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Question 1: PA9 Work

In order to model the circuit with a finite difference model, we need the differential equations which represent the circuit

$V_1 = V_{in}$

$g_1(V_2 - V_1) + C\frac{d(V_2-V_1)}{dt} + G_2V_2 - I_L = 0$

$V_2 - V_3 - L\frac{dI_L}{dt} = 0$

$-I_L + g_3V_3 = 0$

$V_4 - \alpha I_3 = 0$

$G_3V_3 - I_3 = 0$

$G_4(V_O - V_4) + G_OV_O = 0$

Rewritten in the frequency domain these become

$V_1 = V_{in}$

$g_1(V_2 - V_1) + j\omega C(V_2 - V_1) + G_2V_2 - I_L = 0$

$V_2 - V_3 - Lj\omega I_L = 0$

$-I_L + g_3V_3 = 0$

$V_4 - \alpha I_3 = 0$

$G_3V_3 - I_3 = 0$

$G_4(V_O - V_4) + G_OV_O = 0$

the resultant C and G matrices, along with the F vector can be seen in the code block below. the circuit can then be modelled using

$(G + j\omega C)V = F(\omega)$

```
close all
g1=1;
g2=1/2;
g3=1/10;
```

```
g4=1/0.1;
g5=1/1000;
a=100;
c=0.25;
L=0.2;
%V=[V1;V2;Il;V3;I3;V4;V0]
G=[1,0,0,0,0,0,0;
  -g2,g1+g2,-1,0,0,0,0;
  0,1,0,-1,0,0,0;
  0,0,-1,g3,0,0,0;
  0,0,0,0,-a,1,0;
  0,0,0,g3,-1,0,0;
  0,0,0,0,0,-g4,g4+g5]

C=[0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0;
    0,0,-L,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0]


plotV0=[];
plotV3=[];
plotVin=[];
for Vin=-10:10
F=[Vin;0;0;0;0;0;0];
V=G\F;
plotV0=[plotV0,V(7)];
plotV3=[plotV3,V(4)];
plotVin=[plotVin,Vin];
end
figure(1)
hold on
plot (plotVin,plotV0)
plot (plotVin,plotV3)
title("Figure 1: DC Sweep of input voltage")
xlabel("Input Voltage (V)")
ylabel("Output Voltage (V)")
legend("Vout","V3")

plotV0=[];
plotw=[];
Vin=1;
F=[Vin;0;0;0;0;0;0];
for w=0:10000
    V=(G+C*1i*w)\F;
    plotV0=[plotV0,V(7)];
    plotw=[plotw,w];
end
figure (2)
semilogx (plotw,abs(plotV0));
title("Figure 2: Output voltage over an AC sweep")
```

```
xlabel("frequency (rad/s)")
ylabel("Vout (V)")

figure (3)
semilogx (plotw,20*log10(abs(plotV0)))
title("Figure 3: Gain over an AC sweep")
xlabel("frequency (rad/s)")
ylabel("Gain (dB)")

plotgain=[];
plotc=[];
for i=1:2000
    c=0.25+0.05*randn();
    C=[0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0;
    0,0,-L,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0];
    V=(G+C*1i*pi)\F;
    plotgain=[plotgain,20*log10(abs(V(7)))];
    plotc=[plotc,c];
end

figure (4)
histogram(plotgain)
title("Figure 4: Histogram of Gain for random pertubations of
 Frequency")
xlabel("Frequency (rad/s)")
ylabel("Count")


G =

    1.0000         0         0         0         0         0         0
   -0.5000    1.5000   -1.0000         0         0         0         0
         0    1.0000         0   -1.0000         0         0         0
         0         0   -1.0000    0.1000         0         0         0
         0         0         0         0 -100.0000    1.0000         0
         0         0         0    0.1000   -1.0000         0         0
         0         0         0         0         0  -10.0000   10.0010


C =

         0         0         0         0         0         0         0
   -0.2500    0.2500         0         0         0         0         0
         0         0   -0.2000         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

Figure 1: DC Sweep of input voltage

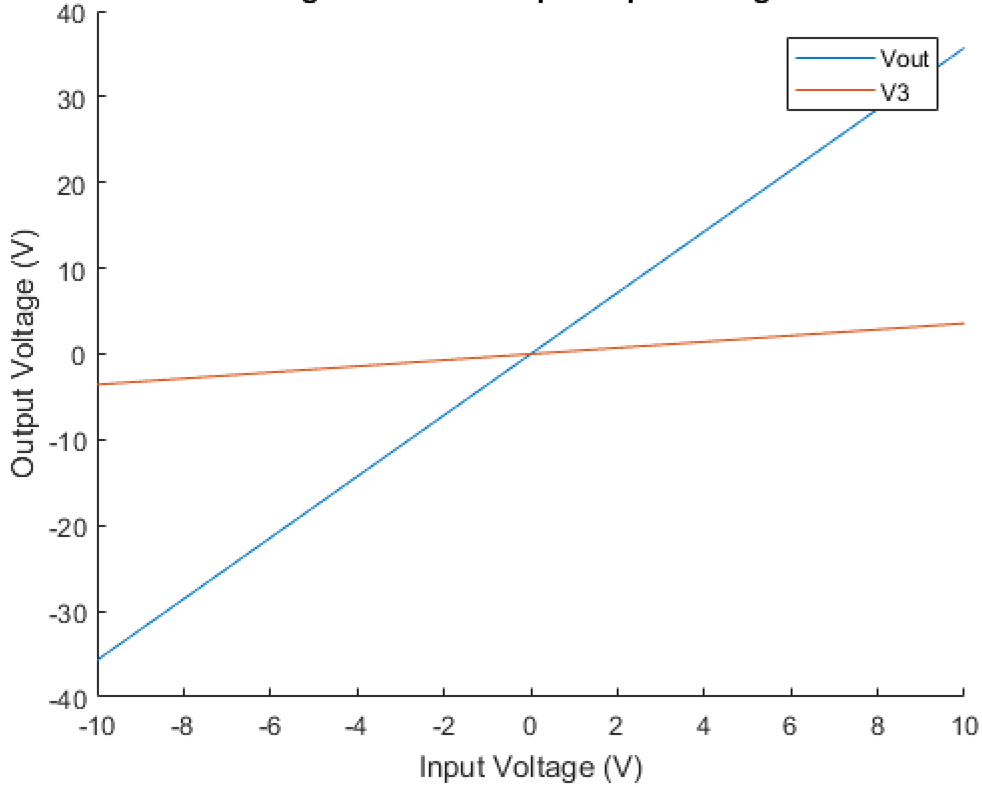


Figure 2: Output voltage over an AC sweep

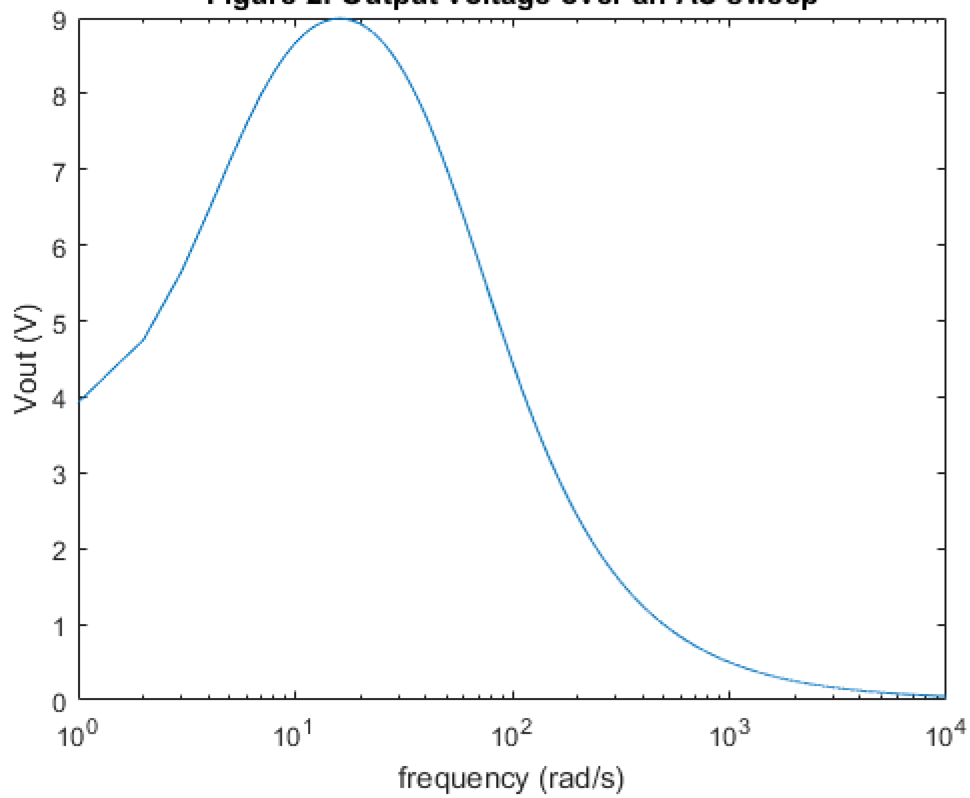


Figure 3: Gain over an AC sweep

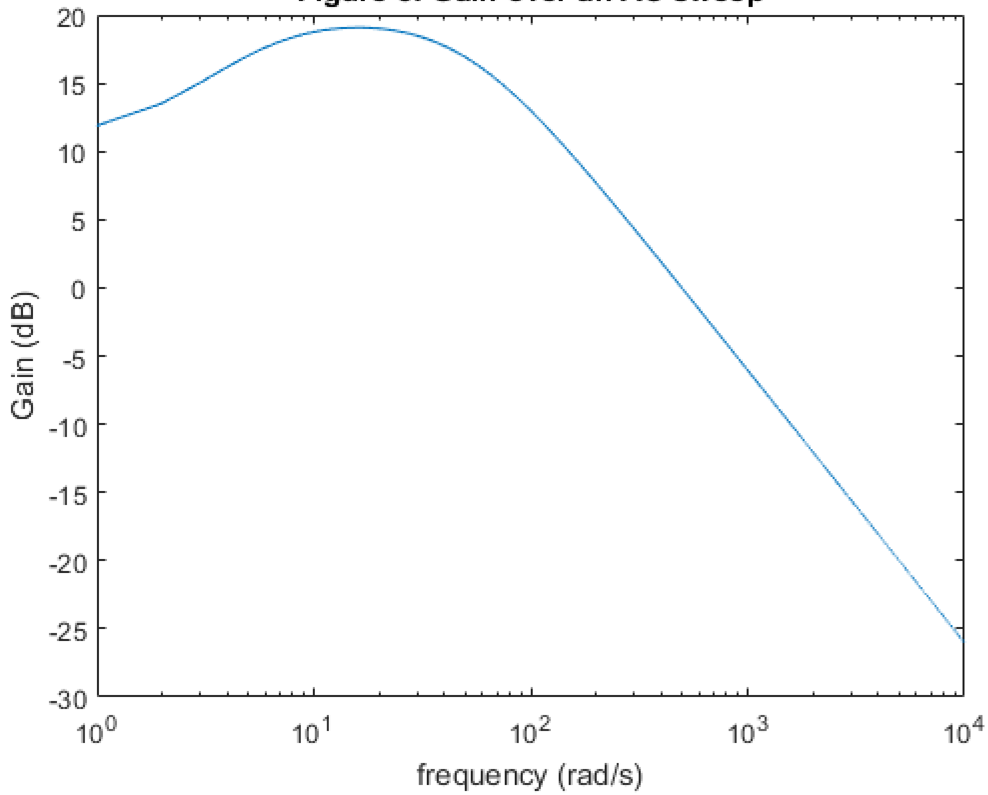
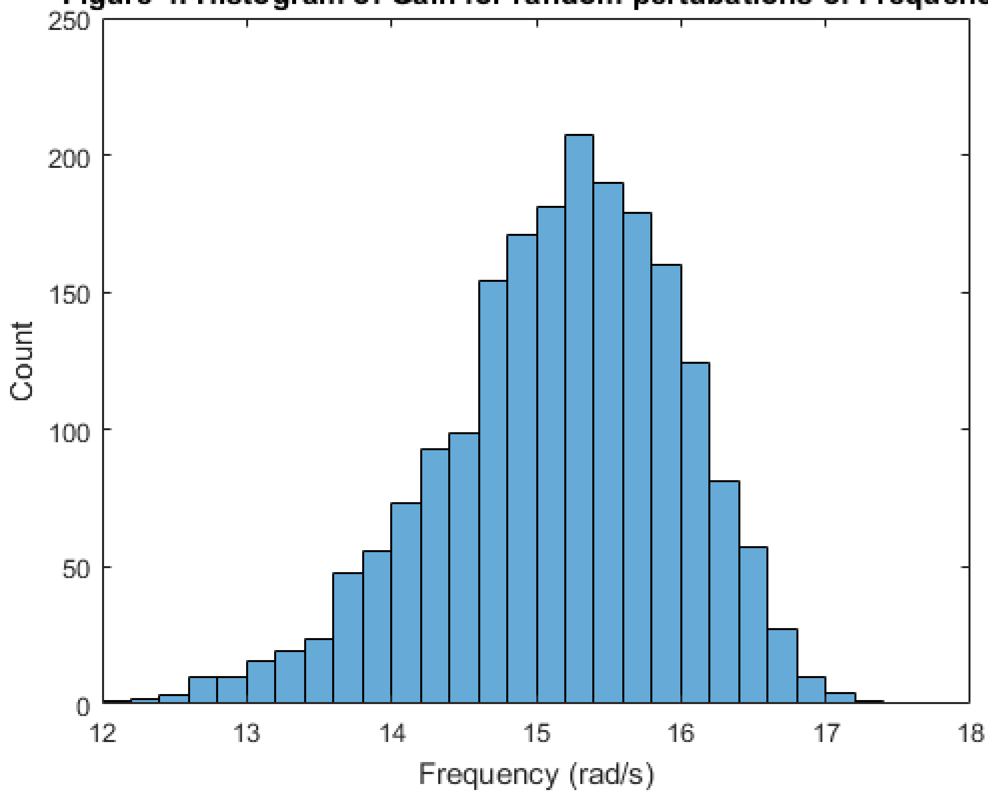


Figure 4: Histogram of Gain for random pertubations of Frequency



Question 2 Transient Circuit Simulation

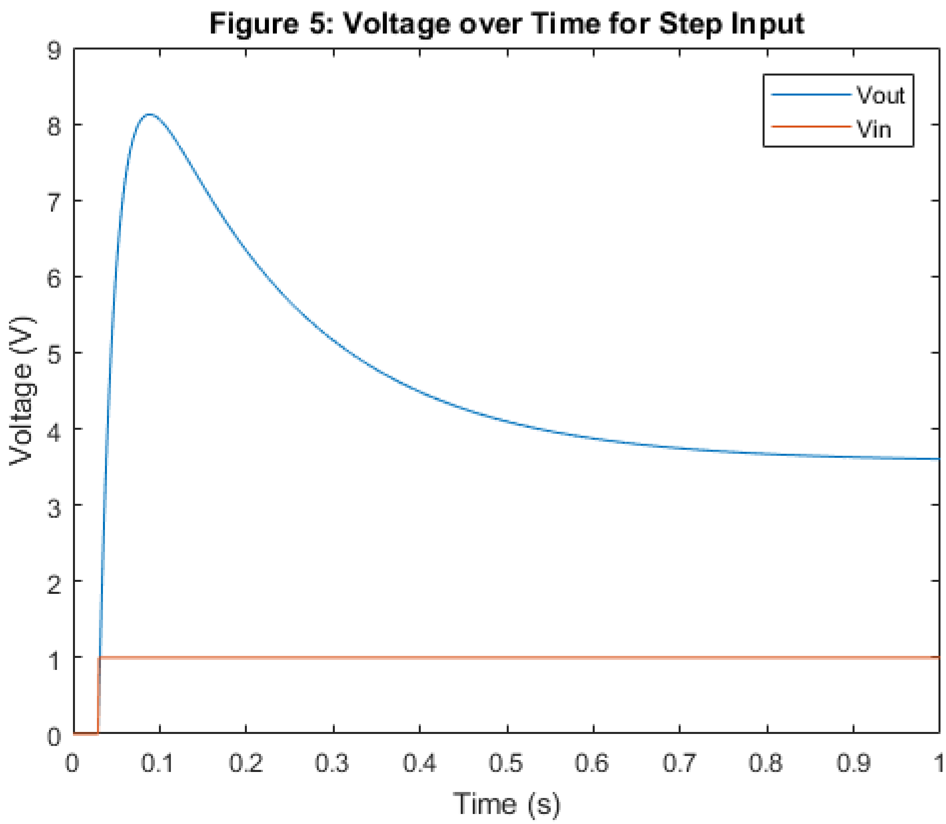
The circuit will act as a sort of Band Pass filter, with the low side rolloff slowed by the R1 in parallel with the capacitor. thus we expect the gain to decrease as the frequency increases above the centre frequency. The current controlled voltage source is a source of gain, giving the circuit the properties of an amplifier as well. Using a numerical finite difference solution, the circuit is simulated in the code block below for a step, sinusiodal, and gaussian input. The input and output voltages for these simulations are shown in figures 5, 7, and 9. The signals were also put through a fourier transform to get their frequency domain representations. The magnitude plots are shown in figures 6, 8, and 10. when the time step is increased by a factor of 10, the simulation becomes very innacurate. This can be seen by comparing figure 11 to figure 9.

