B7: Simulation of Traffic Flow

Jake Pommills
School of Physics and Astronomy, University of Southampton, Southampton, UK

We have carried out a computer simulation of a single lane traffic system with periodic boundary conditions, by treating the motion of individual cars with discreet timesteps. We find evidence of our model matching results found in real life traffic systems, and suggest how seemingly counter-intuitive actions (like reducing the speed limit) can actually improve factors important to successful road traffic systems.

I. Introduction

The motivation to build traffic simulation tools are made necessary because humans introduce a randomisation into any traffic system, which otherwise would cause the system to be completely deterministic, and a simulation would not be appropriate.

The construction of a accurate traffic simulation tools can help understanding of traffic problems, and how to alleviate them. A simulation could show that an underlying problem may lie with the road design, car design, or application or driving laws and rules imposed on the road. More likely however, is finding small improvements to slightly increase some factor like throughput, flow or accident chance.

II. Simulation

The simulation has been built from the important central concept that it should be easy to make additions to the simulation, like change the boundary conditions of the road, or what data is collected about the cars.

We impose Nagel and Schreckenberg's[1] rules for computing the next discreet timestep in the system which are written briefly as:

- **Acceleration** - Increase speed of the car by 1
- **Slowing Down** - Slow down in order to not hit the car in front
- **randomisation** - With probability p, decrease speed of the car by 1
- **Car Motion** - Move the car to the new site.

These rules were designed for a parallel computation, but by ensuring that each rule is applied to each car before moving on to the next rule, this can be alleviated and the model remains the same.

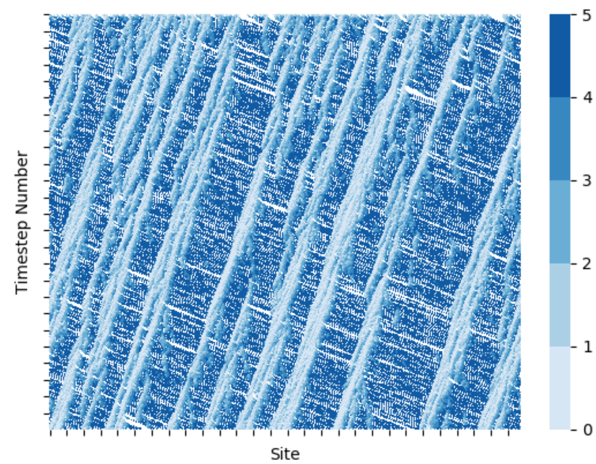

Fig. 1. A plot demonstrating backward propagating traffic jams produced with the model, where traffic is moving right. (colours defines individual car speed)
[Produced with 1000 sites, 250 cars, 500 timesteps, and $p = 0.25$]

One known property of traffic that our model should be able to demonstrate is that traffic jams (regions of stationary traffic) appear to propagate backwards, like a wave. Figure 1 shows the model also exhibits this property, which shows jams are created in two distinct ways - ones created from the initial conditions of the cars being stationary on the first timestep, and also jams caused by multiple applications of the randomisation step slowing the same car.

To ensure jams will still occur naturally and not just from random initial car placements, Figure 2 demonstrates a car having multiple timesteps without moving, causing a large jam. one car is stopped for 12 timesteps causing a large jam, and other car is stopped for 3 timesteps, causing a smaller jam.[1]

III. Investigation & Results

We turn our attention to the investigation of flow ($\boldsymbol{q}$) and density (ρ), which is the number of cars that pass a site per timestep, and the density, which is just the number

[1] with $p = 0.25$, a car stopping for 3 successive timesteps is 1 in 64 and a car stopping for 12 successive timesteps is 1 in 1.66×10^7!

