Elastic Deformation

Hooke's Law $F = kd$

Tensile Engineering Stress $\sigma = \frac{F}{A_0}$

Normal Tensile Strain $\varepsilon_z = \frac{\Delta l}{l_0}$

Lateral Tensile Strain $\varepsilon_x = \frac{\Delta d}{d_0}$

Rigidity $E = \frac{\sigma}{\varepsilon}$

Poisson's Ratio $\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$

Shear Engineering Stress $\tau = \frac{F}{A_0}$

Shear Strain $\gamma = \frac{\Delta x}{y} = \tan(\theta) \approx \theta \; RAD$

Shear Modulus $G = \frac{\tau}{\gamma}$

Shear Modulus given Poisson's Ratio $G = \frac{E}{2\times(1+\nu)}$

Angle of twist due to two moments $\alpha = \frac{32\times M\times l_0}{\pi\times d_0^4\times G}$

Bulk Modulus $P = -K\frac{\Delta V}{V_0}$

Bulk Modulus given Poisson's Ratio $K = \frac{E}{3\times(1-(2\times\nu))}$

UTS $UTS = \frac{P_{\max}}{A_i}$

Fracture Strength $\sigma_f = \frac{P_f}{A_i}$

Strain Hardening Ratio $r_{\mathrm{SH}} = \frac{\sigma_u}{\sigma_o}$

Resilience Modulus $U_r \approxeq \frac{1}{2}\sigma_y\varepsilon_y$

Toughness $U_t = \frac{\text{Energy}}{\text{Volume}} = \int_0^{\varepsilon_f} \sigma dx\varepsilon$

Toughness Approximations $U_t \approx \left(\frac{\sigma_0+\sigma_u}{2}\right)\left(\varepsilon_u - \frac{1}{2}\varepsilon_0\right), \quad U_t \approx$

True Stress $\sigma_t = \sigma_n(1+\varepsilon_n)$

True Strain $\varepsilon_t = \ln(1+\varepsilon_n)$

Percent Elongation $\varepsilon_{pf} = \frac{L_f - L_i}{L_i}$ $\quad U_t \approx \left(\frac{\sigma_0+\sigma_u}{2}\right)\varepsilon_f$

Area Reduction $\%RA = 100\frac{A_i - A_f}{A_i}$

Buckling formula:
I think for a cylinder? $\sigma_c = \frac{\pi^2 E d^2}{(16L^2)}$

Atomic Structure

Atomic Packing Factor $APF = \frac{\text{no. atoms/unit cell}\times\text{volume of atom}}{\text{volume of unit cell}}$

R **of** SC, BCC, FCC **and** HCP $R = \frac{a}{2}, \quad R = \frac{\sqrt{3}a}{4}, \quad R = \frac{\sqrt{2}a}{4}, \quad R = \frac{a}{2}$ and $c = \sqrt{\frac{8}{3}}a$

V **for** HCP $V = 3\sqrt{2}a^3$

Crystalline Material Density $\rho = \frac{\text{Atomic mass of unit cell}}{\text{Volume of unit cell}} = \frac{nA}{V_c N_A}$

Mole Calculation Formula $n = \frac{m}{M}$

Lattice Vacancies Equilibrium Equation $N_v = Ne^{\frac{-Q_v}{kT}}$

Material Properties

Hall-Petch Equation $\sigma_o = \sigma_0 + k_y d^{-\frac{1}{2}}$

Degree of cold working $\%CW = \frac{A_o - A_d}{A_o}\times 100$

Need to know crystalline lattice structures:
SCS:
- Polonium

FCC:
- Aluminium
- Copper
- Gold
- Lead
- Platinum
- Gamma Iron

BCC:
- Chromium
- Alpha Iron
- Beta Iron
- Delta Iron
- Tantalum
- Tungsten

Iron Transition temperatures:
- Beta: 912*C
- Melting point: 1532*C
- Delta: 1532*C, then cooled

