All Formulae Used in First-Year Engineering

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Before you read:

- 1 This formula sheet IS CURRENTLY A DRAFT
- 2 This document does not (yet) contain all formulae.
- 3 There will probably be a few mistakes so if you spot any TELL ME so I can update it
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Mathematical

Vectors

Unit Vectors

$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Vector from vector A to vector B

$$\vec{AB} = \vec{B} - \vec{A}$$

Modulus

$$|\vec{A}| = \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \sqrt{x^2 + y^2 + z^2}$$

Scalar Product

$$\vec{A} \cdot \vec{B} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1b_1 + a_2b_2 + a_3b_3$$

Cross Product

$$\vec{A} \times \vec{B} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$$

Angle between two Vectors

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos(\theta)$$

Compressed Plane Equation

$$r\cdot n=d$$

Standard Plane Equation

$$Ax + By + Cz + D = 0$$

Expanded Plane Equation

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

Distance between a plane and a point

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

General Line Equation given Point \vec{a} and Direction Vector \vec{d}

$$\vec{r} = \vec{a} + \lambda \vec{d}$$

Direction Vector given two Points

$$\vec{d} = \vec{B} - \vec{A}$$

Angle of two Planes

$$\cos\left(\theta\right) = \frac{n_1 \cdot n_2}{|n_1||n_2|}$$

Intersection of two Planes

$$\vec{d} = \vec{n_1} \times \vec{n_2} = \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} \times \begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix}$$

Complex Numbers

The Imaginary Identity

$$j^2 = -1$$

Complex Number

$$z = a + jb$$

Modulus

$$|z| = \sqrt{a^2 + b^2}$$

Polar Form

$$z = r\left(\cos(\theta) + j\sin(\theta)\right)$$

De Moivre's Theorem

$$(\cos(x) + j\sin(x))^n = \cos(nx) + j\sin(nx)$$

 $n^{\rm th}$ Root of Polar Form

$$Z_k = \sqrt[n]{r} \left[\cos \left(\frac{\theta}{n} \right) + j \sin \left(\frac{\theta}{n} \right) \right] \quad \text{or} \quad Z_k = \sqrt[n]{r} \left[\cos \left(\frac{\theta}{n} + \frac{2k\pi}{n} \right) + j \sin \left(\frac{\theta}{n} + \frac{2k\pi}{n} \right) \right]$$

Exponential Form

$$z = re^{\theta j}$$

Matrices

Square Matrix Condition

n of rows = n of columns

Trace of a Matrix

$$tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \dots + a_{nn}$$

Upper Triangular Matrix

$$A_{TU} = \begin{bmatrix} n & n & n \\ 0 & n & n \\ 0 & 0 & n \end{bmatrix}$$

Lower Triangular Matrix

$$A_{TL} = \begin{bmatrix} n & 0 & 0 \\ n & n & 0 \\ n & n & n \end{bmatrix}$$

Diagonal matrix

$$A_D = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix}$$

Null Matrix

$$A_N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transposed Matrix

$$A^{T} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{T} = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Symmetrical Matrix Condition

$$A^T = A$$

The Adjoint

$$\operatorname{adj}(A) = (\operatorname{Cofactor}(A))^T$$

Cofactor

$$C_{ij} = (-1)^{i+j} \det(M_{ij})$$

Matrix of Cofactors

$$\operatorname{Cofactor}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{bmatrix} + \begin{vmatrix} e & f \\ h & i \end{vmatrix} & - \begin{vmatrix} d & f \\ g & i \end{vmatrix} & + \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ - \begin{vmatrix} b & c \\ h & i \end{vmatrix} & + \begin{vmatrix} a & c \\ g & i \end{vmatrix} & - \begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ + \begin{vmatrix} b & c \\ e & f \end{vmatrix} & - \begin{vmatrix} a & c \\ d & f \end{vmatrix} & + \begin{vmatrix} a & b \\ d & e \end{bmatrix}$$

Inverse of a Matrix

$$A^{-1} = \frac{\operatorname{adj}(A)}{\det(A)}$$

Scalar Multiplication

$$qA = q \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \begin{bmatrix} qa_1 & qa_2 \\ qa_3 & qa_4 \end{bmatrix}$$

Matrix Addition

$$A + B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

Matrix Multiplication Condition

for AB n columns in A = n rows in B

 2×2 Matrix Product

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

 2×2 Determinant

$$|A| = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

 3×3 Determinant

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

 2×2 Cross Product

$$\begin{bmatrix} a \\ b \end{bmatrix} \times \begin{bmatrix} c \\ d \end{bmatrix} = \det \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

Simultaneous Equation Matrix Form

Eigenvalue and Eigenvector Equation

$$A\vec{v} = \lambda \vec{v}$$

Eigenvalues of matrix A

$$\det(A - \lambda I) = 0$$

Eigenvectors of matrix A

$$(A - I\lambda)\vec{v} = 0$$

Differentiation

Derivative from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative of the Sum

