Analysis of Airline Overbooking

Summary

As an airline looks to overbook passengers, there are many factors that take place in maximizing revenue. An airline has control over several factors, including how much they overbook, how much compensation is granted to a bumped passenger, and more. Our goal is to develop models to determine how much an airline should overbook passengers by taking these factors into account, and then show the importance of maximizing revenue when an airline is attempting to be profitable.

Before looking into an optimal overbooking rate, an airline needs to look into their compensation granted to a customer and long-term impact a bumped passenger will have on the airline's future revenue. An airline will try to keep the number of involuntarily bumped passengers low because of the dissatisfaction and higher likelihood of not returning with the airline, as well as the initial short-term compensation which will be a much higher cost by following laws to protect passengers. For voluntarily bumped passengers, their compensation is negotiable and will result in lower initial costs and higher potential of staying with an airline, but taking into account hours delayed will impact the loyalty rate and consequently, the longer the passenger is delayed, the higher likelihood they are to not return and therefore costing a higher future revenue. These will be combined into a set compensation value for domestic and international flights, and concluded to be $1793.52 and $5936.91.

These are then used in testing overbooking rates to see what rate would maximize airline revenue. 4 approaches are used for domestic and international each, but then optimal overbooking rates are found when there is a set no-show rate and seat number/prices, split into general percentages for number of seats and each type of seat. For domestic flights, 2.5% overbooking generates a maximum revenue of $62,564.67 per flight and for international flights, 6.25% overbooking gets a maximum revenue of $340,641.28 per flight.

Values aren't set, however, and different scenarios call for different measures. We created a sensitivity analysis as both no-show rate and compensation change, and determined that compensation doesn't impact revenue too much, while no-show rate will allow more overbooking and therefore generate more revenue. For different types of flights then, an airline can change what their overbooking rate should be based on ideas of different scenarios in order to maximize revenue for different flight types.

Maximizing revenue is very important, because even a few hundred dollars in lost revenue or gained from cost can significantly impact profit. Using an industry average of $7 profit per passenger [21], costs are estimated to show what they can look like with each flight revenue, and then shows where different overbooking strategies will significantly impact the amount of profit per passenger, even if that difference is only hundreds of dollars.

Introduction

Flights are one of the most common ways of long-distance travel, and are an unavoidable method or transportation for many people. Whether it be for leisure and vacation, or work and business trips, it provides a safe and quick way of getting to a destination that is generally quite far away. An airline's job is to provide the best experience for their customers while still maximizing their revenue. At the end of the day, an airline is a business that needs to take risks for a greater reward.

There are many things that are out of their control. Some things, like costs with fees, labor, and fuel are unavoidable and there isn't too much room to minimize costs. However, there are ways to maximize revenue through their passengers. The biggest factor in this is the number of passengers an airline should book per flight. There are many factors that play into it, like how many seats there are or even what type of seats, how many people won't show up for their flight, and many others. This is called overbooking passengers, and is a common airline practice in order to maximize their revenue. This seems greedy, as some passengers may be denied boarding on their flight, but an airline needs to compensate for this case. They may have to pay the customer, but they also are trying to keep customers so that they keep on using their airline, because that is an essential piece of future revenue.

We are tasked to: *build a mathematical model that examines the effects that different overbooking schemes have on the revenue received by an airline company in order to find an optimal overbooking strategy, i.e., the number of people by which an airline should overbook a particular flight so that the company's revenue is maximized. Ensure that the model reflects the issues above, and consider alternatives for handling "bumped" passengers.*

We will consider many of the above factors, and our goal will be to find an optimal overbooking rate an airline should choose for their flights. An airline can also choose different strategies depending on the type of flight or passenger type, which we will briefly explore. While there are a lot of details that can be more researched, we will develop models for the overarching problem and answer our main question: how can an airline maximize revenue through overbooking passengers?

Model 1: Compensation

Description

The first part of finding optimal overbooking strategies is to first find the compensation granted towards a passenger when they are overbooked. Compensation is granted in many different ways and has many different factors to it. When understanding what compensation to a passenger means, we will develop models and methods to explain the short-term cost per passenger, but also the long-term implications based on loyalty factors and the likelihood of a passenger staying with an airline if they are overbooked. As we dive deeper, we will look into ethical approaches and legal requirements given by the US Department of Transportation[5]. From there, we will develop our short term cost of the airline, where probabilistic models are used as part of delay times for overbooked passengers, as well as the compensation granted for these delay hours. There is a difference for when passengers volunteer to be bumped in exchange for compensation, versus when they are involuntarily bumped, which will have more results as we go. After a short term, we will use similar ideas in modeling long-term impact, both the theoretical aspect and a developed model to find monetary value. The Customer Lifetime Value (CLV) equation will be used, where future customer loyalty plays an impact in future airline revenue, and the likelihood of continuing to use the airline will be modeled with a logistic curve. All these models will eventually give a compensation value for domestic and international flights, which will play an important role in modeling airline revenue for different overbooking strategies.

Ethics for Compensation

Airlines have to follow general guidelines and legalities when they overbook passengers and what they do to bumped passengers. It is perfectly legal for airlines to do this, but there are some regulations for consumer protection. The general structure for how airlines go about bumping starts is relatively set and specific for their protection as well.

Airlines will first ask if anyone will voluntarily be bumped to the next flight, and will still receive compensation. Once an airline tells them all the necessary information about being bumped, which includes any restrictions on compensation like reduced/free tickets or vouchers, then the passenger can make a decision, had they volunteered to potentially be bumped. This is negotiable, and airlines will probably try to give the least amount possible and work up from there, especially since there is technically no limit on compensation for those who voluntarily give up their seat. These things may be money or vouchers, but can even include meals or hotel rooms while waiting for the next flight, tickets for future flights, and more.

If there are not enough volunteers, the airline has the right to select passengers to give up their seats, called denied boarding. Each airline has specific policies on who is selected, which can include a passenger's check-in time, fare paid, or flyer status. They cannot choose by discrimination or prejudice.

When an airline selects the passenger, they are required to give them a reason as to why they have been bumped, following their set criteria. From there, they will decide compensation. There are cases where the involuntarily bumped passenger is not eligible for compensation, like if a different plane was needed to be used, the flight is too small, and more unusual circumstances. However, in most cases, you are eligible for compensation. If you were bumped because of any of the criteria in the previous paragraph.

Compensation is calculated on the time it will take to get to your next destination. According to the domestic and international Denied Boarding Compensation (DBC), no compensation granted if its less than a 1 hour delay, 200% of the one-way fare up to $1075 for a 1 to 2 hour delay (1-4 hour delay for international), and 400% of the one-way fare up to $2150 for a 2+ hour delay (4+ hour delay for international), as well as the next available flight if a passenger chooses so[1]. These are the minimum that the airline has to give by law if they are involuntarily bumped, and they are free to give the passenger more.

These ethical approaches, as well as additional compensation ideas, will provide a basis for generating a model. These minimum requirements aren't the only thing given as they want to keep passengers happy, while still minimizing the amount of costs for them. As we develop a model for this short term compensation, we will find averages that will provide a general basis for how much compensation should be granted for an overbooked passenger.

Model Assumptions

Assumptions and averages are key in a compensation model because there are no specific values that can be given as compensation, other than the one required by law for involuntarily bumped passengers. In the short term revenue model, we are going to include compensation as part of the losses. Compensation will be made up of 3 factors: next flight as the airline is essentially paying for the passenger to get onto the next flight, cash/check compensation, and vouchers such as meals, travel, or hotels. Both domestic and international flights consist of economy and business class seat compensation.

Assumptions are as follows, in order of each subsection:

- Waiting ranges from 1-6 hours, with 2 hours as the mode for domestic flights, and 4-24 hours, with 8 hours as the mode for international flights, so that a triangular distribution can be used (These waiting times are averages)
    - For an airline, these are going to be the main time ranges that a passenger will experience delays. Outside of a few outliers, a popular destination will be generally a shorter waiting period, while a less popular destination may take longer
    - In the case where outliers exist, when testing a large number of flights over a large number of days or simulations, both ends will have respective compensation rates that may be more or less than the given values below, but will ultimately converge to the expected results.
- Involuntarily overbooked passengers will follow a real values in terms of laws based on waiting times, as well as the cost of the next flight
- Voluntarily overbooked passengers will have set values for compensation. In a small case, it's not reasonable, but over a large number of overbooked passengers in a simulation, the amount of variability decreases and will likely converge towards a set monetary value, which is determined throughout our model.
- 3% of overbooked passengers are involuntary, 97% voluntary[2]
- 90% of overbooked passengers and compensation granted will be for economy class, and 10% will be business class. This will carry over to how many in each seat class are overbooked for a flight.
- Flight costs will be explained in the revenue section, but for simplicity, the prices for flights:
    - Domestic: $200 economy, $800 business [3]
    - International: $600 economy, $2400 international [4]

<u>Model 1a: Probabilistic Model</u>

Triangular Distribution

When deciding on the type of distribution to use for the model, we ended up choosing a triangular distribution due to its features that allow simplicity through the lack of real data. By having a minimum and maximum waiting time, as well as a mode which gives the waiting time that occurs the most often, it provides a simplistic model that can give an estimate of how long an overbooked passenger is going to wait. In this case, we split up the time into ranges, and these ranges correspond to a specific compensation value. We used basic probability methods such as expected values to eventually calculate the short term compensation loss of an airline, both for domestic flights and international flights.

To calculate the probabilities for a range of values, we will use the cumulative distribution function (CDF) of a triangular distribution as follows:

$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & \text{if } a < x \leq b \\ 1 - \frac{(c-x)^2}{(c-a)(c-b)} & \text{if } b < x < c \\ 1 & \text{if } x \geq c \end{cases}$$

Where a is the minimum value, b is the mode, and c is the maximum.

Using the CDF of a triangular distribution, we can calculate the probability of each range:

- Domestic – a=1, b=2, c=6

$$P(x \leq 2) = \frac{(2-1)^2}{(2-1)(6-1)} = \frac{1}{5} = 0.2$$

$$P(2 < x \leq 6) = P(x \leq 6) - P(x \leq 2) = 1 - 0.2 = 0.8$$

- International – a=4, b=8, c=24

$$P(x \leq 8) = \frac{(8-4)^2}{(8-4)(24-4)} = \frac{16}{80} = 0.2$$

$$P(8 < x \leq 12) = P(x \leq 12) - P(x \leq 8) = \left(1 - \frac{(24-12)^2}{(24-8)(24-4)}\right) - 0.2 = (1 - 0.45) - 0.2 = 0.35$$

$$P(12 < x \leq 24) = P(x \leq 24) - P(8 < x \leq 12) - P(x \leq 8) = 1 - 0.35 - 0.2 = 0.45$$

These probabilities will be used in modeling the amount of compensation needed to be granted, depending on the hours a passenger will be delayed.

Model 1b: Short Term Cost

Involuntarily Bumped Passengers

We will calculate wait times using a triangular distribution and then the amount of compensation provided for those wait times. However, compensation is granted differently between passengers who are voluntarily bumped versus involuntarily bumped. If a passenger is involuntarily bumped, the airline has to follow specific laws, according to the US DOTS, Department of Transportation Services. It will be shown through the structure of compensation granted for economy and business class seats, for domestic and international flights. This compensation is granted through cash or checks, and not up for negotiation. This means that no vouchers or hotels will be granted, and a passenger is forced to take this compensation whether they like it or not. Fortunately, only a very small percentage of overbooked passengers are involuntary, at 3%. We have found data from the US Department of Transportation Services that contains certain data. It has found that .1% of all airline passengers are bumped, and only .003% are involuntarily bumped. That means 3% (.00003 / .001 = .03 or 3%) of bumped passengers are involuntarily bumped, while the other 97% are voluntarily bumped[2]. We will later see that airlines would want and prefer passengers to volunteer to be bumped as this typically will save an airline money due to less direct compensation costs to the passenger, as well as more positive long-term implications compared to being forced off when bumped. It will be explained that involuntarily bumping hurts the airline due to passenger complaints/loyalty issues, and have greater long-term implications than passengers who volunteer to get bumped. For now, we will focus on the short term impact, starting with involuntarily bumped passengers. All of these percentage fares below are according to US DOTS [2].

Domestic Economy (min = 1 hr, mode = 2 hr, max = 6 hr)

- *0 – 1 hour – no compensation*
- *1 – 2 hour – 200% fare, next flight*
    - *$400 + $200 flight = $600*
- *2 – 6 hour – 400% fare, next flight*
    - *$800 + $200 flights = $1000*

Domestic Business (min = 1 hr, mode = 2 hr, max = 6 hr)

- *0 – 1 hour – no compensation*
- *1 – 2 hour – 200% fare, next flight*
    - *$1075 limit + $800 flight = $1875*
- *2 – 6 hour – 400% fare, next flight*
    - *$2150 limit + $800 flight =$2950*

International Economy (min = 4 hr, mode = 8 hr, max = 24 hr)

- *4 – 8 hour – 400% fare, next flight*
    - *$2150 limit + $600 flight = $2750*
- *8 – 12 hours – 400% fare, next flight*
    - *$2150 limit + $600 flight = $2750*
- *12 – 24 hours - 400% fare, next flight*
    - *$2150 limit + $600 flight = $2750*

International Business (min = 4 hr, mode = 8 hr, max = 24 hr)

- *4 – 8 hour – 400% fare, next flight*
    - *$2150 limit + $2400 flight = $4550*
- *8 – 12 hours – 400% fare, next flight*
    - *$2150 limit + $2400 flight = $4550*
- *12 – 24 hours - 400% fare, next flight*
    - *$2150 limit + $2400 flight = $455*

Using the probabilities from the triangular distribution, as well as the compensation amount for each range of waiting time, we can calculate the expected value of the compensation for domestic and international flights. Expected value is calculated as the sum of each value times its probability.

