Using Dynamic Systems Theory To Model The Mind

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Progressive research into major depressive disorder uses cognitive modelling to study the disease when patient-based studies fail. Through qualitative data, a dynamic computational model of goal directed decision making has been constructed and optimised. The decisions being modelled are between engaging and disengaging in a positive activity. Bayesian decision theory and decision field theory build the decision making, expectation-maximisation and Bayesian inference build the learning process. The change mechanisms of the system are explored under forced low mood or forced disengagement, and courses of behavioural and cognitive therapy (CBT). A key focus in this work is on the extent of which apathy is a symptom of depression and the effect that has on the success of treatments.

This model shows that behavioural focused CBT was more successful than its cognitive counterpart in treating long term depression. The simulations ran show that shorter periods of disengagement are needed to produce stable MDD than low mood. Alterations to the apathy dependence (AD) to mood dependence ratio of MDD revealed that stable MDDs are not possible with only mood dependant MDD and are unlikely with high mood dependence. Stability of depressive episodes increases with increasing AD and expected treatment success for MDD occurs at $AD = 0.3$. Treatment resistant depression occurs at higher AD values which can only be treated by combining the two CBT models. At $AD > 0.75$ the combined treatment became less successful highlighting the importance for alternate treatments such as medication or somatic therapies. This suggests that high apathy levels are a cause of treatment resistant depression.

This model supports the use of the computational techniques mentioned to investigate the change mechanisms of MDD and its treatments. It is hoped that clinical research into the role of apathy as a symptom of MDD and a mechanism of treatment resistant depression will be inspired.

Contents

1. DEPRESSION AND APATHY

1.1. Depression the Ongoing Battle

Major Depressive Disorder (MDD, also known as depression) is a mental health disorder associated with sadness, hopelessness, rumination, and disengagement. Although there has been significant research into understanding the mechanisms and treatments of MDD, the rates of depression have not lowered since the early 2000's. It is estimated in 2021 that over 52 million adults over 16 years of age displayed symptoms of depression with a significant increase due to the Covid-19 pandemic in the UK [1–3]. A recent joint study by LSE and the Mental Health Foundation conservatively estimates the annual cost of mental health in the UK to be $£118$ billion, concentrated in a loss of employment [4]. To compare, the cost of the NHS in 2020 was $£150.4$ billion. Common treatments of MDD involve the prescription of antidepressants, and psychotherapy. These two treatments can ease depressive symptoms. However, some patients are unreactive to these treatments, for reasons that are not completely understood. In light of these unfortunate statistics, it is vitally important for the good of the people, of the economy, and of society as a whole to gain a better understanding of this disease in order to best treat and prevent it. This work hopes to contribute to such an understanding, and provide insight into suitable treatments for a patient.

1.2. Diagnosis and Course of Action

1.2.1. Diagnostic Techniques

MDD has a variances in the active symptoms, onset age, trigger events, and family history which makes it a heterogeneous disease. A definition of MDD was required for consistent clinical diagnosis. Hence, it was introduced to the DSM-3 in 1980. Currently, to be diagnosed with MDD, a patient must fit the criteria in the DSM-5. Over a two week period, a patient must show five symptoms from: depressed mood, loss of interest or pleasure (anhedonia), weight loss or gain, slowed thinking or less physical movement, fatigue, feeling of worthlessness or guilt, inability to concentrate, and recurring thoughts of death. One of the first two symptoms must be present for diagnosis. These symptoms must have an effect on a patient's daily functioning, and must not be explainable by: substance use, a schizophrenic disorder, or a manic episode [5]. Any five (or more) symptoms out of the nine necessary for diagnosis can be present in a patient, in any combination. This inconsistency in the disease is what makes MDD a difficult disorder to understand, diagnose, and treat.

There are different diagnostic techniques used to evaluate if a patient is displaying MDD symptoms. In primary care, the PHQ-9, a self report, is used. It has nine items corresponding to the DSM-5 criteria and a maximum score of 27; a score above 10 indicates MDD. In the hospital setting, the Hamilton Rating Scale for Depression is used. Other scales exist such as the Beck Depression Inventory and the Zung Self-Rating Depression Scale [6]. These are all interpretive evaluation techniques. This can lead to human errors in diagnosis, either from a biased self report, or improper clinical practice.

There are some measurement based methods for evaluating the symptoms of MDD. For

example, toxicology screening can be used to rule out organic or medical causes of depression [6]. Functional magnetic resonance imaging (fMRI), has become popular in MDD research. fMRI has classified MDD as abnormal functioning between networks in the brain. Studies have found in MDD there is hyper-connectivity and hypo-connectivity both within and between certain brain networks which can offer an explanation towards the perturbed cognitive processes of a patient with MDD. However, there are inconsistencies in the location and nature of the connections between samples. When coupled with the costs of fMRI scans, the inconsistencies make this an unsuitable method for diagnosis but it remains a useful tool for research [7].

1.2.2. Recurrence and Stuck States

If a patient fits the criteria of MDD, they are having a major depressive episode (MDE). If the patients recovers from these symptoms, they are still medically diagnosed with MDD but the MDE has stopped. MDD is a highly recurrent disease, with at least 50% of those who recover from one MDE having further episodes later in life. On average individuals who suffer from MDD will have five to nine depressive episodes during their lifetime [8]. This is explained by conflicting hypotheses. Scar theory suggests that MDD leaves a lasting effect on a patient which causes future vulnerability. Vulnerability theory suggests that depressed individuals have pre-existing traits which make them vulnerable to MDEs [9]. Research shows that cognitive behavioural therapy (CBT) alongside depression medication can significantly reduce the rate of relapse in patients recovering from a major depressive episode [10].

If a patient is successfully recovering from a MDE, and experiences minimal symptoms for three weeks, they are in a state of remission. Remission is often the end goal of CBT. The remission state can be lost if symptoms return, this is a relapse and describes the return to the original MDE the patient escaped from. After four months of remission, a state of recovery is entered. If symptoms of MDD return after recovery, a recurrence has occurred, this describes a new episode, this process is illustrated in Fig.1 [11, 12].

Alternate theories of depression describe MDD as the tendency to enter into and the inability to escape from a negative mood, not as a condition described by a state of negative mood. This stuck state or description is offered as the reason behind the failure of antidepressants and other treatments which focus on easing the symptoms. The heterogeneity of depression as it

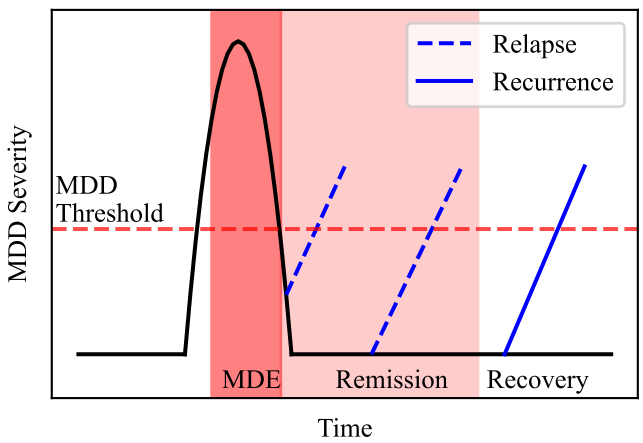

FIG. 1: Illustration of a possible treatment course of MDD. The scales are arbitrary but the figure distinctly separates remission from recovery and relapse from recurrence. MDD threshold represents a level of symptoms that diagnoses MDD.

is currently defined suggested that MDD could be an umbrella term for multiple diseases with distinct neurobiology, which can be grouped by the causing of a stuck state [13, 14].

1.2.3. Treatment

The most common methods of treatment for MDD are antidepressants (ADMs) and CBT. Somatic techniques are available in hard to treat cases such as electro-convulsive therapy (ECT), transcranial magnetic stimulation (TMS), and vagus nerve stimulation (VNS). ADMs treat MDD from the outside-in. They change how external factors (such as serotonin and norepinephrine) affect the internal functioning of a patient. The success of ADM can be investigated using TMS to study the neuro-chemistry of drug treatment [15, 19]. A study of MDD through cognitive modelling is an inside-out approach, focused on how the internal processes of a patient affect the world around them [16]. This is suited to a study of CBT. CBT is based on Beck's cognitive theories, "It is not a situation in and of itself that determines what people feel but rather the way in which they construe a situation" [24]. CBT aims to restructure the patient's perspective and approach to situations to better deal with them [17]. CBT has two areas of focus. First, supporting healthy behaviour in a patient through engagement and noticing the consequences of actions. Possible methods are activity scheduling or reflection of current events. Second, is a focus on healthy cognition in a patient. Techniques include working on assumptions or beliefs while combating negative thoughts. These methods are often integrated together and overlap within sessions of CBT therapy for an integrated approach to therapy [18].

If courses of ADM and CBT fail, MDD is classed as treatment resistant depression (TRD). It is estimated that $10\% - 30\%$ of patients exhibit conventional TRD symptoms. In this case, a patient can switch treatment, combine multiple strategies, or try somatic treatments [20, 21]. The somatic therapies target the body. ECT has the largest success rate and involves electrically stimulating the brain. TMS is similar to ECT but uses rapidly alternating magnetic fields to stimulate specific areas. VNS involves implanting an electrical pulse generator under the skin in the patient's chest that provides intermittent electrical stimulation to the vagus nerve in the neck. Each of the somatic therapies have their own pros and cons and are reserved for TRD due to the invasive nature of the treatment [23]. TRD is not as well-defined or understood as MDD, and its diagnosis is not as precise. However, research has revealed links between TRD and childhood trauma, which causes deep negative memory integration and complex interpersonal issues. Advanced methods of CBT have been developed to improve self-regulation in TRD. It is postulated that representations of the self develop mainly in childhood, and to a lesser extent later in adolescence or adulthood. If there is a lack of positive self representations, vulnerability to depression can increase. This leads to impaired motivation, passive life goals, intrusive memories, and disengagement. Without advanced CBT to regulate these representations, the chance to fall into a depressed state can rise, with the chance of escape falling [22].

