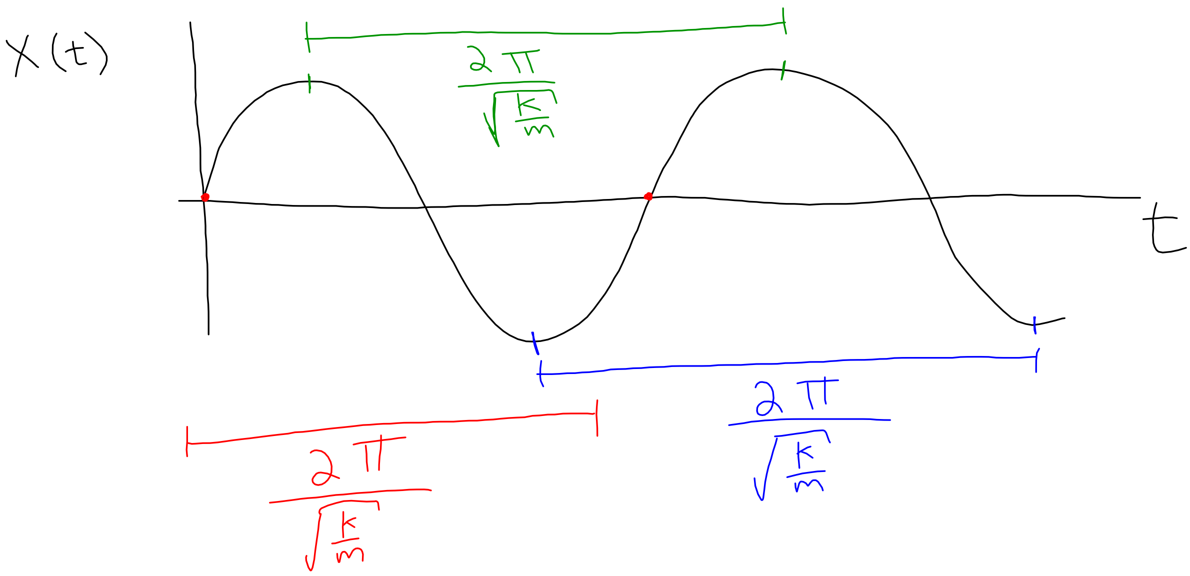
plug in:

$$-\cancel{m}X_{max}\left(\frac{k}{\cancel{m}}\right)\sin\left(\sqrt{\frac{k}{m}}\,t+\phi\right) = -kX_{max}\sin\left(\sqrt{\frac{k}{m}}\,t+\phi\right)$$



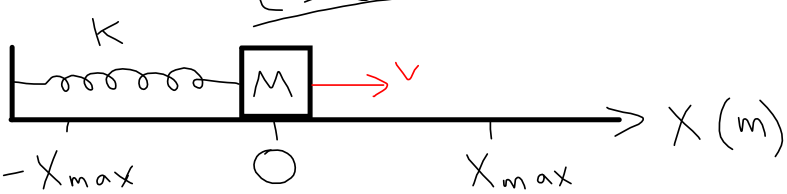
(for now, assume $\phi = 0$)

$$X(t) = X_{max} \sin\left(\sqrt{\frac{k}{m}}\, t\right)$$



Example

$t = 0$



Given:

$k = 12\ N/m$

$M = 3\ kg$

$X_{max} = 0.5\ m$

1. How much time to return back to $x = 0$?

2. What is x when $t = 2.5\,s$?