```
clear
g1=1;
g2=1/2;
g3=1/10;
g4=1/0.1;
g5=1/1000;
a=100;
c=0.25;
L=0.2;

G=[1,0,0,0,0,0,0;
  -g2,g1+g2,-1,0,0,0,0;
  0,1,0,-1,0,0,0;
  0,0,-1,g3,0,0,0;
  0,0,0,0,-a,1,0;
  0,0,0,g3,-1,0,0;
  0,0,0,0,0,-g4,g4+g5];

C=[0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0;
    0,0,-L,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0;
    0,0,0,0,0,0,0];

%V=[V1;V2;Il;V3;I3;V4;V0]
t=linspace(0,1,1000);
dt=t(2)-t(1);
V=zeros(7,1000);
plotV0=[];
plotVin=[];
Vin=0;
for i=1:1000
    if t(i)>0.03
    Vin=1;
    end
    F=[Vin;0;0;0;0;0;0];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
```

```
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(5)
plot(t,plotV0)
hold on
plot(t,plotVin)
title("Figure 5: Voltage over Time for Step Input")
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")

figure(6)
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotVin))))
title("Figure 6: Magnitude in Frequency Domain for Step Input")
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
legend("output","input")

V=zeros(7,1000);
plotV0=[];
plotVin=[];
Vin=0;
f=1/(0.03); %frequency of sin
for i=1:1000
    Vin=sin(2*pi*f*t(i));
    F=[Vin;0;0;0;0;0;0];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(7)
plot(t,plotV0)
hold on
plot(t,plotVin)
title("Figure 7: Voltage over Time for sin Input")
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")

figure(8)
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotVin))))
title("Figure 8: Magnitude in Frequency Domain for sin Input")
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
```

```
legend("output","input")

V=zeros(7,1000);
plotV0=[];
plotVin=[];
for i=1:1000
    Vin=exp(-((t(i)-0.06)^2)/(2*0.03^2));
    F=[Vin;0;0;0;0;0;0];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(9)
plot(t,plotV0)
hold on
plot(t,plotVin)
title("Figure 9: Voltage over Time for Gaussian Input")
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")

figure(10)
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotVin))))
title("Figure 10: Magnitude in Frequency Domain for Gaussian Input")
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
legend("output","input")
V=zeros(7,500);
plotV0=[];
plotVin=[];
Vin=0;
f=1/(0.03); %frequency of sin
for i=1:100
    Vin=exp(-((t(i)-0.06)^2)/(2*0.03^2));
    F=[Vin;0;0;0;0;0;0];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/(10*dt) + G)^-1)*(C*(V(:,i-1)/(10*dt))+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(11)
newt=linspace(0,1,100);
plot(newt,plotV0)
hold on
plot(newt,plotVin)
```