1.3. The Role of Apathy

The DSM-5 states that "apathy is typically characterised by diminished motivation and reduced goal-directed behaviour, accompanied by decreased emotional responsiveness" [5].

There are disagreements about whether apathy is a symptom of depression, or an interconnected disorder with similar effects [25–27]. Anhedonia, a symptom of MDD, describes a lack of interest and pleasure. MDD is also linked to reduced energy to engage in activities, and a bias towards viewing opportunities as negative. These symptoms of MDD, if present in a patient, can lead to ”diminished motivation and reduced goal-directed behaviour”. It is likely that apathy plays a complicated role in the development of MDD. However, treating apathy as a symptom of MDD can still provide insight into how depressed individuals make decisions about activity engagement. Since MDD is a heterogeneous disease, the symptoms that link MDD and apathy may not always be present within a patient. This research will focus on when the symptoms are present in an individual.

1.4. Cognitive Models in Depression

1.4.1. Beck's theory

Cognitive theories address the role of thoughts and feelings in MDD. The internal processes of patients suffering from a MDE are different compared to healthy individuals. Beck developed a cognitive theory of depression based around cognitive bias, negative self-schemas and the cognitive triad. The cognitive bias describes how information is negatively distorted during a MDE. An example is the over-generalisation of a negative event, if one exam is failed, it is assumed that all future exams are bound to fail. Negative self-schemas are often developed from negative childhood experiences and involve negative beliefs about the self and the world. The negative triad is maintained by the previous two aspects of Beck's theory. It is comprised of negative views of the self, world, and future [24]. It is thus important to take an inside-out approach to understand how the mechanisms of MDD influence the actions of an individual. Qualitative conclusions from experimental studies will be used to create a computational model to identify the key mechanisms in the decision making process. Beck's research will play an important role in this work. Both explicitly as the motivation for cognitive modelling and implicitly within the research that the model is grounded in.

1.4.2. Memory Bias, Effort, and Learning

Interpretive bias refers to tendency to view events and experiences in a negative light [28]. This was first noticed by Beck who suggested focusing on these biases in CBT. A review of research into modern cognitive models reveals an increase in the recollection of negative events, and the interpretation of ambiguous events as negative, in patients with MDD [29]. Experimental work concluded that the magnitude of negative bias is directly related to the severity of MDD [30]. MDD also affects long term memory retrieval, with a diminished recollection of positive memories and an increase in negative memories. This memory bias is also apparent in patients who have suffered a MDE in the past but are now in a state of recovery [31]. The cognitive biases in patients with MDD are the negative self-schemas and will maintain the negative triad. This keeps an individual within a stuck state unless these views can be overcome.

In the DSM-5, it states that patients with depression often report that even the smallest tasks

seem to require substantial effort, while the efficiency with which tasks are accomplished may be reduced [5]. Fatigue is a symptom used in the diagnosis of MDD. Research involving the computational modelling of participant's reactions to behavioural tasks revealed a significant sensitivity to effort for participants with MDD [32]. This suggests that tasks require more effort for depressed individuals, and individuals with MDD are more likely to avoid effortful tasks. The effort studied in this model is mental effort. Studies involving the use of working memory to gain reward have found that patients with MDD are less willing to expend energy for expected reward [33]. A long deliberation process when making a decision indicates that a patient is struggling between the decision options. This mental struggle will be more intense the more difficult it is to choose between the two decisions. This can lead to a patient choosing to disengage as a default option, rather than battle with the decision making process further.

Reinforced learning (RL) is the process of maximising rewards and minimising harm by altering behaviour as a consequence of an experience [34]. Several studies have revealed that participants with MDD struggle to learn rules within games, in order to maximise rewards [35, 36]. Model free RL will be used in the learning process. Model free RL involves the updating of parameters that minimise prediction errors between predicted and experienced values, it is a slow learning process. Model Based learning involves the building of cognitive maps that model the environment, these maps are explored for every decision made in order to reach a result of maximum reward. Model based learning is far more complex than model free, and will not be required in this study [37]. However, it is a useful model for investigating rumination within the decision making process or for studying anxiety related disorders. Model free learning is also linked to the actions of dopamine. Dopamine neurons encode the information of prediction errors, and transport it to the brain regions involved in RL [38]. Studies suggest there is a deficit of dopamine in MDD, significantly so when anhedonia is present [39].

The mechanism for the effect of anhedonia on RL is not completely understood. It is postulated that anhedonia could both effect the sensitivity to reward (hence prediction errors) and/or learning rate. Computational modelling of participant's learning, and subsequent reviews, suggests that sensitivity to reward plays a far more important role in a diminished RL process than learning rate. However, it will be important to consider both mechanisms [40, 41].

2. COMPUTATIONAL MODELS OF THE MIND

2.1. Dynamic Systems

The first computational modelling of the mind occurred in 1943 when it was first suggested that something similar to the Turing machine could provide a suitable model for the brain [42]. This led to the development of the classical computational theory of the mind. This theory has influenced research in psychology, economics, and neuroscience. Over time, various theories and models have become popular [43]. In this paper, the main computational techniques used to model the brain are dynamic systems theory and reinforced learning.

MDD is a dynamic process, a state which changes over time. Dynamic models govern how a state evolves in time and can lead to unexpected behaviour emerging from a system that is not explicitly programmed in. The system's behaviour, and reactions to its environment decide the

future trajectory of the system [44, 45]. This suggests that it is possible to understand the time evolution of MDD without a complete understanding of all the internal mechanisms. Within dynamic systems, the effect of a perturbation on the system will depend on the systems' current state. This explains why treatment may be effective for some patients but not for others or why it may only work at specific stages of MDD [46].

2.1.1. Decision Theory

Bayesian Decision Theory (BDT) is a method of logically mapping decision making in the domain of uncertainty. In BDT, choices are made by a subject to gain maximum utility [47]. The key features of BDT are the state and prior. The state has two parts. First, the objective state of the world which determines all the future outcomes for a subject, and the utilities of the decisions they will make, this is not generally known by the subject. The second is the probabilistic summary of the knowledge the subject has on the objective state. The prior distribution is made up of this knowledge and is used to make decisions [48]. Decisions are made by the subject with the goal of maximising reward (or minimising punishment) using their probabilistic knowledge of the possible outcomes.

Decision field theory (DFT) provides the framework for a dynamic and stochastic model of decision making. DFT has a broad range of application when it comes to understanding human decision making. In this report, the focus will be on making decisions under uncertainty. Hence, the model will be a hybrid of DFT and BDT. Adaptations will be made to the standard DFT model to better suit this goal.

First a brief description of DFT will be provided, then the adaptations will be presented. It is best to present this theory in the case of two decisions (L and R) with two random events (E^1 and E^2). The valence of each of the two decisions is:

$$V^i = p(E^{1i}) \times U(E^{1i}) + p(E^{2i}) \times U(E^{2i}) \tag{1}$$

with $i = L, R$. $p(E^{ji})$ is the probability of event j occurring after decision i, and $U(E^{ji})$ the corresponding utility of this event. The probability of an event occurring and its utility can differ depending on the decision made. The preference state is then defined as:

$$P = V^R - V^L \tag{2}$$

a positive value of P indicates decision R should be taken, a negative value indicates decision L. In subjective expected utility theory (SEU), $p(E^{ji})$ are subjective probability weights on the possible decisions. This means they are continuous random variables which can differ for repeated decisions based on the subject's preference of R or L. Continuous SEU builds on this theory such that a decision is not made after one deliberation or sample. Multiple samples are taken, the probability weights can differ between samples, and the total preference state is the sum of all preference states in the decision. Decisions are made over a period of time. Hence, $p(E^{ji})$ has some form of sample dependence or time dependence. This process continues until P reaches the threshold value P^R_{th} or P^L_{th} which indicates which decision to take [49].

In BDT, a subject has probabilistic knowledge of the possible rewards of a decision. This is a continuous distribution of utility described by a probability density function. Also, a subject

has no knowledge of random events that can occur after making a decision because it is stored in the objective state of the world. This means that only the expected outcome, not the actual outcome of a future event, can influence the current decision. Hence, the valence of a decision is sampled from the probability density function of its expected utility. SEU is adapted so that the utility functions are probability density functions that encode the utility and the probability of occurrence. Next, if the two motivations for making decision R or L are independant of each other, it does not make sense to have one preference state whose signum describes the decision direction. Instead, under continuous SEU, each decision should have its own preference state which increases if the current deliberation is in its favour, and is otherwise stationary. A probability density function of the expected utilities is shown in Fig.4 and the multiple preference state approach in Fig.3.

2.1.2. Learning Processes

Expectation-maximisation (EM) is an optimising procedure for minimising loss functions. Loss functions encode the difference in actual and predicted values in model free reinforced learning. Under the adapted DFT and BDT, a decision is made based on the probability distribution of its utility. This distribution is based on the knowledge a subject has, not the objective state of the world. This means that the utility sampled can be different to the actual utility gained from a decision. This is sampled from the objective state. The loss function is:

$$L = A - PV \tag{3}$$

for actual value A and predicted value PV. The predicted value is the average valence sampled for a decision. Parameters in the decision making process are then optimised to reduce this loss function. This depends on both the magnitude of L and its signum.

Bayesian inference (BI) is a learning technique which stems from Bayes's law

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}. \tag{4}$$

Framing this law in terms of probability distributions, results, and information (θ) gives :

$$P(\theta|results) \propto P(results|\theta)P(\theta) \tag{5}$$

$P(\theta)$ is the prior distribution, $P(\theta|results)$ is the posterior, and $P(result|\theta)$ is the likelihood of a result given the information a subject has. The posterior is used as the prior in the next decision so that subject learns from the results of making a decision. If the prior and likelihood distributions can be modelled by two Gaussians, the posterior will also be a Gaussian due to their self conjugate property. The product of two Gaussians with parameters μ_1, σ_1 and μ_2, σ_2 results in a Gaussian with:

$$\mu = \frac{\mu_1{\sigma_2}^2 + \mu_2{\sigma_1}^2}{{\sigma_1}^2 + {\sigma_2}^2}. \tag{6}$$

Under the multiplication of two Gaussians, the variance is also affected and over many multiplications it is minimised. The choice has been made to keep σ constant to avoid overconfidence, and maintain an accurate representation of human learning.

