Counting Problems

1. One unique subset of 5 letters
$\{A, L, N, S, U\}$
Different Strings $= 5! \rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$

2. 2 different values: $\binom{13}{2}$
Has to be 2 different suits: $\binom{4}{2}\binom{4}{2}$
Choose value of last (fifth) ~~card~~: $\binom{11}{1}\binom{4}{1}$ (and its suit)
$$P = \frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$$

3. # of ways $= 16_C \times 15_{C_6} \rightarrow \frac{16!}{1! \times (16-1)} \times \frac{15!}{6! \times (15-6)}$
$= 16 \times 5{,}005$
$= \boxed{80{,}080}$ given 7 couples and 16 songs

4.
3
1,2 — 9
4,5,6,7,8 — 10,11,12
2 values
5 values
3 values

$\frac{2nC_n}{n+1}$
$C_5 = 42$
$C_3 = 5$
$C_2 = 2$

Possible Binary Trees
$42 \times 5 \times 2$
$= \boxed{420}$

5. How many diff combinations?
$${}^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}$$
$= 10 \cdot 9 \cdot 8 \cdot 7$
$= \boxed{5040}$ combinations