```
title("Figure 11: Voltage over Time for Gaussian Input with Reduced
 Timestep")
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")
```



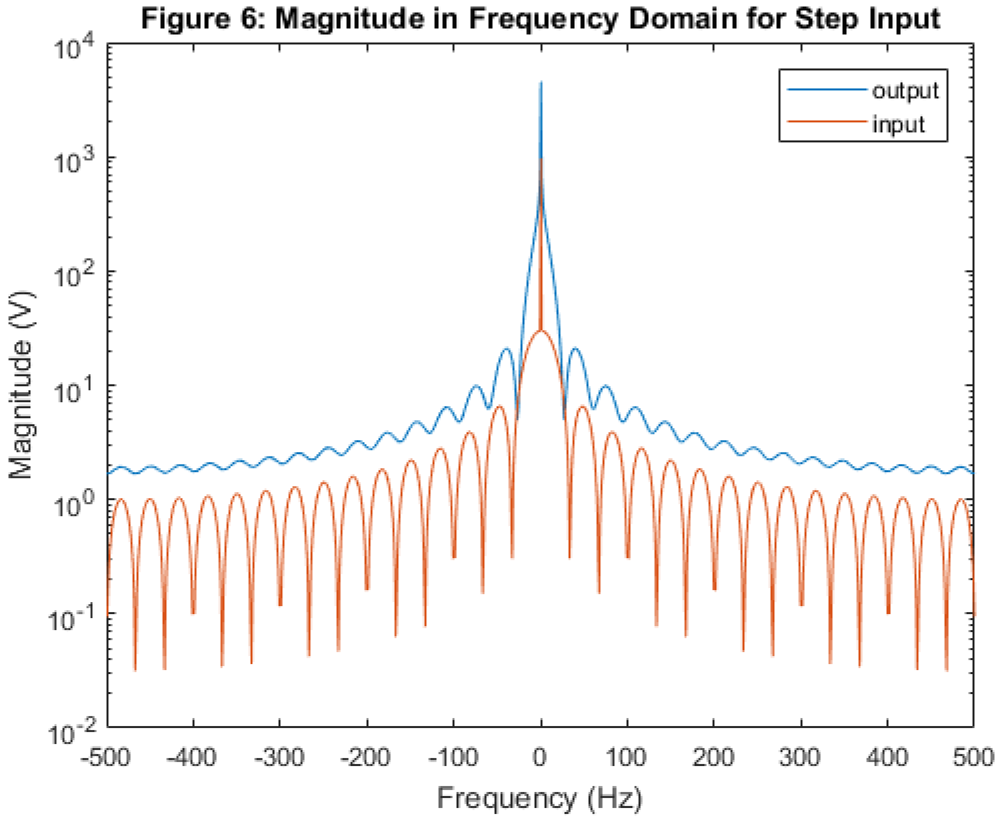


Figure 6: Magnitude in Frequency Domain for Step Input
output
input
Magnitude (V)
Frequency (Hz)
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
-500
-400
-300
-200
-100
0
100
200
300
400
500

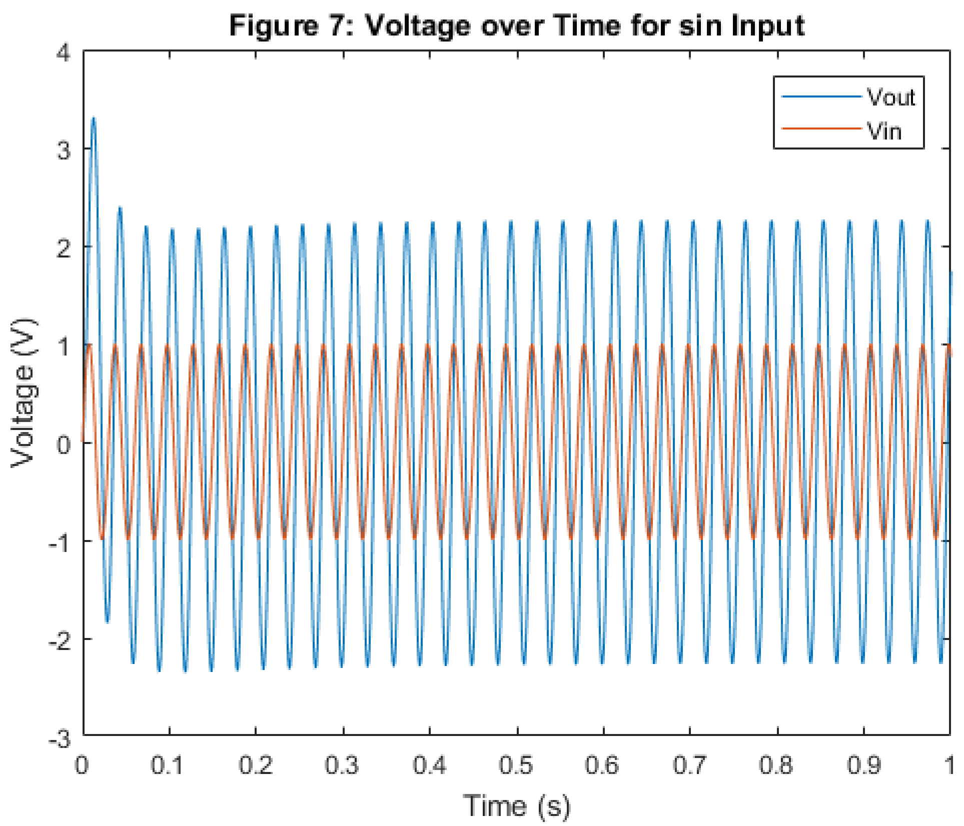


Figure 7: Voltage over Time for sin Input
Vout
Vin
Voltage (V)
Time (s)
4
3
2
1
0
-1
-2
-3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

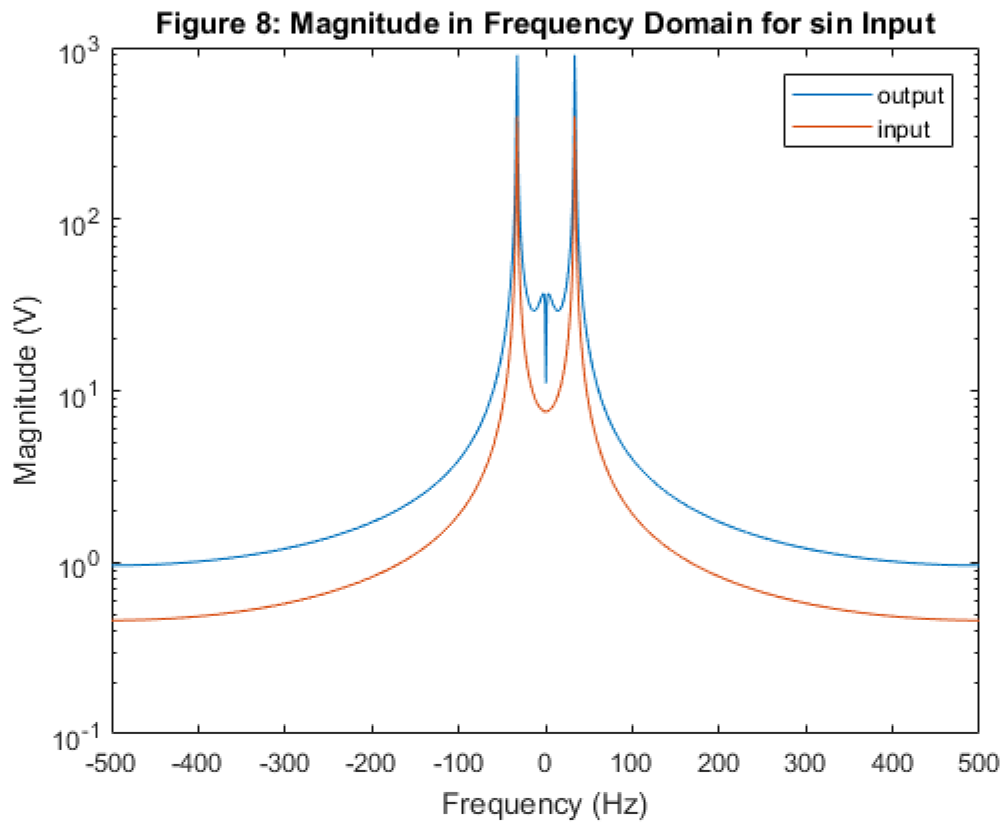


Figure 8: Magnitude in Frequency Domain for sin Input
output
input
Magnitude (V)
10^3
10^2
10^1
10^0
10^{-1}
-500
-400
-300
-200
-100
0
100
200
300
400
500
Frequency (Hz)

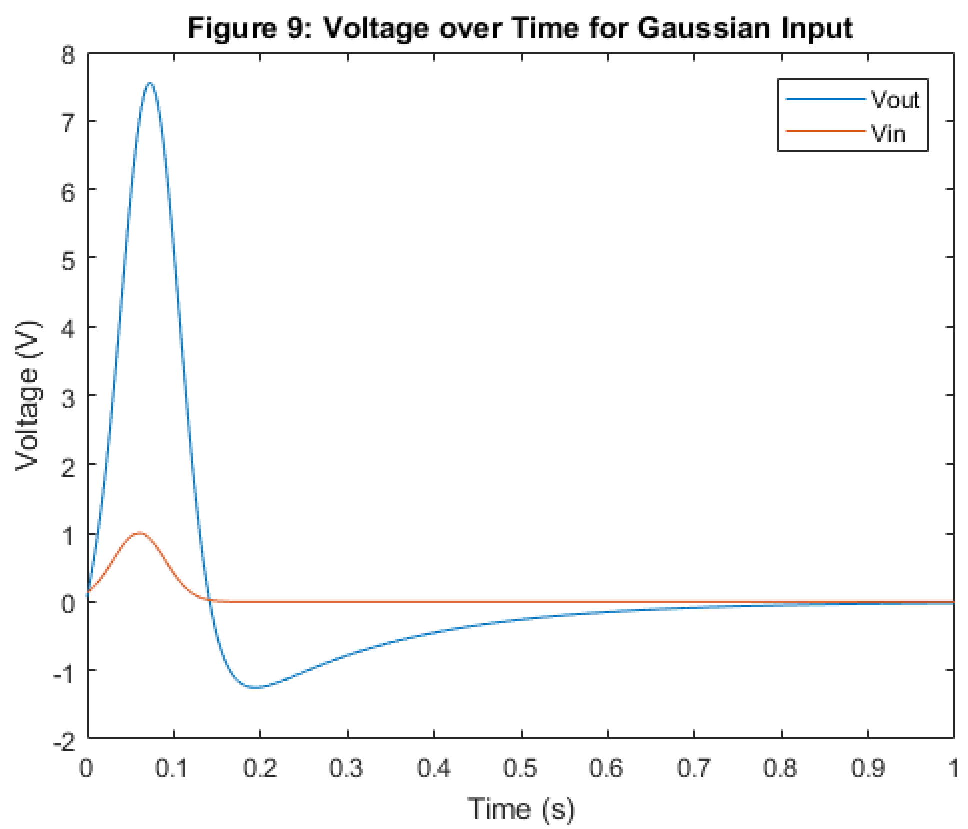


Figure 9: Voltage over Time for Gaussian Input
Vout
Vin
Voltage (V)
8
7
6
5
4
3
2
1
0
-1
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time (s)

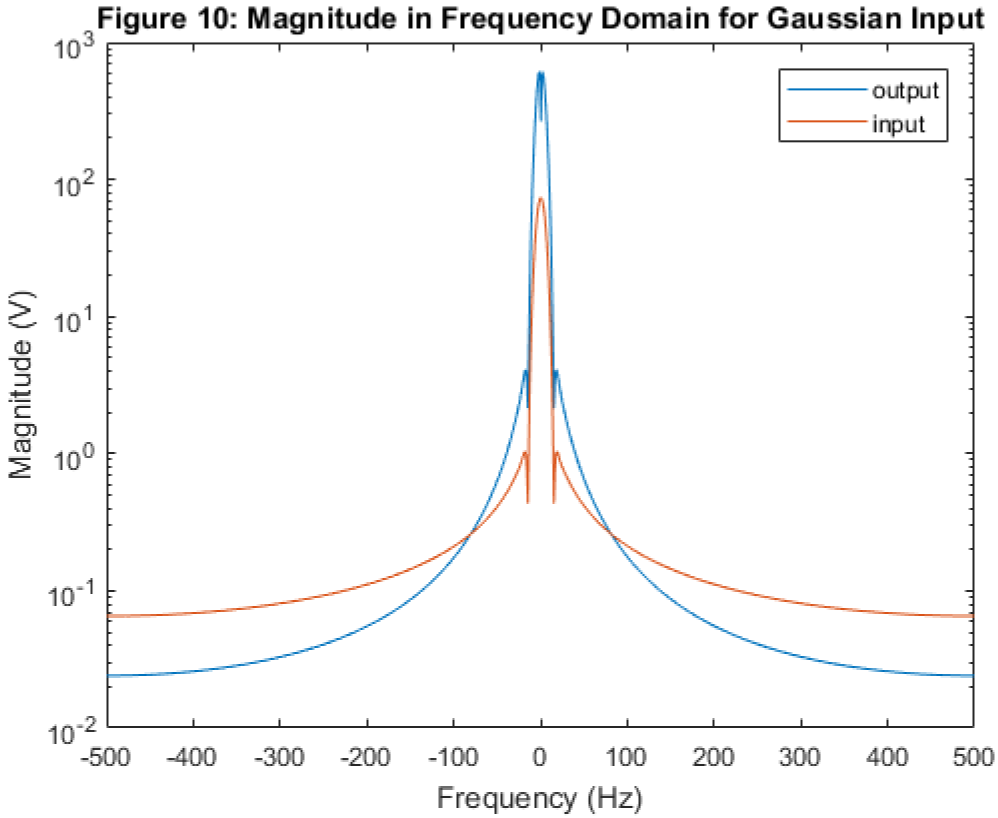


Figure 10: Magnitude in Frequency Domain for Gaussian Input