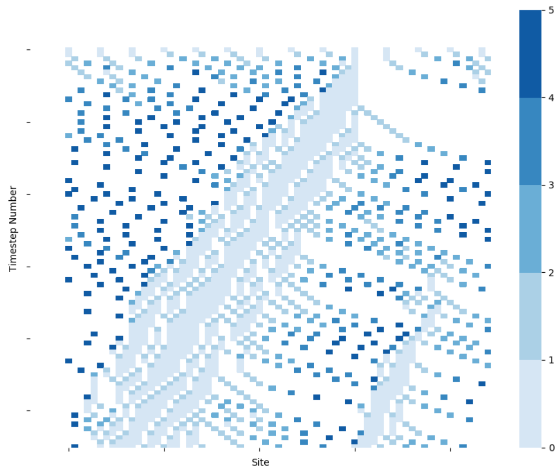

Fig. 2. A zoom in of one large jam, and one small jam. This plot has been produced where cars have been evenly distributed on the road.
[Produced with 400 sites, 80 cars, 400 timesteps, and $p = 0.25$]

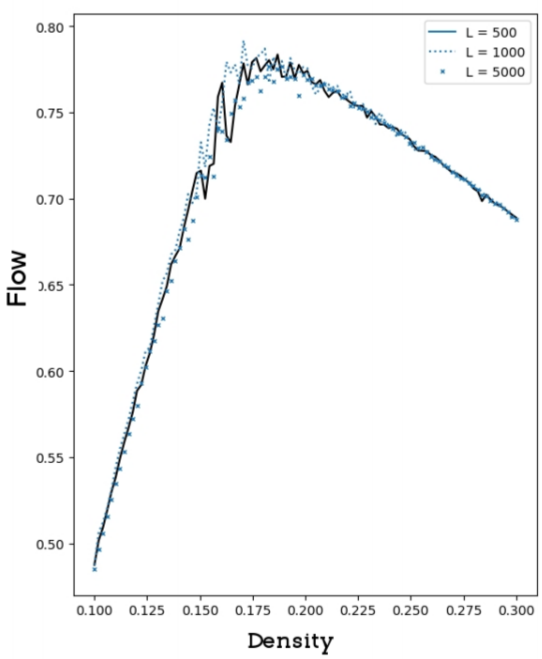

Fig. 3. A plot demonstrating maximum flow has negligible dependence road size L when randomisation factor $p = 0$
[Produced with 1000 timesteps]

of cars per site on the road. The flow is calculated in the simulation by taking the average amount every timestep has been visited, instead of just one timestep, to reduce errors.

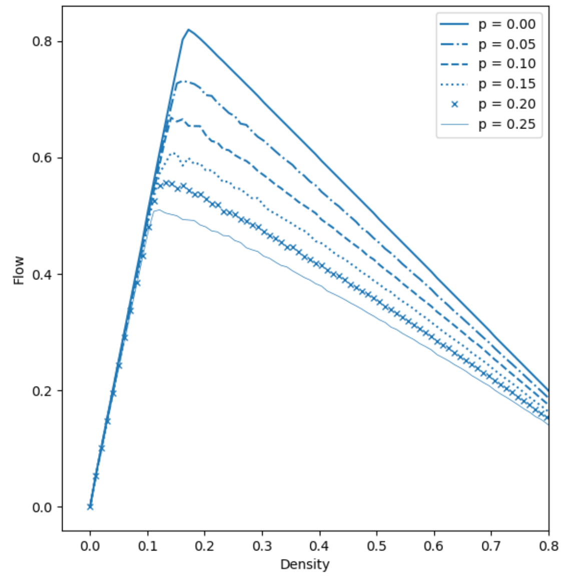

Fig. 4. A plot demonstrating the dependence of randomisation on flow.
[Each produced with 1000 timesteps, 1000 sites]

We begin by showing that with no randomisation factor p, the maximum flow is independent of the length of the road L. It can be seen graphically in Figure 3 that the largest error will be around the maximum flow, and for numerical analysis it will be necessary to perform multiple repeats around the range $0.125 < \rho < 0.225$, where our maximum clearly lies for $p = 0$. We also find that this independence of L upon the maximum flow holds for non-zero randomisation factors. We focus on p values between the range $0 < p < 0.25$ because for the simple randomisation model, any $p > 0.50$ is nonsensical and values in between are unrealistic for these systems.

Continuing this investigation of flow and density, we find (shown in Figure 4) that the maximum flow decreases with a higher randomisation value, and that the density which the maximum flow is achieved is shifted towards a density of 0. This is an expected result as more jams will occur with increased p values.

To find values for maximum flow, several repeats of the simulation in the same conditions was necessary. The result of maximum flow from Figure 5 is extracted and at several other randomisation factors have been tabulated, and at which density it is achieved at. A clear trend that reducing p increases our maximum flow, and permits this flow to happen at a higher density.

