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MAT 243 Project 1

Prove:

When a white square and a black square are removed from a standard 8 x 8 checkerboard, it is possible to tile the remaining squares using dominoes.

Proof:

Multiple combinations were used to demonstrate that it is possible to tile a checkerboard with dominoes after removing 1 black and 1 white square. While there may be redundancy in the methods for tiling the remaining squares given a combination of missing squares, the additional combinations can be used to quickly identify tiling combinations without relying on more complicated algorithms. However, certain situations do require a more complicated algorithm to tile the remaining squares.

Combination 1 – Removing 2 Adjacent Squares in the Same Column/Row Leaving Even Parts in the Column/Row

If one removes 2 adjacent squares in the same column leaving even parts that total 6 squares within the column, as shown in **Figure 1**, then one can tile the remaining squares with vertically oriented dominoes. Note that even parts totaling 6 can be 2 squares + 4 squares where the 2 adjacent squares in the column are separated from the rest of the 4 squares in the column by the space left by the two removed squares. Similarly, if one removes 2 adjacent squares in the same row leaving even parts totaling 6 remaining squares in the row, one can tile the remaining squares with horizontally oriented dominoes. (It should be noted that for similar combinations involving the removal of squares from the same columns or rows, the figures shown as examples will show the removal of squares from columns alone. This is sufficient to demonstrate the removal of squares from rows due to a rotational symmetry.)

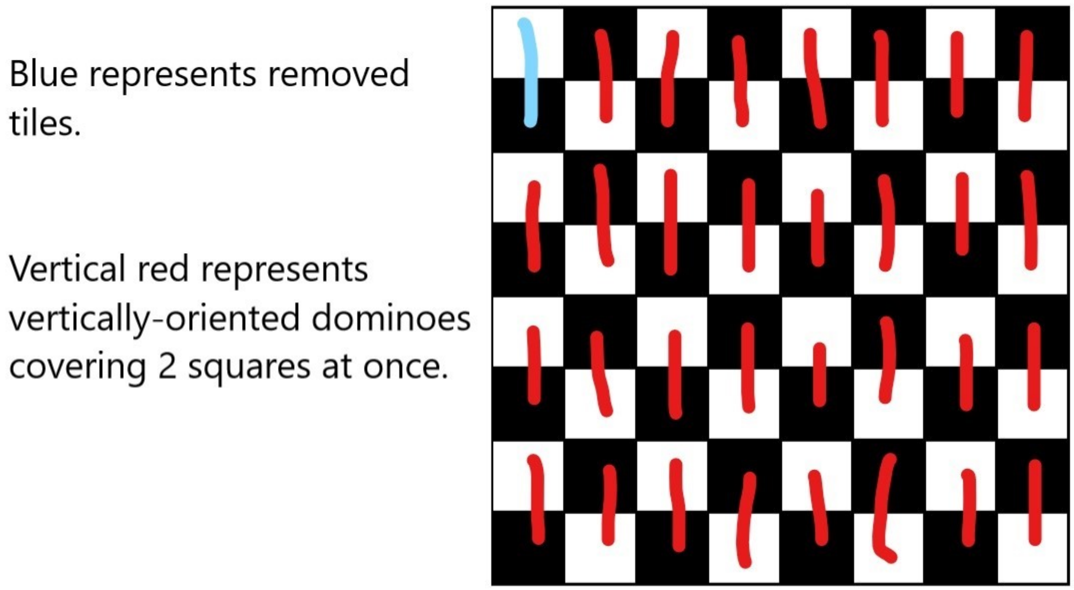

Figure 1. Note that there are 6 unseparated remaining squares in the column. This makes it an even part.

Combination 2 – Removing 2 Adjacent Squares in the Same Column/Row Leaving Odd Parts in the Column/Row

If one removes 2 adjacent squares in the same column leaving odd parts in the same column, e.g. 1 square + 5 squares = 6 squares or 3 squares + 3 squares = 6 squares, then one can tile the top and bottom rows with horizontally oriented squares while tiling the remaining squares with vertically oriented dominoes. Similarly, if one removes 2 adjacent squares in the same row leaving odd parts in the row, one can tile the far left and far right rows with vertically oriented dominoes while tiling the remaining squares with horizontally oriented dominoes. **Figure 2** demonstrates Combination 2.