output
input
Magnitude (V)
Frequency (Hz)

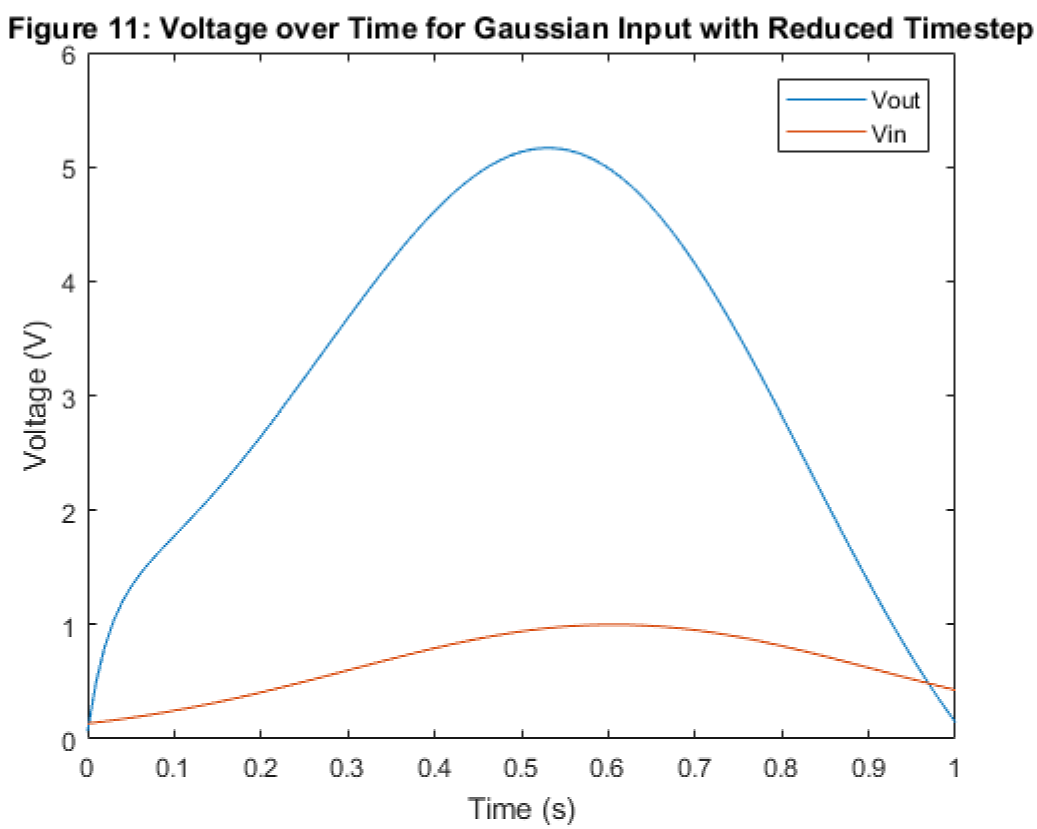


Figure 11: Voltage over Time for Gaussian Input with Reduced Timestep
Vout
Vin
Voltage (V)
Time (s)

Question 3: Circuit with Noise

To simulate thermal noise in R3, we add a current source and capacitor in parralel to it. The current source simulates the noise, while the capacator serves to bandwidth limit that noise. This requires an extra equation to be added, adding an extra line to the C and G matricies. the new matricies are included in the code block below, and output into the report. Figure 12 shows the input and output voltage for a gaussian input. Figure 13 shows the fourier transforms of those signals. increasing Cn lowers the noise level, this can be most clearly seen in the fourier transform plots in figures 13, 15, and 17. The voltage plots are in figures 12, 14, and 16. Increasing the time step once again causes the signals to be very innacurate. It also has the additional effect of lessening the frequency of the noise, as can be seen by comparing figure 18 to figure 12.