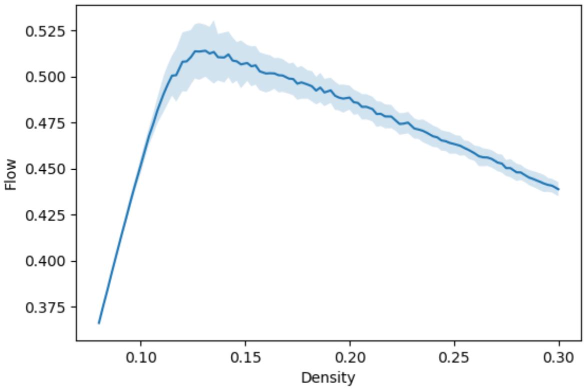

Fig. 5. A Flow-density plot with 1σ error bands. As predicted the highest errors are around the maximum.
[Produced with 1000 timesteps, 1000 sites, 100 repeats $p = 0.25$]

p	q_{max}	$\rho(q_{max})$
0	0.8155 ± 0	0.168
0.05	0.7353 ± 0.0090	0.167
0.10	0.6711 ± 0.0105	0.159
0.15	0.6119 ± 0.0132	0.142
0.20	0.5606 ± 0.0139	0.137
0.25	0.5140 ± 0.0141	0.131

The table shows that with a higher randomisation factor p, the maximum point shifts closer to 0, and the maximum itself is lowered.

We can see from the flow-density plots that the region before the maximum flow appears to be straight, and we find gradients (with forced line through origin) for two particular results as:

- $p = 0.00: \ m = 4.997 \pm 0.004$
- $p = 0.25: \ m = 4.729 \pm 0.031$

These results are the within the range of the average speeds the cars would achieve, when there is no car-car interactions. ("free flow" regime) For instance at $p = 0$ we would expect the average speed to be V_{max} which is set to 5, but for $p = 0.25$ we expect 25% of the time cars will be on speed of $V_{max} - 1$, and an easy calculation gives an average speed of 4.75. Both these results are in the range of the found gradients.

Another variable which has been held at a constant to this point is the maximum speed. From Figure 6, we can show increasing the maximum speed will increase the maximum flow, and shift the density at which this occurs to the left. In reality, $V_{max} = 5$ is closest to the speed limit in the UK using estimates from Nagel and Schreckenberg, and $V_{max} = 10$ is not a speed achievable

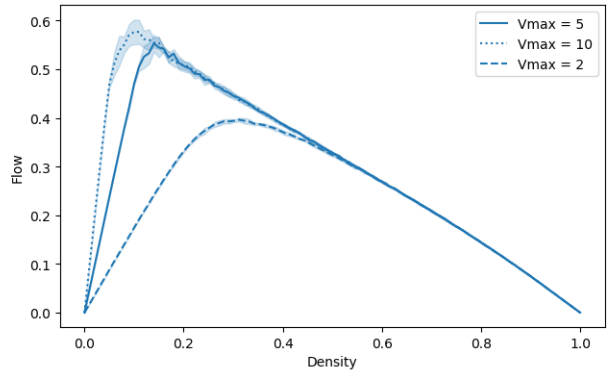

Fig. 6. A Flow-density plot showing results from different maximum speeds. We may imply diminishing returns from increasing V_{max}
[Produced with 1000 timesteps, 500 sites, 100 repeats $p = 0.25$]

for many cars, so investigating the model at these speeds wouldn't lead to any practical application. Despite this the plot still shows that within the mode, increasing the maximum speed would increase the maximum flow, but it is only making difference between the free flow regime, which can be seen from $V_{max} = 5$ and $V_{max} = 10$, where there is only a noticeable difference with a density of 17% or less. The figure may suggest that raising the speed limit for roads that have low densities will increase the flow, and make no difference on high density roads, as cars are not able to achieve the speed at all.

