$F_x = 100\cos(30)$
$F_y = 100\sin(30)$
$\alpha = \tan^{-1}\left(\frac{F_{ytotal}}{F_{xtotal}}\right)$
$|F_{res}| = \sqrt{F_{x\,total}^2 + F_{y\,total}^2}$

Moment = torque a force imposes about a point. Product of force mag. x lever arm (perp. Dist.)

$F_x = 50\cos(20 \times \pi/180)$
$F_y = 50\sin(20 \times \pi/180)$
$M = \underbrace{(47\times 6)}_{F_x} - \underbrace{(17.1\times 3)}_{F_y} = 230.6\,Nm$

Couples: Equal mag., // LoA, Act in opposite direct.
$M_{couple} = F_a x z = F_b x z$
Equivalents, 2 systems:

1. Sum of all forces in x is equal in both
$\Sigma F_{x,A} = \Sigma F_{x,B}$
2. Same for y
$\Sigma F_{y,A} = \Sigma F_{y,B}$
3. Moments in A = Moments in B
$\Sigma M_A = \Sigma M_B$

Build-in/Encastre/Cantilever
Restrained against:
Vert motion
Horiz } Reaction forces can be gen
Rotational0

Pin
Resists:
Vert
Horiz } Reaction can be gen
No resist to rotation, no moment gen

Roller
Resists:
Linear in 1 axis
Allows move in horiz, no vert. no resistance to rotation

Statically indeterminate
$\Sigma F_y = 0$
$V_A + V_B = 75$
$\Sigma M_A = 0$
$\Sigma F_x = 0$
$H_A = 0$
$-M_A + (4.5\times 5) + (30\times 4) - (V_B \times 15) = 0$
$-M_A - 15V_B = -405$

$\Sigma M_A = 0$
$(80\times 5) - (V_B \times 11) + (125 \times 11.5) + 40 = 0$
$V_B = 170.7\,kN$
$\Sigma F_y = 0$
$V_A - 80 - 125 + 170.7 \ldots$

$H_A = 37.5\,kN$
$H_C = 37.5\,kN$
$V_C = 50\,kN$

Assumptions:
Nodes/joints behave as pins
Members subject to axial loads only – no bending

$T_{BC}\sin(53.1)$
$\Sigma F_y = 0$
$T_{BC}\sin(53.1) - 50 = 0$
$T_{BC} = 62.5\,kN$
$T_{BA} = -37.5\,kN$ ∴ compression.
$\Sigma F_x = 0$
$-T_{BA} - T_{BC}\cos(53.1) = 0$

Method of Sections:
Cant cut thru >3 unknowns

$\Sigma F_y = 0$
$40.3 - 40.3 + T_{LE}\sin(57.3) + T_{LD} = 0$
$T_{LD} = -0.84\,T_{LE}$
$\Sigma F_x = 0$
$-77.4 + T_{LK} + T_{LE}\cos(57.3) = 0$
$T_{LK} + 0.537\,T_{LE} = 77.4$
$\Sigma M_A = 0$
$(77.4\times 7) + 40.3 \times (3\times 5.74) - T_{LD} \times (3\times 5.74) - T_{LE}\sin(57.3) \times (3\times 5.74)$
$T_{LD} = -0.84\,T_{LE}$

Bending +
Strain, $\varepsilon = \frac{\Delta L}{L}$
b = thickness (into page)
$\sigma = E\varepsilon$
compression force, $F_c = \underbrace{\sigma_c \times \frac{1}{2}}_{avg\ stress} \times \underbrace{b \times \frac{h}{2}}_{area}$

External Loading on Structure + Deflection = 2 forces:
Equal in mag (equilib)
// (perp to cut face)
Act opposite direct.
Separated by dist/lever arm, Z
$M = F_c \times Z = F_t \times Z$

Shear: $\tau_{avg} = \frac{V}{A}$ for V = shear at this point in structure.
$\tau_{max} = \frac{3}{2}\tau_{avg}$ ... lv thru depth of section

Stat. Indet.
2 x J
2J – 3 = M
If 2J – 3 < M = stat. indet.
M + 3 – 2J = 0 - Det.
M + 3 – 2J = -1