2.2. Modelling the Mind

2.2.1. Model Overview

The decisions being modelled are to engage in a positive activity or not. Activity examples are: reading, exercising, studying, meeting a friend, and other small beneficial day to day activities. Each day in the subject's life will consist of ten possible decisions. The processes that define the model will evolve in time based on the decisions made and their outcomes. The depression level (dep) indicates if a subject is in a MDE or a healthy state. It is calculated from the mood and apathy levels ($mood$ and ap). A $dep < 0.5$ indicates a subject is experiencing a MDE. A diagram of the model is available in Fig.2. The $mood$ is sampled from a Gaussian at the start of each day. It then evolves depending on the decisions made. The ap continuously evolves throughout the simulation. At the end of the day, the mood and apathy levels update dep.

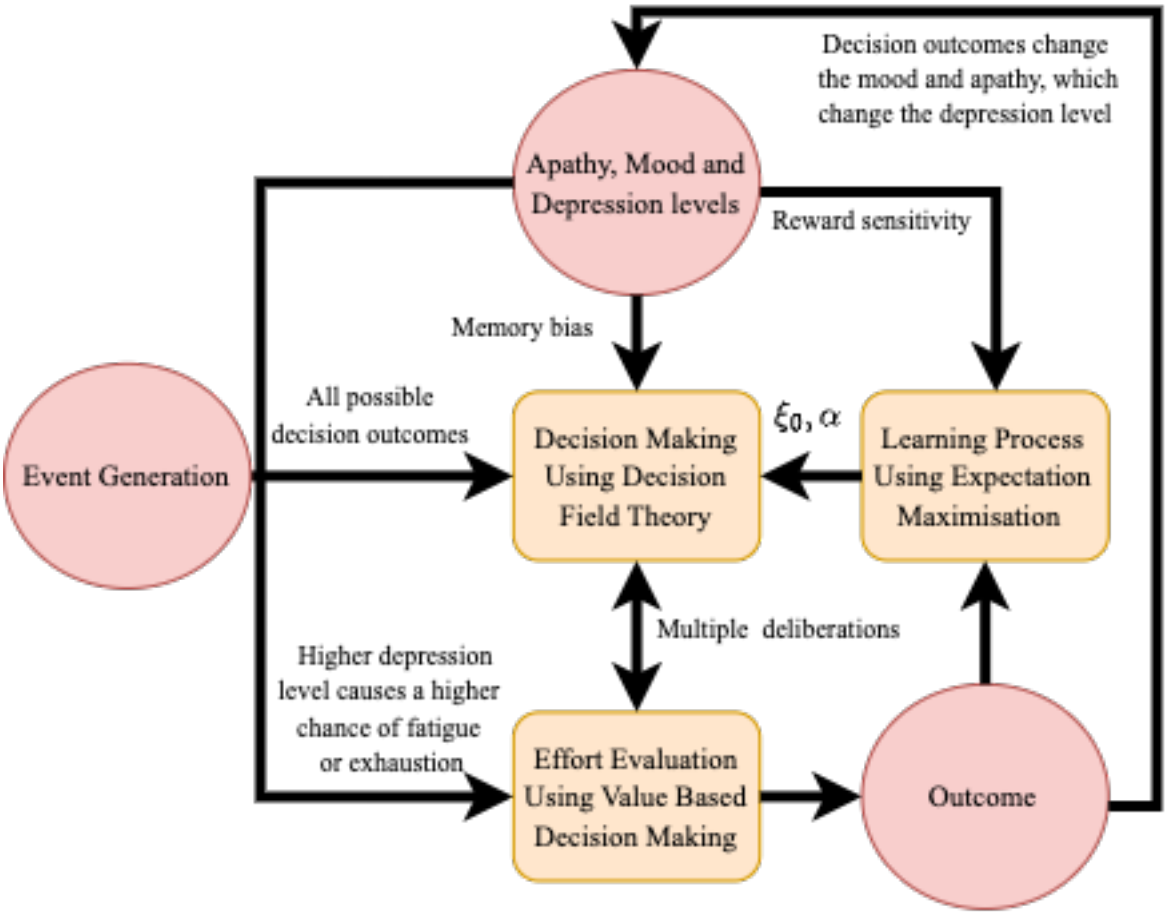

FIG. 2: Components of the model in this work. The red ovals are variables while the yellow rectangles are processes modelled through computational psychology. The arrows show the flow of inputs, outputs and change mechanisms.

2.2.2. Decision Making

Part one of the model is the decision making process. It will take an input of the event generator and the subject's current state. It will output to the effort evaluation stage using DFT and BDT. In Eq.2 L and R become Dis and En representing disengaging and engaging. The valences are given by the predicted positivity of each decision. The predicted values are sampled from two Gaussians that contain the probabilistic knowledge that a subject has on the benefit of engaging and not engaging. If a sampled value is outside the range $-1 < V^{Dis/En} < 1$, the sample is repeated. The disengage Gaussian is fixed at zero with a small variance of 0.1 and will not evolve in time. The engage Gaussian has a larger mean, and variance of 0.2 but takes on some level of skewness. The mean and skew of this Gaussian is free to evolve in time depending on the depression level and learning stage. The preference are:

$$\begin{aligned} V^{Dis} &\sim \frac{1}{0.1\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x-0}{0.1}\right)^2\right) \\ V^{En} &\sim \frac{1}{0.2\pi} exp\left(-\frac{1}{2}\left(\frac{(x-\xi)}{0.2}\right)^2\right) \int_{-\infty}^{(\alpha(\frac{x-\xi}{0.2}))} exp\left(-\frac{t^2}{2}\right) dt \end{aligned} \tag{7}$$

with $\sim$ representing the act of sampling from the distribution, α is skew, and ξ is location. Location is analogous of mean for a skewed Gaussian, and will be given by,

$$\xi = \xi_0 - (1-m)\delta\xi \tag{8}$$

where m is the mood and $\delta\xi$ is a fixed value, this creates a memory bias for individuals with low mood. ξ_0 is free to evolve from the learning process so that a subject who has entered remission still suffers from a memory bias, but one that can improve with continuous recovery. Although the mathematical term for ξ is the location, within this report it will be referred to as the mean for ease of comparison. Using continuous SEU, the total preference state is the sum of the individual preference values at each deliberation in the decision making process. With the adaptations, the decision to engage and not engage have individual preference states which are only updated during samples which favour the respective decision. Hence

$$P^{En/Dis} = \sum p^{En/Dis} \tag{9}$$

$p^{En} = |V^{En} - V^{Dis}|$ when $V^{En} > V^{Dis}$ otherwise it is 0, the reverse holds for p^{Dis}. The sum continues, with repeated samples (or deliberations), until one of the preference state reaches its threshold value, P_{th}^{En} or P_{th}^{Dis}. The decision is made in favour of whichever preference reaches its threshold first. While a decision is being made, ξ and α increase or decease by small incremental values. They increase if the current deliberation is to engage, and decrease for disengage. These small changes do not effect the next decision. The changes while making a decision reflect the subjects current preference in the decision outcome which do not affect the next decision which is a different decision subject entirely.

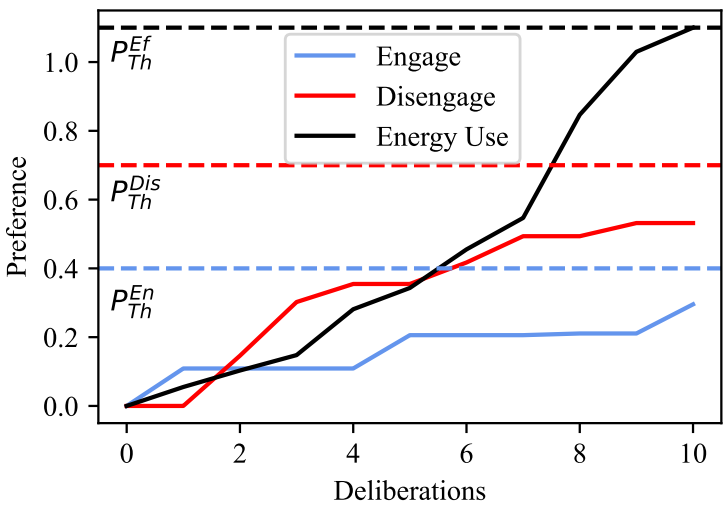

FIG. 3: Diagram of the evolution of the preference states of engage and disengage, included is the effort evaluation which is also computed by decision field theory. In this particular decision, the subject is exhausted after ten deliberations and the decision is to disengage via exhaustion.

2.2.3. Effort Evaluation

Part two of the model is the effort evaluation. The effort evaluation is intertwined with the decision making process, as can be seen in Fig.3 and Fig.2. There is an effort penalty at each deliberation within the decision making process given by

$$p^{Ef} = \frac{(0.1 + \frac{dep}{4})}{p^{En/Dis}}. \tag{10}$$

p^{Ef} has a maximum value of 1.5 for cases when $|V^{En} - V^{Dis}|$ is very small. Two similar sized valences will provide a more difficult choice than if one was larger and a far better option. This is likely to lead to a longer deliberation and a larger effort usage. This effect is magnified by a high dep. This effort penalty is summed over the deliberations in the same manner as the engage and disengage preference states. However, there is an effort penalty at every deliberation. If the total effort expended reaches a maximum allowed effort value (P^{Ef}_{th}), then the decision defaults to not engage. Within the model the not engage by exhaustion is treated similar to not engaging by decision but with damped learning. There is also a maximum daily effort threshold (P^{DEf}_{th}). If the sum of all effort expended in a day exceeds this threshold value, which depends on dep, then the rest of the decisions in the day default to not engage. This is fatigue. Any disengagements occurring due to fatigue are treated the same as disengagement due to exhaustion in the next steps of the model. Fatigue represents too many difficult decisions in a day overwhelming an individual to the the point of avoiding future decisions for the rest of the day.