Horizontal yellow represents horizontally tiled dominoes.

Blue represents the removed squares.

Vertical red represents vertically tiled dominoes.

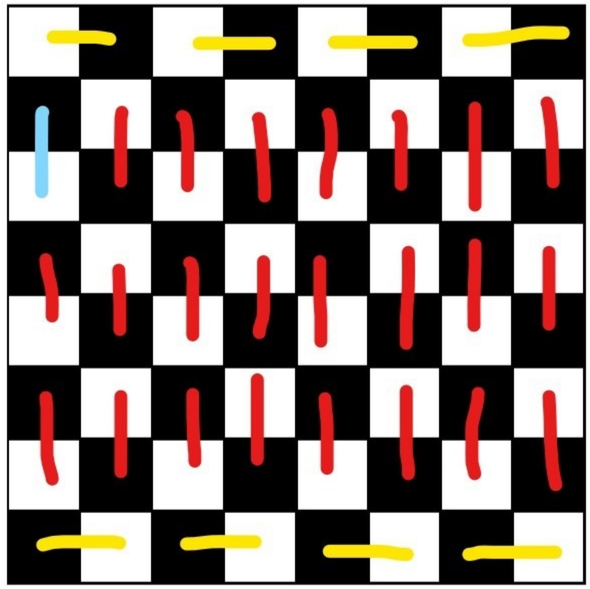

Figure 2. Note how 1 square is separated from the remaining 5 squares in the column. The individual sets of squares totaling 6 squares are both odd in the number of squares.

Combination 3 – Removing 2 Non-Adjacent Squares in a Column/Row Leaving 6 Squares between the Removed Squares

If one removes 2 non-adjacent squares in a column leaving 6 squares between the spaces left by the removed squares, it is possible to tile the remaining squares with vertically oriented dominoes. Similarly, if one removes 2 non-adjacent squares in the same row leaving 6 squares between the two spaces, it is possible to tile the remaining squares with horizontally oriented dominoes. **Figure 3** demonstrates this combination.

Blue represents the removed squares.

Vertical red represents vertically tiled dominoes.

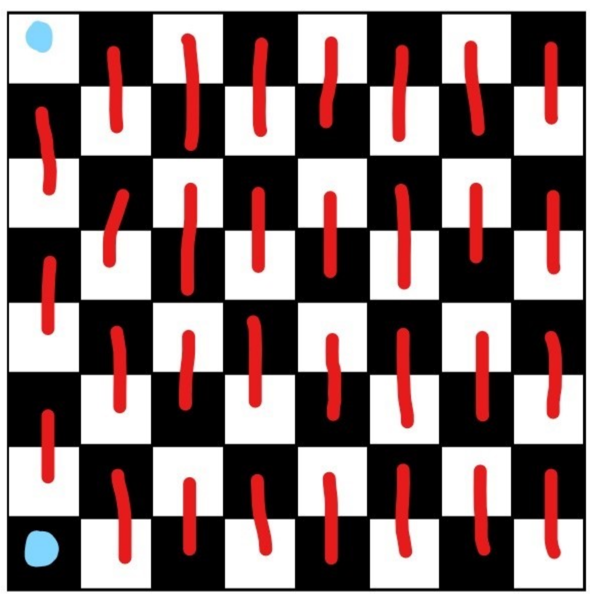

Figure 3.

Combination 4 – Removing 2 Non-Adjacent Squares in the Same Column/Row Leaving 4 Squares Between the Spaces and Leaving Even Parts

If one removes 2 non-adjacent squares in the same column leaving 4 squares between the spaces and leaving even parts, e.g. 2 squares + 4 squares = 6 squares, it is possible to tile the remaining squares with vertically oriented dominoes. Similarly, if one removes 2 non-adjacent squares in the same row leaving 4 squares between the spaces and leaving even parts, it is possible to tile the remaining squares with horizontally oriented dominoes. **Figure 4** demonstrates this combination.