This plot is one of the first insights into why variable speed limits (VSL) are used on motorways. VSLs in general reduce the speed limit on different regions on a road, attempting to redistribute the cars into more favourable positions, so attempt to make jams "dissipate". It has been shown that these systems improve maximum throughput, reduced accidents, reduced fuel consumption and reduced noise pollution[2] on the M25 in the UK. A problem with the systems is driver compliance, which is difficult to get data for, and is difficult to model (our simple randomisation factor would not suffice - are more detailed modern ruleset for a model would have to be used)

In Figure 7 we find a plot of average velocity of the cars, plotted against the density. The first thing to notice is that there appears to be some critical values where cars no longer travel at a constant speed but instead a decay towards zero speed at maximum density. The two points of interest from the figure is the value of the average speed when stable, and the critical point where some decay begins. We find that the average speeds for the

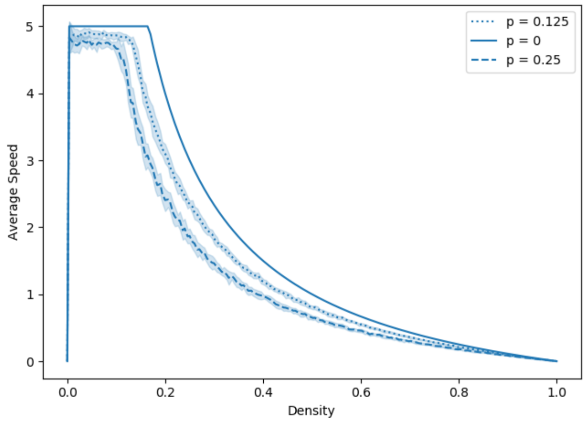

Fig. 7. An average velocity-density plot showing results from different randomisation s. There appears to be some critical value for each randomisation where ”free flow” ends.
[Produced with 1000 timesteps, 500 sites, 100 repeats $p = 0.25$]

$p = 0$ and $p = 0.25$ plots give values within the range of previously found values for gradients in the flow-density plots, and also that the critical point of this figure matches the region of changeover from the previous flow-density plots.

IV. Discussion

The Nagel-Schrekenberg model we have used is one of the simplest cellula automanon traffic models. While the model is able to show some key features of traffic, like spontaneous traffic jams emerging, the model is not commonly used in real implementations of traffic prediction systems or traffic monitoring. The relationships of variables like speed, road density, randomisation and road flow are quite clear in this model, which is helpful to make clear statements about what a change of one variable will have on the system as a whole. When considering applying the model to a real system, it is important to know what can and cannot be changed, biggest example being the randomisation factor p, which no alterations of rules or road design can change. The randomisation factor also is the most simplified variable in this model, as in reality all cars will have their own p value, and also will likely have their own temporal and spacial dependence. This model is therefore best suited for investigating changing of boundary conditions, or rules of the road.

One advantage of our model is our object-orientated approach. This allows for setting exact and complex situations easily, as each car can easily be assigned their own variables, like the randomisation factor p. This is helpful as it separates car behaviour and the rules of the road. This approach also allows easily following one car by its object reference, which is useful for visualization. Changing boundary conditions (like using a section of a long road instead of a circular one). While this the method of building the simulation is good for initial investigation, for simulations with very high values parameters (e.g. $timesteps > 10^4$, $\#Cars > 10^4$) this model will struggle. With high values in the system, we find that many parameters in the system stop behaving with linear time complexity. When two (or more) variables ceases to be able to be approximated with linear time complexity, the program will take an unfeasible amount of time. Very fast algorithms can be made using better parallelisation, Boolean operations and bitwise manipulation[3] to achieve the same thing. These programs written in a faster language than python would be able to handle much larger parameters which would not be feasible in this model. Our model tries to find some balance in the trade-off between accessibility and speed, so that it can produce useful results, and also be repurposed if necessary.

V. Conclusion

We have shown that our model based from Nagel and Schreckenberg's rules is able to demonstrate many features of real traffic, such as the generation of spontaneous traffic jams, and their backwards propagation. Our model has demonstrated that there is two distinct phases of traffic flow, free flow and congested traffic. We have found that the average car speed is equal gradient of the flow-density plots, at a low density free flow regime.

We have also shown that it makes sense within our model that maximum road speed could be lowered depending on the current density of the road, with negligible change, which is desirable for real traffic systems as this decreases accident chance and increases maximum throughput.

While the model has been able to produce consistent results that mimic real scenarios, the rules the model is based on is simple by design and without a simple randomisation factor, the results would be completely deterministic[2]. As real systems that collect data about cars are being installed and developed across the world, there will be more insight on driver behaviour, which is the least understood aspect of the field of traffic

[2]In fact, without randomisation, this is Wolfram's Rule 184

control. This will allow improvements on the models, which consequently will allow control over how to balance the many variables involved with traffic systems.

References

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