```
clear
g1=1;
g2=1/2;
g3=1/10;
g4=1/0.1;
g5=1/1000;
a=100;
c=0.25;
L=0.2;
cn=0.00001;

G=[1,0,0,0,0,0,0,0;
  -g2,g1+g2,-1,0,0,0,0,0;
  0,1,0,-1,0,0,0,0;
  0,0,-1,g3,0,0,0,-1;
  0,0,0,0,-a,1,0,0;
  0,0,0,g3,-1,0,0,0;
  0,0,0,0,0,-g4,g4+g5,0;
  0,0,0,0,0,0,0,1]

C=[0,0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0,0;
    0,0,-L,0,0,0,0,0;
    0,0,0,cn,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0]

%V=[V1;V2;Il;V3;I3;V4;V0;In]
t=linspace(0,1,1000);
dt=t(2)-t(1);
V=zeros(8,1000);
plotV0=[];
plotVin=[];
for i=1:1000
    Vin=exp(-((t(i)-0.06)^2)/(2*0.03^2));
    In=0.001*randn();
    F=[Vin;0;0;0;0;0;0;In];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
```

```
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(12)
plot(t,plotV0)
hold on
plot(t,plotVin)
title({"Figure 12: Voltage over Time for Gaussian Input with
 noise", "Cn=0.00001"})
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")
figure(13)
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotVin))))
title({"Figure 13: Magnitude in Frequency Domain for Gaussian Input
 with noise","Cn=0.00001"})
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
legend("output","input")

cn=0.00005;
C=[0,0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0,0;
    0,0,-L,0,0,0,0,0;
    0,0,0,cn,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0];
V=zeros(8,1000);
plotV0=[];
plotVin=[];
for i=1:1000
    Vin=exp(-((t(i)-0.06)^2)/(2*0.03^2));
    In=0.001*randn();
    F=[Vin;0;0;0;0;0;0;In];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(14)
plot(t,plotV0)
hold on
plot(t,plotVin)
title({"Figure 14: Voltage over Time for Gaussian Input with
 noise","Cn=0.00005"})
```