2.2.4. Learning

Part three is the learning process of the model. This process takes an input of the outcome of the previous two stages. This includes: if the decision was to engage with the predicted positiveness of engaging, or if it was to disengage, and if that was deliberate or through effort expenditure. If the decision is made to engage, the model learns via EM. Eq.3 becomes

$$L = A * \epsilon - PV \tag{11}$$

where ϵ is the sensitivity to reward which linearly depends on dep. ξ_0 is then shifted by $L \times w$ with w a weighing on the influence of recent events on a subject's knowledge. The actual reward value is sampled from an unperturbed initial V^{En} with $\xi = 0.2$ and $\alpha = 0$. This is just a Gaussian with $\mu = \sigma = 0.2$. If the decision is to disengage, the learning is different. There will be no difference between expected and actual value, nor is there a significantly reward or punishment to learn from. Instead, the system is further pushed into a state of disengagement as it is the only experience for the subject to draw from. In this case, the mean is updated under modified Bayesian inference. With prior distribution V^{en} and likelihood distribution V^{Dis}. Approximations made under this modified BI are: there is no change in α or σ, and that locations takes the place of the mean in Eq.6. Under these assumptions,

$$\begin{aligned} \xi_0^{new} &= \xi_0^{old} + w \times (\mu - \xi_0^{old}) \\ \mu &= \frac{\xi_0^{old} \times 0.1^2}{0.2^2 + 0.1^2}. \end{aligned} \tag{12}$$

The evolution of the means and skew during an induced MDE and treatment can be seen in Fig.4.

After a day of decisions, the skew of the distribution is updated. The change in α takes into account the number of engages and increases by a factor proportionate to $engages - 5$. The ap is also updated at the end of a day. The ap is a property of the subject, any changes in ap are

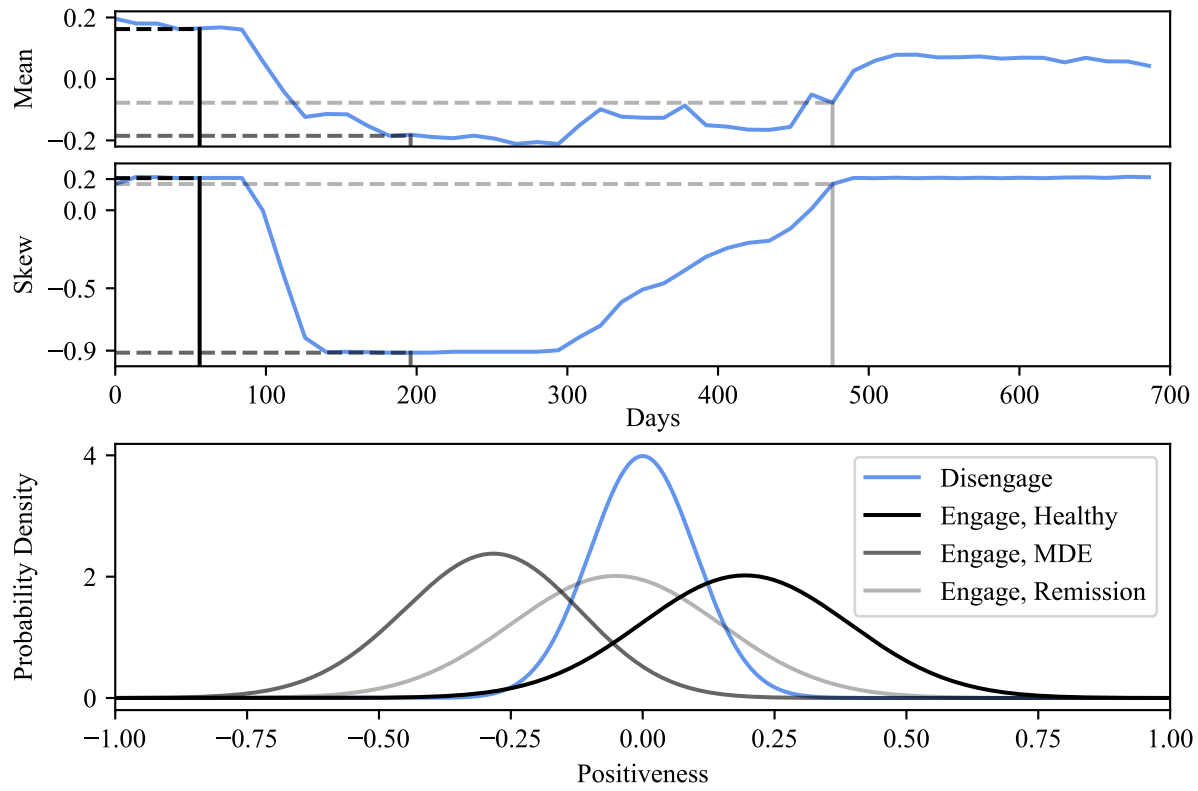

FIG. 4: The mean and skew of the engage skewed Gaussian distribution, V^{En}, over 800 days with a MDE starting at day 1200. Remission is entered around day 500. The healthy V^{En} has $\xi = 0.16$ and $\alpha = 0.21$, the remission V^{En} has $\xi = -0.01$ and $\alpha = 0.16$, the MDE V^{En} has $\xi = -0.19$ and $\alpha = -0.91$. The disengage Gaussian, V^{Dis}, and σ are defined as in Eq.7

not a part of learning but a part of updating the state. The ap changes depending on the current and previous decision. If the subject does not engage ap increases, if the subject engaged in the last two it decreases, otherwise (disengage followed by engage) ap does not change. This is to differentiate goal directed behaviour from random engagement. The changes in α and ap are examples of sequential updating. Each change mentioned above occurs by some factor $w < 1$ which is set to 5×10^{-4}. This takes into account the neural-plasticity of the individual and the difference in efficiency between humans and computational models. The effect the learning has on ξ and α and hence the behaviour of the model is illustrated in Fig.4 through a MDE induced by low mood and later treated by behavioural CBT.

With the learning techniques used, ξ and α approximate a closed system of allowed values. Through engagement, ξ_0 will approach a value of 0.2. It will approach 0 through disengagement. EM stops the mean from being significantly larger than the mean that actual reward values are sampled from. BI causes the mean of engaging to approach the mean of disengaging. This almost closed systems prevents extreme cases of euphoria from being modelled and places a limit on the severity of the memory bias. Boundaries on α create a similar bound state for the skew. The maximum value is 0.2 and the minimum is -0.9. A large change in α or ξ can violate their boundaries for a limited period of time. The maximum value of α an also be violated by cognitive focused CBT which is what gives the modelled treatment its effectiveness.

2.3. Model Expectations

The success of the model will be measured by its behaviour. Certain behaviour will be hard coded into the model such as an increased use of effort occurs at high dep. However it is expected that emergent behaviour will occur that is expected from a patient with MDD. Expected behaviours are shown in Tbl.I which is a marker that the model can successfully represent the behaviour of a patient with MDD. Not all of these behaviours may emerge as expected. For example, if a subject reaches exhaustion any further decisions will not have a high deliberation time because they default to not engage. If the subject takes a very negative outlook for future events, long deliberation will not be necessary to decide not to engage because it is obviously the better option. Insight of the inner workings of MDD will be gained from observing why some behaviours dominate.

Behaviour	Mechanism
Less engages during a MDE	Decision Making and Effort Evaluation
Longer deliberation time during a MDE	Decision Making
A MDE is a stuck state	Learning Process

TABLE I: Significant aspects of the model, seeing the emergent behaviour marks the model as a success.

2.4. Inducing Depression

In order to study the effects of depression and its treatments, MDEs will be induced upon a model displaying healthy behaviour. This is necessary because the model will not spontaneously enter a state of depression. A MDE is often pre-dated by some adverse life event whether recent, or in childhood with a trigger that activates the episode. Spontaneous depression is not common in a clinical setting. The two approaches to inducing a MDE were forced periods of low mood (ILM), and forced periods of disengagement (ID). Both of these techniques fulfil one of the two necessary diagnostic criteria for MDD if the symptoms persist for at least two weeks. During ILM, ξ is directly reduced through Eq.8 causing less engagement. Through ID, the ap will directly increase and the mood will decrease due to disengagement. Both techniques cause reduced $mood$ (hence reduced ξ) and increased ap but ILM directly causes the former and ID the latter. This will result in an increase in dep. Disengagement will cause a decrease in the value of ξ_0 and α during the learning process. If these values decrease sufficiently, a stable episode will be induced, preventing the subject from entering remission immediately after the induction is stopped.

2.5. Modelling Treatments

Inducing a MDE allows treatments to be tested on the model. Spontaneous recovery from stable MDEs are not expected from clinical patients or from the model. Recovery without treatment is unusual in MDD and more common in mild depressive disorders. Self-recovery is likely to occur in cases where a depressive episode is short lived, with minor symptoms, and undiagnosed in the first place. Patients may have fit the criteria for a mild depressive disorder but recovered through their own behaviour. CBT will be modelled as treatment in this system. The focus of CBT will be split into two parts, a focus on behavioural improvements (CBT-b) and a focus on positive cognition (CBT-c). The treatments are represented by modelling their desired effects. In clinical cases, many factors (the patient's current state, the competence of the therapist, the therapist-client relationship) can prevent these effects from being administrated. This model looks at ideal treatments, which means they have a lower chance of failure compared to clinical practice. An ideal treatment fails because it is not a suitable method for the subject, not because of any other factors.