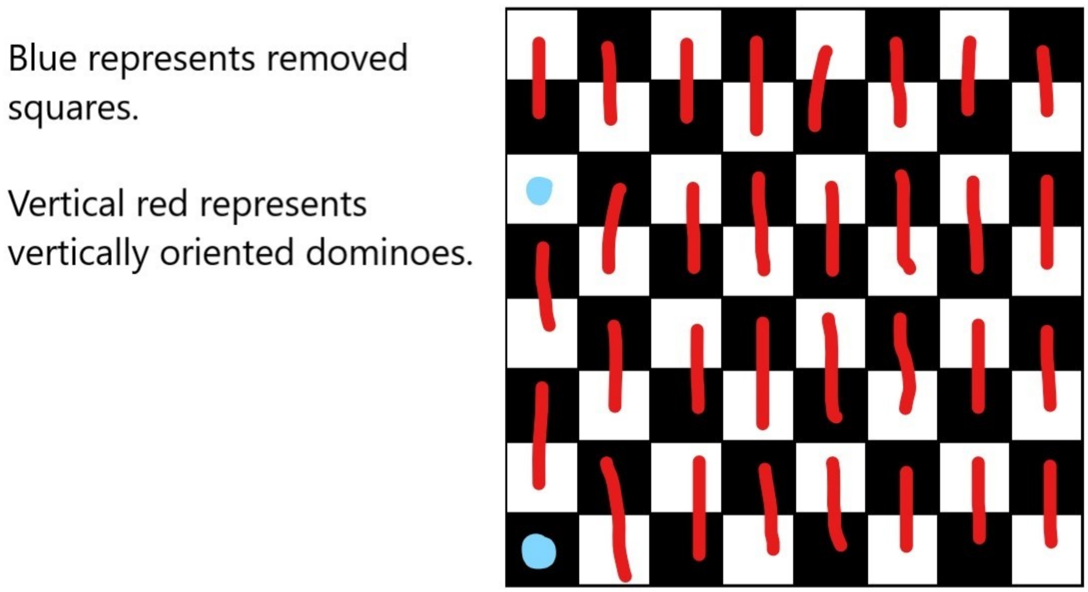

Figure 4. Again, note the even parts, separated in this case by the upper space. The set of 2 squares and the set of 4 squares are both even sets of squares.

Combination 5 – Removing 2 Non-Adjacent Squares in the Same Column/Row Leaving 4 Squares Between the Spaces and Leaving Odd Parts

If one removes 2 non-adjacent squares in the same column leaving 4 squares between the spaces and leaving odd parts, e.g. 1 square (odd) + 4 squares + 1 square (odd) = 6 squares, it is possible to tile the remaining squares by tiling the top and bottom rows with horizontally oriented dominoes while tiling the remaining squares with vertically oriented dominoes. Similarly, if one removes 2 non-adjacent squares in the same row leaving 4 squares between the spaces and leaving odd parts, it is possible to tile the remaining squares by tiling the far left and far right columns with vertically oriented dominoes while tiling the remaining squares with horizontally oriented dominoes. **Figure 5** demonstrates this combination.

