

# Descongelen a Victor Moreno

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## 1 Estructuras de Datos

### 1.1 Unordered Map

```
1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3
4 struct custom_hash {
5     static uint64_t splitmix64(uint64_t x) {
6         // http://xorshift.di.unimi.it/splitmix64.c
7         x += 0x9e3779b97f4a7c15;
8         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
9         x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
10        return x ^ (x >> 31);
11    }
12
13    size_t operator()(uint64_t x) const {
14        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().
15            time_since_epoch().count();
16        return splitmix64(x + FIXED_RANDOM);
17    }
18 };
19
20 gp_hash_table<int, int, custom_hash> m1;
21
22 //Funcion count
23 m1.find(x)!=m1.end()
```

### 1.2 Segment tree Recursivo

```
1 %%
2 %% This is file `tex',
3 %% generated with the docstrip utility.
4 %%
5 %% The original source files were:
6 %%
7 %% fileerr.dtx (with options: `return')
8 %%
9 %% This is a generated file.
10 %%
11 %% The source is maintained by the LaTeX Project team and bug
12 %% reports for it can be opened at https://latex-project.org/bugs/
13 %% (but please observe conditions on bug reports sent to that address!)
```

```

14 %%
15 %%
16 %% Copyright (C) 1993-2021
17 %% The LaTeX Project and any individual authors listed elsewhere
18 %% in this file.
19 %%
20 %% This file was generated from file(s) of the Standard LaTeX `Tools Bundle
21 %%
22 %%
23 %% -----
24 %%
25 %% It may be distributed and/or modified under the
26 %% conditions of the LaTeX Project Public License, either version 1.3c
27 %% of this license or (at your option) any later version.
28 %% The latest version of this license is in
29 %% https://www.latex-project.org/lppl.txt
30 %% and version 1.3c or later is part of all distributions of LaTeX
31 %% version 2005/12/01 or later.
32 %%
33 %% This file may only be distributed together with a copy of the LaTeX
34 %% `Tools Bundle'. You may however distribute the LaTeX `Tools Bundle'
35 %% without such generated files.
36 %%
37 %% The list of all files belonging to the LaTeX `Tools Bundle' is
38 %% given in the file `manifest.txt'.
39 %%
40 \message{File ignored}
41 \endinput
42 %%
43 %% End of file `.tex'.

```

### 1.3 Segment Tree Iterativo

```

1 //Para procesar queries de tipo k-esimo es necesario crear un arbol binario
2 //perfecto (llenar con 0's)
3 template<typename T>
4 struct SegmentTree{
5     int N;
6     vector<T> ST;
7
8     //Creacion a partir de un arreglo 0(n)
9     SegmentTree(int N, vector<T> & arr): N(N){

```

```

9     ST.resize(N << 1);
10     for(int i = 0; i < N; ++i)
11         ST[N + i] = arr[i]; //Dato normal
12     ST[N + i] = creaNodo(); //Dato compuesto
13     for(int i = N - 1; i > 0; --i)
14         ST[i] = ST[i << 1] + ST[i << 1 | 1]; //Dato normal
15         ST[i] = merge(ST[i << 1], ST[i << 1 | 1]); //Dato compuesto
16     }
17
18     //Actualizacion de un elemento en la posicion i
19     void update(int i, T value){
20         ST[i += N] = value; //Dato normal
21         ST[i += N] = creaNodo(); //Dato compuesto
22         while(i >= 1)
23             ST[i] = ST[i << 1] + ST[i << 1 | 1]; //Dato normal
24             ST[i] = merge(ST[i << 1], ST[i << 1 | 1]); //Dato compuesto
25         }
26
27     //query en [l, r]
28     T query(int l, int r){
29         T res = 0; //Dato normal
30         nodo resl = creaNodo(), resr = creaNodo(); //Dato compuesto
31         for(l += N, r += N; l <= r; l >= 1, r >= 1){
32             if(l & 1) res += ST[l++]; //Dato normal
33             if(!(r & 1)) res += ST[r--]; //Dato normal
34
35             if(l & 1) resl = merge(resl, ST[l++]); //Dato compuesto
36             if(!(r & 1)) resr = merge(ST[r--], resr); //Dato compuesto
37         }
38         return res; //Dato normal
39         return merge(resl, resr); //Dato compuesto
40     }
41
42     //Para estas queries es necesario que el st tenga el tam de la siguiente
43     //potencia de 2
44     //ll nT = 1;
45     // while(nT<n) nT<=1;
46     //vector<int> a(nT,0);
47
48     //Encontrar k-esimo 1 en un st de 1's
49     int Kth_One(int k) {
50         int i = 0, s = N >> 1;
51         for(int p = 2; p < 2 * N; p <= 1, s >= 1) {

```

```

51     if(k < ST[p]) continue;
52     k -= ST[p++]; i += s;
53 }
54 return i;
55 }
56
57 //i del primer elemento >= k en todo el arr
58 int atLeastX(int k){
59     int i = 0, s = N >> 1;
60     for(int p = 2; p < 2 * N; p <= 1, s >= 1) {
61         if(ST[p] < k) p++, i += s;
62     }
63     if(ST[N + i] < k) i = -1;
64     return i;
65 }
66
67 //i del primer elemento >= k en [l,fin]
68 //Uso atLeastX(k,l,1,nT)
69 int atLeastX(int x, int l, int p, int s) {
70     if(ST[p] < x or s <= 1) return -1;
71     if((p < 1) >= 2 * N)
72         return (ST[p] >= x) - 1;
73     int i = atLeastX(x, l, p < 1, s >> 1);
74     if(i != -1) return i;
75     i = atLeastX(x, l - (s >> 1), p < 1 | 1, s >> 1);
76     if(i == -1) return -1;
77     return (s >> 1) + i;
78 }
79 };

```

## 1.4 Segment Tree Lazy Recursivo

```

1 %%
2 %% This is file `tex',
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```

```

12 %% reports for it can be opened at https://latex-project.org/bugs/
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33 %% `Tools Bundle'. You may however distribute the LaTeX `Tools Bundle'
34 %% without such generated files.
35 %%
36 %% The list of all files belonging to the LaTeX `Tools Bundle' is
37 %% given in the file `manifest.txt'.
38 %%
39 \message{File ignored}
40 \endinput
41 %% End of file `tex'.

```

## 1.5 Segment Tree Lazy Iterativo

```

1 //Lazy propagation con incremento de u en rango y minimo
2 //Hay varias modificaciones necesarias para suma en ambos
3 template<typename T>
4 struct SegmentTreeLazy{
5     int N,h;
6     vector<T> ST, d;
7

```