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

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$$\frac{d}{dx}(e^x) = e^x$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule

if
$$y = uv$$
 then $\frac{dy}{dx} = \frac{du}{dx}v + \frac{dv}{dx}u$

Quotient Rule

if
$$y = \frac{u}{v}$$
 then $\frac{dy}{dx} = \frac{\frac{du}{dx}v - \frac{dv}{dx}u}{v^2}$

Curvature

$$\kappa = \left| \frac{f''(x)}{(1 + [f'(x)]^2)^{3/2}} \right|$$

Newton-Raphson Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Maclaurin Series

$$f(x) = \sum_{m=0}^{\infty} \frac{x^m}{m!} f^{(m)}(0)$$

Taylor Series

$$f(x) = \sum_{m=0}^{\infty} \frac{(x-a)^m}{m!} f^{(m)}(a)$$

Nabla Operator

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$$

Rate of change along a unit vector

$$\frac{\partial \phi}{\partial \vec{a}} = \operatorname{grad} \phi \cdot \hat{a}$$

Div

$$\operatorname{div}(F) = \nabla \cdot F = \frac{dF_x}{dx} + \frac{dF_y}{dy} + \frac{dF_z}{dz}$$

Curl

$$\operatorname{curl}(F) = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ F_x & F_y & F_z \end{vmatrix}$$

Integration

Integral of a function

for
$$f(x) \to \int f(x)dx$$

Definite Integral

$$\int_{a}^{b} f(x) dx = [A(x)]_{a}^{b} = A(b) - A(a)$$

Constant Multiple Rule

$$\int kf(x) \, dx = k \int f(x) \, dx$$

Integration by Parts

$$\int \frac{dv}{dx} u \, dx = uv - \int \frac{du}{dx} v \, dx$$

Rational Function Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Line section length with a function

$$s = \int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx$$

Line section length with a parametric equation

$$s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Line section length with a polar equation

$$s = \int_{\varphi_1}^{\varphi_2} \sqrt{r^2 + \left(\frac{dr}{d\varphi}\right)^2} \, d\varphi$$

Surface Area of a Revolved Graph

$$A = \int_{a}^{b} 2\pi y \sqrt{1 + (f'(x))^{2}} dx$$

Volume of a Revolved Graph

$$V = \int_{a}^{b} \pi(y)^{2} dx$$

Trapezium Rule

$$\int_{a}^{b} f(x) dx \approx \frac{\Delta x}{2} [f_0 + 2(f_1 + F_2 + \dots + f_{n-2} + f_{n-1}) + f_n]$$

Simpson Rule

$$\int_{a}^{b} f(x) dx \approx \frac{\Delta x}{3} [y_0 + 4(y_1 + y_3 + \dots + y_{n-2} + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-4} + y_{n-2}) + y_n], \quad \Delta x = \frac{b - a}{n}$$

Centroid Position

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}, \bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$

First Moment of Area

$$S = A \times d$$

The Jacobian

for
$$f(x,y) = x + \sin(y), y + \sin(x),$$

$$J = \begin{bmatrix} \frac{f_1(x,y)}{dx} & \frac{f_1(x,y)}{dy} \\ \frac{f_2(x,y)}{dx} & \frac{f_2(x,y)}{dy} \end{bmatrix}$$

Gauss' Theorem

$$\iiint\limits_{V} \overrightarrow{divF} dV = \iint\limits_{S} \overrightarrow{F} \cdot \overrightarrow{dS}$$

Green's Theorem

$$\oint_C (P \, dx + Q \, dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA$$

Differential Equations

Transforms

Statistics

Mechanical

Forces, Moments and Systems

Force given Acceleration

$$F = ma$$

Force given Components (2D)

$$F = \sqrt{F_x^2 + F_y^2}$$

Force's Angle from base Axis

$$\alpha = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$

Moment

$$M = |F| \times \lambda$$

System Equilibrium Equations

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$$

Trusses

Degree of Indeterminacy

M - 2J + 3 = degree of inderterminacy

Torsion

Shear Strain at r

$$\gamma_{\rm max} = r\theta$$

Shear Strain given L

$$\gamma_{\rm max} = \frac{r\phi}{L}$$

Shear Modulus Torsion Equation

$$\tau = Gr\theta$$

Shear Stress at an internal radius ρ

$$\tau = \frac{\rho}{r} \tau_{\text{max}}$$

Torsion Formula

$$\tau_{\rm max} = \frac{T_r}{I_p}$$

Torsional Rigidity Equation

$$\theta = \frac{T}{GI_P}$$

Polar Moment of Inertia

$$I_P = \int_A \rho^2 dA$$

Polar Moment of Inertia for a circle

$$I_P = \frac{\pi d^4}{32}$$

Fluids

Shear Stress / Viscosity

$$\tau = \mu \frac{du}{dy}$$

Mass Density

$$\rho = \frac{m}{V}$$

Specific Weight

$$w = \rho g$$

Specific Gravity

$$\sigma = \frac{\rho}{\rho_{\rm standard}}$$

Compressibility

$$K=\rho\frac{dp}{d\rho}$$

Pressure

$$p = \frac{F}{A}$$

Pressure at Depth

 $p = \rho g h + p_{\text{atm}} = \text{Gauge pressure} + \text{Atmospheric Pressure}$

