

(constant) rates of change

Ex 1: Pay rate = \$12/hr. You have \$100 initially, and then you work a 4 hour shift. How much money do you have?

$$\begin{aligned}\text{Answer: } \$_{\text{final}} &= \$_{\text{initial}} + (\text{pay rate}) \cdot \text{time} \\ &= \$100 + (\$12/\text{hr}) \cdot 4 \text{ hours} \\ &= \$148\end{aligned}$$

Ex 2: Price of share = \$50/share. You want to buy 5 shares. How much will it cost?

$$\text{Answer: } (\text{price per share}) \cdot (\text{number of share}) = (\$50/\text{share}) \cdot (5 \text{ shares}) = \$250$$

Ex 3: Velocity = 20 m/s. You are already 30 meters from your house and driving away from your house. How far away are you after 7 more seconds?

$$\begin{aligned}\text{Answer: } \text{Distance}_{\text{final}} &= \text{Distance}_{\text{initial}} + (\text{velocity}) \cdot (\text{time}) \\ &= 30 \text{ m} + (20 \text{ m/s}) \cdot (7 \text{ s}) \\ &= 170 \text{ m}\end{aligned}$$

Ex 4: You are stopped at a red light ($v = 0 \text{ m/s}$), but then it turns green. One second later you look at your speedometer and it says 3 m/s; another second later it says 6 m/s; yet another second later it reads 9 m/s; etc.

<u>time (s)</u>	<u>v (m/s)</u>
0	0
1	3
2	6
3	9

Velocity is changing at a rate of 3 m/s every second, i.e. 3 (m/s)/s or 3 m/s^2 .

What is the velocity at 11s?

$$\begin{aligned}\text{Answer: } \text{velocity}_{\text{final}} &= (\text{rate of change of velocity}) \cdot \text{time} \\ &= (3 \text{ (m/s)/s}) \cdot 11 \text{ s} = 33 \text{ m/s}\end{aligned}$$

Assuming constant acceleration:

$$V_f = V_i + a t$$

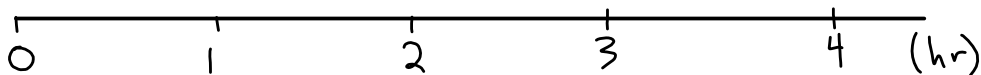
\uparrow acceleration \equiv rate of change of velocity

Velocity \equiv rate of change of position
(m/s)

$$V_{ave} = \frac{\Delta x}{\Delta t} \quad , \quad V_{instantaneous} = \frac{dx}{dt}$$

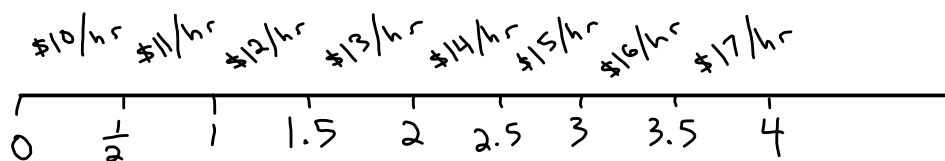
$$a_{ave} = \frac{\Delta v}{\Delta t} \quad , \quad a_{instantaneous} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

\$10/hr \$12/hr \$14/hr \$16/hr



How much money did you earn?

$$(\$10/hr)(1hr) + (\$12/hr)(1hr) + \dots = \$52$$



How much do you earn?

$$(\$10/hr)(0.5h) + (\$11/hr)(0.5h) + (\$12/hr)(0.5h) + \dots = \$54$$

What about working 16 1/4-hour shifts where you get a raise of \$0.50/hr every 1/4 hour?

What about working 32 1/8-hour shifts where you get a raise of \$0.25/hr every 1/8 hour?

etc.