E(inv. domestic economy) = 600.2 + 1000*.8 = $920*
E(inv. domestic business) = 1875.2 + 2950*.8 = $2735*
E(inv. International economy) = 2750.2 + 2750*.35 + 2750*.45 = $2750*

E(inv. international business) = 4550.2 + 4550*.35 + 4550*.45 = $4550*

According to our model assumptions, 10% of seats being overbooked will be business class, and 90% will be economy seats. So, when finding the expected value of compensation for domestic and international total, we will multiply each expected value by its seat status, and get the sum of each type of flight.

E(inv. domestic compensation) = 920.9 + 2735*.1 = $1101.5*
E(inv. international compensation) = 2750.9 + 4550*.1 = $2930*

This solely gives the expected amount of compensation granted to an involuntarily bumped passenger. Next, we will find how the compensation differs for voluntarily bumped passengers.

Voluntarily Bumped

When an airline overbooks passengers, they want passengers to be voluntarily bumped because it saves them money due to decreased compensation costs, and also that passengers have the option to receive negotiated compensation and a choice to be overbooked rather than forced, which helps long-term customer satisfaction.

Typically, an airline will ask for a passenger, like an auction. They will start off with low compensation, and work their way up until a passenger bites and chooses that given compensation. This can include vouchers such as meal/travel, hotel rooms, etc. These values will typically end up cheaper than being involuntarily bumped, but more than the value of the flight that the passenger will receive, plus the airline's cost of putting them on the next flight. All of the meal vouchers are estimated according to US DOT [6].

Domestic Economy (min = 1 hr, mode = 2 hr, max = 6 hr)

- *1 – 2 hour – next flight, $20 meal voucher, $300 travel voucher*
  - *$200 flight + $320 vouchers = $520*
- *2 – 6 hour – next flight, $20 meal voucher, $500 travel voucher*
  - *$200 flight + $520 vouchers = $720*

Domestic Business (min = 1 hr, mode = 2 hr, max = 6 hr)

- *1 – 2 hour – next flight, $20 meal voucher, $600 travel voucher*
  - *$800 flight + $620 vouchers = $1420*
- *2 – 6 hour – next flight, $20 meal voucher, $1200 travel voucher*
  - *$800 flight + $1220 vouchers = $2020*

International Economy (min = 4 hr, mode = 8 hr, max = 24 hr)

- *4 – 8 hour – next flight, $20 meal voucher, $900 travel voucher*
  - *$600 flight + $920 vouchers = $1520*
- *8 – 12 hours – next flight, $50 meal voucher, $1200 travel voucher, $200 hotel 50% of time*
  - *$600 flight + $1350 vouchers/hotel = $1950*

- *12 – 24 hours - next flight, $100 meal voucher, $1500 travel voucher, $200 hotel*
    - *$600 flight + $1800 vouchers/hotel = $2400*

International Business (min = 4 hr, mode = 8 hr, max = 24 hr)

- *4 – 8 hour – next flight, $20 meal voucher, $1800 travel voucher*
    - *$2400 flight + $1820 vouchers = $4220*
- *8 – 12 hours – next flight, $50 meal voucher, $2000 travel voucher, $200 hotel 50%*
    - *$2400 flight + $2150 vouchers/hotel = $4550*
- *12 – 24 hours - next flight, $100 meal voucher, $2400 travel voucher, $200 hotel*
    - *$2400 flight + $2700 vouchers/hotel = $5100*

Using the probabilities from the triangular distribution, as well as the compensation amount for each range of waiting time, we can calculate the expected value of the compensation for domestic and international , separated by volunteering status and seat status. Expected value is calculated as the sum of each value times its probability.

E(v. domestic economy) = 520.2 + 720*.8 = $680*
E(v. domestic business) = 1420.2 + 2020*.8 = $1900*
E(v. International economy) = 1520.2 + 1950*.35 + 2400*.45 = $2066.5*
E(v. international business) = 4220.2 + 4550*.35 + 5100*.45 = $4731.5*

According to our model assumptions, 10% of seats being overbooked will be business class, and 90% will be economy seats. So, when finding the expected value of compensation for domestic and international flights, we will multiply each expected value by its seat status, and get the sum of each type of flight.

E(v. domestic compensation) = 680.9 + 1900*.1 = $802*
E(v. international compensation) = 2066.5.9 + 4731.5*.1 = $2333*

Now that we have the compensation for voluntary/involuntary overbooked passengers, we can calculate the overall domestic and international compensation for overbooked passengers in the short term. Using that voluntarily overbooked passengers make up 97% of the total, we can calculate the expected compensation as follows:

E(domestic short term compensation) = 1101.5.03 + 802*.97 = $810.99*
E(international short term compensation) = 2930.03 + 2333*.97 = $2350.91*

These values, in summary, are composed of the short-term impact on revenue from airline losses, including the value of the next flight that the passenger is essentially getting on for free by being bumped and the compensation granted to the overbooked passenger by law or through negotiation, depending on whether the passenger involuntarily was bumped or voluntarily gave up their seat in exchange for negotiated compensation. All these values and estimated waiting times are general averages, where in the long term after 100s of days, or in our case, simulations, will roughly converge to these values.

Model 1a: Long-term Implications

Qualitative Analysis

Short term impacts are given in monetary values, so developing models can easily be done to find the compensation for a customer. However, there is no set way to give a value for how much an airline will lose in the long term for overbooking a passenger. This aspect plays a huge role and may arguably be more important, because reputation and reviews mean a lot. Let's first dive into the theoretical aspect of what the long-term impact can mean, and then a model will be developed to determine what the long term "cost" could be.

The long term impact can be both positive and negative when overbooking. Of course, when done correctly, the long term revenue will be greater, and the goal is to find what "correctly" means in the end, or how much an airline should overbook. For now, the focus will be on the negative aspects of when a passenger is overbooked, and what it can mean for the future of the airline.

The first idea is simple: when too many people are overbooked, the short-term compensation costs will outweigh the ticket revenue, and will result in a decrease of profit (Also seen in figure 2.3 and 2.4). Now, the amount of overbooked passengers would have to be relatively high for this to happen, if there were no long term implications.

The second has to do with internal issues. While this may not directly impact any costs, if many passengers are constantly needing to be overbooked and the system is therefore unreliable, it may be a constant hassle to deal with all that comes with it, like granting a passenger compensation, or providing accommodations. Also, this extra strain may impact employees, who may see a morale decrease and added stress if there are too many overbooked passengers.

The final reasons have to do with passenger impacts, and these will directly impact revenue. Customer loyalty can be one of the greatest factors for airline revenue, so if a passenger is overbooked, there is a chance that they may be dissatisfied with the airline and no longer decide to fly with them. An airline should want a high customer retention and loyalty rate to beat out the competition and maximize revenue from passengers. There is a twist with overbooking, where an airline, if they provide very generous compensation, the people who volunteer to be overbooked may not be as mad with the airline and less likely to switch airline, but that also comes with an airline providing more in short term compensation to make up for the potential long term loss. Additionally, airlines need to handle situations clearly with the overbooked passenger, because passengers can always have an impact on others, like word of mouth or leaving bad reviews, which can impact the amount of people who fly with the airline in the future.

In any case though, there can be so many factors that can impact long term revenue of an airline. A model will be able to give an estimation of the long term impact based on individual loyalty of a passenger being bumped.

Quantitative Analysis

While long-term impact has a lot of theoretical scenarios because human behavior is not exactly measurable from a specific event happening, meaning that if they are overbooked, the amount of delay isn't going to correspond to a specific value. However, we can generate a model to potentially describe a long term impact and assign a monetary long-term compensation value per overbooked passenger, which will show the dollar amount of a passenger being overbooked based on the fact that they won't return and generate revenue for the airline in the future.

To assign a value, we used an equation denoted as the Customer Lifetime Value (CLV), which gives the amount of revenue they will generate for the airline for a number of years, based on their loyalty with the company. Again, measuring the loyalty for an individual passenger isn't specific, but we can give a general estimate based on their flying behavior. Seat prices will be given as an average of business and economy seats, where the flight will consist of 10% business class and 85% economy, and also separated between domestic and international flights[7]. We can then separate people who fly into different classes. Of the 50% of people who fly in the US [8], 75% are occasional flyers, meaning they will fly roughly 2 times per year. Most of these flyers will probably look more often than not for the cheaper flight rather than familiarity, so they will be assigned 2 loyalty years. Another 20% are moderate flyers, which average around 4 flights per year[9]. These flyers may stay with a more familiar airline due to more consistent travel, but may be more volatile due to airline policy or price change. They are assigned 5 loyalty years. The last 5% of flyers are considered frequent flyers, averaging 6 flights per year[9]. These passengers often get loyalty rewards and frequent flyer points, and will stay with the same airline for long periods of time. They are assigned 10 loyalty years. We can calculate each CLV based on the status of type of flight and flyer status, and then use the percent of flyers in each group to calculate the CLV of a passenger for a domestic and international flight.

We have to find constants for business class and economy class using the Customer Lifetime Value.

*CLV = (Avg. Flights per year) * (Seat price) * (Retention Years)*

- *Seat price is average between business and economy (business is 10%, economy 85%)*
    - *Domestic: 800*.10 + 200*.85 = $250*
    - *International: 2400*.10 + 600*.85 = $750*
- *Occasional – 2 flights per year, 2 loyalty years (75% of flyers)*
    - *CLV(Dom,Occ) = 2*250*2 = $1000*
    - *CLV(Int,Occ) = 2*750*2 = $2500*
- *Moderate – 4 flights per year, 5 loyalty years (20% of flyers)*
    - *CLV(Dom,Mod) = 4*250*5 = $5000*
    - *CLV(Int,Mod) = 4*750*5 = $15000*
- *Frequent – 6 flights per year, 10 loyalty years (5% of flyers)*
    - *CLV(Dom,Freq) = 6*250*10 = $15000*
    - *CLV(Int,Freq) = 6*750*10 = $45000*

To convert to just domestic and economy CLV, we will multiply each group's percentage of flyers' rate times its CLV.

CLV(Domestic) = 1000.75 + 5000*.2 + 15000*.05 = $2500*

CLV(International) = 2500.75 + 15000*.2 + 45000*.05 = $7125*

These are the customer loyalty values, and whatever percentage of the loss depending on the logistical curve's hours delayed times the CLV will be the amount of loss per overbooked passenger. We will use separate models depending on whether the customer is involuntarily bumped or voluntarily, and separating domestic and international.

We can now develop a model to calculate the long-term impact of an overbooked passenger. Utilizing the same approach with a triangular distribution for the type of flight, as well as consistent data that 3% of the overbooked passengers are involuntarily bumped, we can equate the number of hours that a passenger is delayed to the likelihood that they are to be a loyalty loss, meaning that they stop flying with the airline. When this happens, the airline will "lose" the future revenue from the customer, based on the average CLV value calculated. A logistic curve will be used, with the max value being at one that the passenger will 100% result in a loyalty loss. This type of curve is used because certain time values, or ranges, like if a customer is delayed 16 hours vs 24 hours, may not make too much of a difference in the chance they will leave, but a smaller 8 hour difference like 4 hours vs 12 hours will have a lot higher of a likelihood of leaving. Also, the steepness of a curve can be changed to apply to the type of flight, where domestic will need a steeper curve due to the delay range being shorter. Finally, the inflection point can be modified so that the starting percentage of customers leaving, like if they are involuntarily bumped, will start a lot higher. This will be shown further in the graphical representation.

The code (explained in Appendix A.1) utilized for each type of flight and whether they were voluntarily or involuntarily bumped will use the information below for the logistic equation.

$$\frac{L}{1 + e^{-k(h-h_0)}}$$

Where L is the maximum loyalty loss, k is the steepness, h is the hours delayed, and h is the inflection point .