$V_a = V_b = \frac{wL}{2}$
$V_a = \frac{wL}{2}$; M_L, M_R = internal bending moments $= \frac{wL}{2}$; V_L, V_R = internal shear forces

Must remain equilib under influence of:
External distrib. Load
LHS reaction VA (mag. Remains unchanged)
Internal bending mom @ cut, ML. Internal shear force @ cut, VL

$\Sigma M_{cut} = 0$
$\frac{wL}{2}x - wx\frac{x}{2} - M(x) = 0$
$M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$
$M(x) = \frac{wx}{2}(L - x)$

Location + Mag of max. Mom:
$\frac{d}{dx}M(x) = \frac{wL}{2} - wx$
$\frac{wL}{2} - wx = 0$ ∴ $x = \frac{L}{2}$

Sign Convention:

+ve Bending Mom Elongate bottom Compress top	**-ve** Elongate top Compress bottom
+ve Shear CW rotation	**-ve** CCW

UDL:
Infinitesimal inc in moment and shear
Shear: $\Sigma F_y = 0$
$V - (V + dV) - w\,dx = 0$
∴ $\frac{dV}{dx} = -w$
Slope of SFD @ a point = -load intensity @ that point

If load intensity inc linearly from 0 to w, at cut, load intensity is wx/L
$\Sigma F_y = 0$
$V(x) = \frac{wL}{6} - \frac{wx^2}{2L}$
$\frac{dV}{dx} = -\frac{wx}{L}$

Integrating both sides between arbitrary lim, A, B
$\frac{dV}{dx} = -w$
$\int_A^B dV = -\int_A^B w\,dx$
$V_B - V_A = -\int w\,dx$ = area under loading diag. between A + B.

Bending Moment:
$\Sigma M_{LHS} = 0$
$M - (M + dM) + (V + dV)(dx) + w \cdot dx \cdot \frac{dx}{2} = 0$
∴ $\frac{dM}{dx} = V$
Slope of BMD @ point = SF @ that point
When Sf = 0, slope BMD = 0, local max.

Point force loads:
Beam subjected to point load P at dx/2
Internal SF and BM inc. by finite amount V1 and M1, rather than infinitesimal.
$\Sigma F_y = 0$
$-P + V - (V + V_1) = 0$ ∴ $V_1 = -P_1$
$\Sigma M_{LHS} = 0$
$M - (M + M_1) + P\frac{dx}{2} + (V + V_1)dx = 0$
$M_1 = P\frac{dx}{2} + (V + V_1)dx$ ∴ $M_1 \approx 0$
Point load doesn't change value of BM infinitesimally small
V to V-P ∴ $\frac{dM}{dx} = V$

Point Mom Loading:
SF:
$\Sigma F_y = 0$
$V - (V + V_1) = 0$ ∴ $V_1 = 0$
does not change w/ application of moment. SFD
BM:
$\Sigma M = 0$
$M + M_0 - (M + M_1) + (V + V_1)dx = 0$
infinitesimally small ∴ $M_0 = M_1$
BMD
Step change in BMD = to mag. of moment applied

Rotational Hinge/Pins:
Limit of unknowns inc. to 4
indet. to 1°
$\Sigma M_B = 0$
$F_{EL} \times 7.5 - 30 \times 6 = 0$
$F_{EL} = 24\,kN$
Stat. det.
$\Sigma F_{yA} = 0$
$H_A = 0$
$V_A = 69\,kN$
$V_B = 6\,kN$
$F_{EL} = 24\,kN$
$\Sigma M_A = 0$
$M_A = (45\times 3) + (24\times 7.5) = 315\,kNm$ CCW
No BM thru rotational hinge, BMD show 0 moment @ location of hinge

Torsion:
BM about longitudinal axis. RH Screw Rule
Torsional Deformations:
Assumptions:
Bar is circular in CS and prismatic
Angle of twist θ between both ends is small
Applied torque is constant throughout length of bar and ∴ pure

App torque, T induces θ such that q moves to q' at free end. Θ varies linearly between x = 0 and x = L
Subsections, length dx. Right CS rotates relative to left by amount dθ. Abcd becomes ab'c'd. Lengths ab' and dc' remain unchanged. Deformation of abcd defined by dec in angle BAD. Pure shear, only pure strain.