```

8 //Creacion a partir de un arreglo
9 SegmentTreeLazy(int n, vector<T> &a): N(n){
10     //En caso de inicializar en cero o algo similar, revisar que la
        construccion tenga su respectivo neutro mult y 1
11     ST.resize(N << 1);
12     d.resize(N);
13     h = 64 - __builtin_clzll(n);
14
15     for(int i = 0; i < N; ++i)
16         ST[N + i] = a[i];
17     //Construir el st sobre la query que se necesita
18     for(int i = N - 1; i > 0; --i)
19         ST[i] = min(ST[i << 1] , ST[i << 1 | 1]);
20 }
21
22 //Modificar de acuerdo al tipo modificacion requerida, +,*,|,^,etc
23 void apply(int p, T value) {
24     ST[p] += value;
25     if(p<N) d[p] += value;
26 }
27
28 // Modifica valores de los padres de p
29 //Modificar de acuerdo al tipo modificacion requerida, +,*,|,^,etc y a la
        respectiva query
30 void build(int p){
31     while(p>1){
32         p >>= 1;
33         ST[p] = min(ST[p << 1], ST[p << 1 | 1]) + d[p];
34         //ST[p] = (ST[p << 1] & ST[p << 1 | 1]) | d[p]; Ejemplos con bitwise
35     }
36 }
37
38 // Propagacion desde la raiz a p
39 void push(int p){
40     for (int s = h; s > 0; --s) {
41         int i = p >> s;
42         if (d[i] != 0) {
43             apply(i << 1, d[i]);
44             apply(i << 1 | 1, d[i]);
45             d[i] = 0; //Tener cuidado si estoy haciendo multiplicaciones
46         }
47     }
48 }

```

```

49
50 // Sumar v a cada elemento en el intervalo [l, r)
51 void increment(int l, int r, T value) {
52     l += N, r += N;
53     int l0 = l, r0 = r;
54     for (; l < r; l >>= 1, r >>= 1) {
55         if(l & 1) apply(l++, value);
56         if(r & 1) apply(--r, value);
57     }
58     build(l0);
59     build(r0 - 1);
60 }
61
62 // min en el intervalo [l, r)
63 T range_min(int l, int r) {
64     l += N, r += N;
65     push(l);
66     push(r - 1);
67     T res = LLONG_MAX;
68     //T res = (1 << 30) - 1;    Requerir operacion and
69     for (; l < r; l >>= 1, r >>= 1) {
70         if(l & 1) res = min(res, ST[l++]);
71         //if(res >= mod) res -= mod;
72         if(r & 1) res = min(res, ST[--r]);
73         //if(res >= mod) res -= mod;
74     }
75     return res;
76 }
77
78 };

```

## 1.6 Rope

```

1 #include <ext/rope>
2 using namespace __gnu_cxx;
3 rope<int> s;
4 // Sequence with O(log(n)) random access, insert, erase at any position
5 // s.push_back(x);
6 // s.insert(i,r) // insert rope r at position i
7 // s.erase(i,k) // erase subsequence [i,i+k)
8 // s.substr(i,k) // return new rope corresponding to subsequence [i,i+k)
9 // s[i] // access ith element (cannot modify)
10 // s.mutable_reference_at(i) // acces ith element (allows modification)

```

```

11 // s.begin() and s.end() are const iterators (use mutable_begin(),
    mutable_end() to allow modification)

```

## 1.7 Ordered Set

```

1 #include<ext/pb_ds/assoc_container.hpp>
2 #include<ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> ordered_set;
5 // find_by_order(i) -> iterator to ith element
6 // order_of_key(k) -> position (int) of lower_bound of k

```

## 1.8 Union Find

```

1 vector<pair<int,int>>ds(MAX,{-1,0});
2 // Solo siu requieres los elementos del union find, utiliza
3 // ds[0] en caso contrario borrarlo
4 list<int>dsext[MAX];
5 void init(int n){
6     for(int i=0;i<n;i++)dsext[i].push_back(i);
7 }
8 int find(int x){
9     if(-1==ds[x].first) return x;
10    return ds[x].first=find(ds[x].first);
11 }
12 bool unionDs(int x, int y){
13     int px=find(x),py=find(y);
14     int &rx=ds[px].second,&ry=ds[py].second;
15     if(px==py) return false;
16     else{
17         if(rx>ry){
18             ds[py].first=px;
19         }
20         else{
21             ds[px].first=py;
22             if(rx==ry) ry+=1;
23         }
24     }
25     return true;
26 }

```

## 2 Geometria

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using ld = long double;
4 const ld eps = 1e-9, inf = numeric_limits<ld>::max(), pi = acos(-1);
5 // For use with integers, just set eps=0 and everything remains the same
6 bool geq(ld a, ld b){return a-b >= -eps;} //a >= b
7 bool leq(ld a, ld b){return b-a >= -eps;} //a <= b
8 bool ge(ld a, ld b){return a-b > eps;} //a > b
9 bool le(ld a, ld b){return b-a > eps;} //a < b
10 bool eq(ld a, ld b){return abs(a-b) <= eps;} //a == b
11 bool neq(ld a, ld b){return abs(a-b) > eps;} //a != b
12
13 struct point{
14     ld x, y;
15     point(): x(0), y(0){}
16     point(ld x, ld y): x(x), y(y){}
17
18     point operator+(const point & p) const{return point(x + p.x, y + p.y);}
19     point operator-(const point & p) const{return point(x - p.x, y - p.y);}
20     point operator*(const ld & k) const{return point(x * k, y * k);}
21     point operator/(const ld & k) const{return point(x / k, y / k);}
22
23     point operator+=(const point & p){*this = *this + p; return *this;}
24     point operator-=(const point & p){*this = *this - p; return *this;}
25     point operator*=(const ld & p){*this = *this * p; return *this;}
26     point operator/=(const ld & p){*this = *this / p; return *this;}
27
28     point rotate(const ld & a) const{return point(x*cos(a) - y*sin(a), x*sin(
        a) + y*cos(a));}
29     point perp() const{return point(-y, x);}
30     ld ang() const{
31         ld a = atan2l(y, x); a += le(a, 0) ? 2*pi : 0; return a;
32     }
33     ld dot(const point & p) const{return x * p.x + y * p.y;}
34     ld cross(const point & p) const{return x * p.y - y * p.x;}
35     ld norm() const{return x * x + y * y;}
36     ld length() const{return sqrtl(x * x + y * y);}
37     point unit() const{return (*this) / length();}
38
39     bool operator==(const point & p) const{return eq(x, p.x) && eq(y, p.y);}
40     bool operator!=(const point & p) const{return !(*this == p);}
41     bool operator<(const point & p) const{return le(x, p.x) || (eq(x, p.x) &&
        le(y, p.y));}