```
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")
figure(15)
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotVin))))
title({"Figure 15: Magnitude in Frequency Domain for Gaussian Input
 with noise","Cn=0.00005"})
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
legend("output","input")

cn=0.0005;
C=[0,0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0,0;
    0,0,-L,0,0,0,0,0;
    0,0,0,cn,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0];
V=zeros(8,1000);
plotV0=[];
plotVin=[];
for i=1:1000
    Vin=exp(-((t(i)-0.06)^2)/(2*0.03^2));
    In=0.001*randn();
    F=[Vin;0;0;0;0;0;0;In];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(16)
plot(t,plotV0)
hold on
plot(t,plotVin)
title({"Figure 14: Voltage over Time for Gaussian Input with
 noise","Cn=0.0005"})
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")
figure(17)
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,1000),fftshift(abs(fft(plotVin))))
title({"Figure 17: Magnitude in Frequency Domain for Gaussian Input
 with noise","Cn=0.0005"})
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
```

```
legend("output","input")

cn=0.00001;
C=[0,0,0,0,0,0,0,0;
    -c,c,0,0,0,0,0,0;
    0,0,-L,0,0,0,0,0;
    0,0,0,cn,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0];
V=zeros(8,200);
t=linspace(0,1,200);
dt=t(2)-t(1);
plotV0=[];
plotVin=[];
for i=1:200
    Vin=exp(-((t(i)-0.06)^2)/(2*0.03^2));
    In=0.001*randn();
    F=[Vin;0;0;0;0;0;0;In];
    if i==1
        V(:,i)=((C/dt + G)^-1)*(F);
    else
        V(:,i)=((C/dt + G)^-1)*(C*(V(:,i-1)/dt)+F);
    end
    plotV0=[plotV0,V(7,i)];
    plotVin=[plotVin,Vin];
end
figure(18)
plot(t,plotV0)
hold on
plot(t,plotVin)
title({"Figure 18: Voltage over Time for Gaussian Input with
 noise","Reduced Timestep"})
xlabel("Time (s)")
ylabel("Voltage (V)")
legend("Vout","Vin")
figure(19)
semilogy(linspace(-500,500,200),fftshift(abs(fft(plotV0))))
hold on
semilogy(linspace(-500,500,200),fftshift(abs(fft(plotVin))))
title({"Figure 19: Magnitude in Frequency Domain for Gaussian Input
 with noise","Reduced Timestep"})
xlabel("Frequency (Hz)")
ylabel("Magnitude (V)")
legend("output","input")


G =

  Columns 1 through 7

    1.0000         0         0         0         0         0         0
   -0.5000    1.5000   -1.0000         0         0         0         0
```

```
         0    1.0000         0   -1.0000         0         0         0
         0         0   -1.0000    0.1000         0         0         0
         0         0         0         0 -100.0000    1.0000         0
         0         0         0    0.1000   -1.0000         0         0
         0         0         0         0         0  -10.0000   10.0010
         0         0         0         0         0         0         0

  Column 8

         0
         0
         0
   -1.0000
         0
         0
         0
    1.0000


C =

  Columns 1 through 7

         0         0         0         0         0         0         0
   -0.2500    0.2500         0         0         0         0         0
         0         0   -0.2000         0         0         0         0
         0         0         0    0.0000         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Column 8