Mag. of shear strain:
$\gamma_{max} = \frac{bb'}{dx}$
For small angles:
$\gamma_{max} = \frac{r\,d\phi}{dx}$
dθ/dx = rate of twist/unit length:
$\theta = \frac{d\phi}{dx}$; $\gamma_{max} = r\theta$
For elements length L in pure torsion θ = Φ/L and for pure only
$\gamma_{max} = \frac{r\phi}{L}$

Linearly Elastic:
Shear stress linearly propor to shear strain
τ = Gγ   τ @ radius r: τ = Grθ
To det shear stress @ some internal radius ρ:
τ = (ρ/r)τ_{max} τ_{max} = outer surface
Shear stress varies linearly w/ radius
Torsion formula related app torque T w/ shear stress produced
$dM = \tau\rho\,dA$
$dM = \frac{\tau_{max}}{r}\rho^2 dA$
$T = \int dM = \frac{\tau_{max}}{r}\int \rho^2 dA$
Polar moment of inertia of circular CS:
$I_P = \int \rho^2 dA$
Circle w/ diameter d:
$I_P = \frac{\pi d^4}{32}$ ∴ $T = \frac{\tau_{max}}{r}I_P$
$\tau_{max} = \frac{Tr}{I_P}$
$\theta = \frac{T}{GI_P}$ torsional rigidity. Analogous to flexural rigidity EI.

Limitations:
Assume mat. remains linearly elastic – at no time is T such that the propor lim for mat is exceeded.
Derivations only valid in absence of stress conc. that result from discontinuities in geometry or @ points of load app.

Fluid: substance that deforms continuously under action of SF. Will take shape of container/surface restraining it. Fluid restrained by surface will exert force that acts perp to surface. Fluid offer no resistance to a SF. No SF acting = stationary
Fluid element ABCD w/ thickness, S (into page) on top side of element, SF acts over area B x C x S
τ = F / (BCS)   τ = GΦ
As long as F is app, Φ will continue to inc as fluid flows
Shear stress propor to rate of shear strain.
Over period, t, particle moves p to p' thru x. small angles corresponds to Φ: Φ = x / y
Rate of shear deformation, Φr = Φ/t = x/(yt)
if travelling at velo, u = x/t: Φr = u/y
τ = μ du/dx   μ = dynamic viscosity
Fluids that obey Newton's law of viscosity =
Shear stress linearly propor to velo grad.
Fluids in contact w/ boundary have same velo as boundary. as dist from boundary, y, inc, velo, u, inc.

Mass Density, ρ: Mass of a substance per unit vol.
ρ = mass/vol [kgm^{-3}] [water = 1000] [air = 1.225]
Specific Weight, w: Weight of a substance per unit vol.
w = (m x g)/vol = ρg   [Nm^{-3}]
Specific Gravity, σ: Ratio of mass density to standard mass density. σ = ρ/$\rho_{standard}$ [water = 1] [oil = 0.9]
Viscosity, μ: shear stress req. to drag 1 layer of fluid another a unit dist away from it within the body.
μ = F/A x Dist/Velo = (F x t)/A
$\tau = \mu\frac{du}{dy}$ $\mu = \frac{\tau}{\frac{du}{dy}}$

Kinematic Viscosity, υ: Ratio of dynamic viscosity to mass density. υ = μ/ρ
Compressibility: Relationship between change in pressure and resulting change in vol. Depends on Bulk modulus
$K = \frac{dp}{\frac{-dV}{V}} = -V\frac{dp}{dV}$
$\rho = \frac{mass}{V}$ → $V_\rho = 1$
$V\,d\rho + \rho\,dV = 0$
$dV = -\frac{V}{\rho}d\rho$ $K = \rho\frac{dp}{d\rho}$
Bulk mod inc, changes in density w/ inc pressure decrease.
assume fluids incompressible as BM is high that observe negligible changes in density for typical pressure changes.

