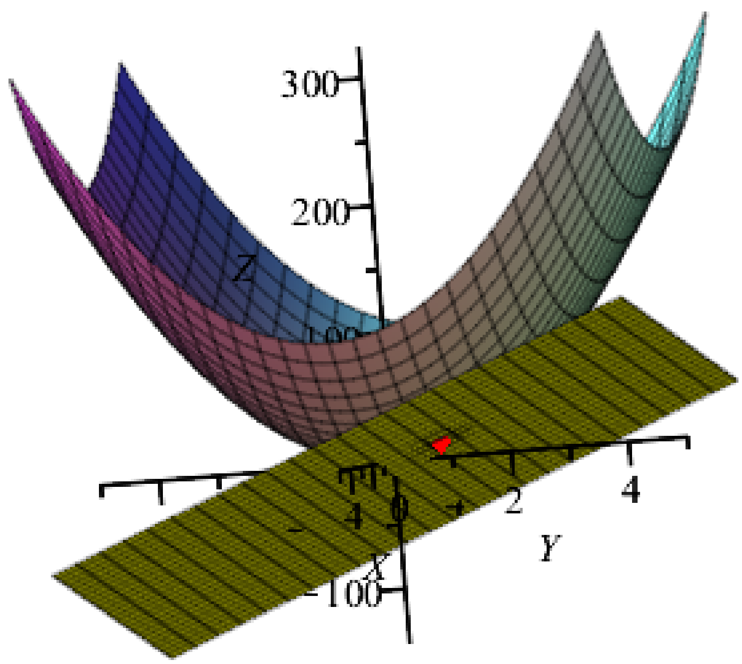
> $with(plots):$

> $f(x,y) := 4\cdot x^2 + 9\cdot y^2$

$$f := (x, y) \mapsto 4\,x^2 + 9\,y^2 \qquad (1)$$

> $S := plot3d(f(x,y), x=-5..5, y=-5..5, axes=normal, labels=[X, Y, Z]):$

> $display(S, pt, TP)$



> $pt := pointplot3d([1, 1, 13], symbol=soliddiamond, symbolsize=20, color=red):$

> $TP := plot3d(8\cdot x + 18\cdot y - 13, x=-5..5, y=-5..5, color=yellow):$

> **1. Gradient of f(x,y) is <Partial of f with respect to x, Partial of f with respect to y>**

> $\frac{\partial}{\partial x}(f(x,y))$

$$8\,x \qquad (2)$$

> $8\cdot 1$

$$8 \qquad (3)$$

> $\frac{\partial}{\partial y}(f(x,y))$

$$18\,y \qquad (4)$$

> $18\cdot 1$

(5)

$$18 \tag{5}$$

Gradient at (1,1) is <8,18>

The slope on the surface moving in a Northwest direction from (1, 1) is the Directional Derivative, which is represented by the dot product of the gradient of f and the directional unit vector.

The directional unit vector, described by Northwest, when drawn on the unit circle, shows that the angle that produces that vector is 3Pi/4. The x-component is -sqrt(2)/2. The y-component is sqrt(2)/2.

Thus, the dot product of the two is described as <8,18>dot<-sqrt(2)/2,sqrt(2)/2>.

The result is $5\sqrt{2}$.

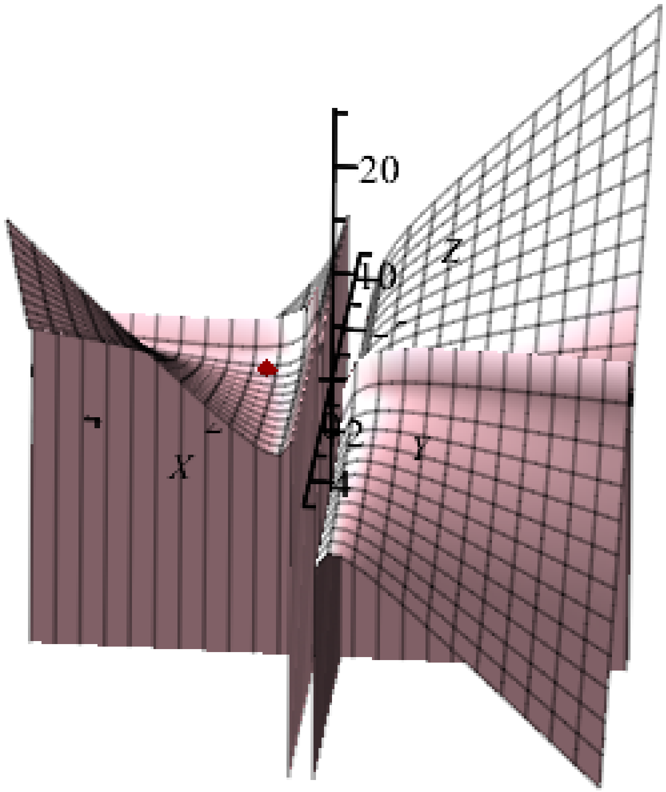
This is the slope on the surface at the point (1,1) moving in a Northwest direction.