Resultant Hydrostatic Force

$$R = \rho g A \bar{y}$$

Depth of Pressure

$$D = \sin^2(\phi) \frac{I_O}{A\bar{y}}$$

Parallel Axis Theorem

$$I_O = I_G + A\bar{d}^2$$

Buoyancy

$$R_n = \rho_n g V_n$$

First Moment of Area

$$Q_{x/y} = \int_{A} (y/x) dA$$

Position of a Centroid

$$\bar{x} = \frac{Q_y}{A}, \qquad \bar{y} = \frac{Q_x}{A}$$

Position, Motion and Acceleration

The five suvat equations

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{t}{2}(u+v)$$

$$v^2 = u^2 + 2as$$

Polar Velocity

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

Polar Acceleration

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_{\theta}$$

Polar Velocity to Cartesian Velocity

$$v_x = \dot{r}\cos\theta - r\dot{\theta}\sin\theta, \quad v_y = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$$

Electronic

Current

Current

$$I = \frac{Q}{t}$$

Kirchhoff's First Law

$$\sum_{i=1}^{n} I_i = 0$$

Electric Fields

Energy of a Charge in an Electric Field

$$E=Vq$$

Electric Field Strength

$$E = \frac{V}{d}$$

Force on a charge in a field

$$F = qE$$

Resistance

Current and Voltage through a Resistor

$$V = IR$$

Power dissipated through a Resistor

$$P = IV, \quad P = I^2R, \quad P = \frac{V^2}{R}$$

Resistors in Series

$$R_T = R_1 + R_2 + \dots + R_n$$

Resistors in Parallel

$$R_T = (R_1^{-1} + R_2^{-1} + \dots + R_n^{-1})^{-1}$$

Impedance

Complex Form

$$Z = R + jX$$

Absolute Value

$$|Z| = \sqrt{R^2 + X^2}$$

Net Reactance

$$X = X_L - X_C$$

Current and Voltage given Impedance

$$V = IZ$$

Admittance

$$Y=Z^{-1}$$

Capacitance

Capacitor Defining Equation

$$I = C \times \frac{dV}{dt}$$

Capacitance of a Parallel Plate Capacitor

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Charge stored

$$Q = CV$$

Energy Stored

$$E = \frac{1}{2}CV^2$$

Capacitors in Series

$$C_T = (C_1^{-1} + C_2^{-1} + \dots + C_n^{-1})^{-1}$$

Capacitors in Parallel

$$C_T = C_1 + C_2 + \dots + C_n$$

RC cutoff frequency

$$f_c = \frac{1}{2\pi RC}$$

RC filter quality

$$Q = \frac{1}{\omega RC}$$

Capacitor Charging quantity X

$$X_t = X_0 \times \left(1 - e^{-\frac{t}{RC}}\right)$$

Capacitor Discharging quantity X

$$X_t = X_0 \times e^{-\frac{t}{RC}}$$

Capacitor Reactance

$$X = \frac{1}{2\pi fC}$$

RC phase difference

$$\Delta\theta = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Inductance

Inductor Defining Equation

$$V = L \frac{dI}{dt}$$

Inductance of a coil

$$L = \frac{\mu N^2 A}{l}$$

Inductor Reactance

$$X_L = 2\pi f L$$

Inductors in Parallel

$$L_{total} = (L_1^{-1} + L_2^{-1} + \dots + L_n^{-1})^{-1}$$

Inductors in Series

$$L_{\text{total}} = L_1 + L_2 + \dots + L_n$$

RL cutoff frequency

$$f_c = \frac{R}{2\pi L}$$

RL filter quality

$$Q = \frac{\omega L}{R}$$

Transformers

Induced E.M.F

$$V = -N \frac{\Delta \Phi}{\Delta t}$$

Power and current

$$\frac{V_{\rm secondary}}{V_{\rm primary}} = \frac{I_{\rm primary}}{I_{\rm secondary}} = {\rm Power}$$

Turn Ratio

$$n = rac{N_{
m primary}}{N_{
m secondary}} = rac{V_{
m primary}}{V_{
m secondary}}$$