The behavioural focus will be modelled by not allowing a subject to actively disengage. This represents activity scheduling which hopes to help a patient to engage in scheduled activities even if they do not feel like doing so in the moment. Within the model, decision field theory was adapted to the random walk model. This allows for an initial preference state not equal to zero. During CBT-b, the decision making process can occur in loops if P^{Dis} reaches P^{Dis}_{th}. If this occurs, P^{Dis} returns to zero, $P^{En} = P^{En}_0 \times n$ for n occurrences of this loop, and P^{Ef} continues into the next loop. This continues until P^{En}_{th} or P^{Ef}_{th} are reached. P^{En}_0 is a subject's initial preference to engaging. CBT-b prevents active disengagement, it is expected to cause more decisions to engage but also more exhaustion and fatigue. CBT-c will be focused on the distribution. This is akin to breaking down the negative patterns that cause low mood and disengagement in MDD. A focus on preventing a cognitive bias and an improvement to

negative self-schemas aims to stop the maintenance of Beck's negative triad, specifically the negative views of the future. Overcoming these negative views is expected to cause more goal directed behaviour. This will be modelled by increasing α. The choice to change α and not ξ was made to stop the treatment from being overly effective. While a subject is being treated with CBT-c, α is increased by a value sampled from a Gaussian with $\mu = 75w, \alpha = 30w$. The sampling adds some randomness to the treatment process. Consistent progress is not expected in treatment. This increase in α is also offset by the change at the end of the day, which can take a maximally negative value of $-5w$. Cognitive treatment can increase α to a maximum value of 1.3, which ensures an expected treatment success rate while preventing a large positive memory bias (when considering negative value of ξ).

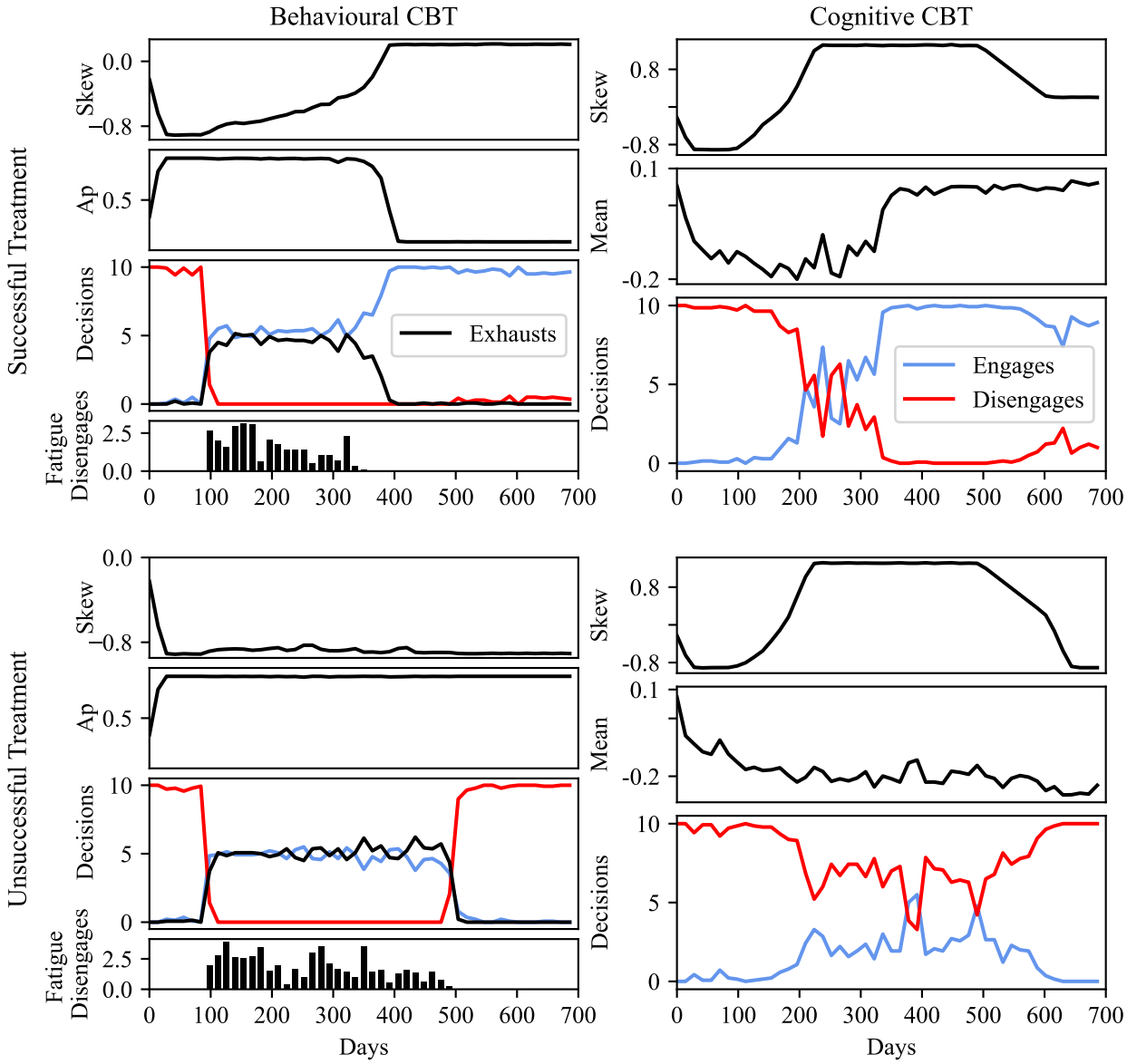

FIG. 5: Plot of the evolution of selected parameters of the engage Gaussian distribution, and selected decisions made over a 700 day period. Treatments start on day 100 and last for 400 days. Values are averaged over a 14 day period.

Within CBT-b, the difference between success and failure for a given treatment length and start date is the current state of the subject, this can be seen in Fig.5. If a subject is more likely to engage rather than suffer from exhaustion or fatigue, them the treatment will succeed. This depends on the current state of the engage Gaussian. An average of more than five engages a day allows for a gradual increase in α during the treatment. If this increase is significant it allows for engagement when the therapy is stopped. On the other-hand, if the average daily engages is five or bellow, there is fatigue and exhaustion throughout the treatment which causes no improvements in the distribution, and continued disengagement after CBT-b ends. Throughout the course of CBT-b, ap does not significantly decrease until continuous engagement occurs, or until the average engagement is greater than six. Similarly, for CBT-c, the only difference between two treatments is ξ when α reaches its max value. The value of ξ must be large enough to cause more engagement than disengagement to allow for positive reinforced learning and an increase in $mood$ and ap. This is supportive of research that the success of CBT depends on the current state of a patient which is well reflected in dynamic systems.

3. PARAMETRISING THE MODEL

3.1. Mood, Apathy, and the Depression Level

To describe the current state of the model, whether that is in a MDE or a healthy state, the depression level dep is used. The $mood$ and ap are also useful indicators of state. These two levels set the value of the depression level. All levels are in-between zero and one. Values of ap and dep close to zero indicates a healthy subject, as does $mood$ close to one.

$$dep = ap \times AD + (1 - mood) \times (1\text{–}AD) \tag{13}$$

where AD is the apathy dependence of the depression, it is set to a value of 0.3. The apathy level depends on engagement. The chosen characterisation of apathy causes ap to be consistent over time, there are few random fluctuations and the only changes occur when there are significant changes in a subject's behaviour. If dep only depended on apathy it would stay at consistent levels, and the only response to treatment would be sudden shifts from one extreme to the other. This is not consistent with known behaviour within MDD. Treatment is a long process with improvements and declines in MDD symptoms, with a chance of relapse throughout remission. Therefore another parameter that affects dep was needed, one that is prone to fluctuations, such as the mood.

$$mood \sim \mathcal{N}\left((\mu_0^M - dep \times 0.8), 0.1^2\right) \tag{14}$$

where $\mathcal{N}(\mu, \sigma^2)$ is a Gaussian and $\mu_0^M = 0.9$. At the start of each day, $mood$ is sampled and then evolves depending in the outcome of the decisions made. If mood is sampled outside of $0 < mood < 1$ the sample is repeated. The mood changes by $0.1A \times \epsilon$ during engagement or $0.1V^{Dis}$, which tends to zero. This causes either an increase in mood when engaging or a perpetuation of low moods when not engaging. At the end of a day, after ten decisions, the depression level is updated from the final values of mood and apathy levels as of Eq.13. AD influences the evolution of the model over time. A higher weighting on the significance of ap will cause more stable states of MDD that take longer perturbations to enter but are then harder to reach remission from.

3.2. Defining a Healthy State

A healthy state of cognitive function provides a point to induce a MDE from and to compare a MDE to. It is expected, that of the ten decisions in a day that a healthy individual would engage in at least nine of them. It was possible to create this base healthy state by running a simulation with no induced MDE or treatments, with an initial high mood, low apathy, and low depression level. Parameters within the model could then be explored to bound the simulation to at least nine engages per day. For maximum efficiency, parameters to explore were chosen either logically because they would have an obvious effect on engagement at low MDD (such as P_{th}^{En}) or through rough rudimentary testing. The chosen parameters of investigation were ξ_0, P_{th}^{En}, P_{th}^{Dis} and μ_0^M. With low dep any deviation from the base means are insignificant. The average daily engagement, over 100 days, was found while changing two parameters at a time and keeping the rest fixed on values which were known to allow the simulation to run in an expected way. These are: $\xi_0 = 0.2$, $P_{th}^{En} = 0.4$, $P_{th}^{Dis} = 0.7$, and $\mu_0^M = 0.9$. Parameter values outside of the boundary are values that we expect for a depressed individual which are shown in Fig.6. Parameter values far outside the boundaries are likely to represent treatment resistant depression. For example $P_{th}^{En} > 1.5$ and a $P_{th}^{Dis} < 0.2$ would make a subject very unlikely to decide to engage unless treatment was focussed on braking down the preference to risk avoidance behaviour and instead promoted reward based behaviour. Another insight is that parameter values close to the boundaries are a prerequisite to MDD. These values allow negligible changes to have a large effect on the engagement causing a MDE. This supports vulnerability theory as an explanation for the high recurrence rates in MDD. The model's initial state must be vulnerable to a MDE to experience one, which is the state it will return to after remission. This still leaves it vulnerable to future MDEs. Subjects with parameter values far within the boundaries are unlikely to enter into a stable MDE. For example, a subject who doesn't need to think much about engaging, modelled by a low P_{th}^{En}, will almost always pursue constructive and beneficial behaviour, even after an episode is induced.