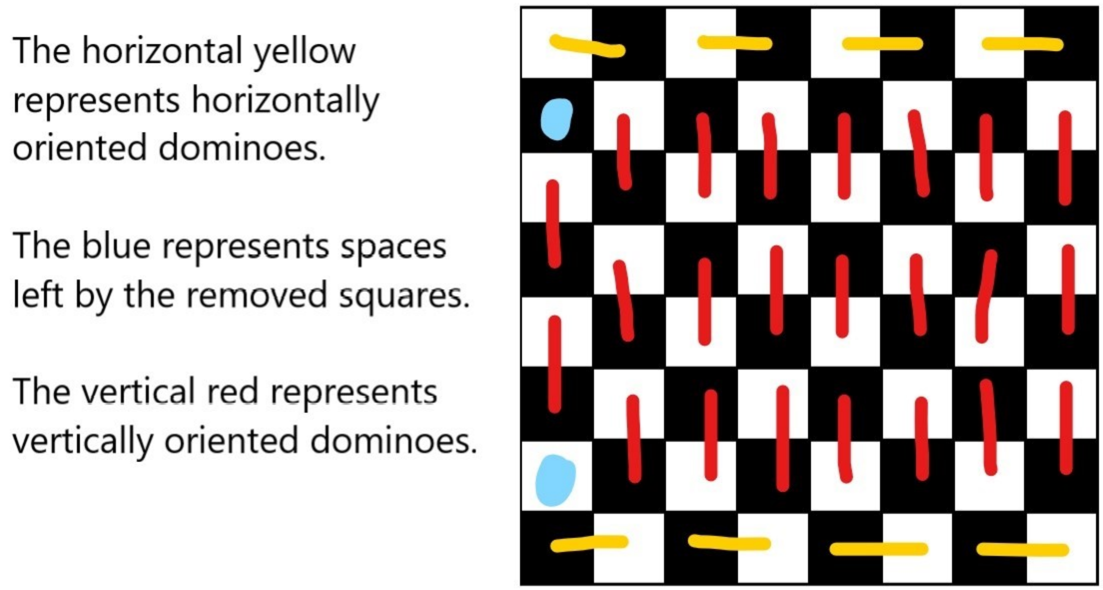

Figure 5. Note that there are only 2 odd parts, namely, the 2 individual sets of 1 square each.

Combination 6 – Removing 2 Non-Adjacent Squares in the Same Column/Row Leaving 2 Squares Between the Spaces and Leaving Even Parts

If one removes 2 non-adjacent squares in the same column leaving 2 squares between the spaces and leaving even parts, e.g. 2 squares + 2 squares + 2 squares = 6 squares or 2 squares + 4 squares = 6 squares, it is possible to tile the remaining squares with vertically oriented dominoes. Similarly, if one removes 2 non-adjacent squares in the same row leaving 2 squares between the spaces and leaving even parts, it is possible to tile the remaining squares with horizontally oriented dominoes. **Figure 6** demonstrates this combination.

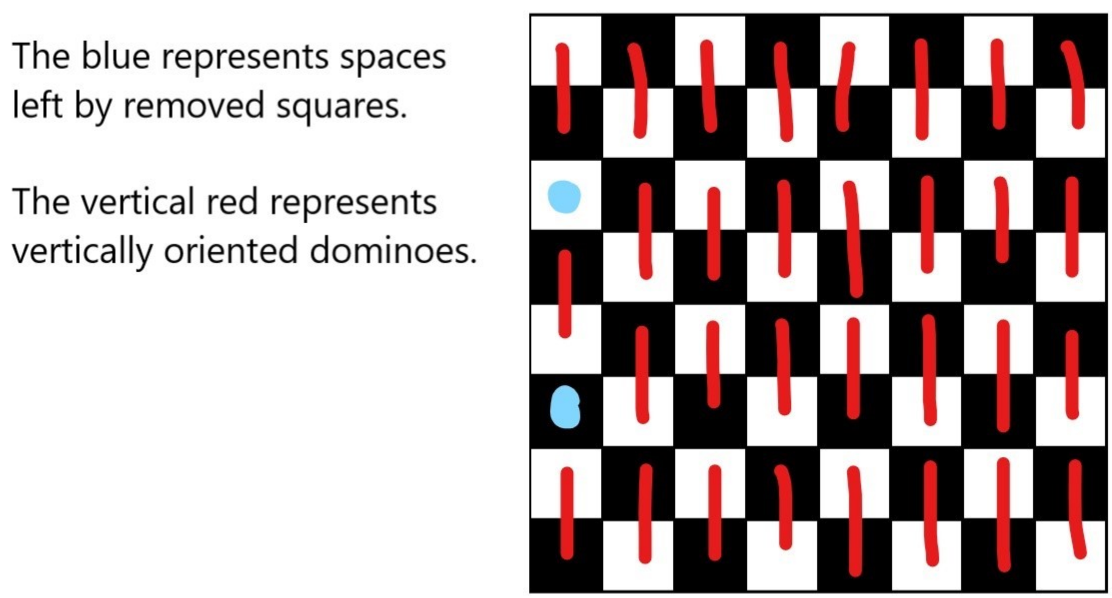

Figure 6.

