Problem 3: Runtime Analysis Part a

in	out
1	2
2	4
3	8
4	16
5	

$2^{2^k} > n$

$2^k = \log_2 n \rightarrow \log_2(\log_2(n))$

Runtime $= \log_2(\log_2(n))$

Part b. K L i

i % $\sqrt{n}$

in	# iterations
1	1
2	8
3	27
4	64

i	n=1	n=9
1	✓	X
2	✓	X
3	✓	✓

$$\sum_{i=1}^{n}\left(\Theta(1) + \sum_{k=1}^{\sqrt{n}} i^3\right) \rightarrow \sum_{i=1}^{n}\Theta(1) + \sum_{i=1}^{n}\sum_{k=1}^{\sqrt{n}}(k\sqrt{n})^3$$

$$(\sqrt{n})\sum_{k=1}^{3\sqrt{n}} k^3 = \left(\frac{\sqrt{n}(\sqrt{n}+1)}{2}\right)^2 \cdot \sqrt{n}^3 + \Theta(n)$$

highest degree

$$= \frac{n^2 + 2\sqrt{n} + n}{4} \cdot n\sqrt{n} + \Theta(n) \rightarrow \frac{n^3\sqrt{n} + 2n^2 + n^2\sqrt{n}}{4}$$

Thus, $\Theta(n^{3/2})$

Part C

First two for loops run n times.
if statement is true n times at most.
$n = n + m$, $m \leq n$, runs $\log n$ times.
$n = 1, 2, 4, 8, 16, 32$, pattern is 2^k
$2^k = n$, $k = \log n$

$$= \sum_{i=1}^{n} \sum_{k=1}^{n} \theta(1) + \sum_{n=1,2,4,8}^{n} \theta(1)$$

$$= \sum_{i=1}^{n} \theta(n) + \sum_{k=1}^{n} \theta(\log n)$$

$$= \theta(n^2) + \theta \log n$$

$$= \theta(n^2)$$

Part d

First for loop is $\theta(n)$, runs n times

$$\sum_{i=0}^{n} \left(\theta(1) + O\left(\sum_{j=1}^{10 \times \frac{7i}{2}} \theta 1\right)\right) \qquad i = \log_{\frac{3}{2}} \frac{35}{2}$$

$$= \sum_{i=0}^{n} \theta(1) + \sum_{j=2}^{10 \times \frac{3i}{2}} \theta(1)$$

$$= \theta(n) + \sum_{k=0}^{\log_{\frac{3}{2}} \frac{n}{10}} \sum_{j=0}^{10 \times \frac{3k}{2}} \theta(1) \to \theta(n) + \sum_{k=0}^{\log_{\frac{3}{2}} \frac{n}{10}} \theta\left(10 \times \frac{3}{2}^k\right)$$

$$= \theta(n) + 10\left(\frac{3}{2}\right)^{\log_{\frac{3}{2}} \frac{n}{10}} \to \theta(n) + \theta\left(10\left(\frac{n}{10}\right)\right)$$

$$= \theta(2n) \to \theta(n)$$