Jake Marillo

2. a) ipynb

b) $\overbrace{\sigma_1 = 7, \quad \sigma_2 = 1}^{\text{Singular values}}$

Left sing. vectors $(\vec{u}_1, \vec{u}_2)$: $U = \begin{array}{cc} \left[\begin{array}{c|c} -\frac{\sqrt{2}}{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{array}\right] \\ \vec{u}_1 \quad \vec{u}_2 \end{array} \in \mathbb{R}^{2\times 2}$

Right sing Vectors: $(\vec{v}_1, \vec{v}_2)$

$$V^T = \begin{array}{cc} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \hline -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} & \begin{array}{c} \vec{v}_1 \\ \vec{v}_2 \end{array} \end{array} \in \mathbb{R}^{2\times 2}$$

Note: Right Singular vectors are rows of V^T due to transposition.

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c) $\text{rank}(A) = 2$, # of nonzero σ_i Values.

d) $$A = U\Sigma V^T \rightsquigarrow A^{-1} = (U\Sigma V^T)^{-1} = \underset{\underset{V^{T^T}}{\parallel}}{(V^T)^{-1}} \; (\Sigma)^{-1} \; \underset{\underset{U^T}{\parallel}}{(U)^{-1}} = V^{T^T}\left(\frac{1}{\Sigma}\right)U^T$$

b/c orthogonal

$$\boxed{A^{-1} = V\Sigma_2 U^T \quad \text{w/ } \Sigma_2 = \begin{bmatrix} 1/\sigma_1 & 0 \\ 0 & 1/\sigma_2 \end{bmatrix} = \begin{bmatrix} 1/7 & 0 \\ 0 & 1 \end{bmatrix}}$$

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e) $\det(A - \lambda I) = \det\begin{pmatrix} 3-\lambda & 4 \\ -4 & -3-\lambda \end{pmatrix} = 0 \rightarrow (3-\lambda)(-3-\lambda) + 16 = \lambda^2 + 7 = 0$

$$\boxed{\Rightarrow \lambda_{1,2} = \pm i\sqrt{7}}$$

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f) $\det(A) = -9-(-16) = 7 \rightsquigarrow \lambda_1\lambda_2 = (i\sqrt{7})(-i\sqrt{7}) = -(-1)(7) = 7 = \det(A)$

$|\det(A)| = 7 = 7 \cdot 1 = \sigma_1 \cdot \sigma_2$ ✓