$z := \frac{1}{x} + \frac{1}{y} + x \cdot y$

$$z := \frac{1}{x} + \frac{1}{y} + x\,y \tag{6}$$

$S := plot3d(z, x = -5 ..5, y = -5 ..5, axes = normal, color = pink, labels = [X, Y, Z]) :$

$display(S, pts)$



> $pts := pointplot3d([[1, 1, 3]], symbol = soliddiamond, symbolsize = 20, color = red)$:

> **2. When determining if puddles will form on the surface $z = \frac{1}{x} + \frac{1}{y} + x \cdot y$, we start by finding the critical points, which is done by finding the first partial of the function with respect to x and the first partial of the function with respect to y. One then sets the two equal to 0. From here, I found that $y = x^{-2}$, allowing me to find the values of x where the first partials are equal to 0. This meant that x could equal 0, or 1. I could not use the 0 x value since 1/0 is undefined. Judging by the surface produced, water would fall in the valley where x = 0, but it's unclear if a puddle ever forms. Where x = 1, y is equal to 1 and z is equal to 3. At this point, a puddle can clearly form based on the lip produced by the surface.**

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\> 3.

\> a) If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$, then f(x,y) is a constant. FALSE. First, a case where this is true is f(x,y) = 3. $\frac{\partial}{\partial x}(f(x,y)) = 0$ and $\frac{\partial}{\partial y}(f(x,y)) = 0$, meaning both partials are equal to each other. However, let f(x, y) = x + y. Then $\frac{\partial}{\partial x}(f(x,y)) = 1$ and $\frac{\partial}{\partial y}(f(x,y)) = 1$. The two partials are equal, but f(x,y) is not a constant.

\> $\frac{\partial}{\partial x}(x+y)$

$$1 \tag{7}$$

\> $\frac{\partial}{\partial y}(x+y)$

$$1 \tag{8}$$

\>

\> b) If $f(x,y) = x^2 + y^2$ then ∇f is perpendicular to the graph of $f(x,y) = x^2 + y^2$. FALSE. The gradient of f points in the direction of steepest ascent on the surface of the graph, not perpendicular to the surface of the graph.

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\> c) There are exactly two functions with the following partial derivatives,

\> $\frac{\partial f}{\partial x} = 4x^3y^2 - 3y^4$ and $\frac{\partial f}{\partial y} = 2x^4y - 12xy^3$. FALSE. One function that produces both partials is $f = x^4y^2 - 3xy^4$. One can add constants to the function to produce far more than two different functions. For example, in addition to the function I provided you could have $f = x^4y^2 - 3xy^4 + 300$ or $f = x^4y^2 - 3xy^4 - 1000$.

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\> d) Given f(x,y) is a smooth surface at (a,b) then there exists a unit vector u such that $|\nabla f(a,b)| < \nabla f(a,b) \cdot u$. FALSE. The directional derivative, which is the dot product of the unit vector, is maximized when equal to the gradient of the function. One can consider $|\nabla f| \cdot |u| \cdot \cos(\theta)$, which is the formula for the directional derivative. The magnitude of the unit vector is always 1. The maximum value of $\cos(\theta)$ is 1. Thus, the maximum value of the directional derivative is equal to the gradient of the function. The directional derivative can never be greater than the gradient of the function.

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\> e) (0,0) is a critical point for the function $f(x,y) = \sin(x)\cos(y)$. FALSE.

\> $\frac{\partial}{\partial x}(\sin(x) \cdot \cos(y))$

$$\cos(x)\cos(y) \tag{9}$$

> $\cos(0)\cos(0)$

$$1 \tag{10}$$

> $\frac{\partial}{\partial y}(\sin(x)\cdot\cos(y))$

$$-\sin(x)\sin(y) \tag{11}$$

> $-\sin(0)\sin(0)$

$$0 \tag{12}$$

> **The partial of with respect to x at (0,0) results in 1. The partial cannot equal 0 at (0,0). Thus, (0,0) is not a critical point.**

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