Combination 7 - Removing 2 Non-Adjacent Squares in the Same Column/Row Leaving 2 Squares Between the Spaces and Leaving Odd Parts

If one removes 2 non-adjacent squares in the same column leaving 2 squares between the spaces and leaving odd parts, e.g. 1 square (odd) + 2 squares + 3 squares (odd) = 6 squares, it is possible to tile the remaining squares by tiling the top and bottom rows with horizontally oriented dominoes while tiling the remaining squares with vertically oriented dominoes. Similarly, if one removes 2 non-adjacent squares in the same row leaving 2 squares between the spaces and leaving odd parts, it is possible to tile the remaining squares by tiling the far left and far right columns with vertically oriented dominoes while tiling the remaining squares with horizontally oriented dominoes. **Figure 7** demonstrates this combination.

The horizontal yellow represents horizontally oriented dominoes.

The blue represents spaces left by removed squares.

The vertical red represents vertically oriented dominoes.

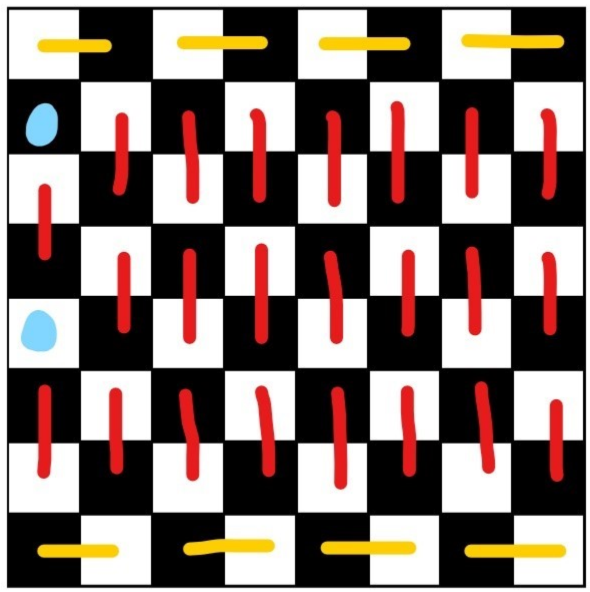

Figure 7. Note the two odd parts are the set including only 1 square and the set including 3 squares.

Combination 8 – Removing 2 Non-Adjacent Squares, 1 Black and 1 White, That Are Not in the Same Row or Column Algorithm #1

If one removes 2 non-adjacent squares, 1 black and 1 white, that are not in the same row or column, one can tile the remaining squares using the following algorithm. First, if there are columns to the left of the column containing the far-left space, tile those columns with vertically oriented dominoes. If there are columns to the right of the column containing the far-right space, tile those columns with vertically oriented dominoes. Next, start at the top row and begin tiling the row with horizontally oriented dominoes from left to right. If the dominoes completely cover that row, move to the row directly below and tile again from left to right. If the dominoes do not entirely cover that row but leave 1 square at the right of the row, tile that square and the square below with a vertically oriented domino, then move down to the next row. From there, tile the row from the right side of the row to the left with horizontally oriented dominoes. If that row is filled with no remaining squares, move to the next row and tile that row normally, with horizontally oriented dominoes from left to right. However, if that row has one remaining square at the left side of the row, tile that square and the square below it with a vertically oriented domino and proceed to tile the next row with horizontally oriented dominoes from left to right. Repeat this process of tiling the remaining rows as stated until the checkerboard is completely tiled. **Figure 8** demonstrates this algorithm on one checkerboard combination.