```

```

42 bool operator>(const point & p) const{return ge(x, p.x) || (eq(x, p.x) &&
    ge(y, p.y));}
43 bool half(const point & p) const{return le(p.cross(*this), 0) || (eq(p.
    cross(*this), 0) && le(p.dot(*this), 0));}
44 };
45
46 istream &operator>>(istream &is, point & p){return is >> p.x >> p.y;}
47 ostream &operator<<(ostream &os, const point & p){return os << "(" << p.x
    << ", " << p.y << ")};
48
49 int sgn(ld x){
50     if(ge(x, 0)) return 1;
51     if(le(x, 0)) return -1;
52     return 0;
53 }
54
55 void polarSort(vector<point> & P, const point & o, const point & v){
56     //sort points in P around o, taking the direction of v as first angle
57     sort(P.begin(), P.end(), [&](const point & a, const point & b){
58         return point((a - o).half(v), 0) < point((b - o).half(v), (a - o).cross
            (b - o));
59     });
60 }
61
62 bool pointInLine(const point & a, const point & v, const point & p){
63     //line a+tv, point p
64     return eq((p - a).cross(v), 0);
65 }
66
67 bool pointInSegment(const point & a, const point & b, const point & p){
68     //segment ab, point p
69     return pointInLine(a, b - a, p) && leq((a - p).dot(b - p), 0);
70 }
71
72 int intersectLinesInfo(const point & a1, const point & v1, const point & a2
    , const point & v2){
73     //lines a1+tv1 and a2+tv2
74     ld det = v1.cross(v2);
75     if(eq(det, 0)){
76         if(eq((a2 - a1).cross(v1), 0)){
77             return -1; //infinity points
78         }else{
79             return 0; //no points

```

```

80     }
81 }else{
82     return 1; //single point
83 }
84 }
85
86 point intersectLines(const point & a1, const point & v1, const point & a2,
    const point & v2){
87     //lines a1+tv1, a2+tv2
88     //assuming that they intersect
89     ld det = v1.cross(v2);
90     return a1 + v1 * ((a2 - a1).cross(v2) / det);
91 }
92
93 int intersectLineSegmentInfo(const point & a, const point & v, const point
    & c, const point & d){
94     //line a+tv, segment cd
95     point v2 = d - c;
96     ld det = v.cross(v2);
97     if(eq(det, 0)){
98         if(eq((c - a).cross(v), 0)){
99             return -1; //infinity points
100         }else{
101             return 0; //no point
102         }
103     }else{
104         return sgn(v.cross(c - a)) != sgn(v.cross(d - a)); //1: single point,
            0: no point
105     }
106 }
107
108 int intersectSegmentsInfo(const point & a, const point & b, const point & c
    , const point & d){
109     //segment ab, segment cd
110     point v1 = b - a, v2 = d - c;
111     int t = sgn(v1.cross(c - a)), u = sgn(v1.cross(d - a));
112     if(t == u){
113         if(t == 0){
114             if(pointInSegment(a, b, c) || pointInSegment(a, b, d) ||
                pointInSegment(c, d, a) || pointInSegment(c, d, b)){
115                 return -1; //infinity points
116             }else{
117                 return 0; //no point

```

```

118     }
119     }else{
120         return 0; //no point
121     }
122     }else{
123         return sgn(v2.cross(a - c)) != sgn(v2.cross(b - c)); //1: single point,
124         0: no point
125     }
126 }
127 ld distancePointLine(const point & a, const point & v, const point & p){
128     //line: a + tv, point p
129     return abs(v.cross(p - a)) / v.length();
130 }
131
132 ld perimeter(vector<point> & P){
133     int n = P.size();
134     ld ans = 0;
135     for(int i = 0; i < n; i++){
136         ans += (P[i] - P[(i + 1) % n]).length();
137     }
138     return ans;
139 }
140
141 ld area(vector<point> & P){
142     int n = P.size();
143     ld ans = 0;
144     for(int i = 0; i < n; i++){
145         ans += P[i].cross(P[(i + 1) % n]);
146     }
147     return abs(ans / 2);
148 }
149
150 vector<point> convexHull(vector<point> P){
151     sort(P.begin(), P.end());
152     vector<point> L, U;
153     for(int i = 0; i < P.size(); i++){
154         while(L.size() >= 2 && leq((L[L.size() - 2] - P[i]).cross(L[L.size() -
155             1] - P[i]), 0)){
156             L.pop_back();
157         }
158         L.push_back(P[i]);

```

```

159     for(int i = P.size() - 1; i >= 0; i--){
160         while(U.size() >= 2 && leq((U[U.size() - 2] - P[i]).cross(U[U.size() -
161             1] - P[i]), 0)){
162             U.pop_back();
163         }
164         U.push_back(P[i]);
165     }
166     L.pop_back();
167     U.pop_back();
168     L.insert(L.end(), U.begin(), U.end());
169     return L;
170 }
171
172 bool pointInPerimeter(const vector<point> & P, const point & p){
173     int n = P.size();
174     for(int i = 0; i < n; i++){
175         if(pointInSegment(P[i], P[(i + 1) % n], p)){
176             return true;
177         }
178     }
179     return false;
180 }
181
182 bool crossesRay(const point & a, const point & b, const point & p){
183     return (geq(b.y, p.y) - geq(a.y, p.y)) * sgn((a - p).cross(b - p)) > 0;
184 }
185
186 int pointInPolygon(const vector<point> & P, const point & p){
187     if(pointInPerimeter(P, p)){
188         return -1; //point in the perimeter
189     }
190     int n = P.size();
191     int rays = 0;
192     for(int i = 0; i < n; i++){
193         rays += crossesRay(P[i], P[(i + 1) % n], p);
194     }
195     return rays & 1; //0: point outside, 1: point inside
196 }
197
198 //point in convex polygon in O(log n)
199 //make sure that P is convex and in ccw
200 //before the queries, do the preprocess on P:
201 // rotate(P.begin(), min_element(P.begin(), P.end()), P.end());