Pressure: Perp force applied to a surface per unit area.
P = F/A[Nm^{-2}, Pa]

Pascal's Law: P @ point is the same in all directions
Characterised by θ. P on each face px, py, ps.
Consider horiz + vert force equilib, can show ps = px = py
force equilib in x direction:
Force due to px = px . δy . δz
Horiz compo of force due to ps = -ps . δs . δz . sinθ
$\sin\theta = \frac{\delta y}{\delta s}$
$p_x \delta y \delta z = p_s \delta s \delta z \frac{\delta y}{\delta s}$
∴px = ps

Variation w/ Depth
Element in a cylinder
Circular ends = A
Forces acting on ends = p1A and p2A
Force due to mass of element: ρg x vol = ρgA.(z2 – z1)
Vert force equilib:
p1A – p2A – ρgA.(z2 – z1) = 0
p1 = p2 + ρg.(z2 – z1)
∴ as depth inc, p inc

Equality of Pressures: 2 pts of same level in a fluid at rest experience same p

Forces p1A @ P, p2A @ Q
Horiz force equilib = p1 = p2   F1/A1 = F2/A2
if A2 = 5A1, then F2 = 5F1 – hydraulic lift
Force amp factor applies to dist thru which F1, F2 move. If F1 moves 100mm, F2 moves

Pressure + Head: If fluid has constant ρ, can det p at any depth z as: p = -ρgz+constant.
Measure depth from surface ∴ let z = -h; p = ρgh + constant
Assume surface of liquid is experiencing Atm p; p = ρgh + p_{atm}

Hydrostatic Paradox: p doesn't depend on weight of fluid present.
4 diff containers, same base A, filled to same h. Each case, pressure @ bottom of container is p = ρgh ∴ force exerted by fluid on base of container is F = pA =

Manometer + P Measurement: Tube into a vessel containing fluid/gas. Fluid rises in tube w/ height used to cal. p of liquid in vessel. if tube open to atm, p measured is gauge.
Pressure @ A: pA = ρgh1   Pressure @ B: pB = ρgh2

U-Tube: Liquids or Gases. manometer liquid that is of greater density + immiscible w/ liquid/gas in
Pressure @ C, pC, B, pB are equal
Pressure @ C: pC = $\rho_{man}gh_2$

pC = pA + ρga
pB = pB + ρg(b-h) + ρ_{man}gh
pA – pB = ρg(b-a) + gh(ρ_{man} – ρ)

Pipes

Submerged Bodies: Submerged plane area A. infinitesimal element of area δA, subject to pressure p1.   p1δA
Force acts perp to element.
Sum all forces on all elements of δA across all A, obtain RHSF. Also acts perp to plan and at CoP.

RF and CoP
Plane PQ, in liquid ρ
element of area δA
Total Area A
δA located, y, below free surface
Element of area δA
$\delta F = \rho g y\,\delta A$
$R = \rho g \boxed{\Sigma y\,\delta A}$ 1st moment of area of PQ
$\Sigma y\,\delta A = A\bar{y}$
A = total area of plane
$\bar{y}$ = dist. from axis to centroid of area.
RHSF = $R = \rho g A\bar{y}$

Centroid: Function of shape of plane. Geometric centre of area
CoP: position thru which RHSF acts, depends on how the HS p varies across area.
To find CoP, take mom about O:
$\delta F = \rho g y\,\delta A$
Lever arm for δF about O is s:
$\delta F = \rho g s \sin\phi\,\delta A$
$M = \rho g \sin\phi \boxed{\Sigma s^2 \delta A}$ 2nd moment of area.   lever arm
Axis runs thru O, perp to page.
$M = R \times \frac{D}{\sin\phi}$
$M = \rho g A\bar{y}\frac{D}{\sin\phi}$
$\rho g \sin\phi\,\Sigma s^2 \delta A = \rho g A\bar{y}\frac{D}{\sin\phi}$ let $\Sigma s^2 \delta A = I_O$
$D = \sin^2\phi\frac{I_O}{A\bar{y}}$
p will always be below centroid of area except when plane is horiz (Φ=0)

// Axis Theorem:
$I_O = I_G + Ad^2$
I_G = 2nd MoA about centroidal axis
d = dist. between 2 axes
$d = \frac{\bar{y}}{\sin\phi}$
$D = \sin^2\phi\frac{I_G}{A\bar{y}} + \bar{y}$
As depth of plane ^, CoP moves closer to centroid. Variation in p across area reduces in comparison to mean p ∴ more uniform