Inductive Reactance

$$X_L = 2\pi f \frac{\mu N^2 A}{l}$$

Mutual Inductance

$$L_M = k\sqrt{L_1 L_2}$$

Reflected Resistance

$$R_{\mathrm{pri}} = \frac{R_L}{n^2}$$

Efficiency

$$\eta = \left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) 100\%$$

RLC circuits

Impedance

$$|Z_T| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Q-Factor

$$Q = \frac{E_{\text{Stored}}}{E_{\text{Lost per cycle}}}$$

Parallel Circuit Q-Factr

$$Q = \frac{R}{X_L}$$

Resonant Frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Material

Elastic Deformation

Hooke's Law

$$F = kd$$

Tensile Engineering Stress

$$\sigma = \frac{F}{A_0}$$

Normal Tensile Strain

$$\varepsilon_z = \frac{\Delta l}{l_0}$$

Lateral Tensile Strain

$$\varepsilon_x = \frac{\Delta d}{d_0}$$

Rigidity

$$E = \frac{\sigma}{\varepsilon}$$

Poisson's Ratio

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$$

Shear Engineering Stress

$$\tau = \frac{F}{A_0}$$

Shear Strain

$$\gamma = \frac{\Delta x}{y} = \tan(\theta) \approx \theta \ RAD$$

Shear Modulus

$$G = \frac{\tau}{\gamma}$$

Shear Modulus given Poisson's Ratio

$$G = \frac{E}{2 \times (1 + \nu)}$$

Angle of twist due to two moments

$$\alpha = \frac{32 \times M \times l_0}{\pi \times d_0^4 \times G}$$

Bulk Modulus

$$P = -K\frac{\Delta V}{V_0}$$

Bulk Modulus given Poisson's Ratio

$$K = \frac{E}{3 \times (1 - (2 \times \nu))}$$

UTS

$$UTS = \frac{P_{\text{max}}}{A_i}$$

Fracture Strength

$$\sigma_f = \frac{P_f}{A_i}$$

Strain Hardening Ratio

$$r_{\rm SH} = \frac{\sigma_u}{\sigma_o}$$

Resilience Modulus

$$U_r \approxeq \frac{1}{2} \sigma_y \varepsilon_y$$

Toughness

$$U_t = \frac{\text{Energy}}{\text{Volume}} = \int_0^{\varepsilon_f} \sigma dx \varepsilon$$

Toughness Approximations

$$U_t \approx \left(\frac{\sigma_0 + \sigma_u}{2}\right) \left(\varepsilon_u - \frac{1}{2}\varepsilon_0\right), \quad U_t \approx \left(\frac{\sigma_0 + \sigma_u}{2}\right)\varepsilon_f$$

True Stress

$$\sigma_t = \sigma_n (1 + \varepsilon_n)$$

True Strain

$$\varepsilon_t = \ln(1 + \varepsilon_n)$$

Percent Elongation

$$\varepsilon_{pf} = \frac{L_f - L_i}{L_i}$$

Area Reduction

$$\%RA = 100 \frac{A_i - A_f}{A_i}$$

Atomic Structure

Atomic Packing Factor

$$APF = \frac{\text{no. atoms/unit cell} \times \text{volume of atom}}{\text{volume of unit cell}}$$

R of SC, BCC, FCC and HCP

$$R = \frac{a}{2}$$
, $R = \frac{\sqrt{3}a}{4}$, $R = \frac{\sqrt{2}a}{4}$, $R = \frac{a}{2}$ and $c = \sqrt{\frac{8}{3}}a$

$$V$$
 for HCP

$$V = 3\sqrt{2}a^3$$

Crystalline Material Density

$$\rho = \frac{\text{Atomic mass of unit cell}}{\text{Volume of unit cell}} = \frac{nA}{V_c N_A}$$

Mole Calculation Formula

$$n = \frac{m}{M}$$

Lattice Vacancies Equilibrium Equation

$$N_v = Ne^{\frac{-Q_v}{kT}}$$

Material Properties

Degree of cold working

$$\%CW = \frac{A_o - A_d}{A_o} \times 100$$

Hall-Petch Equation

$$\sigma_o = \sigma_0 + k_y d^{-\frac{1}{2}}$$

Module Independent

Energy

Power

$$P = \frac{E}{t}$$

Work Done

$$W = F \times d$$

Periodic Functions and Waves

Angular Frequency

$$\omega = 2\pi f$$

RMS of a sinusoidal wave

$$RMS = \frac{\text{Peak Amplitude}}{\sqrt{2}}$$

RMS of any wave

$$RMS = \sqrt{\frac{1}{T} \int_{T_1}^{T_2} [f(t)]^2 dt}$$

General Wave-Function Equation

$$v(t) = A\cos(\omega t + \theta)$$

Average Wave Power

$$P_{average} = \frac{\int_0^T p(t) dx}{T}$$