```

```

201 // int right = max_element(P.begin(), P.end()) - P.begin();
202 //returns 0 if p is outside, 1 if p is inside, -1 if p is in the perimeter
203 int pointInConvexPolygon(const vector<point> & P, const point & p, int
    right){
204     if(p < P[0] || P[right] < p) return 0;
205     int orientation = sgn((P[right] - P[0]).cross(p - P[0]));
206     if(orientation == 0){
207         if(p == P[0] || p == P[right]) return -1;
208         return (right == 1 || right + 1 == P.size()) ? -1 : 1;
209     }else if(orientation < 0){
210         auto r = lower_bound(P.begin() + 1, P.begin() + right, p);
211         int det = sgn((p - r[-1]).cross(r[0] - r[-1])) - 1;
212         if(det == -2) det = 1;
213         return det;
214     }else{
215         auto l = upper_bound(P.rbegin(), P.rend() - right - 1, p);
216         int det = sgn((p - l[0]).cross((l == P.rbegin() ? P[0] : l[-1]) - l[0])
            ) - 1;
217         if(det == -2) det = 1;
218         return det;
219     }
220 }
221
222
223
224
225
226 vector<point> cutPolygon(const vector<point> & P, const point & a, const
    point & v){
227     //returns the part of the convex polygon P on the left side of line a+tv
228     int n = P.size();
229     vector<point> lhs;
230     for(int i = 0; i < n; ++i){
231         if(geq(v.cross(P[i] - a), 0)){
232             lhs.push_back(P[i]);
233         }
234         if(intersectLineSegmentInfo(a, v, P[i], P[(i+1)%n]) == 1){
235             point p = intersectLines(a, v, P[i], P[(i+1)%n] - P[i]);
236             if(p != P[i] && p != P[(i+1)%n]){
237                 lhs.push_back(p);
238             }
239         }
240     }

```

```

241     return lhs;
242 }
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259 point centroid(vector<point> & P){
260     point num;
261     ld den = 0;
262     int n = P.size();
263     for(int i = 0; i < n; ++i){
264         ld cross = P[i].cross(P[(i + 1) % n]);
265         num += (P[i] + P[(i + 1) % n]) * cross;
266         den += cross;
267     }
268     return num / (3 * den);
269 }
270
271 vector<pair<int, int>> antipodalPairs(vector<point> & P){
272     vector<pair<int, int>> ans;
273     int n = P.size(), k = 1;
274     auto f = [&](int u, int v, int w){return abs((P[v%n]-P[u%n]).cross(P[w%n]
        ]-P[u%n]));};
275     while(ge(f(n-1, 0, k+1), f(n-1, 0, k))) ++k;
276     for(int i = 0, j = k; i <= k && j < n; ++i){
277         ans.emplace_back(i, j);
278         while(j < n-1 && ge(f(i, i+1, j+1), f(i, i+1, j)))
279             ans.emplace_back(i, ++j);
280     }
281     return ans;
282 }

```



```

283 pair<ld, ld> diameterAndWidth(vector<point> & P){
284     int n = P.size(), k = 0;
285     auto dot = [&](int a, int b){return (P[(a+1)%n]-P[a]).dot(P[(b+1)%n]-P[b]);};
286     auto cross = [&](int a, int b){return (P[(a+1)%n]-P[a]).cross(P[(b+1)%n]-P[b]);};
287     ld diameter = 0;
288     ld width = inf;
289     while(get(dot(0, k), 0)) k = (k+1) % n;
290     for(int i = 0; i < n; ++i){
291         while(get(cross(i, k), 0)) k = (k+1) % n;
292         //pair: (i, k)
293         diameter = max(diameter, (P[k] - P[i]).length());
294         width = min(width, distancePointLine(P[i], P[(i+1)%n] - P[i], P[k]));
295     }
296     return {diameter, width};
297 }
298
299 pair<ld, ld> smallestEnclosingRectangle(vector<point> & P){
300     int n = P.size();
301     auto dot = [&](int a, int b){return (P[(a+1)%n]-P[a]).dot(P[(b+1)%n]-P[b]);};
302     auto cross = [&](int a, int b){return (P[(a+1)%n]-P[a]).cross(P[(b+1)%n]-P[b]);};
303     ld perimeter = inf, area = inf;
304     for(int i = 0, j = 0, k = 0, m = 0; i < n; ++i){
305         while(get(dot(i, j), 0)) j = (j+1) % n;
306         if(!i) k = j;
307         while(get(cross(i, k), 0)) k = (k+1) % n;
308         if(!i) m = k;
309         while(get(dot(i, m), 0)) m = (m+1) % n;
310         //pairs: (i, k) , (j, m)
311         point v = P[(i+1)%n] - P[i];
312         ld h = distancePointLine(P[i], v, P[k]);
313         ld w = distancePointLine(P[j], v.perp(), P[m]);
314         perimeter = min(perimeter, 2 * (h + w));
315         area = min(area, h * w);
316     }
317     return {area, perimeter};
318 }
319
320 ld distancePointCircle(const point & c, ld r, const point & p){
321

```

```

322 //point p, circle with center c and radius r
323 return max((ld)0, (p - c).length() - r);
324 }
325
326 point projectionPointCircle(const point & c, ld r, const point & p){
327     //point p (outside the circle), circle with center c and radius r
328     return c + (p - c).unit() * r;
329 }
330
331 pair<point, point> pointsOfTangency(const point & c, ld r, const point & p)
332 {
333     //point p (outside the circle), circle with center c and radius r
334     point v = (p - c).unit() * r;
335     ld d2 = (p - c).norm(), d = sqrt(d2);
336     point v1 = v * (r / d), v2 = v.perp() * (sqrt(d2 - r*r) / d);
337     return {c + v1 - v2, c + v1 + v2};
338 }
339
340 vector<point> intersectLineCircle(const point & a, const point & v, const
341     point & c, ld r){
342     //line a+tv, circle with center c and radius r
343     ld h2 = r*r - v.cross(c - a) * v.cross(c - a) / v.norm();
344     point p = a + v * v.dot(c - a) / v.norm();
345     if(eq(h2, 0)) return {p}; //line tangent to circle
346     else if(le(h2, 0)) return {}; //no intersection
347     else{
348         point u = v.unit() * sqrt(h2);
349         return {p - u, p + u}; //two points of intersection (chord)
350     }
351 }
352
353 vector<point> intersectSegmentCircle(const point & a, const point & b,
354     const point & c, ld r){
355     //segment ab, circle with center c and radius r
356     vector<point> P = intersectLineCircle(a, b - a, c, r), ans;
357     for(const point & p : P){
358         if(pointInSegment(a, b, p)) ans.push_back(p);
359     }
360     return ans;
361 }
362
363 pair<point, ld> getCircle(const point & m, const point & n, const point & p
364     ){