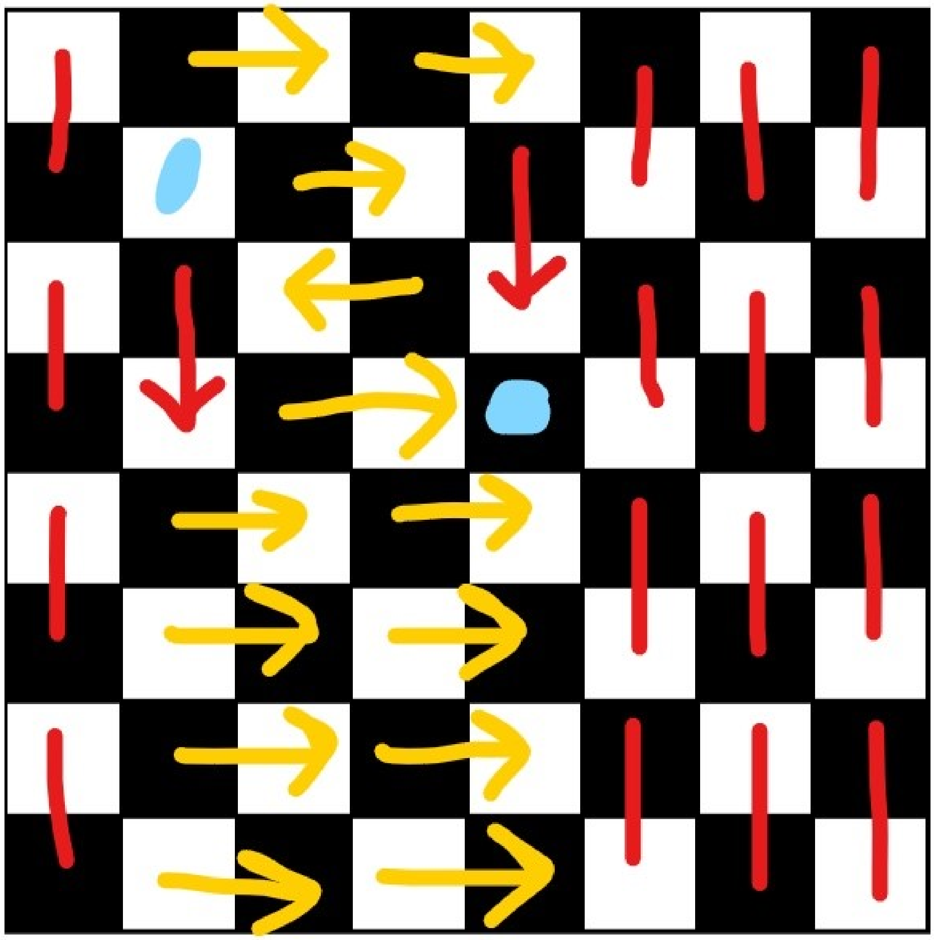

Figure 8. Arrows indicate direction of tiling.

Combination 9 Removing 2 Non-Adjacent Squares, 1 Black and 1 White, That Are Not in the Same Row or Column Algorithm #2

If the previous algorithm does not work, one may instead need this algorithm to tile the remaining squares. Like the previous algorithm, one begins by tiling the columns to the left of the column containing the far-left space and the columns to the right of the column containing the far-right space with vertically oriented dominoes. Then, to tile the remaining rows, start by tiling the first row from left to right with horizontally oriented dominoes. If the row is completely tiled, begin tiling the next row with horizontally oriented dominoes, only this time, tile from right to left. Whether the row has an even or odd number of squares, one must always alternate tiling from left to right and from right to left. Like the previous algorithm, if there is one remaining square in the row, tile that square and the square below it with a vertically oriented domino. **Figure 9** demonstrates this algorithm.