Geometric Properties of Area:
For det 2nd MoA wrt to an axis that passes thru centroid

General triangle $(x_1, y_1), (x_2, y_2), (x_3, y_3)$
Centroid: $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

	Area A	2nd MoA abt axis GG thru centroid I_{GG}
Rectangle	bd	$\frac{bd^3}{12}$
Triangle	$\frac{bh}{2}$	$\frac{bh^3}{36}$
Circle	πR^2	$\frac{\pi R^4}{4}$
Semicircle	$\frac{\pi R^2}{2}$	$0.1102R^4$

Chain Rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Product Rule if $y = uv$ then $\frac{dy}{dx} = \frac{du}{dx}v + \frac{dv}{dx}u$

Quotient Rule if $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{\frac{du}{dx}v - \frac{dv}{dx}u}{v^2}$

Integration by Parts $\int \frac{dv}{dx}u\,dx = uv - \int \frac{du}{dx}v\,dx$

Rational Function Integration $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

Forces, Moments and Systems

Force given Acceleration $F = ma$

Force given Components (2D) $F = \sqrt{F_x^2 + F_y^2}$

Force's Angle from base Axis $\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

Moment $M = |F| \times \lambda$

System Equilibrium Equations $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$

Trusses

Degree of Indeterminacy $M - 2J + 3 = \text{degree of inderterminacy}$

Torsion

Shear Strain at r $\gamma_{\max} = r\theta$

Shear Strain given L $\gamma_{\max} = \frac{r\phi}{L}$

Shear Modulus Torsion Equation $\tau = Gr\theta$

Shear Stress at an internal radius ρ $\tau = \frac{\rho}{r}\tau_{\max}$

Torsion Formula $\tau_{\max} = \frac{T_r}{I_p}$

Torsional Rigidity Equation $\theta = \frac{T}{GI_P}$

Polar Moment of Inertia $I_P = \int_A \rho^2 dA$

Polar Moment of Inertia for a circle $I_P = \frac{\pi d^4}{32}$

Fluids

Shear Stress / Viscosity $\tau = \mu\frac{du}{dy}$

Mass Density $\rho = \frac{m}{V}$

Specific Weight $w = \rho g$

Specific Gravity $\sigma = \frac{\rho}{\rho_{\text{standard}}}$

Compressibility $K = \rho\frac{dp}{d\rho}$

Pressure $p = \frac{F}{A}$

Pressure at Depth $p = \rho gh + p_{\text{atm}} = \text{Gauge pressure} + \text{Atmospheric Pressure}$

Resultant Hydrostatic Force $R = \rho gA\bar{y}$

Depth of Pressure $D = \sin^2(\phi)\frac{I_O}{A\bar{y}}$

Parallel Axis Theorem $I_O = I_G + A\bar{d}^2$

Buoyancy $R_n = \rho_n gV_n$

First Moment of Area $Q_{x/y} = \int_A (y/x)dA$

Position of a Centroid $\bar{x} = \frac{Q_y}{A}, \qquad \bar{y} = \frac{Q_x}{A}$

Mass Continuity Equation $A_1\bar{u}_1 = A_2\bar{u}_2 = Q$

Mass flow of Fluid $\dot{m} = \rho A(v - u)\cos\theta$

Jet Impact Equation $v_{\text{normal}} = v\cos(\theta)$

Position, Motion and Acceleration

Newton's Second Law $F = ma$

The five suvat equations $v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$s = \frac{t}{2}(u + v)$

$v^2 = u^2 + 2as$

Velocity (Circular Motion) $v = \omega r$

Acceleration (Circular Motion) $a = \frac{mv^2}{r} = \omega^2 r$

Force (Circular Motion) $F = \frac{mv^2}{r} = m\omega^2 r$

Polar Velocity $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

Polar Acceleration $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

Polar Velocity to Cartesian Velocity $v_x = \dot{r}\cos\theta - r\dot{\theta}\sin\theta, \quad v_y = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$

Density of water: $1000kg/m^3$

Density of Sea water: $1025kg/m^3$

One Atmosphere: $101.325kPa$

Density of Mercury: 13.5336