Domestic – Involuntary

- *L = 1, k = .5, h0 = 0 – the inflection point is at 0 so that the graph will start at a 50% loyalty loss. It makes sense because if a customer is involuntarily bumped, the dissatisfaction rate will be a lot higher*
- *Expected Loss = E(Dom,Inv) = $2011*

Domestic – Voluntary

- *L = 1, k = .5, h0 = 4 – if they voluntarily chose to get bumped, the likelihood that they leave is significantly less. Since the max is at 6, the point at which is at 6 hours won't be super close to the max of 100% chance of leaving*
- *E(Dom,Vol) = $960*

International– Involuntary

- *L = 1, k = .3, h0 = 0 – because of the longer time frame, the steepness will be less. It still shows a greater increase initially, but as time goes on and the chance of a loss is greater, the number of hours doesn't impact as much*
- *E(Int,Inv) = $6787*

International– Voluntary

- *L = 1, k = .3, h0 = 12 – same reason as domestic voluntary*
- *E(Int,Vol) = $3487*

The logistic curve for each of the 4 models can then be graphed:

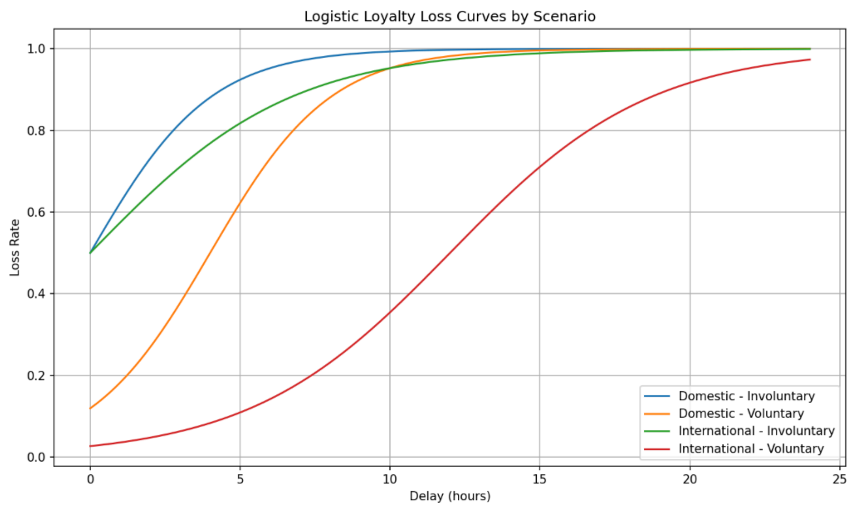


Figure 1.1

The values for the logistic curve had a reason as to why they are chosen as such, and the expected loss for each of the 4 sections gives the amount of future revenue loss from the airline when a customer is overbooked. Each curve in figure 1.1 shows each of these sectors, and as delay time increases, each curve shows how much the likelihood that an overbooked passenger will leave, resulting in a loyalty loss. The simulation is run 100,000 times to accurately simulate the expected loss for each sector.

Now we can use these expected losses and put it into domestic and international long-term loss per overbooked passenger. Since the proportion of involuntarily bumped passengers is 3%, that will be used in calculating expected value.

E(Domestic Long Term Comp) = 2011.03 + 960*.97 = $991.53*
E(International Long Term Comp) = 6787.03 + 3487*.97 = $3586*

As a reminder, short term compensation costs per overbooked passenger are as follows:

E(domestic short term comp) = $810.99
E(international short term comp) = $2350.91

Now, we can combine both short term compensation costs and long term revenue impact into a singular monetary value for each type of flight, denoted as the total compensation. When we create our revenue model in the next section, this will play a factor in the amount of seats an airline should overbook.

E(domestic compensation) = $1802.52
E(international compensation) = $5936.91

<u>Comprehensive Compensation Model</u>

We can put together a comprehensive model that takes into account short-term compensation, long-term loyalty loss, voluntary vs involuntary distinctions, and domestic vs international flights.

The variables for this model can be defined as:

p_{vol} - % of passengers voluntarily bumped
p_{inv} - % of passengers involuntarily bumped
C_s^{vol} , C_s^{inv} - short-term cost per overbooked passenger
C_L^{vol} , C_L^{inv} - long-term loyalty loss per overbooked passenger
C_{total} - total compensation per overbooked passenger

The total compensation per overbooked passenger model can be written as:

$$C_{total} = p_{inv} \cdot (C_s^{inv} + C_L^{inv}) + p_{vol} \cdot (C_s^{vol} + C_L^{vol})$$

Through the previous analysis, we have obtained the expected values for the % of passengers voluntarily and involuntarily bumped, the short-term cost per overbooked passenger, and the long-term loyalty loss per overbooked passenger. If we plug these values for both domestic and international flights into our compensation model we obtain:

Domestic: $C_{total} = 1802.52$
International $C_{total} = 5936.91$

As we can see these values calculated through the model are the same as our previously calculated compensation amounts of both domestic and international flights. This model may be too simplistic, but hopefully it can be implemented to give an airline company an idea of total compensation amounts needed for a certain flight.

<u>Model 2: Revenue</u>

Our main goal and objective function is the airline to maximize revenue using overbooking strategies. After finding the compensation for an overbooked passenger, it will ultimately be subtracted from the revenue generated from ticket prices.

Assumptions

Four main approaches will be used to gather initial data – a conservative approach where there is no overbooking, and a moderate, aggressive, and extra aggressive approach. Because there are two types of flights, domestic and international, there is also varying no-show rates between the two that will make the strategies have different percentages between the type of flight. An optimal approach will also be found

Our first assumption is that the no-show rate for domestic flights is 4%, and for international flights, 7.5%. This is determined based on the general range that domestic flights have a 3-5% no show rate, and international has a 5-10% no show rate[10]. While these ranges would have a uniform distribution, in a large number of simulations, the average no show rate would converge to the mean. The assumption is also that the no show rate between economy class and business class is constant, so there is no higher chance of a no show rate for one class or the other. Also, it is assumed that the no show rate is constant each day, and isn't impacted by time of day, season, etc. This assumption may seem unreasonable, but in modeling revenue over a large number of days or simulations, the average no show rate would still converge to these values, and modeling each separately would be unnecessary as the results would not change significantly, if any.

Based on these no show rates, we assigned the percentage of overbooking for each model as follows:

Domestic – Conservative = 0%; Moderate = 3%; Aggressive = 5%; Extra aggressive = 10%
International - Conservative = 0%; Moderate = 5%; Aggressive = 10%; Extra aggressive = 15%

This is done to take into account no show rates, so that the conclusions for each approach depending on the type of flight won't vary significantly, i.e. the same approach is the best model.

We are also assuming a constant number of seats on a flight at 200 for domestic and 400 for international. This isn't entirely accurate, but the number of seats won't matter in finding the best approach – it will only show a difference in revenue if compared to real data. Since our goal is to find the best approach, and we are using percentages for other values, this assumption is reasonable.

One of the percentages we are using is the amount of type of seats being overbooked. In a general flight, 85% of seats are economy, 10% are business class, and 5% are first class seats for domestic flights.[7] For international flights, 90% are economy, 7.5% are business, and 2.5% are first class[11]. Since first class seats are never/very rarely overbooked, we assumed 90% of the seats overbooked are economy seats, and 10% are business class[12]. For each approach, below shows the amount of seats sold per class:

Conservative (0%)

- *Domestic - # Seats Sold: 200 - Economy: 170, Business: 20, First Class: 10*
- *International - # Seats Sold: 400 - Economy: 360, Business: 30, First Class: 10*

Moderate (3% domestic, 5% international)

- *Domestic - # Seats Sold: 206 - Economy: 176, Business: 20, First Class: 10*
- *International - # Seats Sold: 420 - Economy: 378, Business: 32, First Class: 10*

Aggressive (5% domestic, 10% international)

- *Domestic - # Seats Sold: 210 - Economy: 179, Business: 21, First Class: 10*
- *International - # Seats Sold: 440 - Economy: 396, Business: 34, First Class: 10*

Extra Aggressive (10% domestic, 15% international)

- *Domestic - # Seats Sold: 220 - Economy: 188, Business: 22, First Class: 10*
- *International - # Seats Sold: 460 - Economy: 414, Business: 36, First Class: 10*

To put a price on the seats, we are going to assume round trip tickets are $400 for domestic flights [3] and $1200 for international[4], which equates to one way values of $200 and $600. Also, we are assuming that business class seats are 4 times more expensive than economy, and first class is 6 times more expensive. These ratios reflect general pricing trends observed in commercial airline pricing. To calculate ticket revenue, it is just the sum ticket price per seat times the number of its respective seat.

Model 2a: Monte Carlo Simulation

Description

Our model will use Monte Carlo simulation to calculate each flight's revenue, and simulate that there will be 1000 flights per day for domestic and 100 for international. The two different types of flights will have their own respective models. In addition to having each approach, we will include optimal overbooking rate and show all overbooking percentages to its respective revenue.

This model uses a Monte Carlo simulation in Python to evaluate four different airline overbooking strategies in terms of net revenue, and to determine the optimal overbooking percentage for both domestic and international flights.

We simulate multiple scenarios using defined flight parameters and financial assumptions, with the goal of helping airlines maximize revenue while accounting for no-show passengers and costly compensation fees for overbooked travelers.

For each trial, out of 1000, we simulate 1,000 domestic flights and 100 international flights. We calculate the net revenue by adding all of the ticket prices together(including the overbooked passengers) and then subtract compensation costs for any passengers bumped due to overbooking. Then, we compute and return the revenue per flight, and the average number of overbooked passengers per day. We then simulate each of our four overbooking strategies for both domestic and international flights.

As this code is run, the simulations will provide the probability that over the course of 1000 flights (100 for international), how many will fall into each range(Appendix B.1). The ranges are calculated around the industry average of .1% customers overbooked, so each approach will find the likelihood of calculating below .05% passengers overbooked per day, in between .05% to .15%, and above .15%. Additionally, it will calculate the actual overbooking rate using that approach and by doing the reciprocal of that percentage, 1 in every how many passengers will be overbooked. (OB = overbooked)

Domestic: 200,000 pass.	Conservative (0%)	Moderate (3%)	Aggressive (5%)	Extra Aggressive (10%)
Prob. of 100 - 300 OB	0.00%	73.50%	0.00%	0.00%
Prob. of > 300 OB	0.00%	26.50%	100.00%	100.00%
Prob. of < 100 OB	100.00%	0.00%	0.00%	0.00%
Approx. OB rate per pass.	0.00%	0.142%	1.013%	5.092%
OB likelihood	None	1 in 702	1 in 99	1 in 20

Figure 2.1

International: 40,000 pass.	Conservative (0%)	Moderate (5%)	Aggressive (10%)	Extra Aggressive (15%)
Prob. of 20 - 60 OB	0.00%	0.00%	0.00%	0.00%
Prob. of > 60 OB	0.00%	0.00%	100.00%	100.00%
Prob. of < 20 OB	100.00%	100.00%	0.00%	0.00%
Approx. OB rate per pass.	0.00%	0.005%	1.663%	5.542%
OB likelihood	None	1 in 21363	1 in 60	1 in 18

Figure 2.2

For both domestic and international, as more people are overbooked using the set no-show rate and compensation per overbooked passenger, the estimated overbooking rate per passenger increases significantly, some not even close to the .1%. We can also see how many people are overbooked and the likelihood of each range occurring. For the moderate approach in domestic (Figure 2.1), we can see that around ¾ of the days will fall into the acceptable range of passengers overbooked, and also see that the estimated overbooking rate per passenger is relatively close to the .1% at .142%. Seeing this, we can assume a moderate approach will be more likely than not the correct rate. However, these results don't exactly provide us with specific enough values. Will one of the 4 approaches actually be the optimal overbooking rate that gives the highest revenue?

To further improve our understanding, we simulated a wide range of overbooked passenger counts. To do this, we used the simulation code (explained thoroughly in Appendix B.2) we already made, stored the revenue for each flight, and graphed it to make a line graph.

Domestic:

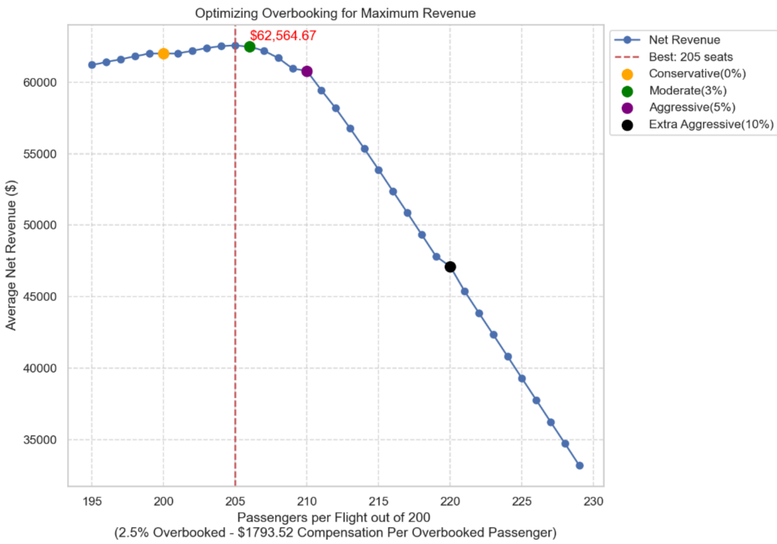


Figure 2.3

International:

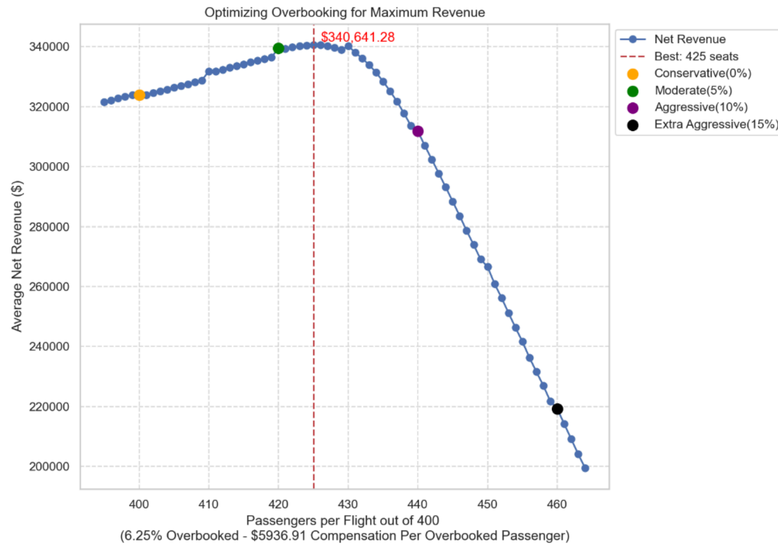


Figure 2.4

For domestic flights, the highest revenue of $62,564.67 was achieved by booking 205 passengers per flight, which is 2.5% overbooked. For international flights (figure 2.4), the optimal number was 425 passengers, or 6.25% overbooked, resulting in a maximum revenue of $340,641.28. Each approach is labeled on the graphs as well to visualize how they are in comparison to the optimized. In both flight cases, the moderate approach is very similar in revenue. Look at the bar graphs in the appendix section B.2, figures B.2.1 and B.2.2.