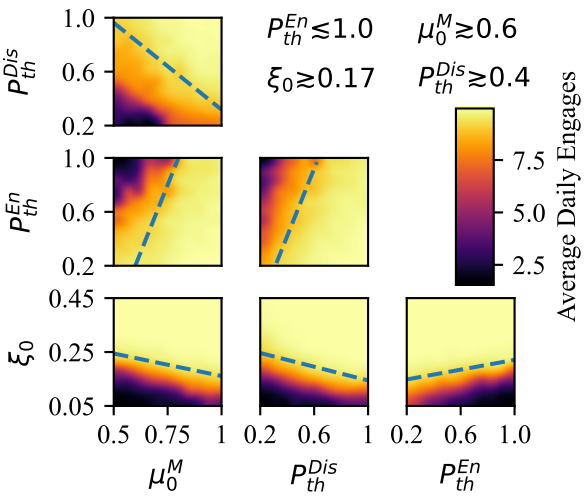

FIG. 6: Density plots showing the alteration of key model parameters and the effect they have on the daily number of engagements for a healthy subject. Blue dotted lines show the approximate boundary between more and less than nine engages or a healthy and an unhealthy subject. When a variable is not being altered, it is kept at a constant known value that does not display extreme behaviour.

3.3. Weighting of new Information

Learning in the model is proportional to a weighting value, w, which represents neural-plasticity and distinguishes human learning from computational learning. The magnitude of w causes the model to behave in different ways. If w is too small, the model will remain in the state it is initialised in, unless acted upon for a large time period. This causes a subject to resist stable induced MDD because the learning process does not significantly change any parameters during the induction time frame. This creates unrealistic behaviour. ILM or ID should create a stable MDE after 50 days, this length of time more than fits the criteria for diagnosing a MDE (14 days). Too high a weighting value causes spontaneous depression. This is a highly unstable model and better represents manic depression. There is a range of $2 \times 10^{-4} < w < 6 \times 10^{-3}$ which causes no resistance to MDE and no spontaneous episodes or recoveries, seen in Fig.7. It is worth noting that values of w do not describe prerequisites to MDD as was extracted from Fig.6. It instead ensures the model is representative of realistic clinical patients. However, if a subject enters into a MDE, and then the value of w is significantly decreased it is likely that the subject will remain within this MDE indefinitely even with the treatment modelled. This is modelling chronic TRD which is usually initiated in childhood and can span decades. Chronic depression is linked to levels of low neural-plasticity blocking any opportunity to learn and recover from this state. Growing research looks to manipulate the synapses through drug treatment to increase or decrease neural-plasticity [50, 51]. Manipulations of neural-plasticity could be modelled with a w that is variable. A w that depends on dep could also have been used to explore the effect MDD has on the learning rate in RL. However, research suggests that reward sensitivity has a larger effect than learning rate and it was decided that reward sensitivity is a more interesting study. Error bars are not included in Fig.7 or similar plots because the patterns is important not the values. However, the sample size used ensures that noise from the errors will not impact the behaviour shown.

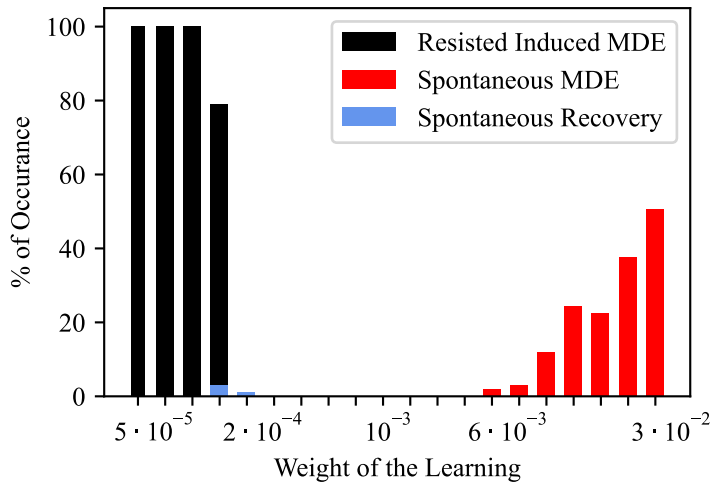

FIG. 7: Bar charts showing the occurrences of spontaneous recovery and resisted MDE after 50 days of ID, and spontaneous MDE for different w values on a logarithmic scale. The sample size is 100, error bars are not included but take a maximum of 5%.

4. RESULTS AND DISCUSSION

4.1. Model Functionality

4.1.1. Success of the Model

The behaviour that the model is expected to display is presented in Tbl.I, if the model displays this behaviour, it is considered functionally successful. To test this, the engages, and deliberations per decision can be measured before and after induced depression while monitoring for spontaneous recovery.

In Fig.8 values before day 150 describe a subject not in a MDE and those after day 214 describe a MDE. For low dep Fig.8 shows an average of ten engages per day. While the MDE is induced and immediately after, there are less than ten average engages per day. This drops to zero later into the MDE. It is assumed that more deliberations within the model represent a longer deliberation time for a clinical patient. The least deliberations occur at low dep, there is an average of 2.85 ± 0.10 deliberations per decision, averaged from day zero to 199. The deliberations are highest when a subject enters into a MDE and in the period after, peaking at a value of 4.50 ± 0.11. The longer the subject is in the MDE the lower the deliberation time. However, it does not reach as low as the pre MDE value, even after day 400. From the perspective of the model, this occurs because ξ_0 approaches 0 in the learning process, this causes $V^{Dis} > V^{En}$ hence most increases in the preference state occur to P^{Dis}. Aoon after the MDE begins ξ_0 is not as small, causing an increase in both preference states and more deliberations. During the later stages of a MDE, this model describes a patient in a state of disengagement. These disengagements occur due to a large memory bias and pessimistic view of the world. This causes less deliberations because there is little perceived benefit to engaging. This behaviour is representative of a diminished ability to think about engaging at all. If a stable state with more engagement could be found, without spontaneous recovery, it would cause longer deliberation times such as those around day 200. This would better represent the indecisiveness experienced by a patient attempting to engage during a MDE. Both an inability to concentrate and indecisiveness are symptoms defined within the DSM-5. These are two different states of functioning, reflecting the heterogeneity of MDD. Through carefully chosen parameters, it should be possible to create stable indecisive state instead of a stable diminished ability to think.

There is no spontaneous recovery in Fig.8 and Fig.7 shows how the chosen value of $w =$

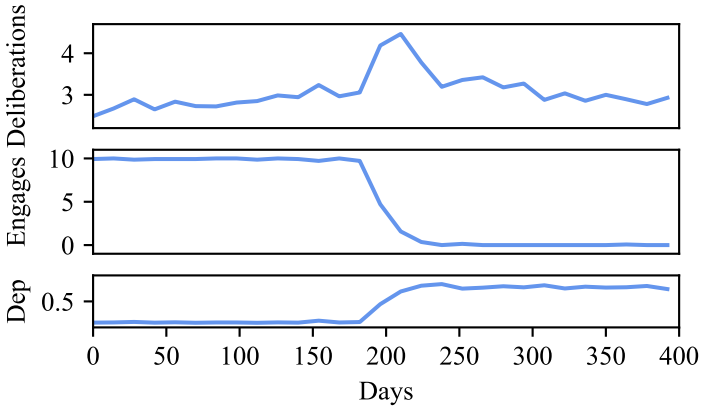

FIG. 8: Plot of the depression level (dep), the average deliberation per ten decisions in a day, and the engages in a day. All values are averaged over a 14 day period. The plots are produced with ILM starting at day 200 and lasting 50 days.

5×10^{-4} does not allow for any such recovery. This describes an inability to escape the MDE without correct treatment, a stuck state. The model meets the criteria of less engages during a MDE and, a MDE is a stuck state. However, all disengages are actively chosen, the effort evaluation is less important to meeting this criteria than predicted. This model does not focus on rumination in the decision making process, which is where mental effort is likely to occur. The model best displays a longer deliberation time immediately after a MDE is induced. Later into the MDE there is a state that describes a diminished ability to concentrate. This was not expected to emerge from this model but still describes a patient with MDD.

4.1.2. Over-Fitting and Over-Parametrising

It was important to avoid overfitting of the model. It would have been easy to add functions that ensure the model behaves as expected but this would not allow for emergent behaviours. The amount of key functions within the model evolved from the initial plan during its creation to ensure it worked, two key examples are $mood$ and reward sensitivity. Initially the model was completely focused on ap, any model functions that depended on $mood$ or dep initially depended on ap such as Eq.8. This created a very rigid model which resisted ID and treatments. More randomness hence a parameter that is prone to fluctuations was necessary. The logical candidate was $mood$. Similarly, the reward sensitivity ϵ in Eq.11 was initially not included, this caused spontaneous recovery. If an engagement occurred during a MDE, PV was significantly lower than A because ξ was small. This caused a large L close to 0.2, leading to a large positive shift in ξ_0. This change was too large in magnitude and caused the model to spontaneously enter a state of remission. Once this part of the model was identified as the issue, it was clear from both the model and literature that reward sensitivity was needed. Without ϵ and $mood$, the model could not display the behaviour in Fig.I. Reduced reward sensitivity, and low mood are fundamental mechanisms within MDE.

The issue with having many functions within the model is the amount of free parameters necessary, and the different ways parameters can be combined to create the functions. Many parameters can be as easily bounded as those in Fig.6. Minor tweaks in the parameters can change the course of the model. Some parameters must be set to create the behaviour in Tbl.I but some are still left free. Changing these parameters can change values such as the success percentage of treatments within the model. That is why the relative behaviours (MDE vs no MDE, changes in treatment length) are important to analyse not the numerical values drawn from this model. Overall, this is a model of least functionality which can show the internal mechanisms of MDD and its treatment.