```

```

361 //find circle that passes through points p, q, r
362 point c = intersectLines((n + m) / 2, (n - m).perp(), (p + n) / 2, (p - n
    ).perp());
363 ld r = (c - m).length();
364 return {c, r};
365 }
366
367 vector<point> intersectionCircles(const point & c1, ld r1, const point & c2
    , ld r2){
368     //circle 1 with center c1 and radius r1
369     //circle 2 with center c2 and radius r2
370     point d = c2 - c1;
371     ld d2 = d.norm();
372     if(eq(d2, 0)) return {}; //concentric circles
373     ld pd = (d2 + r1*r1 - r2*r2) / 2;
374     ld h2 = r1*r1 - pd*pd/d2;
375     point p = c1 + d*pd/d2;
376     if(eq(h2, 0)) return {p}; //circles touch at one point
377     else if(1e(h2, 0)) return {}; //circles don't intersect
378     else{
379         point u = d.perp() * sqrt(h2/d2);
380         return {p - u, p + u};
381     }
382 }
383
384 int circleInsideCircle(const point & c1, ld r1, const point & c2, ld r2){
385     //test if circle 2 is inside circle 1
386     //returns "-1" if 2 touches internally 1, "1" if 2 is inside 1, "0" if
        they overlap
387     ld l = r1 - r2 - (c1 - c2).length();
388     return (ge(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
389 }
390
391 int circleOutsideCircle(const point & c1, ld r1, const point & c2, ld r2){
392     //test if circle 2 is outside circle 1
393     //returns "-1" if they touch externally, "1" if 2 is outside 1, "0" if
        they overlap
394     ld l = (c1 - c2).length() - (r1 + r2);
395     return (ge(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
396 }
397
398 int pointInCircle(const point & c, ld r, const point & p){
399     //test if point p is inside the circle with center c and radius r

```

```

400     //returns "0" if it's outside, "-1" if it's in the perimeter, "1" if it's
        inside
401     ld l = (p - c).length() - r;
402     return (le(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
403 }
404
405 vector<vector<point>> tangents(const point & c1, ld r1, const point & c2,
    ld r2, bool inner){
406     //returns a vector of segments or a single point
407     if(inner) r2 = -r2;
408     point d = c2 - c1;
409     ld dr = r1 - r2, d2 = d.norm(), h2 = d2 - dr*dr;
410     if(eq(d2, 0) || 1e(h2, 0)) return {};
411     point v = d*dr/d2;
412     if(eq(h2, 0)) return {{c1 + v*r1}};
413     else{
414         point u = d.perp()*sqrt(h2)/d2;
415         return {{c1 + (v - u)*r1, c2 + (v - u)*r2}, {c1 + (v + u)*r1, c2 + (v +
            u)*r2}};
416     }
417 }
418
419 ld signed_angle(const point & a, const point & b){
420     return sgn(a.cross(b)) * acosl(a.dot(b) / (a.length() * b.length()));
421 }
422
423 ld intersectPolygonCircle(const vector<point> & P, const point & c, ld r){
424     //Gets the area of the intersection of the polygon with the circle
425     int n = P.size();
426     ld ans = 0;
427     for(int i = 0; i < n; ++i){
428         point p = P[i], q = P[(i+1)%n];
429         bool p_inside = (pointInCircle(c, r, p) != 0);
430         bool q_inside = (pointInCircle(c, r, q) != 0);
431         if(p_inside && q_inside){
432             ans += (p - c).cross(q - c);
433         }else if(p_inside && !q_inside){
434             point s1 = intersectSegmentCircle(p, q, c, r)[0];
435             point s2 = intersectSegmentCircle(c, q, c, r)[0];
436             ans += (p - c).cross(s1 - c) + r*r * signed_angle(s1 - c, s2 - c);
437         }else if(!p_inside && q_inside){
438             point s1 = intersectSegmentCircle(c, p, c, r)[0];
439             point s2 = intersectSegmentCircle(p, q, c, r)[0];

```

```

440     ans += (s2 - c).cross(q - c) + r*r * signed_angle(s1 - c, s2 - c);
441 }else{
442     auto info = intersectSegmentCircle(p, q, c, r);
443     if(info.size() <= 1){
444         ans += r*r * signed_angle(p - c, q - c);
445     }else{
446         point s2 = info[0], s3 = info[1];
447         point s1 = intersectSegmentCircle(c, p, c, r)[0];
448         point s4 = intersectSegmentCircle(c, q, c, r)[0];
449         ans += (s2 - c).cross(s3 - c) + r*r * (signed_angle(s1 - c, s2 - c)
450             + signed_angle(s3 - c, s4 - c));
451     }
452 }
453 return abs(ans)/2;
454 }
455
456 pair<point, ld> mec2(vector<point> & S, const point & a, const point & b,
457     int n){
458     ld hi = inf, lo = -hi;
459     for(int i = 0; i < n; ++i){
460         ld si = (b - a).cross(S[i] - a);
461         if(eq(si, 0)) continue;
462         point m = getCircle(a, b, S[i]).first;
463         ld cr = (b - a).cross(m - a);
464         if(le(si, 0)) hi = min(hi, cr);
465         else lo = max(lo, cr);
466     }
467     ld v = (ge(lo, 0) ? lo : le(hi, 0) ? hi : 0);
468     point c = (a + b) / 2 + (b - a).perp() * v / (b - a).norm();
469     return {c, (a - c).norm()};
470 }
471
472 pair<point, ld> mec(vector<point> & S, const point & a, int n){
473     random_shuffle(S.begin(), S.begin() + n);
474     point b = S[0], c = (a + b) / 2;
475     ld r = (a - c).norm();
476     for(int i = 1; i < n; ++i){
477         if(ge((S[i] - c).norm(), r)){
478             tie(c, r) = (n == S.size() ? mec(S, S[i], i) : mec2(S, a, S[i], i));
479         }
480     }
481     return {c, r};

```

```

481 }
482
483 pair<point, ld> smallestEnclosingCircle(vector<point> S){
484     assert(!S.empty());
485     auto r = mec(S, S[0], S.size());
486     return {r.first, sqrt(r.second)};
487 }
488
489 bool comp1(const point & a, const point & b){
490     return le(a.y, b.y);
491 }
492
493 pair<point, point> closestPairOfPoints(vector<point> P){
494     sort(P.begin(), P.end(), comp1);
495     set<point> S;
496     ld ans = inf;
497     point p, q;
498     int pos = 0;
499     for(int i = 0; i < P.size(); ++i){
500         while(pos < i && geq(P[i].y - P[pos].y, ans)){
501             S.erase(P[pos++]);
502         }
503         auto lower = S.lower_bound({P[i].x - ans - eps, -inf});
504         auto upper = S.upper_bound({P[i].x + ans + eps, -inf});
505         for(auto it = lower; it != upper; ++it){
506             ld d = (P[i] - *it).length();
507             if(le(d, ans)){
508                 ans = d;
509                 p = P[i];
510                 q = *it;
511             }
512             S.insert(P[i]);
513         }
514     }
515     return {p, q};
516 }
517
518 struct vantage_point_tree{
519     struct node
520     {
521         point p;
522         ld th;
523         node *l, *r;
524     }*root;