Now that we found the optimal overbooking rate, we can show again the likelihood of the amount of overbooked passengers falling into the acceptable range, as well as the estimated rate per passenger.

Domestic: 200,000 pass.	Optimal (2.5%)	International: 40,000 pass.	Optimal (6.25%)
Prob. of 100 - 300 OB	97.40%	Prob. of 20 - 60 OB	47.90%
Prob. of > 300 OB	0.00%	Prob. of > 60 OB	0.10%
Prob. of < 100 OB	2.60%	Prob. of < 20 OB	52.00%
Approx. OB rate per pass.	0.064%	Approx. OB rate per pass.	0.053%
OB likelihood	1 in 1558	OB likelihood	1 in 1894

Figure 2.5

Both of these overbooking rates, since they provide the highest revenue, should be a reasonable overbooking rate per passenger, and it is. The values are slightly lower than the .1%, but this may happen due to the constraints of compensation and no-show rate impacting overbooking. Later, using a sensitivity analysis, we can see what happens if those values slightly change, but since these are the averages, a normal flight will be recommended to overbooking 2.5% for domestic and 6.25% for international to maximize revenue.

Interestingly, both of these optimal overbooking percentages fall slightly below their respective no show rates of 4% and 7.5%.This may seem counterintuitive at first, since you would expect the overbooking level to match or exceed the no show rate to fully fill each flight and generate the most amount of revenue. We will explain why this is not always true, using the international model, but the same thing occurs for domestic. It is not guaranteed that exactly 400 for international flights will show up to the flight, even if the airline books 432 passengers, or 7.5% overbooked. This is because we use a binomial distribution, meaning sometimes 397 passengers will show up, or 405, or even 410+. But, when the flight has more than 400 passengers, the airline has to pay a hefty compensation fee, heavily affecting the revenue. Therefore, slightly under booking compared to the no show rate, or overbooking only 6.25%, will generate more revenue daily. We can see that for overbooking past 6.25% or 425 passengers, the net revenue starts to decrease significantly. This is because after the maximum revenue and the optimal overbooking level, the number of overbooked passengers gets high, and the compensation per passenger outweighs the revenue that the airline is bringing in. As a result, the most profitable approach is to overbook cautiously at 2.5% overbooked for domestic flights and 6.25% for international flights.

Should these be the overbooking rates set in stone for every flight? We can see how optimal overbooking rates may change as no-show rates and compensation changes using a sensitivity analysis, and then provide examples and scenarios of when it could be used. Maybe there are different overbooking rates depending on the scenario, and that will be the best way to maximize revenue.

Model 2b: Sensitivity Analysis, Flight Scenarios

We have results for these general approaches and with the assumptions for values of no-show and compensation, but they aren't 100% accurate. While compensation was modeled thoroughly, it was modeled within our constraints through data found, but isn't entirely accurate and may deviate a little. No-show rates are the same. For this reason, we can model a sensitivity analysis to show what will happen to optimal overbooking rates and revenue if these values change. This is important due to events that may change these values, or in the future if additional data comes out, a new model doesn't need to be created – it can just be found through this model.

The graphs below show the expected amount of seats overbooked and revenue using a contour graph:

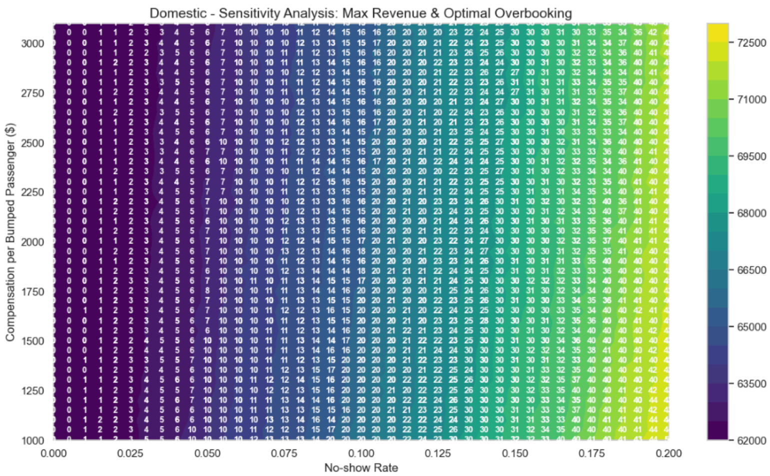


Figure 2.6

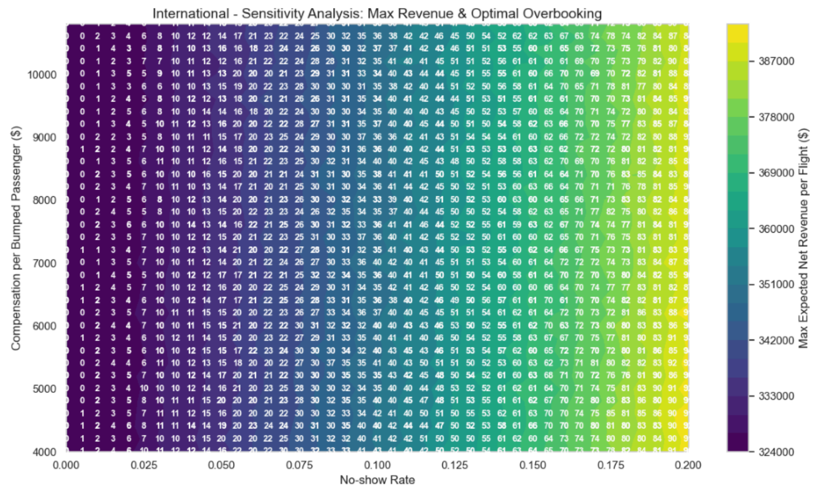
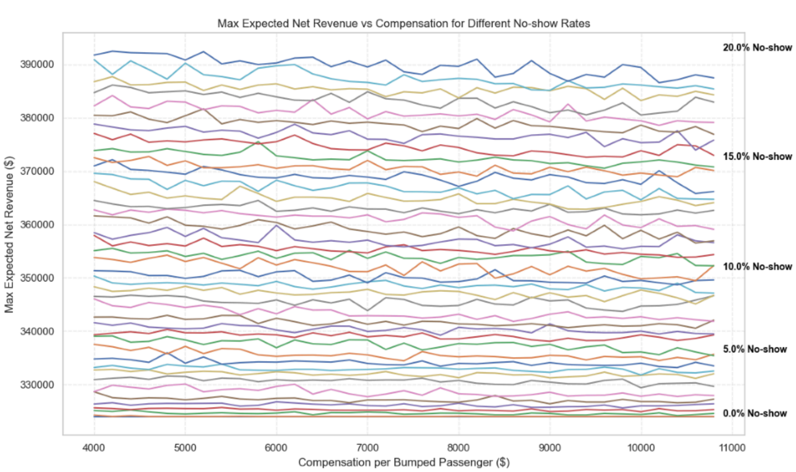


Figure 2.7

The graphs as sensitivity models can be analyzed to determine what variables change the optimal overbooking and by how much. The no-show rates change the optimal overbooking by a lot, and it nearly follows that for every 1% increase in no-show rate, the optimal overbooking increases by about 1% (2 seats for domestic and 4 seats for international). However, the optimal overbooking is not very sensitive to the compensation. The increase in compensation only decreases the number of seats overbooked by a small amount when you stay with the same no-show rate. This is true for both domestic and international flights. We can verify this statement through the next 2 graphs (Figure 2.8 and 2.9), which use international flight values(the domestic graphs can be found in Appendix B.3). In figure 2.8 below, each line is a different compensation ranging from $4000 to $11000, and how it will impact the net revenue as no-show rate increases. Figure 2.9 below shows the impact on revenue as the compensation increases, but through no-show rates range from 0% to 20%.



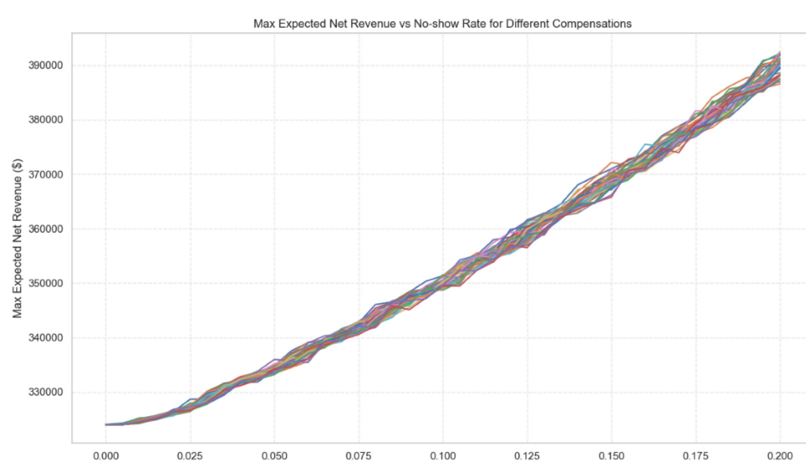


Figure 2.8 *Figure 2.9*

The conclusion from before is backed up by these two models. In the first, each separate compensation value is modeled as a line as it goes through the no-show rate, but there is never a significant difference in the amount of revenue generated. There is a minimum to this, because if the compensation is significantly lower than the seat price, an airline will be able to book more and more because revenue will outweigh the compensation cost, essentially allowing it to overbook infinitely. This is completely unreasonable, which is why compensation is capped at a minimum of $4000. The second graph is the differences in no-show rate, where the highest revenue lines are where the no-show is highest, and you can see the difference as the no-show decreases that the revenue will also decrease significantly.

Going back to the contour graphs (figures 2.6 and 2.7), we can analyze how to extract values. The graphs optimize the overbooked number of seats using the compensation values and the no-show rate. Using a large number of simulations, for every specific case of compensation and no-show rate, by a step size of $50 compensation and .5% no-show rate, it gives the optimum number of seats overbooked and the corresponding revenue generated. After analyzing these graphs, it can be seen that with a lower no-show rate, the overbooking rate will be less so that the compensation granted when a customer is overbooked doesn't outweigh the ticket revenue. Also, for each no-show rate, it can be seen that as the compensation grows, the amount overbooked will be slightly less. This is due to the idea that the impact of a customer being overbooked isn't as severe for a lower compensation, so a few more overbooked customers can generate a little more revenue. Finally, as the no-show rate and the amount overbooked increases, the revenue also grows. This is because more seats are being sold and more ticket revenue is generated, and since the optimal overbooking rate will likely have a bumped passenger around .1%, there is more room to sell additional seats and generate the extra revenue.

Using these figures, we can roughly pinpoint the amount of seats to overbook and the revenue just from looking at the graph. Additionally, we can extract specific values at certain no-show and compensation rates to find the overbooking rate and revenue, which can be used for different flight scenarios.

There are a few different scenarios that an airline might need to maximize revenue, based on what is different from the normal. We have this sensitivity analysis to show what can happen under different no-show rates and compensation granted, but it can also help an airline book different flights situationally. While the overall values may converge to our no-show rate and compensation in the results before, an airline can go even farther to book situationally to maximize revenue. Most times or flights may fall under our assumptions and results before, but we can look at these cases and why they are as they are.