4.2. MDE Stability

ID causes a large and prolonged increase in ap, as can be seen in Fig.9. Comparing successful perturbations, ap reaches 1 quicker for ID than ILM, and it reaches a higher value and stays non zero for longer when comparing unsuccessful or unstable perturbations. Apathy level is more resistant to change than $mood$, directly increasing it is more likely to lead to a stable episode. Perturbations caused by a failed ILM are shorter than those by ID, this is characterised

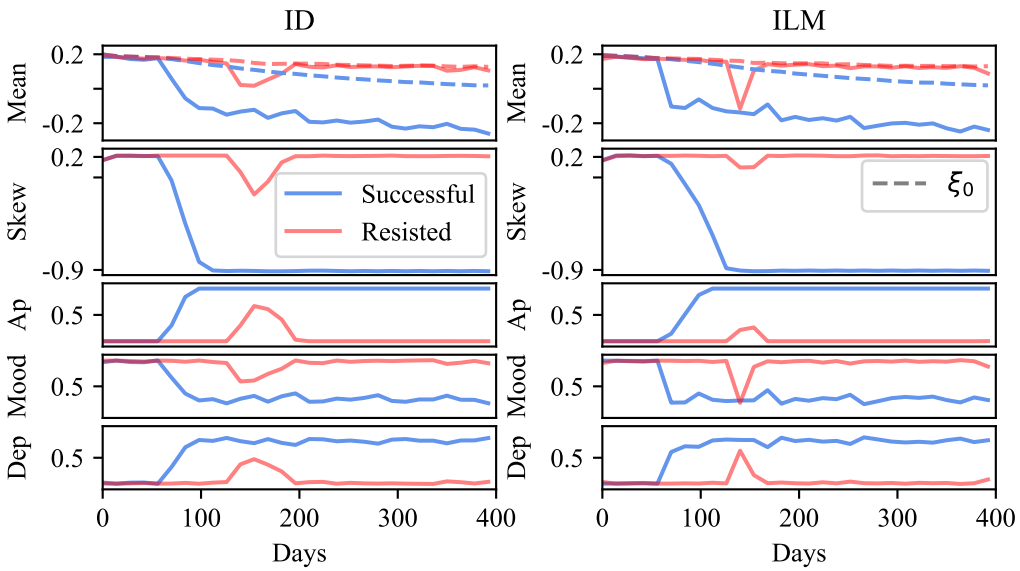

FIG. 9: Plot of the evolution of the mean and skew of the engage Gaussian distribution, and the mood, depression and apathy levels over a 400 day period. ID and ILM beginning on day 70 and lasting for 28 days cause a stable MDE. ID and ILM starting on day 140 and lasting 14 days do not. The dotted line on the mean plot is the mean of the Gaussian distribution neglecting the effect of mood. Values are averaged over a 14 day period.

by sharp peaks in mean and dep, and a small perturbation in ap. Due to $mood$ being a larger influence on dep than ap, it is possible to induce a MDE with a sharp spike in $mood$ which leads to the high count of unstable ILM MDEs in Fig.10. ID causes a sharper decrease in α, while ILM has a larger effect on ξ. Both methods have a similar effect on ξ_0. The difference between a successful and unsuccessful inducement is if the engage Gaussian is perturbed enough to cause disengagement by the end of the inducement.

The DSM-5 states that symptoms must be present for two weeks in order to diagnose MDD. This means that ILM and ID for less than 14 days may not always cause a MDE but longer perturbations should result in a stable MDE. A stable MDE is defined as a MDE that does not self recover after a significant amount of time, in Fig.10 after 150 days. There is no clear data to define how long the average individual can remain in a period of low mood or disengagement without entering a MDE. However, short perturbations for a few days should not cause MDD and long perturbations lasting longer than three weeks should. Short length perturbations, less

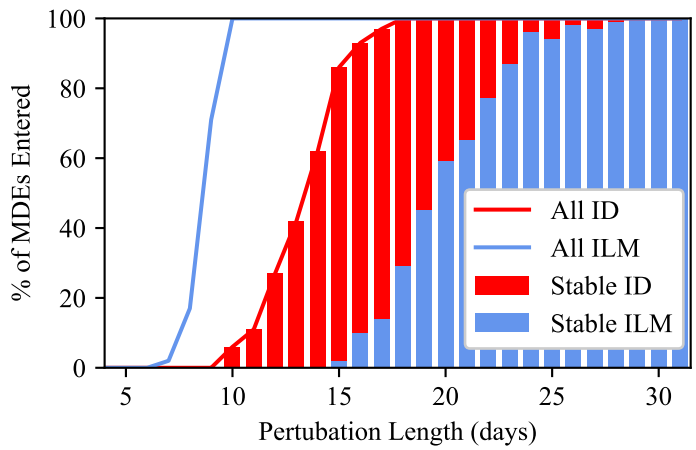

FIG. 10: Plot of the percentage of induced MDEs and stable MDEs by the duration of forced low mood and forced disengagement. Success is marked by a depression level above 0.5 during or after the inducement. The sample size is 100 and error bars take a maximum value of 5%

than five days, cause no MDE, and stable MDEs start after ten days of disengagement. Very long perturbations always cause stable MDEs and after three weeks, over half of ILM cause stable MDEs.

ID does not cause unstable MDEs but requires a longer perturbation time than ILM to cause a MDE. The unstable ILM episodes lasting less then 14 days are better described as minor depressive episodes. From a clinical perspective, periods of complete disengagement are likely to cause lower moods whereas reduced moods may not cause disengagement. Low moods are also a subjective measure of depression whereas disengagement is more measurable. This suggests that disengagement is a better indicator of unhealthy behaviour and there should be a focus on the role of disengagement in MDD rather than the focus on mood. This is in agreement with a 2014 research review of the association of MDD with disengagement specifically when anhedonia is an active symptom [55].

4.3. Treatment Success

It is expected that $51-87\%$ of patients should enter a state of remission through CBT [52]. The parameters within the model can be tweaked to ensure treatment effectiveness within the expected range for a given length and start date. However, the difference in success rates between different treatment lengths and start dates is of interest. The average treatment length of CBT and the average time between onset of depression and treatment varies. A base effective treatment length of 57 weeks was chosen to represent an average treatment length of $1-2$ years [53]. A base time between onset and treatment was chosen as six weeks due to the average waiting times for NHS physiological therapies [54]. In Fig.11 a state of recovery is measured when dep is below 0.5 after 16 weeks of treatment success, only if dep was above 0.5 before the treatment started.

A longer treatment length is more effective. CBT-c has a maximum effectiveness of $59\pm2\%$ but CBT-b reaches almost complete effectiveness. After 20.1 ± 0.3 weeks, the maximum skew value of 1.3 is reached using CBT-c but the effectiveness of the treatment still increases with a longer length. This is because continued treatment maintains the maximum effectiveness and stops α from dropping down to a maximum value of 0.2. This reflects the use of booster sessions in CBT which are used when a patient is in remission to maintain the positive effect achieved and prevent relapse. The model shows that maintaining the positive effect of treatment reduces

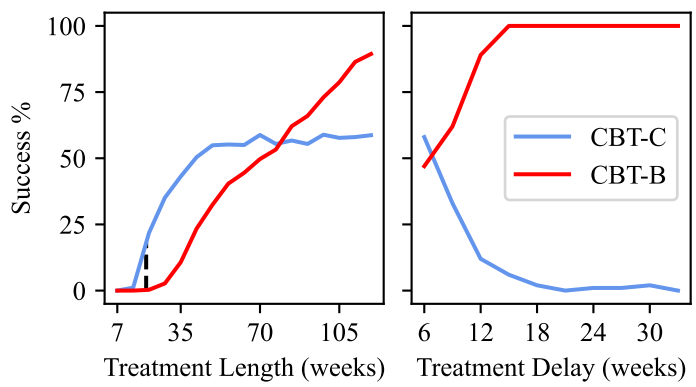

FIG. 11: The percentage of CBT-b's and CBT-c's that cause remission by the length of the treatment and the time in-between the start of the MDE and the start of the treatment in weeks. Sample sizes are 1000 and 100 respectively, maximum errors are 2% and 5%. The dotted line is the weeks taken to reach maximum skew.

remission rates. However, after 49 weeks, there is a negligible increase in the success of CBT-c. The success rates vary within $57 \pm 2\%$, the fluctuations are consistent with the errors due to the sample size. This means the effectiveness of CBT-c reaches a maximum value. If a subject has not recovered after 49 weeks, it is unlikely that future CBT-c treatment will be effective. This means TRD can be diagnosed and alternative treatment or treatment combinations should be used.

CBT-c has a reduced effectiveness the longer a patient has been in a MDE but CBT-b improves in effectiveness. The longer a patient is in a MDE, the smaller the value of ξ_0 and α until the minimum values are reached. A smaller α increases the treatment length needed to reach the maximum skew. A smaller value of ξ_0 will result in less engages for the same maximum α reducing the effectiveness of CBT-c. This same effect increases the effectiveness of CBT-b. If ξ and α are sufficiently small, all deliberations are in favour of P^{Dis} and the effort penalty is small due to $V^{En} << V^{Dis}$. At the start of a MDE this is not the case, both P^{En} and P^{Dis} are increased during the decision making process causing more deliberations, as seen on Fig.8. Also, V^{En} and V^{Dis} are closer together causing a larger effort penalty. This means there is more effort usage when deciding to disengage in the early stages of a MDE. Through CBT-b the model cannot disengage so the preference states are reset. If this loop occurs enough times without exhaustion, and $n \times P_0^{En} > P_{th}^{En}$ the model can engage without ever deliberating to. From a clinical perspective, the decrease in the effectiveness of CBT-c is due to the cementation of the negative self-schemas and the strengthening of a memory bias through further bad experiences. This makes these processes more difficult to overcome which makes it harder to break down the negative triad. The success of CBT-b at a later stage in the MDE can be explained by activity scheduling breaking a subject out of a negative feedback loop which is maintaining the MDE. By forcing themselves to engage in positive activities, a subject can then correct their memory bias and negative views on the world around them creating the cognitive relief that CBT-c is unable to provide.

Fig.11 provides insight on selecting the most effective treatment for a subject. If a subject has been in a MDE for a long time then CBT-b would be more effective than CBT-c. If a patient has intense memory bias and strong negative self-schemas, the best treatment would be to coerce them into engagement in positive activities rather than try and break down those schemas. Naturally, a combination of these techniques would be most effective. As a patient begins to correct their own bias through engaging it would be efficient to assist that through cognitive therapy. The figure also highlights the importance of booster sessions in CBT. Booster sessions occur after the main course of CBT and are less frequent than previous sessions, their main goal is to maintain the positive effect of the CBT and prevent relapse. It is difficult to analyse the usefulness of booster sessions within CBT-b because the effect of the treatment can not be separated from the behaviour of the model. It is likely to not be as important within this model.