```

```

524
525 vector<pair<ld, point>> aux;
526
527 vantage_point_tree(vector<point> &ps){
528     for(int i = 0; i < ps.size(); ++i)
529         aux.push_back({ 0, ps[i] });
530     root = build(0, ps.size());
531 }
532
533 node *build(int l, int r){
534     if(l == r)
535         return 0;
536     swap(aux[l], aux[l + rand() % (r - l)]);
537     point p = aux[l++].second;
538     if(l == r)
539         return new node({ p });
540     for(int i = l; i < r; ++i)
541         aux[i].first = (p - aux[i].second).dot(p - aux[i].second);
542     int m = (l + r) / 2;
543     nth_element(aux.begin() + l, aux.begin() + m, aux.begin() + r);
544     return new node({ p, sqrt(aux[m].first), build(l, m), build(m, r) });
545 }
546
547 priority_queue<pair<ld, node*>> que;
548
549 void k_nn(node *t, point p, int k){
550     if(!t)
551         return;
552     ld d = (p - t->p).length();
553     if(que.size() < k)
554         que.push({ d, t });
555     else if(ge(que.top().first, d)){
556         que.pop();
557         que.push({ d, t });
558     }
559     if(!t->l && !t->r)
560         return;
561     if(le(d, t->th)){
562         k_nn(t->l, p, k);
563         if(leq(t->th - d, que.top().first))
564             k_nn(t->r, p, k);
565     }else{
566         k_nn(t->r, p, k);

```

```

567         if(leq(d - t->th, que.top().first))
568             k_nn(t->l, p, k);
569     }
570 }
571
572 vector<point> k_nn(point p, int k){
573     k_nn(root, p, k);
574     vector<point> ans;
575     for(; !que.empty(); que.pop())
576         ans.push_back(que.top().second->p);
577     reverse(ans.begin(), ans.end());
578     return ans;
579 }
580 };
581
582 vector<point> minkowskiSum(vector<point> A, vector<point> B){
583     int na = (int)A.size(), nb = (int)B.size();
584     if(A.empty() || B.empty()) return {};
585
586     rotate(A.begin(), min_element(A.begin(), A.end()), A.end());
587     rotate(B.begin(), min_element(B.begin(), B.end()), B.end());
588
589     int pa = 0, pb = 0;
590     vector<point> M;
591
592     while(pa < na && pb < nb){
593         M.push_back(A[pa] + B[pb]);
594         ld x = (A[(pa + 1) % na] - A[pa]).cross(B[(pb + 1) % nb] - B[pb]);
595         if(leq(x, 0)) pb++;
596         if(geq(x, 0)) pa++;
597     }
598
599     while(pa < na) M.push_back(A[pa++] + B[0]);
600     while(pb < nb) M.push_back(B[pb++] + A[0]);
601
602     return M;
603 }
604
605 //Delaunay triangulation in O(n log n)
606 const point inf_pt(inf, inf);
607
608 struct QuadEdge{
609     point origin;

```

```

610 QuadEdge* rot = nullptr;
611 QuadEdge* onext = nullptr;
612 bool used = false;
613 QuadEdge* rev() const{return rot->rot;}
614 QuadEdge* lnext() const{return rot->rev()->onext->rot;}
615 QuadEdge* oprev() const{return rot->onext->rot;}
616 point dest() const{return rev()->origin;}
617 };
618
619 QuadEdge* make_edge(const point & from, const point & to){
620     QuadEdge* e1 = new QuadEdge;
621     QuadEdge* e2 = new QuadEdge;
622     QuadEdge* e3 = new QuadEdge;
623     QuadEdge* e4 = new QuadEdge;
624     e1->origin = from;
625     e2->origin = to;
626     e3->origin = e4->origin = inf_pt;
627     e1->rot = e3;
628     e2->rot = e4;
629     e3->rot = e2;
630     e4->rot = e1;
631     e1->onext = e1;
632     e2->onext = e2;
633     e3->onext = e4;
634     e4->onext = e3;
635     return e1;
636 }
637
638 void splice(QuadEdge* a, QuadEdge* b){
639     swap(a->onext->rot->onext, b->onext->rot->onext);
640     swap(a->onext, b->onext);
641 }
642
643 void delete_edge(QuadEdge* e){
644     splice(e, e->oprev());
645     splice(e->rev(), e->rev()->oprev());
646     delete e->rot;
647     delete e->rev()->rot;
648     delete e;
649     delete e->rev();
650 }
651
652 QuadEdge* connect(QuadEdge* a, QuadEdge* b){

```

```

653     QuadEdge* e = make_edge(a->dest(), b->origin);
654     splice(e, a->lnext());
655     splice(e->rev(), b);
656     return e;
657 }
658
659 bool left_of(const point & p, QuadEdge* e){
660     return ge((e->origin - p).cross(e->dest() - p), 0);
661 }
662
663 bool right_of(const point & p, QuadEdge* e){
664     return le((e->origin - p).cross(e->dest() - p), 0);
665 }
666
667 ld det3(ld a1, ld a2, ld a3, ld b1, ld b2, ld b3, ld c1, ld c2, ld c3) {
668     return a1 * (b2 * c3 - c2 * b3) - a2 * (b1 * c3 - c1 * b3) + a3 * (b1 *
        c2 - c1 * b2);
669 }
670
671 bool in_circle(const point & a, const point & b, const point & c, const
    point & d) {
672     ld det = -det3(b.x, b.y, b.norm(), c.x, c.y, c.norm(), d.x, d.y, d.norm()
        );
673     det += det3(a.x, a.y, a.norm(), c.x, c.y, c.norm(), d.x, d.y, d.norm());
674     det -= det3(a.x, a.y, a.norm(), b.x, b.y, b.norm(), d.x, d.y, d.norm());
675     det += det3(a.x, a.y, a.norm(), b.x, b.y, b.norm(), c.x, c.y, c.norm());
676     return ge(det, 0);
677 }
678
679 pair<QuadEdge*, QuadEdge*> build_tr(int l, int r, vector<point> & P){
680     if(r - l + 1 == 2){
681         QuadEdge* res = make_edge(P[l], P[r]);
682         return {res, res->rev()};
683     }
684     if(r - l + 1 == 3){
685         QuadEdge *a = make_edge(P[l], P[l + 1]), *b = make_edge(P[l + 1], P[r])
            ;
686         splice(a->rev(), b);
687         int sg = sgn((P[l + 1] - P[l]).cross(P[r] - P[l]));
688         if(sg == 0)
689             return {a, b->rev()};
690         QuadEdge* c = connect(b, a);
691         if(sg == 1)