Here are 4 of the more common situations of flights, all estimated (based on judgment and previously calculated no-show and compensation rates) with different no-show rates and compensations. Compensations can change based on what should be granted due to passenger behavior, and will be seen below:

- *Holiday weekends, business flights – No-show = 2%, Compensation = $3000 (Dom), $10000 (Int)*

- *These flights are more on the serious side, where business travelers have important obligations and people are going somewhere important for the holidays. No-show rates happen less often because these events are more important*
- *If a passenger here is overbooked, they are more sensitive. To try and keep the customer happy and avoid long-term impacts like bad PR and such, there may be more short-term compensation granted, and potentially still higher long-term impacts. Compensation granted will rise significantly for these flights.*

- *Weekday mornings – No-show = 5%, Compensation = $2000 (Dom), $8500 (Int)*
  - *Because of the timing, no-show rates may be slightly higher. However, international flights may see it lower than normal due to the higher number of business travelers still on these types of flights.*
  - *Compensation may be slightly higher due to business travelers, but is overall not too much different because of the timing and the ability to get to a destination on the same day still.*
- *Bad weather, vacations – No-show = 10%, Compensation = $1600 (Dom), $7000 (Int)*
  - *These flights will likely be missed more often due to the weather issues and the relaxed sense of being on vacation*
  - *The lack of travel urgency due to these factors may not result in higher compensation implications, but still near the average*
- *Red-Eye, budget flights – No-show = 15%, Compensation = $1200 (Dom), $5000 (Int)*
  - *Overnight flights are tough to get to and are often missed, and budget flights are more casual customers that may miss more often*
  - *Compensation is low for these due to the inconvenience of flight time for passengers or that budget flyers won't have as many expectations for compensation.*

As we look at our sensitivity analysis for different no-show rates and compensation, we can pull out these values for the different scenarios listed, and see optimal overbooking rates and the revenue generated from it (code shown in Appendix B.3). If an airline chooses to adapt to these different scenarios, they will be able to generate higher amounts of revenue than simply using the same strategy for every flight.

Flight Type	No-Show	Compensation	Optimal OB	Resulting Revenue
Holiday Weekends, Business flights	2%	$3000 (Dom) $10000 (Int)	.5% 1%	$62140 $326100
Weekday early mornings	5%	$2000 (Dom) $8600 (Int)	3% 3.25%	$63036 $333600
Bad weather flights, Vacations	10%	$1600 (Dom) $7000 (Int)	10% 10.25%	$65809.6 $350200
Red-Eye	15%	$1200 (Dom)	15.5%	$69122.8

(Overnight), Budget flights		\$5000 (Int)	16.5%	\$369900

Figure 2.10

This table shows a few different flight scenarios and what the optimal overbooking rate should be. When recommending to an airline, we can point out cases like these where they should adjust their overbooking rate to maximize revenue. Notice how the resulting revenues are close to or greater than maximum revenue generated found in figures 2.3 and 2.4. This is because at each overbooking rate, and the higher overbooking that's done, the revenue should grow, as shown before. Also, as the compensation is lowered for a higher no-show rate, overbooking can be at or even above the no-show rate because compensation won't outweigh ticket revenue as quickly.

There may even be implications if an airline used the same overbooking strategy for every flight. For example, if we kept our optimal 2.5% overbooking from our general model on the holiday and business flights, we may lose a lot of potential revenue from paying higher compensation values. Likewise, if we used that 2.5% overbooking for red-eye flights that have a very high no-show rate, there will be leftover seats on most of the flights, losing out on a lot of revenue.

Of course, simplifying the process and having one model to fit all flights isn't exactly too big a problem; it will still generate a very high revenue since the average no-show rate of 4% and compensation as is are still averages that will be close to maximum. But of course, there are ways to improve, and overbooking situationally is definitely a step in the right direction in terms of revenue and keeping passengers happy.

Comprehensive Revenue Model

As previously mentioned, it’s difficult to simplify an airline revenue model into a single equation or a single system of equations. This is due to different types of flights, certain specific flight scenarios, and due to the overbooking strategy. However, due to the business impact of overbooking for airlines, we put together the following model in order to offer a comprehensive model that an airline could potentially implement to calculate their revenue.

The variables in the model are:

S: number of seats
p: no show-rate
c: compensation per bumped passenger
γ: baseline compensation
δ: compensation sensitivity
R: average ticket revenue per passenger

The following is a system of equations that can be used to model the net revenue of an airline company using a specific booking strategy.

$$Net\ Revenue = N \cdot R - B \cdot c$$

where $N = S(1 + r)$

$$B = max(0, N(1 - p) - S)$$

and $r = p - \delta \cdot (\frac{c-\gamma}{\gamma})$ or

In this system, N represents the number of passengers booked with an overbooking strategy, B estimates the number of bumped passengers, and r calculates the optimal overbooking rate based on the no show-rate and compensation.

If we implement this model and use the information we previously discussed for a domestic flight, we can set:

S: 200
p: 0.04
c: 1802.52
γ: 1800
δ: 0.2
R: 310

It's important to note that the value, γ, the baseline compensation, is only used if the specific airline company wants to slightly change the compensation obtained from previous simulations and models. In this case, let's consider c to be the baseline compensation, which means p would be the optimal overbooking rate.

For example, If an airline company calculated an optimal simulated compensation for domestic flights to be 1802.52, but company policy forces them to issue 1800 to all overbooked passengers, then this is a scenario where the baseline compensation would be utilized in this model.

If we plug all of these values into the system of equations for the net revenue, we get a total revenue of $64,480. This value is slightly higher than the originally calculated value, which could be due to some variability in the Monte Carlo simulation. However, if airlines want to get an idea of potential revenue with an overbooking strategy, considering compensation, the number of bumped passengers, and the no show-rate, they can utilize this revenue model.

Model 3: Profit

Assumptions

Costs always have a very wide range, and it all depends on flight times, experience, fees, etc. Costs are constantly changing, and it's nearly impossible to estimate every single cost. We can give general estimates for things like fuel and labor costs, but many costs don't have exact values because of the variability depending on the airline and airport. So, some costs we can generalize according to the percentage range that it makes up of the profit. Now, an airline will have these costs if they were to receive a model like so, and can verify our values.

Our goal was to optimize revenue, so including a profit model is only supposed to provide a baseline of what the costs should be, not what they actually are, so that the airline can follow or do better than the $7 average profit per passenger. Throughout this section, we will give general costs for what is available, as well as describe these ranges of what costs can include. Then, we can provide an analysis on what the total cost should be so that it lines up with our optimized overbooking and revenue model. All of the costs are estimated based on the sources provided below.

Category	% of Total Cost	Description
Fuel	20% – 30%	Jet fuel and oil costs
Labor (crew, staff)	20% – 25%	Pilots, flight attendants, mechanics, gate agents, etc.
Aircraft ownership/lease	10% – 15%	Lease payments and depreciation on aircraft
Maintenance	10% – 15%	Additional maintenance done
Airport & navigation fees	5% – 10%	Airport usage, landing fees, airspace usage.
Ground handling & services	5% – 10%	Baggage handling, catering, cleaning
Distribution & sales	3% – 7%	Booking systems, commissions, advertising.
Passenger services	1% – 3%	Food, entertainment, onboard amenities

Figure 3.1

Fuel

Fuel is one of the highest costs per flight of an airline. With costs per gallon averaging $6.25, and flights burning anywhere from 2 tons/hour to 12 tons/hour depending on flight size, costs will quickly pile up. Generally, it can account for almost up to a third of the total costs. We can estimate fuel costs to be $20000 for domestic flights and $113000 for international flights [16][18][19][20].

Labor

Flight crews also have a wide range of how much an airline will be paying them per flight based on experience, size, etc. Real data has the average that for domestic flights, the total cost of the crew is $1152/hour, and for international flights, $2356/hour, but there isn't just the flight crew to account for. Other labor, like people at the gate, customer service, baggage, maintenance crews, engineers, etc. will raise these numbers significantly. We can estimate labor costs to be $13500 per domestic flight and $70000 per international flight [14][17].

Aircraft costs – ownership and maintenance

Each aircraft for the airline is either owned or leased, and there is also additional maintenance needing to be done, whether scheduled or unscheduled, that an airline needs to pay for. Total, this should account for around 25% of the per flight cost. We can estimate this to be around $15000 per domestic flight and $85000 per international flight[14][15].

Fees

Airline fees are always adding up, whether it's to use the airport services like landing, or handling fees like passenger baggage, totaling about 15% of total costs. We can estimate this to be around $9000 for domestic flights and $50000 for international flights. These values are reasonable – for a flight to simply land at an airport, it can be upwards of $5000 in just the landing fee. Of course, with all the additional fees, they will add up[15][16][17].

Marketing, Distribution, Passenger Amenities

Airlines will need to promote their business and have additional amenities that can make them stand out, like the ability to watch movies on-flight. These costs can equate to around 6%, adding $3600 per domestic flight and $20000 per international flight[13][16].

Our total costs will add up as follows:

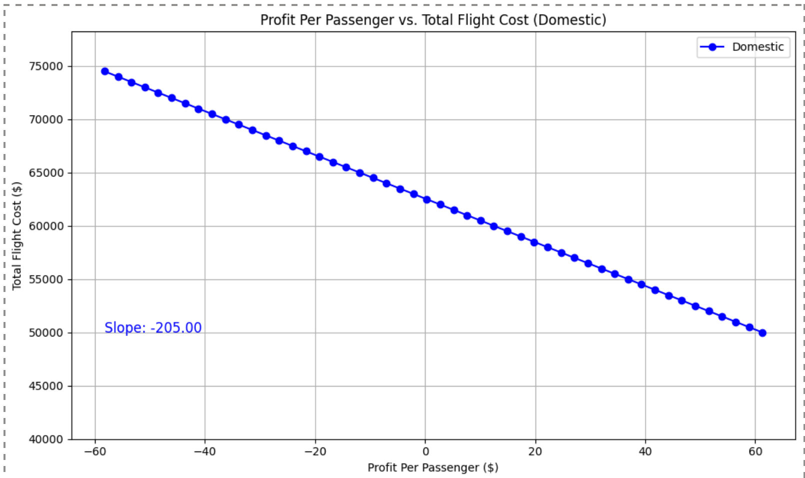
Domestic = 20000 + 13500 + 15000 + 9000 + 3600 = 61100

International = 113000 + 70000 + 85000 + 50000 + 20000 = 338000

Looking back at the real average of $7 profit per passenger, we can compare these rough cost estimates to it. A domestic flight has 200 seats with an optimal overbooking of 2.5%, equating to 205 passengers. That means the total profit of the domestic flight should be 205 * 7 = $1435. Similarly, for an international flight of 400 seats and 6.25% overbooking, or 425 passengers, the total profit of an international flight should be 425 * 7 = $2975.

Finding the difference of the estimated cost to the optimized revenue of a domestic flight, $62564 – $61100 = $1446 (optimized revenue minus costs = profit). This equates to 1446 / 205 = $7.05 profit per passenger. Following the same procedure for an international flight, 340641 – 338000 = $2641 (optimized revenue minus costs = profit), which equates to 2641/425 = $6.21 profit per passenger.

These values were calculated to be what the estimates should be so that the profit follows the average. The costs are all assumptions, but we can show adjustments in the event that costs vary. Since these values are just estimates to see what it could potentially look like for an airline, we can create a sensitivity analysis – what happens to the profit per passenger as the cost per flight changes. The revenue is fixed at where the optimal overbooking amount was. This is essentially a cost versus revenue model, but the cost is subtracted from the revenue, and then divided by the number of passengers from the optimal amount of booked seats, so 205 for domestic and 425 for international. This will give the x-axis profit per passenger. Code for figures 3.2 and 3.3 can be found in appendix C.1.



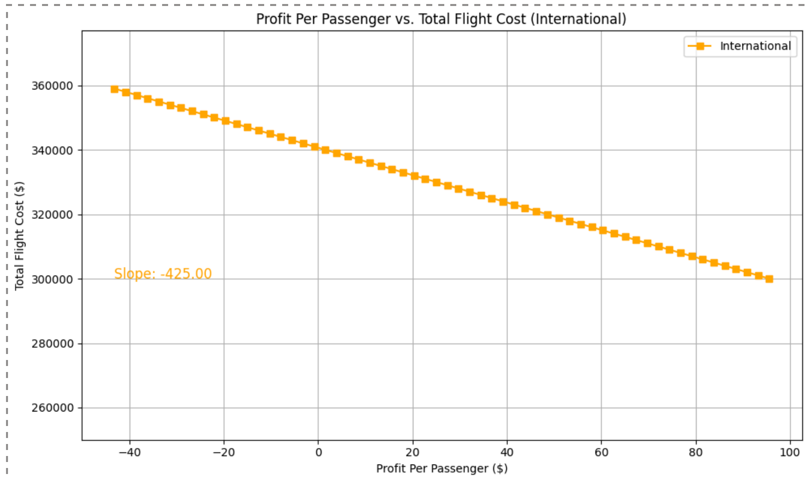


Figure 3.2

Figure 3.3

In both figures, the y-intercept shows a \$0 profit per passenger, which will be where the cost equals the revenue. It also shows a linear relationship where the slope is -\$205 for domestic(figure 3.2) and -\$425 for international(figure 3.3), which means every \$205 and \$425 for international decrease in cost, the increase in profit per passenger will be by \$1. In an entire flight where tens or hundreds of thousands of dollars in cost and in revenue generated, profit is determined by such a small amount.