         0
         0
         0
         0
         0
         0
         0
         0
```

Figure 12: Voltage over Time for Gaussian Input with noise
Cn=0.00001

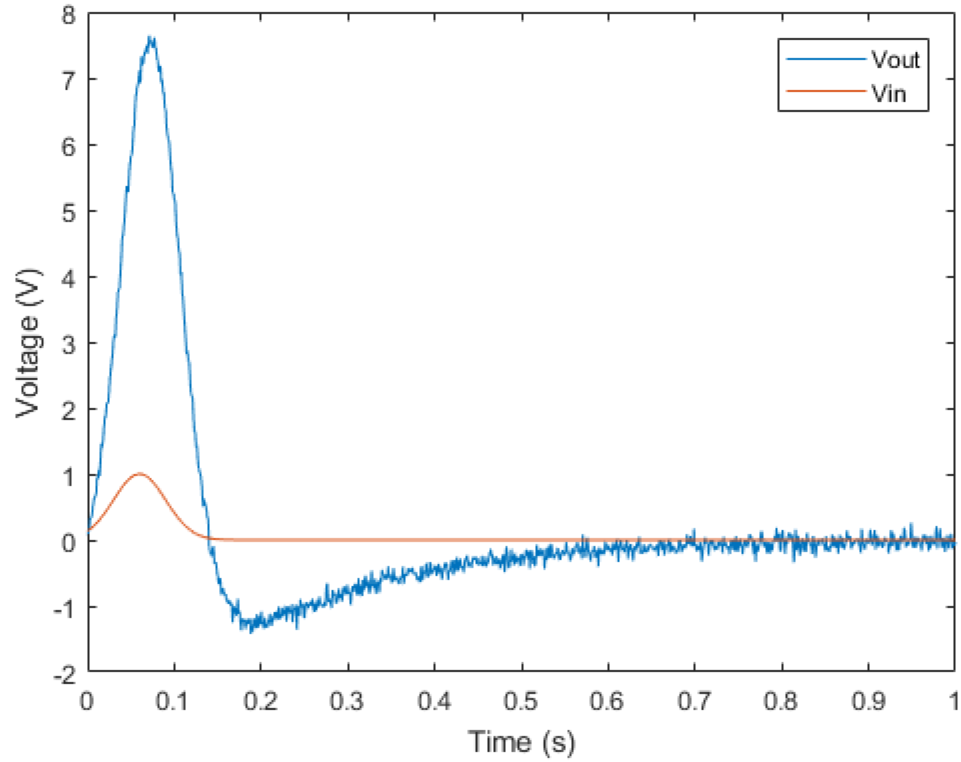


Figure 13: Magnitude in Frequency Domain for Gaussian Input with noise
Cn=0.00001

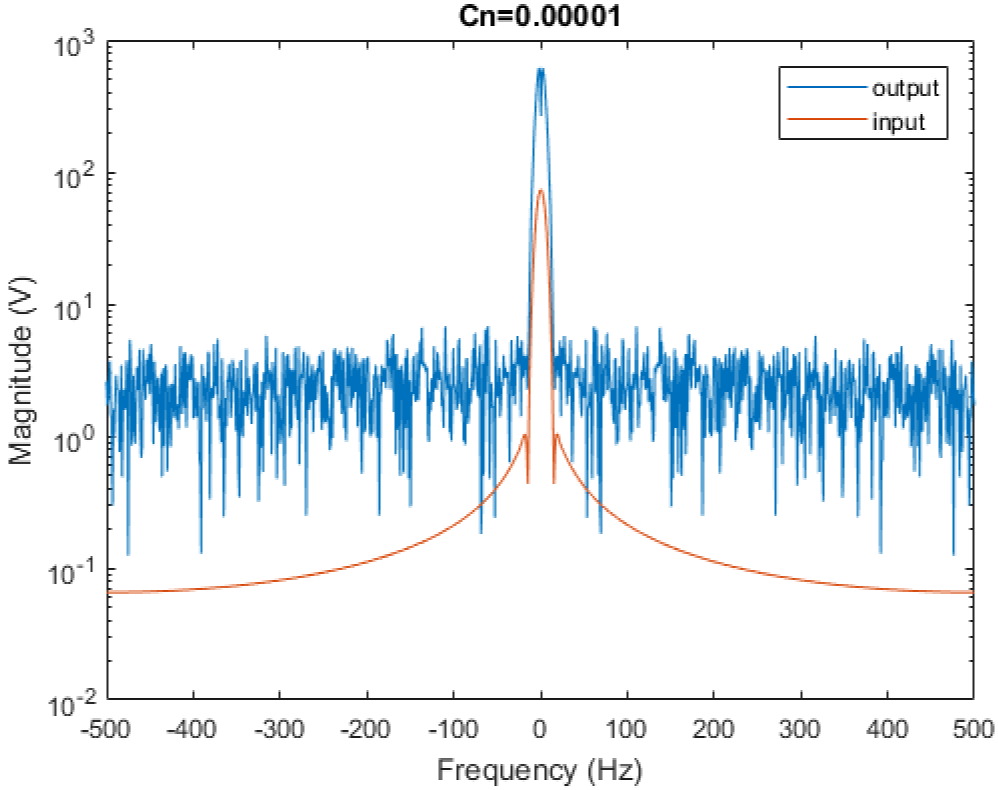


Figure 14: Voltage over Time for Gaussian Input with noise Cn=0.00005

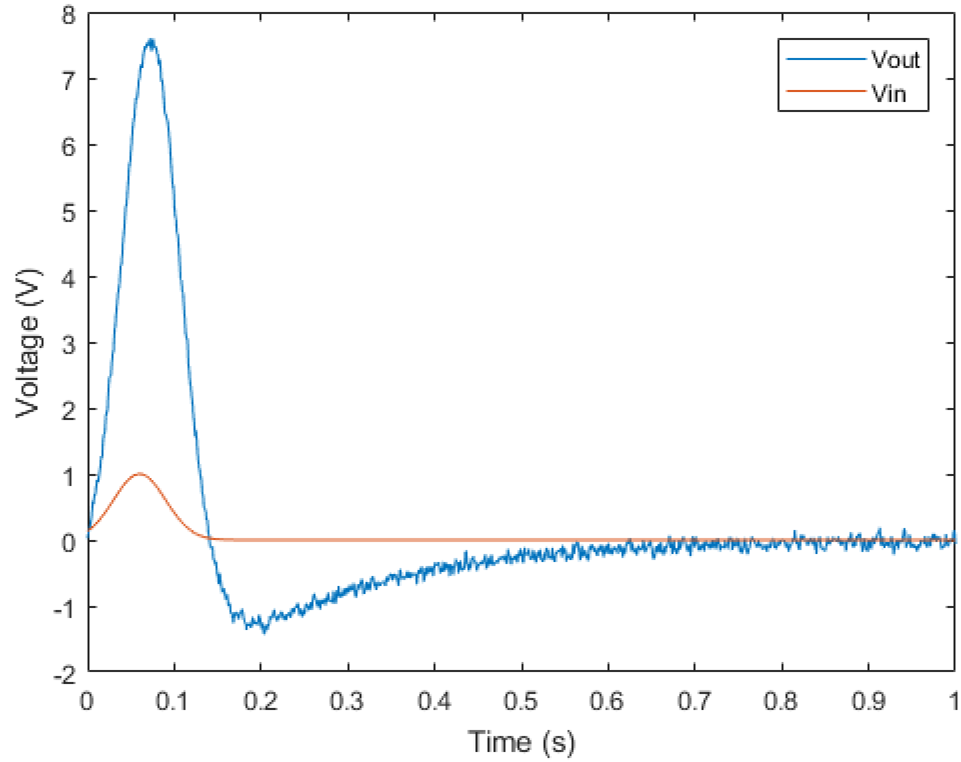


Figure 15: Magnitude in Frequency Domain for Gaussian Input with noise Cn=0.00005

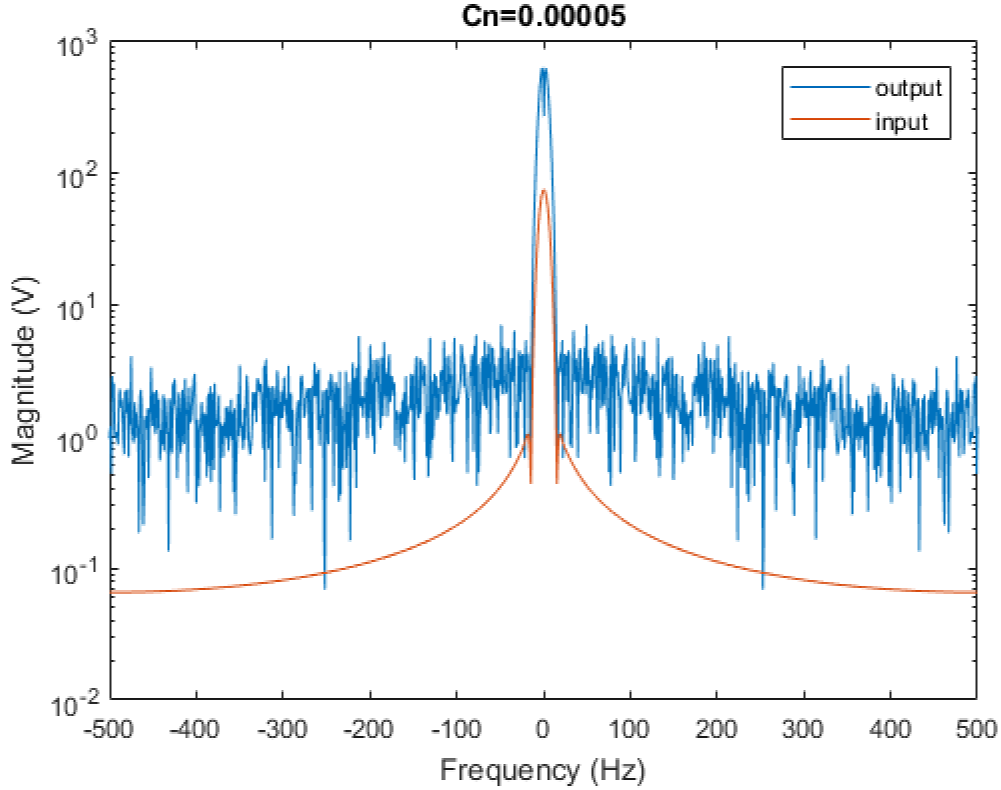


Figure 14: Voltage over Time for Gaussian Input with noise Cn=0.0005

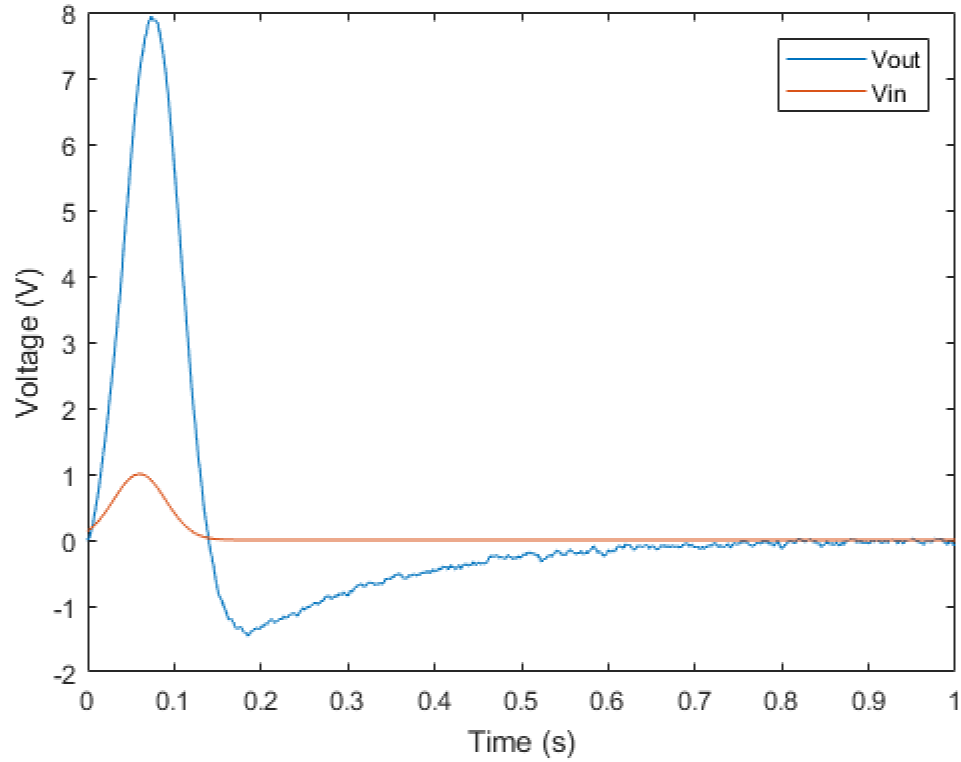


Figure 17: Magnitude in Frequency Domain for Gaussian Input with noise Cn=0.0005

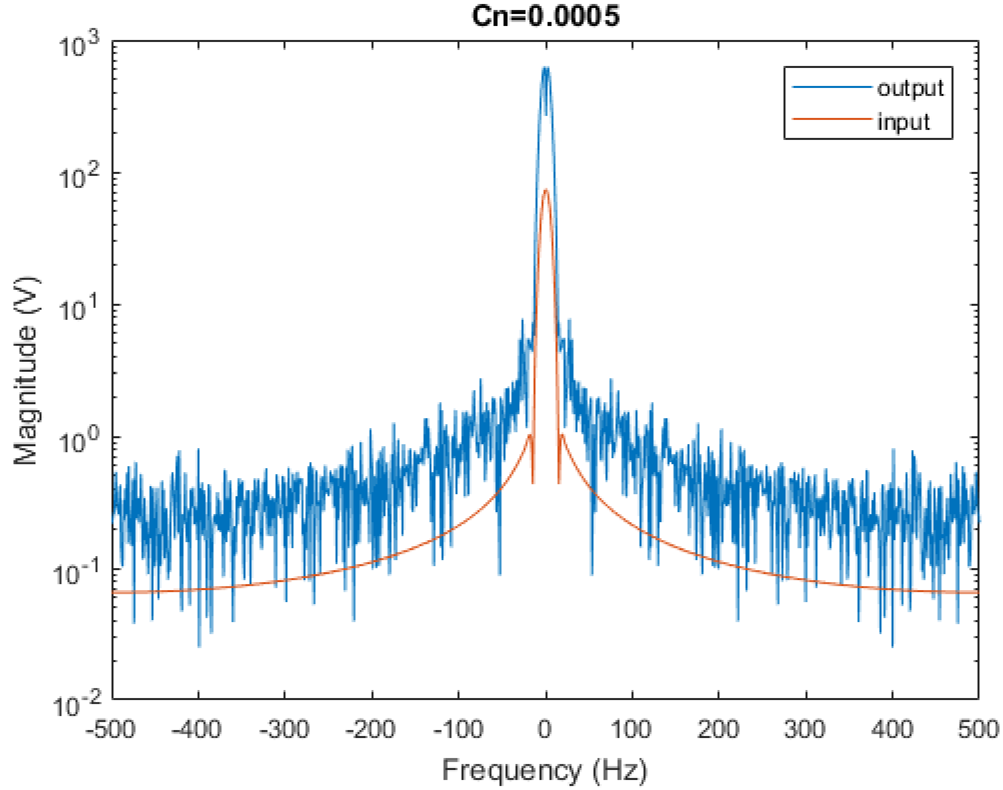


Figure 18: Voltage over Time for Gaussian Input with noise Reduced Timestep