```

```

692     return {a, b->rev()};
693     else
694         return {c->rev(), c};
695 }
696 int mid = (l + r) / 2;
697 QuadEdge *ldo, *ldi, *rdo, *rdi;
698 tie(ldo, ldi) = build_tr(l, mid, P);
699 tie(rdi, rdo) = build_tr(mid + 1, r, P);
700 while(true){
701     if(left_of(rdi->origin, ldi)){
702         ldi = ldi->lnext();
703         continue;
704     }
705     if(right_of(ldi->origin, rdi)){
706         rdi = rdi->rev()->onext;
707         continue;
708     }
709     break;
710 }
711 QuadEdge* basel = connect(rdi->rev(), ldi);
712 auto valid = [&basel](QuadEdge* e){return right_of(e->dest(), basel);};
713 if(ldi->origin == ldo->origin)
714     ldo = basel->rev();
715 if(rdi->origin == rdo->origin)
716     rdo = basel;
717 while(true){
718     QuadEdge* lcand = basel->rev()->onext;
719     if(valid(lcand)){
720         while(in_circle(basel->dest(), basel->origin, lcand->dest(), lcand->
721             onext->dest())){
722             QuadEdge* t = lcand->onext;
723             delete_edge(lcand);
724             lcand = t;
725         }
726     }
727     QuadEdge* rcand = basel->oprev();
728     if(valid(rcand)){
729         while(in_circle(basel->dest(), basel->origin, rcand->dest(), rcand->
730             oprev()->dest())){
731             QuadEdge* t = rcand->oprev();
732             delete_edge(rcand);
733             rcand = t;
734         }
735     }

```

```

733     }
734     if(!valid(lcand) && !valid(rcand))
735         break;
736     if(!valid(lcand) || (valid(rcand) && in_circle(lcand->dest(), lcand->
737         origin, rcand->origin, rcand->dest()))
738         basel = connect(rcand, basel->rev());
739     else
740         basel = connect(basel->rev(), lcand->rev());
741 }
742 return {ldo, rdo};
743 }
744 vector<tuple<point, point, point>> delaunay(vector<point> & P){
745     sort(P.begin(), P.end());
746     auto res = build_tr(0, (int)P.size() - 1, P);
747     QuadEdge* e = res.first;
748     vector<QuadEdge*> edges = {e};
749     while(1e((e->dest() - e->onext->dest()).cross(e->origin - e->onext->dest
750         ()), 0))
751         e = e->onext;
752     auto add = [&P, &e, &edges]() {
753         QuadEdge* curr = e;
754         do{
755             curr->used = true;
756             P.push_back(curr->origin);
757             edges.push_back(curr->rev());
758             curr = curr->lnext();
759         }while(curr != e);
760     };
761     add();
762     P.clear();
763     int kek = 0;
764     while(kek < (int)edges.size())
765         if(!(e = edges[kek++])->used)
766             add();
767     vector<tuple<point, point, point>> ans;
768     for(int i = 0; i < (int)P.size(); i += 3){
769         ans.emplace_back(P[i], P[i + 1], P[i + 2]);
770     }
771     return ans;
772 }
773 struct circ{

```

```

774 point c;
775 ld r;
776 circ() {}
777 circ(const point & c, ld r): c(c), r(r) {}
778 set<pair<ld, ld>> ranges;
779
780 void disable(ld l, ld r){
781     ranges.emplace(l, r);
782 }
783
784 auto getActive() const{
785     vector<pair<ld, ld>> ans;
786     ld maxi = 0;
787     for(const auto & dis : ranges){
788         ld l, r;
789         tie(l, r) = dis;
790         if(l > maxi){
791             ans.emplace_back(maxi, l);
792         }
793         maxi = max(maxi, r);
794     }
795     if(!eq(maxi, 2*pi)){
796         ans.emplace_back(maxi, 2*pi);
797     }
798     return ans;
799 }
800 };
801
802 ld areaUnionCircles(const vector<circ> & circs){
803     vector<circ> valid;
804     for(const circ & curr : circs){
805         if(eq(curr.r, 0)) continue;
806         circ nuevo = curr;
807         for(circ & prev : valid){
808             if(circleInsideCircle(prev.c, prev.r, nuevo.c, nuevo.r)){
809                 nuevo.disable(0, 2*pi);
810             }else if(circleInsideCircle(nuevo.c, nuevo.r, prev.c, prev.r)){
811                 prev.disable(0, 2*pi);
812             }else{
813                 auto cruce = intersectionCircles(prev.c, prev.r, nuevo.c, nuevo.r);
814                 if(cruce.size() == 2){
815                     ld a1 = (cruce[0] - prev.c).ang();
816                     ld a2 = (cruce[1] - prev.c).ang();

```

```

817         ld b1 = (cruce[1] - nuevo.c).ang();
818         ld b2 = (cruce[0] - nuevo.c).ang();
819         if(a1 < a2){
820             prev.disable(a1, a2);
821         }else{
822             prev.disable(a1, 2*pi);
823             prev.disable(0, a2);
824         }
825         if(b1 < b2){
826             nuevo.disable(b1, b2);
827         }else{
828             nuevo.disable(b1, 2*pi);
829             nuevo.disable(0, b2);
830         }
831     }
832 }
833 }
834 valid.push_back(nuevo);
835 }
836 ld ans = 0;
837 for(const circ & curr : valid){
838     for(const auto & range : curr.getActive()){
839         ld l, r;
840         tie(l, r) = range;
841         ans += curr.r*(curr.c.x * (sin(r) - sin(l)) - curr.c.y * (cos(r) -
842             cos(l))) + curr.r*curr.r*(r-l);
843     }
844 }
845 return ans/2;
846 };
847
848 struct plane{
849     point a, v;
850     plane(): a(), v(){}
851     plane(const point& a, const point& v): a(a), v(v){}
852
853     point intersect(const plane& p) const{
854         ld t = (p.a - a).cross(p.v) / v.cross(p.v);
855         return a + v*t;
856     }
857
858     bool outside(const point& p) const{ // test if point p is strictly
859         outside