It can be seen how close the costs are so that there can be a profit per passenger. For reference, a domestic flight earning \$7 profit would only need to increase their costs by \$1,435 to get the profit per passenger to \$0. Similarly for an international flight, a \$2975 cost increase would get the profit per passenger from \$7 to \$0. The increase in costs represents only a 2.35% increase for domestic (62564 / (62564 – 1435) = 1.0235) and just a 0.88% increase for international (340,641 / (340,641 - 2975) = 1.0088).This shows how little wiggle room there is, as small changes in cost can have a large impact on profit. That being said, to go the opposite way for an even higher profit per passenger, the cost difference is the same, only needing a 2.35% (.88% for international) decrease to double the profit to \$14 per passenger due to the linear relationship. This is another way of looking at it, and with this analysis, we can see again how sensitive the cost is to the profit.

We can identify now the importance of maximizing the revenue, even if it is only a small amount. For example, if a domestic flight didn't overbook, and we use the same cost of (62564 – 1435) = \$61129 which gave \$7 profit per passenger before, the difference of cost to revenue to profit per passenger would be down to (62000 – 61129) = \$871 / 200 passengers = \$4.36 profit per passenger. Look at the amount difference just from the revenue being \$564 less.

With all this, it is important that an airline watches their costs, because a small increase in costs can lead to a significant loss in profit. This is also why it is important to maximize revenue, because even a few hundred or thousand dollar increase in revenue per flight can significantly change the amount of profit an airline gets per passenger.

Comprehensive Profit Model:

Taking into account the total costs, the amount of passengers booked per flight, and revenue generated per flight, we can create a comprehensive model that models the profit per passenger for the airline.

$$Profit\ per\ passenger\ =\ \frac{R-C}{n}$$

where *R* is the fixed number of revenue from overbooked passengers, *C* is the total cost for the flight, and *n* is the number of booked passengers

This model is very simple in modeling how much profit an airline generates per passenger, but it can still be used to calculate how much the airline can expect to make with every passenger on the flight. Based on the result from the equation, airlines can compare to the average profit per passenger value, which as previously stated, is $7.

Using the above information for revenue costs, and total costs for a domestic flight: If R = $62,564 and C = $61,100 (estimated costs), and n = 205 passengers

$$Profit\ per\ passenger\ =\ \frac{R-C}{n} = \frac{62654-61100}{205} = 7.14$$

The value that the model gives us is a profit per passenger of $7.14 which is very close to the average amount that airlines can expect to make.

Quality of the models

The model has a higher quality because of the variety of options available to what an airline can do. Throughout the models, multiple ideas are explored and why they may or may not work. However, there is no set recommendation, and it will be up to the airline to choose what strategy will be the best for them, depending on their willingness. Of course, a recommendation can be given, but there is no set way to just say to overbook a certain amount and compensate a certain amount. There's going to be a lot of variety.

This isn't necessarily a limitation of the model, and with communication to an airline, a more narrowed down strategy can be given. After all, just giving a report isn't ideal without additional feedback and back and forth communication. These models, though, will provide a high level of understanding on the possibility of what airlines can do, and the multitude of scenarios and possibilities doesn't leave a lot of room to question what is done.

Of course, assumptions may leave the reader skeptical. However, the assumptions like no-show rate and compensation, because they are always changing and are situational, have a sensitivity analysis to show changes in that. Other assumptions, like seat prices and number of seats, won't impact the optimal overbooking rates, and the revenues will just be proportional. It's okay to make these assumptions without showing differences because they won't change, and an airline can implement their own flight data into the code to see the revenue results, simply by changing the numbers.

The models were created to minimize the amount of variability and to show a multitude of scenarios to eliminate as many "what-ifs" as possible. More data given in some of the assumptions do impact the quality of the model, and receiving this data can allow for adjustments to accurately predict maximized revenue and profitability for the airline.

It's important to put these models to use together. In this way, an airline can put together their ideas of compensation and see how that affects their revenue, which they can then see how it affects their profit. As previously mentioned, these models may become inaccurate based on the circumstances, because of how many different scenarios exist for airline overbooking strategies. However, the models can serve to give an airline a general idea of compensation, revenue, and profit with certain overbooking strategies and under certain circumstances.

Recommendations

There's a lot of quantitative recommendations for an airline, but a lot of what an airline can do depends on their willingness to take risks. There's no set way on what either can do with overbooking and their customers, but there may be a trade-off with customer satisfaction and revenue, or in basic terms, less risky versus more risky.

If an airline wants to do very low overbooking or even no overbooking, there won't be any customer complaints that they are overbooked. However, an airline will be missing out on some revenue because when there's people that don't show up for the flight, the airline is missing out on revenue for that flight. On the other hand, if an airline overbooks a lot of people, and there are constantly overbooked people every flight, there will be unhappy customers because they're getting delayed. Of course with the 2nd option, an airline can give a customer compensation so that that passenger stays loyal, but then the airline is losing money from paying that bumped passenger more and risking that the customer will choose another airline in the future, having a long-term impact on the current airline. There are plenty of conclusions to be drawn from these trade-offs.

For these trade-offs, it's impossible to give exactly what an airline should do. Our general recommendation is that an airline should overbooked situationally, meaning for different flights they should overbooked the flights differently. It is up to the airline to choose what kind of flight it may be, whether that's a red-eye flight, holiday flight, and many more. Of course, not every flight will fall under a type of flight, where the airline can choose the optimal overbooking rate of 2.5% and 6.25% for domestic and international flights. This option will be the safest, but having the situational overbooking will really allow for maximized revenue.

In terms of compensation, an airline should want to keep passengers so that they will use the airline in the future. One way is to minimize the amount of involuntarily bumped passengers, so when an airline is asking for volunteers, they can offer really high compensation amounts. This may result in higher short term costs, but it will allow for less of a long-term impact. Paying more of the short-term compensation may be more incentivized if there are very few bumped passengers, so if this option is chosen, they should also consider overbooking more conservatively.

At the end of the day, the airline needs to understand a trade-off between satisfaction and revenue, and understand a way to maximize both to become successful. This will help more customers choose the airline in the future while still being profitable. It's not as simple to just give what should be done, which is why these models give multiple options instead of just doing one set overbooking rate or compensation.

Conclusion

Airline overbooking is not an easy task, and there is no simple solution to maximize revenue. While there are many factors out of the airline's control in terms of costs or passenger behavior, an airline can use the tangible factors like the amount of seats they book on a plane to maximize their revenue. The common practice will allow an airline to become more profitable and successful, but this strategy needs to be carefully worked so that other factors, like passenger behavior, don't impact their long-term success.

Using these models and recommendations, along with additional feedback and communication with the airline, these models can provide a general basis for predicting airline revenue if they use optimized overbooking strategies as seen in the report. While no strategy is set and necessarily considered right, there are ways to minimize the risk and allow for the most ideal customer experience while maintaining a high level of profitability.

Appendix A: Model 1- Compensation

Add more to the appendix

A.1

The following Python code was used to simulate the expected cost of lost customer loyalty for bumped airline passengers, based on delay and route type.

The code first uses a triangular distribution to model the delays, similarly described in the report:

```
tri_params = {
    'domestic': (1, 2, 6),
    'international': (4, 8, 24)
}
```

To translate the delay hours into loyalty loss, the model uses a logistic curve:

```
def logistic_loyalty_loss(h, L, k, h0):
    return L / (1 + np.exp(-k * (h - h0)))
```

Where L is the maximum loyalty loss, k is the steepness of the curve, h0 is the inflection point, and h is the actual delay in hours.

Passenger Cost Simulation:

```
def simulate_one_passenger(route_type, L, k, h0, clv):
    a, b, c = tri_params[route_type]
    delay = np.random.triangular(a, b, c)
    loss_rate = logistic_loyalty_loss(delay, L, k, h0)
    cost = clv * loss_rate
    return cost, delay, clv, loss_rate
```

For each passenger, the simulation draws a delay from the appropriate triangular distribution. It then calculates the loyalty loss rate for the logistic function, and then multiplies the CLV to the loss rate to get the cost of losing that customer.

Expected Cost Over Many Simulations:

```
def simulate_expected_cost(n_passengers, route_type, L, k, h0, clv):
    costs = [simulate_one_passenger(route_type, L, k, h0, clv)[0] for _ in range(n_passengers)]
    return np.mean(costs), costs
```

Each scenario- domestic involuntary, domestic voluntary, international involuntary, and international voluntary runs the passenger cost simulation for 100,000 passengers and average cost of loyalty loss per bumped passenger.

```
results = {}
for label, params in cases.items():
    mean_cost, cost_list = simulate_expected_cost(100000, params['route'], params['L'], params['k'], params['h0'],
params['clv'])
    results[label] = {
        'mean_cost': mean_cost,
        'costs': cost_list,
        'params': params
    }
    print(f"{label} -> Expected Cost: ${mean_cost:.2f}")
```

After it is run in the simulation above, we get the expected cost, from simulating many passengers and taking the average of the individual loyalty loss costs.
For each scenario, we get these expected costs:

Domestic - Involuntary -> Expected Cost: $2011.13
Domestic - Voluntary -> Expected Cost: $959.61
International - Involuntary -> Expected Cost: $6788.32
International - Voluntary -> Expected Cost: $3486.57

These expected costs are also seen in Model 1a.

To see the full code visit https://github.com/ZachSkiba/Airline-Overbooking - This section of the code is in Compensation/Long_term_cost.py

Appendix B: Model 2- Revenue

B.1- Monte Carlo Simulation

This code uses Monte Carlo simulation to analyze each of our overbooking strategies and shows the probabilities of a certain number of passengers to be overbooked. This code specifically is for domestic flights, but the code is the exact same for the international flights, except for the parameters and overbooking strategies.

Parameters:

```
num_flights = 1000  # Total flights per day
seats_per_flight = 200  # Seats available per flight
no_show_rate = 0.04  # No-show rate
num_simulations = 1000  # Number of Monte Carlo trials
```

These are the parameters of the simulation based on our assumptions stated above in the report.

Overbooking Strategies:

```
booking_levels = {
    "Conservative(0%)": 200,  # 0% overbooking
    "Moderate(3%)": 206,      # 3% overbooking
    "Aggressive(5%)": 210,    # 5% overbooking
    "Extra Aggressive(10%)": 220,  # 10% overbooking
    "Optimal Overbooking(2.5%)": 205  #Based on maximizing revenue in Max_Revenue code
}
```

These are each of the overbooking strategies, which are also defined above in the report.

Acceptable Overbooking Ranges:

```
lower_bound, upper_bound = .05, .15
```

This shows the acceptable range of passengers being overbooked per day (100-300 people out of 200,000 total daily passengers). This is the acceptable range we estimated based on industry average for overbooking rates.

Monte Carlo Simulation:

```
for strategy, booked_per_flight in booking_levels.items():
    overbooked_passengers_per_day = []

    # Monte Carlo simulation
    for _ in range(num_simulations):
        total_overbooked = 0

        for _ in range(num_flights):
            show_up = np.random.binomial(booked_per_flight, 1 - no_show_rate)  # Passengers who show up
            overbooked = max(0, show_up - seats_per_flight)  # Count of overbooked passengers
            total_overbooked += overbooked
```

```
        overbooked_passengers_per_day.append(total_overbooked)
```

For each overbooking strategy, we run 1000 simulations of 1000 flights. For each flight, we simulate the number of passengers showing up using a binomial distribution, which models real world randomness. We used a binomial distribution in this overbooking simulation because each passenger independently has a fixed probability of showing up for the flight. The binomial distribution models the total number of passengers who show up out of all those who booked, treating each as a success/failure event. Since our number of passengers is large and the show-up probability is not extreme, the distribution closely resembles a normal distribution, which allows us to estimate probabilities using normal approximation when needed. If more passengers show up than the number of seats available, then we count them as overbooked. Then the total number of passengers per day is stored.

Analyze Overbooking Results:

```
    #calculate total passengers and overbooking rate per passenger
    overbooked_passengers_per_day = np.array(overbooked_passengers_per_day)
    total_passengers_per_day = num_flights * booked_per_flight
    overbooking_rate_per_passenger = overbooked_passengers_per_day / total_passengers_per_day * 100

    # Calculate probabilities
    prob_within_range = np.mean((overbooking_rate_per_passenger >= lower_bound) & (overbooking_rate_per_passenger
<=   upper_bound))
    prob_above_range = np.mean(overbooking_rate_per_passenger > upper_bound)
    prob_below_range = np.mean(overbooking_rate_per_passenger < lower_bound)
    one_in_x_passengers = round(1 / (avg_overbooking_rate / 100))
```

First we get the total passengers per day and then get the overbooking rate per passenger. Then we compute the probabilities of a certain number of people that get overbooked within the desired range. We then calculate the “ one in x passengers” estimate.
We then store and print out the data, and the output is shown in the report, in figure 2.1 and 2.5.
To see the full code and the code for international flights, visit
https://github.com/ZachSkiba/Airline-Overbooking - This section of the code is in Probability Overbooking/P_Overbooking_d.py and P_Overbooking_i.py for international flights.