4.4. Apathy Dependence

Within this model reduced engagement and goal directed behaviour is effective at inducing a MDE. This shows that the symptom of apathy plays a vital role within MDD. The importance

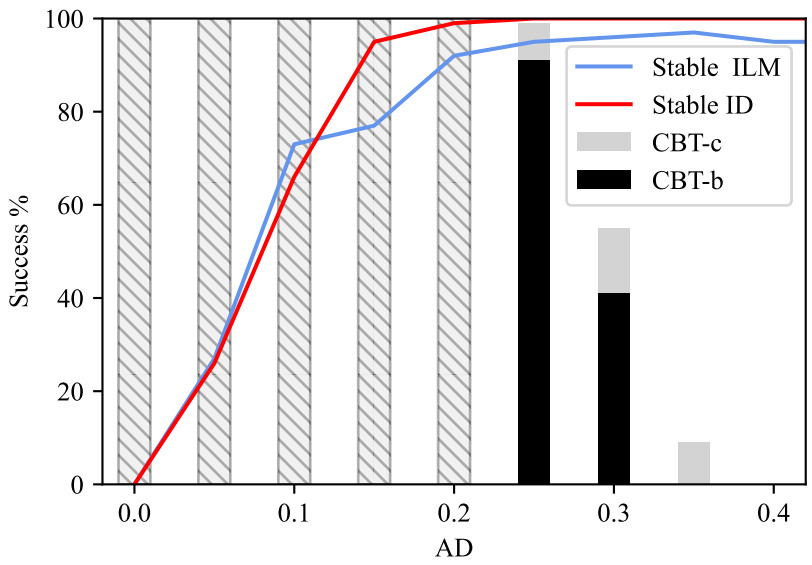

FIG. 12: Plot of the percentage success of inducing stable depression and treating depression by the dependence of depression level on apathy. Treatments lasted 400 days beginning 6 weeks after MDE onset, an ILM of 0.2 lasted 25 days, and ID lasted for 20. For AD values larger than 0.4, there is no treatment success and 100% induction success. The sample size is 100.

of apathy within MDD can be explored through the parameter AD. A higher AD or apathy dependence indicates that the depression level and mechanisms it controls are more dependant on the level of apathy and less on the mood. The allows for the success of inducing and treating a MDE to be explored. As seen in Fig.12, for a low AD, it is difficult to induce a stable MDE and easy to treat it. This is the opposite of is expected for a stuck state suggesting that some apathy dependence is required within MDD. If MDD is only influenced by mood then it would be far more unstable and would change with any fluctuations in mood. As dep becomes more dependant on apathy, the induced MDEs are more likely to be stable and the treatment success begins to fall. Within this model $AD = 0.3$ is a good representation of a patient with MDD. This value results in a high percentage of stable induced MDEs for sensible perturbation lengths, with the success rate of treatments within the expected range. The success of CBT-b reduces quicker than CBT-c. CBT-b attempts to increase the engagement in order to improve the distribution and increase $mood$. In Fig.5 ap only increased once CBT-b reached maximum effectiveness. With a larger AD the improvements in $mood$ have less of an effect and do not cause remission. CBT-c improves the distribution in order to increase engagement, this is focused more on increasing ap than $mood$ so it is less effected by a larger AD. However, as AD increases, the percentage success of both treatments falls, and reaches zero at $AD = 0.4$. In Fig.9 ap tends to a value of 1 during a MDE but $mood$ fluctuates around a non zero value. A higher AD causes a higher and more consistent dep which causes treatment failure. The failure of both treatments suggests that apathy could be a mechanism for TRD. If a patient's depression is due to continuous disengagement it is harder to treat than if it is due to low moods.

If a patient is suffering from TRD, different treatment styles are used. Within this model, the combination of both CBTs can be used. For low AD values, combining two treatments is very effective, this supports the integrated model of CBT which uses different focuses to combat the heterogeneity of MDD. This integrated model is successful when single model CBT fails. However, the success of the combined CBT falls with high AD. For $AD \geq 0.8$ stable MDEs are still induced but treatment is resisted. This describes a level of TRD that resists integrated treatment. In this case, alternate treatments and combinations would be suitable for a patient.

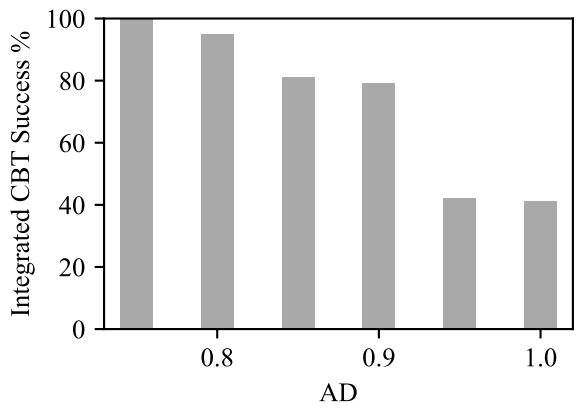

FIG. 13: Plot of the % success of treating depression using a combination of CBTs. For AD values less than 0.7, there is 100% treatment success.

4.5. Model Improvements

This model successfully demonstrates that periods of low mood and disengagement cause a MDE and that altering period length affects the success of inducing a stable episode. Two CBT models, behavioural focused and cognitive focused, are successfully modelled and the behaviour of these treatments explored for different lengths and treatment delay. Overall, this model displays the expected behaviour with no MDE, during a MDE, and during treatment.

The main failure of the model is its inability to maintain a state with a long deliberation time, as shown in Fig.8. When a subject is in a MDE the system approaches a severe memory bias causing a deep state of disengagement. This is due to ξ and α taking their maximum negative values meaning very few deliberations are in favour of V^{En}. This behaviour causes almost zero engages and causes ap to stay at its maximum value of 1. Through alterations of the model's parameters it may be possible to produce a stable MDE that still has some level of engagement. This may be possible by increasing the minimum skew and mean but other parameters would need to change to prevent spontaneous recovery. However, this is a different state of functioning to what this model currently describes. As a heterogeneous disease, MDD is expected to encompass many different states of functioning that cannot be described by one model or one set of parameters. Future work would involve exploring the parameters within this model to find what states of functioning can be represented. This would provide insight to what mechanisms in this model are required to describe different states, and if any mechanisms are currently missing. This would develop the understanding of heterogeneity within MDD and better describe the different states that are under the category of a stuck state. It would also be useful to analyse if apathy is a mechanism of TRD on the different states of MDD.

Currently, this model would not be recommended for clinical usage. The values extracted from this model are all self-consistent. There is no reliable method to connect measurements such as dep or treatment success to current clinical practices or scales. Without quantitative data to ground the model in, the many free parameters cannot be confidently set meaning quantitative data cannot be extracted from the model. Through the use of ecological momentary assessments (EMAs), it could be possible to gain numerical information that could be used to set boundaries on parameters within this model. Information on deliberation times, amount of disengagement, and the effect of disengagement on mood would be useful. If there were more behaviours to meet in Tbl.I the free parameters would be better bound. This would require an EMA designed for the model with defined scales that any quantitative information would output

onto. EMAs face many challenges in MDD. Such as remaining flexible to the disorder's heterogeneity while still having a useful output, accounting for a patient's memory bias, and dealing with missing data. Building confidence in the methods of function creating to ensure their reflection of clinical cases would remain an issue. It is hoped that by having stricter behaviour that this model must display that the possible functioning techniques would be fewer and that by displaying said behaviour the model would inspire confidence. This would allow specific treatment recommendations to be made in clinical practice for patients that can be successfully modelled. This could cut down on the amount of failed treatment a patient goes through before finding one that works.

5. CONCLUSION

Advances in computational psychology has allowed for greater understanding of the mechanisms of major depressive disorder. Such computational work allows one to probe the disease in ways not possible with a patient. However, it is still not a fully understood disease. It is hoped that with the understanding of MDD will come more successful treatments. In this work, a dynamic system was created to explore the overlap of apathy and MDD, investigate the success of treatments, and investigate the cause of treatment resistant depression. The model is based on qualitative conclusions of the effect of MDD on a patient. The success of this model supports the use of computational modelling of MDD based on such data.

The model demonstrates expected behaviour not explicitly coded such as a major depressive episode being described as stuck state and more deliberations in a decision after entering a MDE. The stuck state description is an explanation for the heterogeneity of MDD and could be used as an alternate diagnosis method. The model also displays the unexpected emergent behaviour of reduced deliberations the longer a MDE is active due to a strong memory bias causing a reduced ability to concentrate on engaging. In inducing a MDE, it was shown that forced disengagement is more successful than low mood in creating stable MDEs, suggesting there is scope for a study into the effect of reduced goal orientated behaviour on MDD, most studies focus on mood. Behavioural focused cognitive behavioural therapy was found to be more successful than cognitive focused for treating chronic MDD offering insight to the most suitable treatment for a patient. It was found that stable MDEs were not possible without some influence from apathy. Mood driven MDEs are more prone to fluctuations than the stable apathy driven episodes. This supports the claim that apathy is a symptom of depression. Higher levels of apathy dependence caused treatment resistant depression suggesting that TRD emerges from apathetic behaviour. Treatment could be improved with combined CBT models supporting an integrated model of CBT. High levels of apathy dependence resisted the combined treatment. In this case alternate treatment is necessary such as different CBT models, medication, or somatic therapies.

This work offers insight to the change mechanisms of MDD and its treatments. Further exploration of parameters within the model would allow for specific cases of MDD to be modelled allowing for specific treatment recommendations to be made. It is also hoped that this work can inspire further research into the role of apathy as a symptom of MDD and a mechanism to TRD.

Finally, this model is built on qualitative data hence its outputs are qualitative. Parameters

such as depression level are used to monitor the state of the model and do not lend themselves as a measure of the intensity of MDD within a patient. This does not take away from the conclusions drawn from the mechanisms of the model. However, for case to case modelling and treatment prediction, numerical data, which does not yet exist, is required.

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Appendix Ideas: Parameter list, link to Github, abbrevia-