```



```

858     return le(v.cross(p - a), 0);
859 }
860
861 bool inside(const point& p) const{ // test if point p is inside or in the
862     boundary
863     return geq(v.cross(p - a), 0);
864 }
865
866 bool operator<(const plane& p) const{ // sort by angle
867     auto lhs = make_tuple(v.half({1, 0}), ld(0), v.cross(p.a - a));
868     auto rhs = make_tuple(p.v.half({1, 0}), v.cross(p.v), ld(0));
869     return lhs < rhs;
870 }
871
872 bool operator==(const plane& p) const{ // paralell and same directions,
873     not really equal
874     return eq(v.cross(p.v), 0) && ge(v.dot(p.v), 0);
875 }
876 };
877
878 vector<point> halfPlaneIntersection(vector<plane> planes){
879     planes.push_back({{0, -inf}, {1, 0}});
880     planes.push_back({{inf, 0}, {0, 1}});
881     planes.push_back({{0, inf}, {-1, 0}});
882     planes.push_back({{-inf, 0}, {0, -1}});
883     sort(planes.begin(), planes.end());
884     planes.erase(unique(planes.begin(), planes.end()), planes.end());
885     deque<plane> ch;
886     deque<point> poly;
887     for(const plane& p : planes){
888         while(ch.size() >= 2 && p.outside(poly.back())) ch.pop_back(), poly.
889             pop_back();
890         while(ch.size() >= 2 && p.outside(poly.front())) ch.pop_front(), poly.
891             pop_front();
892         if(p.v.half({1, 0}) && poly.empty()) return {};
893         ch.push_back(p);
894         if(ch.size() >= 2) poly.push_back(ch[ch.size()-2].intersect(ch[ch.size
895             (-1)]));
896     }
897     while(ch.size() >= 3 && ch.front().outside(poly.back())) ch.pop_back(),
898         poly.pop_back();
899     while(ch.size() >= 3 && ch.back().outside(poly.front())) ch.pop_front(),
900         poly.pop_front();

```

```

894     poly.push_back(ch.back().intersect(ch.front()));
895     return vector<point>(poly.begin(), poly.end());
896 }
897
898 vector<point> halfPlaneIntersectionRandomized(vector<plane> planes){
899     point p = planes[0].a;
900     int n = planes.size();
901     random_shuffle(planes.begin(), planes.end());
902     for(int i = 0; i < n; ++i){
903         if(planes[i].inside(p)) continue;
904         ld lo = -inf, hi = inf;
905         for(int j = 0; j < i; ++j){
906             ld A = planes[j].v.cross(planes[i].v);
907             ld B = planes[j].v.cross(planes[j].a - planes[i].a);
908             if(ge(A, 0)){
909                 lo = max(lo, B/A);
910             }else if(le(A, 0)){
911                 hi = min(hi, B/A);
912             }else{
913                 if(ge(B, 0)) return {};
914             }
915             if(ge(lo, hi)) return {};
916         }
917         p = planes[i].a + planes[i].v*lo;
918     }
919     return {p};
920 }
921
922 int main(){
923     /*vector<pair<point, point>> centers = {{point(-2, 5), point(-8, -7)}, {
924         point(14, 4), point(18, 6)}, {point(9, 20), point(9, 28)},
925         {point(21, 20), point(21, 29)}, {point(8, -10), point
926             (14, -10)}, {point(24, -6), point(34, -6)},
927         {point(34, 8), point(36, 9)}, {point(50, 20), point
928             (56, 24.5)}};
929     vector<pair<ld, ld>> radii = {{7, 4}, {3, 5}, {4, 4}, {4, 5}, {3, 3}, {4,
930         6}, {5, 1}, {10, 2.5}};
931     int n = centers.size();
932     for(int i = 0; i < n; ++i){
933         cout << "\n" << centers[i].first << " " << radii[i].first << " " <<
934             centers[i].second << " " << radii[i].second << "\n";
935         auto extLines = tangents(centers[i].first, radii[i].first, centers[i].
936             second, radii[i].second, false);

```



```

931     cout << "Exterior tangents:\n";
932     for(auto par : extLines){
933         for(auto p : par){
934             cout << p << " ";
935         }
936         cout << "\n";
937     }
938     auto intLines = tangents(centers[i].first, radii[i].first, centers[i].
939                             second, radii[i].second, true);
940     cout << "Interior tangents:\n";
941     for(auto par : intLines){
942         for(auto p : par){
943             cout << p << " ";
944         }
945         cout << "\n";
946     }
947 }*/
948
949 /*int n;
950 cin >> n;
951 vector<point> P(n);
952 for(auto & p : P) cin >> p;
953 auto triangulation = delaunay(P);
954 for(auto triangle : triangulation){
955     cout << get<0>(triangle) << " " << get<1>(triangle) << " " << get<2>(
956         triangle) << "\n";
957 }*/
958
959 /*int n;
960 cin >> n;
961 vector<point> P(n);
962 for(auto & p : P) cin >> p;
963 auto ans = smallestEnclosingCircle(P);
964 cout << ans.first << " " << ans.second << "\n";*/
965
966 /*vector<point> P;
967 srand(time(0));
968 for(int i = 0; i < 1000; ++i){
969     P.emplace_back(rand() % 1000000000, rand() % 1000000000);
970 }
971 point o(rand() % 1000000000, rand() % 1000000000), v(rand() % 1000000000,
972     rand() % 1000000000);
973 polarSort(P, o, v);

```

```

971     auto ang = [&](point p){
972         ld th = atan2(p.y, p.x);
973         if(th < 0) th += acosl(-1)*2;
974         ld t = atan2(v.y, v.x);
975         if(t < 0) t += acosl(-1)*2;
976         if(th < t) th += acosl(-1)*2;
977         return th;
978     };
979     for(int i = 0; i < P.size()-1; ++i){
980         assert(1e4*(ang(P[i] - o), ang(P[i+1] - o)));
981     }
982     return 0;
983 }

```

## 3 Varios

### 3.1 Template

```

1  #include<bits/stdc++.h>
2  using namespace std;
3
4  #define forn(i,n)      for(int i=0; i<n; i++)
5  #define forr(i,a,n)    for(int i=a; i<n; i++)
6  #define fore(i,a,n)    for(int i=a; i<=n; i++)
7  #define each(a,b)      for(auto a: b)
8  #define all(v)          v.begin(),v.end()
9  #define sz(a)           (int)a.size()
10 #define debln(a)        cout << a << "\n"
11 #define deb(a)          cout << a << " "
12 #define pb              push_back
13
14 typedef long long ll;
15 typedef vector<int> vi;
16 typedef pair<int,int> ii;
17
18 void sol(){
19
20 }
21
22 int main(){
23     ios::sync_with_stdio(false);cin.tie(0);
24
25     int t=1;

```

```
26     cin>>t;
27     while(t--){
28         sol();
29     }
30
31     return 0;
32 }
```

### 3.2 String a vector<int>

```
1 //Convertir una cadena de numeros separados por " " en vector de enteros
2 //Leer varias de esas querys
3 cin.ignore();
4 while(q--){
5     string s;
6     getline(cin, s);
7     vector<int> qr;
8     stringstream ss(s);
9     int num;
10    while (ss >> num)    qr.push_back(num);
11 }
```

### 3.3 Generar permutaciones

```
1 //Generar todas las permutaciones de un arreglo
2 sort(all(a));
3 do{
4     //hacer lo que quieras con la perm generada
5 }while(next_permutation(all(a)));
```