B.2 - Maximizing Revenue

This Python code uses Monte Carlo simulation to find the optimal number of passengers to book per flight that will maximize an airline's revenue using the desired parameters (Domestic Flights). Below is a detailed explanation of the code that supported Figures 2.3 and 2.4.
Parameters:

```
# Simulation parameters
num_flights = 1000  # Total flights per day
seats_per_flight = 200  # Seats per flight
no_show_rate = 0.04  # No-show rate
num_simulations = 1000  # Number of Monte Carlo trials

compensation_per_passenger = 1793.52  # Compensation for bumped passengers


economy_price = 200   #two-way ticket price / 2
business_price = 4 * economy_price
```

```
first_price = 6 * economy_price
```

These parameters are important in order to optimize revenue and find the number of passengers to overbook. All of these parameters were explained in the report above, in the compensation and revenue section (models 1 and 2).

Main Simulation Function:

```
def run_simulation(booked_per_flight, seats, economy_tickets, business_tickets, first_tickets):
    net_revenues = []

    for _ in range(num_simulations):
        # Show-up count per flight
        show_up = np.random.binomial(booked_per_flight, 1 - no_show_rate, num_flights)

        # Number of passengers overbooked per flight
        overbooked = np.maximum(0, show_up - seats)

        # Daily total bookings and total number of overbooked passengers
        total_booked_passengers = num_flights * booked_per_flight
        total_overbooked_passengers = np.sum(overbooked)

        # Revenue per class
        economy_revenue = economy_tickets * economy_price * num_flights
        business_revenue = business_tickets * business_price * num_flights
        first_revenue = first_tickets * first_price * num_flights
        total_revenue = economy_revenue + business_revenue + first_revenue

        # Total revenue
        revenue = total_revenue
        compensation_cost = compensation_per_passenger * total_overbooked_passengers
        net_revenue = revenue - compensation_cost
        net_revenue_per_flight = net_revenue / num_flights

        net_revenues.append(net_revenue_per_flight)

    return np.mean(net_revenues), np.mean(overbooked)
```

This function takes all of the parameters defined above, and simulates the revenue and overbooking effects for a given number of passengers booked per flight. First, it calculates the number of people that show up per flight using a binomial distribution, as well as the number of overbooked people per flight. Then, based on the passengers booked, it calculates the amount of revenue being generated per flight, by adding the revenue from passengers, minus the compensation per overbooked passenger. This is then repeated for all 1000 trials and returns the average net revenue per flight.

Simulating Specific Overbooking Strategies:

```
def simulate_domestic_flights(models):
    results = {}

    for strategy, params in models.items():
        seats = params["seats"]
        booked_per_flight = params["sold"]

        base_economy = int(0.85 * seats)
        base_business = int(0.1 * seats)
        base_first = int(0.05 * seats)

        # extra overbooked seats allocation
        extra_seats = booked_per_flight - seats
```

```
        extra_business = int(0.10 * extra_seats)
        extra_economy = int(0.90 * extra_seats)

        extra_first = 0

        economy_tickets = base_economy + extra_economy
        business_tickets = base_business + extra_business
        first_tickets = base_first + extra_first

        avg_net_revenue, _ = run_simulation(booked_per_flight, seats, economy_tickets, business_tickets,
first_tickets)
        results[strategy] = avg_net_revenue
```

This function takes all of the 4 overbooking strategies - Conservative, Moderate, Aggressive, and Extra Aggressive, and runs into the run_simulation function described above. It adjusts the amount of passengers in each class (economy, business, and first class) as well as for the overbooked passengers. Then it stores the net revenue for each approach after the simulation was run.

Optimizing Number of Booked Passengers:

```
def simulate_booking_levels(booking_levels):
    best_booking_level = None
    max_net_revenue = float('-inf')
    results = {}

    for booked_per_flight in booking_levels:
        # Base seat allocation before overbooking
        base_economy = int(0.85 * seats_per_flight)
        base_business = int(0.1 * seats_per_flight)
        base_first = int(0.05 * seats_per_flight)

        # Extra overbooked seats allocation
        extra_seats = booked_per_flight - seats_per_flight
        extra_business = int(0.10 * extra_seats)
        extra_economy = int(0.90 * extra_seats)

        extra_first = 0 # No extra first-class seats

        # Ticket revenue
        economy_tickets = base_economy + extra_economy
        business_tickets = base_business + extra_business
        first_tickets = base_first + extra_first

        avg_net_revenue, avg_overbooked_passengers = run_simulation(booked_per_flight, seats_per_flight,
economy_tickets, business_tickets, first_tickets)
        results[booked_per_flight] = {
            "avg_overbooked_passengers": avg_overbooked_passengers,
            "avg_net_revenue": avg_net_revenue,
        }

        if avg_net_revenue > max_net_revenue:
            max_net_revenue = avg_net_revenue
            best_booking_level = booked_per_flight
```

This function tests all of the different overbooking levels, ranging from 195 passengers to 230 (range we selected to test), and finds which level will give the highest revenue. For each level, the code first allocates the number of passengers per class (economy, business, and first class), and then does the run_simulation function described above to get the revenue for each overbooking level. It then finds and stores the overbooking level that corresponds to the highest revenue.

After all of this is run, we can graph them, resulting in Figure 2.3, which is explained in Model 2a. We also produce bar graphs that show the revenue for each of our four defined overbooking strategies. These graphs are shown below:

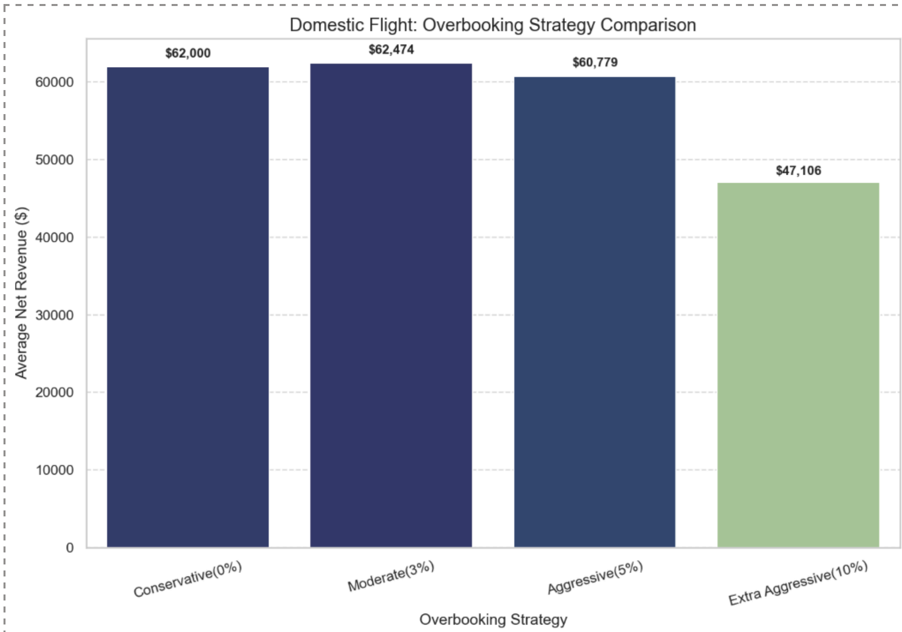


Figure B.2.1

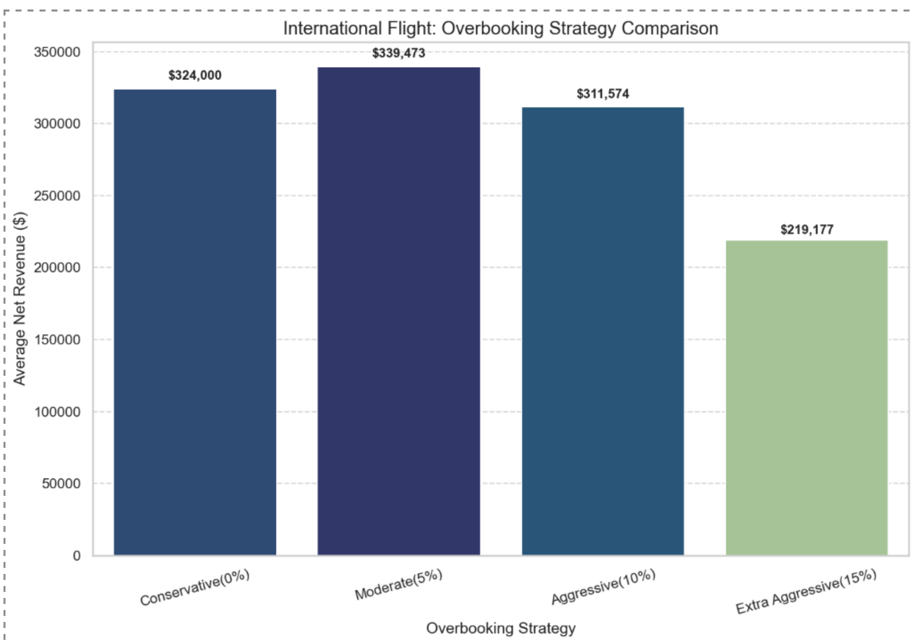


Figure B.2.2

We can see from figures B.2.1 and B.2.2, that overbooking using the moderate approach is the most optimal, for both domestic and international flights with a revenue of $62,474 per flight for domestic, and $339,473 per flight for international, assuming the parameters discussed above.

Conservative strategies follow closely, while the extra aggressive approach generates less revenue, with $47,106 and $219,177 respectively. This follows what was explained in Model 2a in the report above. To see the full code and the code for international flights, visit https://github.com/ZachSkiba/Airline-Overbooking - This section of the code is in Maximize Revenue/Max_Revenue_d.py and Max_Revenue_i.py for international flights.

B.3 - Sensitivity Analysis and Flight Scenarios

This Python code Monte Carlos simulation to make a sensitivity analysis shows what will happen to optimal overbooking rates and revenue if the compensation and no show rates change. To do this, we first need to define our parameters.

```
# Parameters
num_flights = 1000
seats_per_flight = 200
num_simulations = 1000

# Ticket prices
economy_price = 200
business_price = 4 * economy_price
first_price = 6 * economy_price

# Parameters for Sensitivity Analysis
no_show_rates_to_test = np.round(np.linspace(0.00, 0.20, 41), 3)
compensations_to_test = np.arange(1000, 3101, 50)
booking_levels_to_test = range(seats_per_flight, seats_per_flight + 99)
```

First, we define our number of flights, seats per flight, number of simulations, and the price of each seat. Then we select our no show rates to test, from no no shows to 20 percent. Then we select our compensations to test, from 1000 to 3100.

Simulation function:

```
def run_simulation(booked_per_flight, seats, economy_tickets, business_tickets, first_tickets, no_show_rate,
compensation_per_passenger):
    show_up = np.random.binomial(booked_per_flight, 1 - no_show_rate, num_flights)
    overbooked_per_flight = np.maximum(0, show_up - seats)
    total_overbooked_passengers_today = np.sum(overbooked_per_flight)

    # Revenues
    total_economy_revenue = economy_tickets * economy_price * num_flights
    total_business_revenue = business_tickets * business_price * num_flights
    total_first_revenue = first_tickets * first_price * num_flights
    total_potential_revenue_today = total_economy_revenue + total_business_revenue + total_first_revenue

    total_compensation_cost_today = compensation_per_passenger * total_overbooked_passengers_today
    net_revenue_today = total_potential_revenue_today - total_compensation_cost_today

    return net_revenue_today / num_flights
```

This function takes the parameters discussed above and calculates the number of passengers showing up using a binomial distribution. Then it calculates the number of overbooked passengers. It then calculates the revenue and returns the revenue per flight.

Running Simulations:

```
# Sensitivity Analysis Loop
results_list = []
for current_compensation in compensations_to_test:
    for current_no_show_rate in no_show_rates_to_test:
        max_revenue_for_scenario = float('-inf')
```

```
        best_booking_level_for_scenario = seats_per_flight

        for booked_per_flight in booking_levels_to_test:

            # Non-overbooked Seat Allocation
            base_economy = int(0.85 * seats_per_flight)
            base_business = int(0.10 * seats_per_flight)
            base_first = int(0.05 * seats_per_flight)

            # Overbooked Seat Alloaction
            extra_seats = max(0, booked_per_flight - seats_per_flight)
            extra_business_intended = int(0.10 * extra_seats)
            extra_economy_intended = extra_seats - extra_business_intended
            extra_first_intended = 0

            # Ticket Revenue
            current_economy_tickets = base_economy + extra_economy_intended
            current_business_tickets = base_business + extra_business_intended
            current_first_tickets = base_first + extra_first_intended

            avg_net_revenue = run_simulation(
                booked_per_flight,
                seats_per_flight,
                current_economy_tickets,
                current_business_tickets,
                current_first_tickets,
                current_no_show_rate,
                current_compensation
            )
```

This function loops through all the combinations of no show rates and compensation, and then simulates what will happen for each scenario. As seen above, the code uses the previous function run_simulation which will calculate the revenue per flight.

