Orbital Patterns of Martian Moons

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Outline

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- Problem Overview
- Calculations
- Conclusion
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Model Goals

- The goal of our project is to model the interactions of the moons Phobos & Deimos with the planet Mars due to Gravity
- The program we created will simulate an objects path given some initial conditions and a gravitational constant.
- The simulation is run for each moon to simulate their interactions with the planet.

Problem Overview

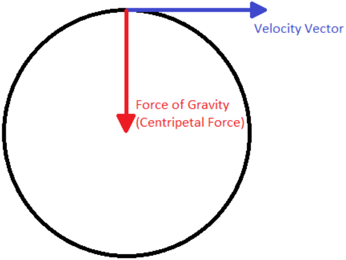


Figure: Orbital Diagram

Calculations

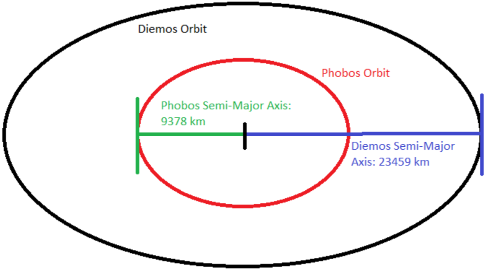
Before performing the calculations to determine an objects new position in space, we must have some initial conditions to go off of. The initial conditions that need to be set are:

- Gravitational Constants for Phobos and Deimos
  - Normalized for each moon
- Initial Position (X and Y Positions)
- Initial Velocity (X and Y Velocity Vectors)
- Time
  - Time Step
  - Initial Time
  - Orbital Period (Max Time)

Calculations - Initial Position

- We set the initial position of each moon to be at the semi-major axis of their orbit.
- The semi-major axis of each moon as defined by Nasa are:
  - Phobos: $9378km \rightarrow 9.378x10^6m$
  - Deimos: $23459km \rightarrow 23.459x10^6m$

Figure: Semi-Major Axis Illustration



Calculations - Gravitational Constants

The calculations will be simplified if we can normalize the gravitational constant for each moon.

Phobos

- $G = 6.683*10^(-11)N*m^2*kg^{-2}$
- $G = 4\pi^2 * \frac{R^3}{Year_{Phobos}*M_{System}}$
- $R = 9.378x10^6 m$
- $M_{System} = M_{Mars} + M_{Phobos}$
- $M_{Phobos} = 10.6 * 10^{15} kg$
- $Year_{Phobos} = 27553.82s$
- $Gravity_{Phobos} = 4.2887 * 10^{13}$

Deimos

- $G = 6.683*10^(-11)N*m^2*kg^{-2}$
- $G = 4\pi^2 * \frac{R^3}{Year_{Moon}*M_{System}}$
- $R = 23.459x10^6 m$
- $M_{System} = M_{Mars} + M_{Moon}$
- $M_{Diemos} = 2.4 * 10^{15} kg$
- $Year_{Diemos} = 109074.81s$
- $Gravity_{Phobos} = 4.283910^{13}$

Calculations - Initial Velocity

Now that we have normalized the gravitational constants, determining each moons initial velocity

Phobos

- $F = ma$
- $F_{Net} = \frac{M_{Phobos} * v^2}{R}$
- $F_{Gravity} = \frac{G_{Phobos}}{R^2}$
- $F_{Net} = F_{Gravity}$
- $V_{Phobos} = \sqrt{\frac{Gravity_{Phobos}}{R_{Phobos}}}$

Deimos

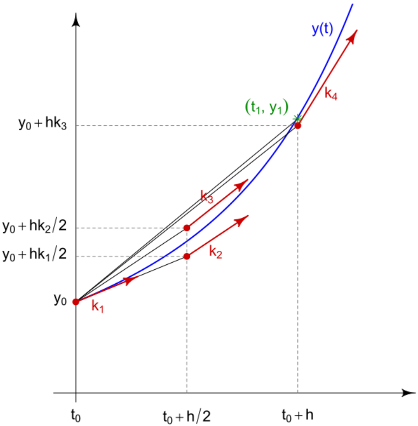
- $F = ma$
- $F_{Net} = \frac{M_{Deimos} * v^2}{R}$
- $F_{Gravity} = \frac{G_{Deimos}}{R^2}$
- $F_{Net} = F_{Gravity}$
- $V_{Deimos} = \sqrt{\frac{Gravity_{Deimos}}{R_{Deimos}}}$

Calculations - Time

- Time Step
  - A time step ($deltaTime = dt$) will be 1 second increments ($dt = 1.0s$)
- Initial Time
  - $t = 0.0s$
- Orbital Periods are constants retrieved from Nasa
  - Phobos: $tmax = 27553.824s$
  - Deimos: $tmax = 109074.816s$

Calculations - Fourth Order Runge Kutta Method

Figure: Runge Kutta



Right Hand Rule

- $\sum_{t=0}^{year} y(t) * dt$

RK4 Expressions

- $k_1 = \Delta t \vec{f}(\vec{y}^n, t)$
- $k_2 = \Delta t \vec{f}(\vec{y}^n + \frac{1}{2}k_1, t + \frac{1}{2}\Delta t)$
- $k_3 = \Delta t \vec{f}(\vec{y}^n + \frac{1}{2}k_2, t + \frac{1}{2}\Delta t)$
- $k_4 = \Delta t \vec{f}(\vec{y}^n + k_3, t + \Delta t)$

Conclusion

Calculations - Fourth Order Runge Kutta Method

RK4

- $k_1 = \Delta t \vec{f}(\vec{y}^n, t)$
- $k_2 = \Delta t \vec{f}(\vec{y}^n + \frac{1}{2}k_1, t + \frac{1}{2}\Delta t)$
- $k_3 = \Delta t \vec{f}(\vec{y}^n + \frac{1}{2}k_2, t + \frac{1}{2}\Delta t)$
- $k_4 = \Delta t \vec{f}(\vec{y}^n + k_3, t + \Delta t)$

Sources and Questions

- "Riemann Sum." Low-Pass Filter - Howling Pixel, howlingpixel.com/i-en/Riemann_sum