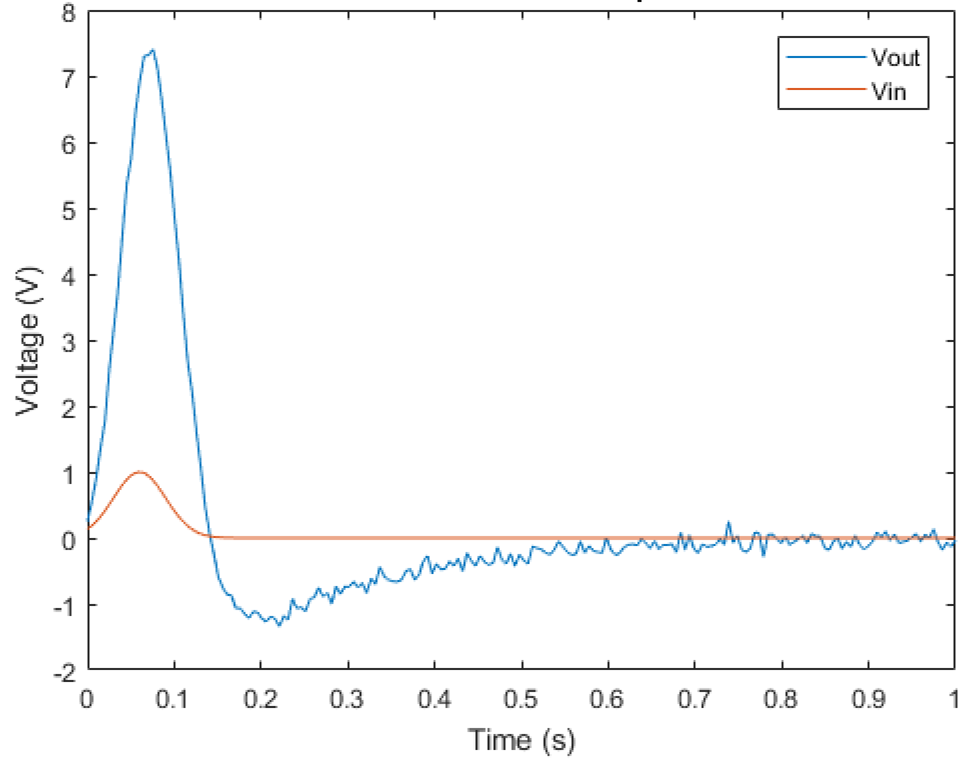
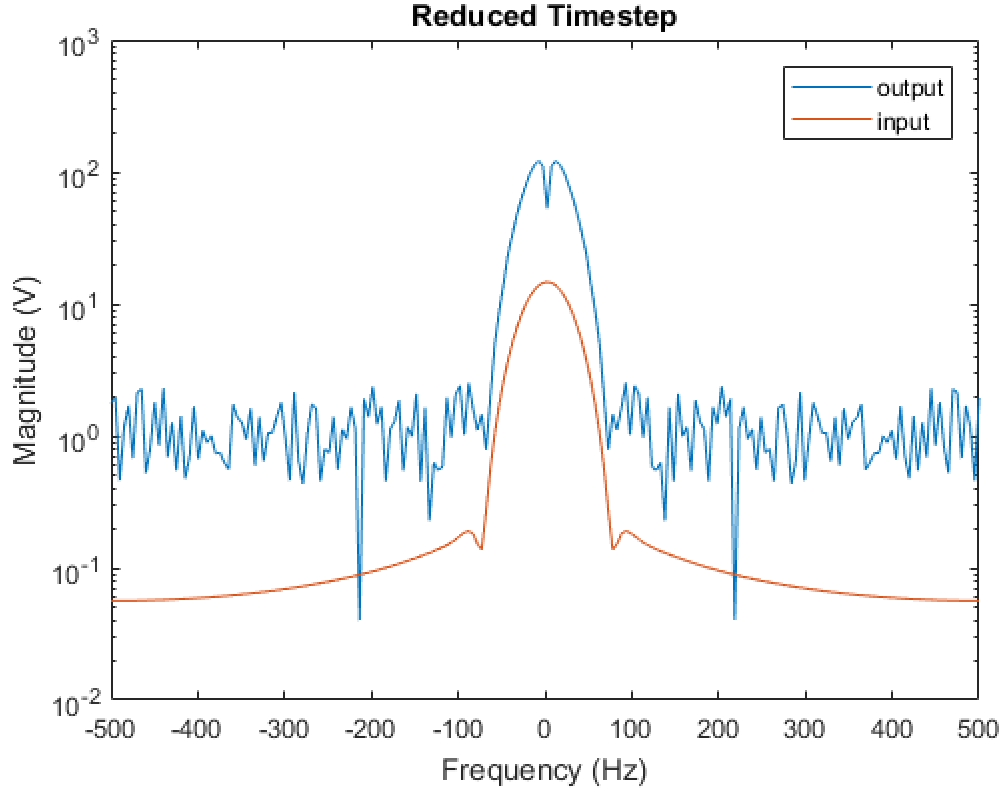


Figure 19: Magnitude in Frequency Domain for Gaussian Input with noise Reduced Timestep



Question 4: Non-linearity

If the current controlled voltage source on the output stage were non-linear, e.g modeled by $V = \alpha I_3 + \beta I_3^2 + \gamma I_3^3$ then an extra non-linear component would need to be added to the equation to account for this. in the matrix solution, a non-linear B-vector would be added such that the time domain equation becomes

$$C\frac{dV}{dt} + GV + B = F$$

with the V4 component of B being $\beta(G_3V_3)^2 + \gamma(G_3V_3)^3$. B is a function of V3 at the current time, and thus in addition to iterating over time, we will need to iterate within each timestep to converge on a value for B.

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