We then make the contour plot with the data we collected, and get Figure 2.6 which is shown in Model 2b.

```
# No show rate and Max Expected revenue for each Compensation
plt.figure(figsize=(12, 7))

for compensation in compensations_to_test:
    revenue_for_compensation = df[df['compensation_($)'] == compensation]
    plt.plot(revenue_for_compensation['no-show_rate'], revenue_for_compensation['max_expected_net_revenue_($)'],
label=f'Compensation = ${compensation}')

plt.title('Max Expected Net Revenue vs No-show Rate for Different Compensations')
plt.xlabel('No-show Rate')
plt.ylabel('Max Expected Net Revenue ($)')
plt.grid(True, linestyle='--', alpha=0.4)
plt.tight_layout()
plt.show()
```

This code generates this figure, which is the same as figure 2.8 in Model 2b, but this is for domestic flights, not international.

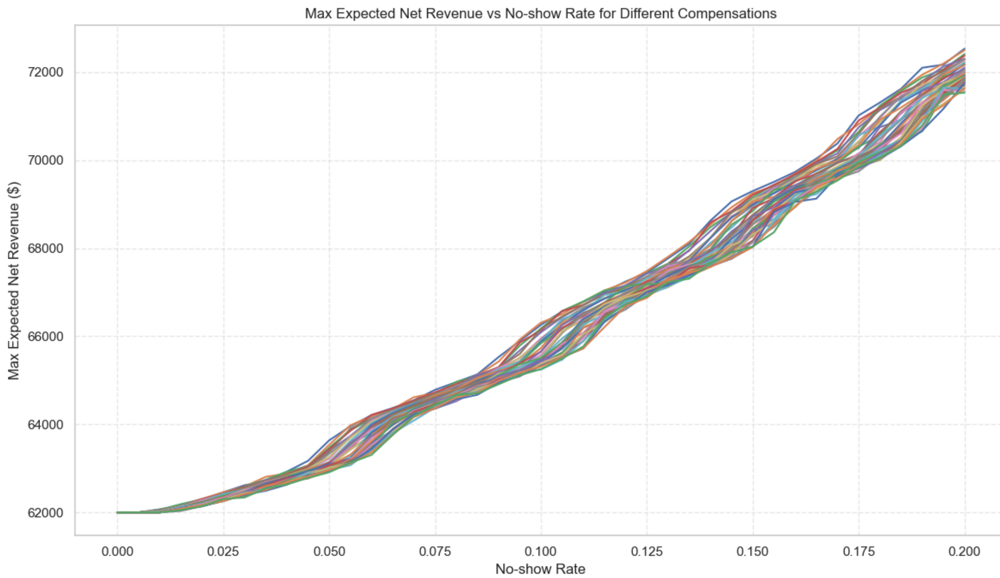


Figure B.2.3

This graph shows how revenue changes with no-show rate for each compensation amount. This is similar to the international one, figure 2.8 in Model 2b, which explains the general trend of the graph.

Code for No show sensitivity analysis:

```
#Compensation and max expected revenue for each No- Show rate
plt.figure(figsize=(12, 7))

for no_show_rate in no_show_rates_to_test:
    revenue_for_no_show_rate = df[df['no-show_rate'] == no_show_rate]
    line, = plt.plot(revenue_for_no_show_rate['compensation_($)'],
revenue_for_no_show_rate['max_expected_net_revenue_($)'])

#Plot labels
highlight_no_show_rates = [0.00, 0.05, 0.10, 0.15, 0.20]
for no_show_rate in highlight_no_show_rates:
    revenue_for_no_show_rate = df[df['no-show_rate'] == no_show_rate]
    max_revenue = revenue_for_no_show_rate['max_expected_net_revenue_($)'].max()

    best_compensation = revenue_for_no_show_rate[revenue_for_no_show_rate['max_expected_net_revenue_($)'] ==
max_revenue]['compensation_($)'].values[0]
    last_compensation = revenue_for_no_show_rate['compensation_($)'].iloc[-1]
    plt.text(last_compensation + 100, max_revenue, f'{no_show_rate*100}% No-show', color='black', fontsize=10,
weight='bold')

plt.title('Max Expected Net Revenue vs Compensation for Different No-show Rates')
plt.xlabel('Compensation per Bumped Passenger ($)')
plt.ylabel('Max Expected Net Revenue ($)')
plt.grid(True, linestyle='--', alpha=0.4)
plt.tight_layout()
plt.show()
```

This code generates this figure, which is the same as figure 2.9 in Model 2b, but this is for domestic flights, not international.

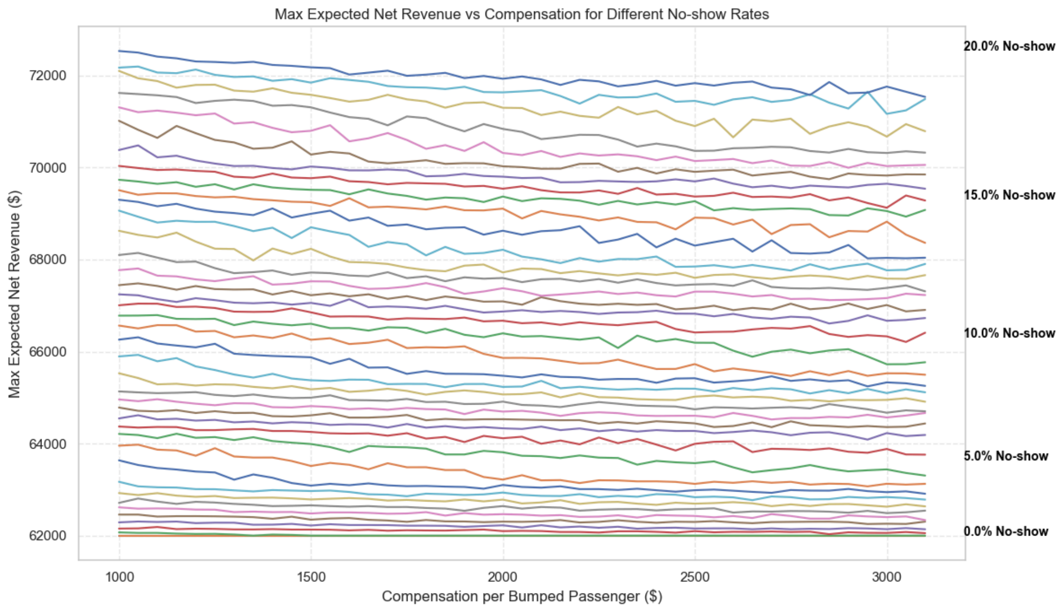


Figure B.2.4

This figure shows how revenue changes with compensation for each no-show rate. This is similar to the international one, figure 2.9 in Model 2b, which explains the general trend of the graph.

From this code, we can also pick out specific scenarios to get a better idea of the revenue from different types of flights.

```
from tabulate import tabulate

# Define the specific no-show rates and compensations to filter for
filters = [
    {'no_show_rate': 0.02, 'compensation': 3000, 'flight_type': 'Holiday Weekends, Business flights'},
    {'no_show_rate': 0.05, 'compensation': 2000, 'flight_type': 'Weekday early mornings'},
    {'no_show_rate': 0.10, 'compensation': 1600, 'flight_type': 'Bad weather flights, Vacations'},
    {'no_show_rate': 0.15, 'compensation': 1200, 'flight_type': 'Red-Eye (Overnight), Budget flights'}
]


results = []

# Extract Fileters
for filter_ in filters:
    filtered_df = df[(df["no-show_rate"] == filter_['no_show_rate']) & (df["compensation_($)"] ==
filter_['compensation'])]

    if not filtered_df.empty:
        flight_type = filter_['flight_type']
        optimal_booking_level = filtered_df["optimal_booking_level"].iloc[0]
        max_revenue = filtered_df["max_expected_net_revenue_($)"].iloc[0]

        # Store the results
        results.append([
            flight_type,
            f"{filter_['no_show_rate']*100}%",
            f"${filter_['compensation']}",
```

```
            optimal_booking_level,
            f"${max_revenue:,.2f}"
        ])


headers = ["Flight Type", "No-show Rate", "Compensation per Overbooked Passenger", "Optimal Overbooking", "Max Revenue"]

# Print Table with different Scenarios
print(tabulate(results, headers=headers, tablefmt="grid"))
```

This code filters out specific scenarios that we want, and displays the maximum revenue based on those scenarios.

Flight Type	No-show Rate	Compensation per Overbooked Passenger	Optimal Overbooking	Max Revenue
Holiday Weekends, Business flights	2.0%	$3000	201	$62,140.00
Weekday early mornings	5.0%	$2000	207	$63,100.00
Bad weather flights, Vacations	10.0%	$1600	220	$65,843.20
Red-Eye (Overnight), Budget flights	15.0%	$1200	231	$69,106.00

This is the output of the code, which shows the different domestic flight scenarios, similar to figure 2.10 in Model 2b, which contains the information for both domestic and international flights. To see the full code and the code for international flights, visit https://github.com/ZachSkiba/Airline-Overbooking - This section of the code is in Sensitivity Analysis/values_analysis_d.ipynb and values_analysis_d.ipynb for international flights.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Domestic Flights

#Simulation Parameters
revenue_domestic = 62564  #Revenue Based from Max_Revenue Code
passengers_domestic = 205 #Passangers Based on Max_Revenue Code
cost_range_d = np.arange(50000, 75000, 500) #Range of costs to test

# Calculate profit per passenger
profit_per_passenger_domestic = (revenue_domestic - cost_range_d) / passengers_domestic


df = pd.DataFrame({
    'Total Cost d': cost_range_d,
    'Profit/Passenger (Domestic)': profit_per_passenger_domestic,
})

# Compute slopes using linear regression
slope_domestic, intercept_domestic = np.polyfit(profit_per_passenger_domestic, cost_range_d, 1)

# Plotting Domestic
plt.figure(figsize=(10, 6))
plt.plot(df['Profit/Passenger (Domestic)'], df['Total Cost d'], color='blue', label='Domestic', marker='o')
```

```
plt.axhline(0, color='gray', linestyle='--')
plt.title('Profit Per Passenger vs. Total Flight Cost (Domestic)')
plt.ylabel('Total Flight Cost ($)')
plt.xlabel('Profit Per Passenger ($)')
plt.legend()
plt.grid(True)
plt.ylim(40000)

plt.text(
    x=profit_per_passenger_domestic[-1],
    y=cost_range_d[0],
    s=f"Slope: {slope_domestic:.2f}",
    fontsize=12,
    color='blue'
)

plt.tight_layout()
plt.show()
```

Appendix C: Model 3 - Profit

C.1- Profit

This code shown below was used to generate figure 3.2. First, we needed to define our parameters, and the range of cost that we are going to use to analyze our profit.

```
#Simulation Parameters
revenue_domestic = 62564  #Revenue Based from Max_Revenue Code
passengers_domestic = 205 #Passengers Based on Max_Revenue Code
cost_range_d = np.arange(50000, 75000, 500) #Range of costs to test
```

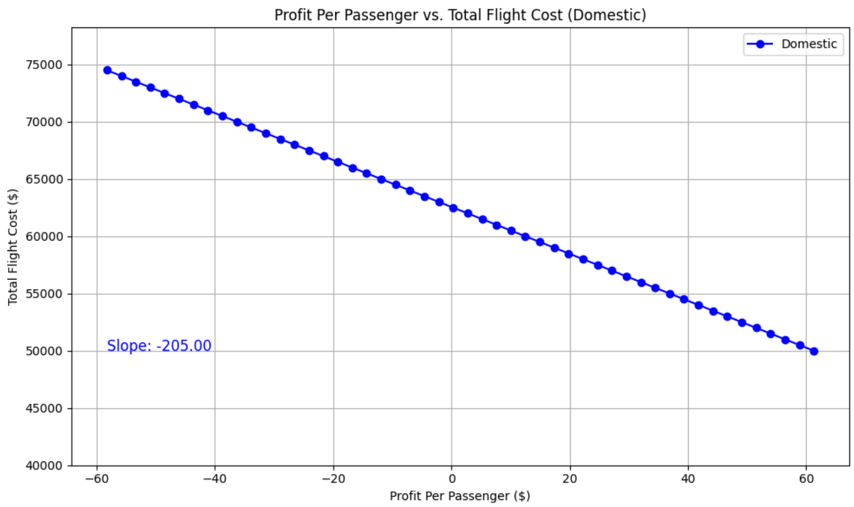
The revenue here is from maximizing the revenue in our revenue model, in model 2a. Also based on the maximizing revenue model, was the optimal number of passengers, which is 205. Then we needed a range of costs to test, based around the average cost calculated in Model 3 of $61100.

Profit per passenger:

```
# Calculate profit per passenger
profit_per_passenger_domestic = (revenue_domestic - cost_range_d) / passengers_domestic


df = pd.DataFrame({
    'Total Cost d': cost_range_d,
    'Profit/Passenger (Domestic)': profit_per_passenger_domestic,
})
```

We then calculate the profit per passenger by subtracting the revenue by each of the costs, and then dividing it by the number of passengers. It is then stored in a data frame used to output the graph.



We then get this graph as the output, which is the same as figure 3.1 in the report above. To see the full code and the code for international flights, visit https://github.com/ZachSkiba/Airline-Overbooking - This section of the code is in Profit/Profit_d_i.ipynb.

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