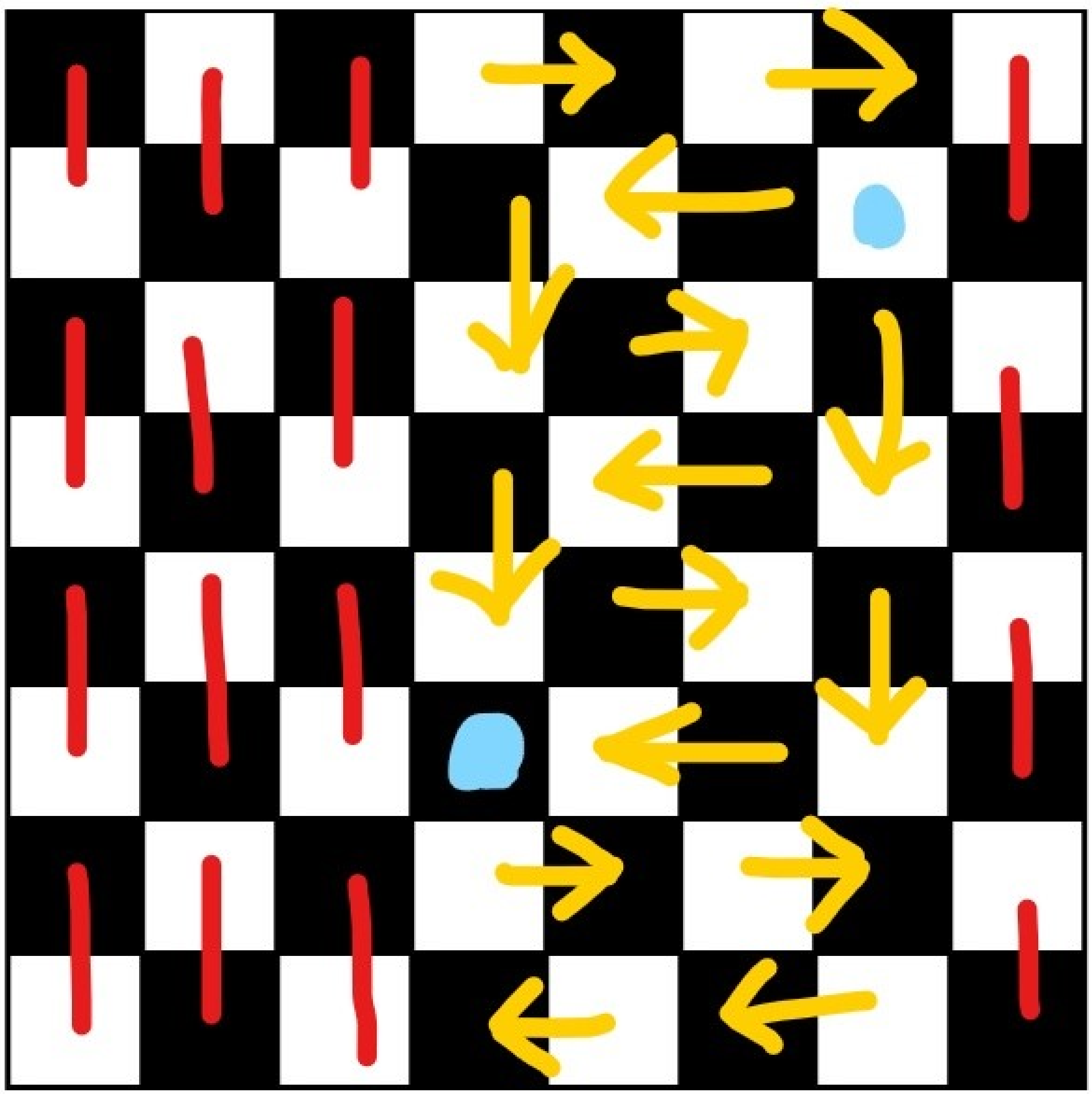

Figure 9. Note that the board has been rotated for the algorithm to work.

If neither algorithm works, try rotating the checkerboard by 90-degree intervals and performing each algorithm between rotations.

Using these combinations and algorithms to demonstrate that every variation of a checkerboard missing a black and white square can be tiled by dominoes, we have effectively proven that it is possible to tile the remaining squares of a checkerboard after removing 1 black and 1 white square.