Approx metal shear : elastic modulus G = 0.4E

1. **Engineering Stress:** $\sigma = \frac{F}{A_0}$, σ: stress (Pa), F: force (N), A_0: original cross-sectional area (m²)
2. **Engineering Strain:** $\varepsilon = \frac{\Delta L}{L_0}$, ε: strain (unitless), ΔL: elongation (m), L_0: original length (m)
3. **Hooke's Law:** $\sigma = E\varepsilon$, E: Young's modulus (Pa), ε: strain
4. **Elastic Elongation:** $\Delta L = \frac{FL_0}{EA_0}$, ΔL: elongation (m), F: force (N), L_0: length (m), E: modulus, A_0: area (m²)
5. **Shear Stress:** $\tau = \frac{F}{A}$, τ: shear stress (Pa), F: shear force (N), A: sheared area (m²)
6. **Shear Strain:** $\gamma = \frac{x}{y}$, γ: shear strain, x: displacement (m), y: height (m)
7. **Shear Modulus:** $G = \frac{\tau}{\gamma}$, G: shear modulus (Pa), τ: shear stress, γ: shear strain
8. **Relation Between E and G:** $G = \frac{E}{2(1+\nu)}$, ν: Poisson's ratio (unitless)
9. **Bulk Modulus:** $K = -V\frac{\Delta P}{\Delta V}$, K: bulk modulus (Pa), ΔP: pressure change, ΔV: volume change
10. **Poisson's Ratio:** $\nu = -\frac{\varepsilon_{\text{trans}}}{\varepsilon_{\text{axial}}}$, $\varepsilon_{\text{trans}}$: transverse strain, $\varepsilon_{\text{axial}}$: axial strain
11. **True Strain:** $\varepsilon_t = \ln(1+\varepsilon_n)$, ε_t: true strain, ε_n: engineering strain
12. **True Stress:** $\sigma_t = \sigma_n(1+\varepsilon_n)$, σ_t: true stress, σ_n: engineering stress
13. **Ultimate Tensile Strength:** $\sigma_u = \frac{P_{\max}}{A_0}$, σ_u: ultimate stress, $P_{\max}$: max load (N), A_0: original area (m²)
14. **Fracture Stress:** $\sigma_f = \frac{P_{\text{fracture}}}{A_0}$, σ_f: stress at fracture, P_{fracture}: final load
15. **Strengthening Ratio:** $r_{SH} = \frac{\sigma_u}{\sigma_y}$, σ_y: yield strength (Pa)
16. **Fracture Toughness:** $K = F\sigma\sqrt{\pi a}$, K: stress intensity factor (MPa√m), F: geometry factor, σ: stress, a: crack length
17. **Max Allowable Stress:** $\sigma = \frac{K_{IC}}{F\sqrt{\pi a}}$
18. **Max Crack Size:** $a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{F\sigma}\right)^2$
19. **Theoretical Density:** $\rho = \frac{nA}{V_c N_A}$, ρ: density (kg/m³), n: atoms/unit cell, A: atomic mass (g/mol), V_c: unit cell volume (m³), N_A: Avogadro's number
20. **Unit Cell Volume (Cubic):** $V_c = a^3$, a: lattice parameter (m)
21. **Hall-Petch Equation:** $\sigma_y = \sigma_0 + k_y d^{-1/2}$, σ_0: friction stress, k_y: constant, d: grain diameter (mm)
22. **Percent Reduction in Area:** $\%RA = \frac{A_0 - A_f}{A_0}\times 100$, A_f: final cross-sectional area
23. **Elastic and Plastic Strain:** $\varepsilon_e = \frac{\sigma}{E}$, $\varepsilon_p = \varepsilon - \varepsilon_e$, ε: total strain
24. **Percent Cold Work:** $\%CW = \left(\frac{A_0 - A_f}{A_0}\right)\times 100$
25. **Modulus of Resilience:** $U_r = \frac{1}{2}\frac{\sigma_y^2}{E}$, U_r: elastic energy per unit volume (J/m³)
26. **Toughness (approx.):** Area under stress–strain curve = $\int \sigma d\varepsilon$
27. **Axial Stress (Thin-Walled Pressure Vessel):** $\sigma_{\text{axial}} = \frac{Pr}{2t}$, P: internal pressure, r: radius, t: wall thickness
28. **Vacancy Concentration:** $n_v = N\exp\left(\frac{-Q_v}{kT}\right)$, Q_v: activation energy, k: Boltzmann constant, T: temp (K)
29. **Volume Change in Phase Transition:** $\%\Delta V = \frac{V_{\text{new}} - V_{\text{old}}}{V_{\text{old}}}\times 100$
30. **Dislocation Density:** $\rho_d = \frac{L}{V}$, L: total dislocation length, V: volume
31. **Fatigue Load Ratio:** $R = \frac{\sigma_{\min}}{\sigma_{\max}}$
32. **Fatigue Mean Stress:** $\sigma_{\max} = \sigma_{\text{mean}} + \sigma_a$, σ_a: stress amplitude
33. **Weight Percent:** $\text{wt}\%_1 = \frac{m_1}{m_1+m_2}\times 100$, m: mass
34. **Atomic Percent:** $\text{at}\%_1 = \frac{(m_1/A_1)}{(m_1/A_1+m_2/A_2)}\times 100$, A: atomic mass
35. **Percent Elongation:** $\%\text{Elongation} = \left(\frac{L_f - L_0}{L_0}\right)\times 100$, L_f: length after fracture, L_0: original length
36. **Geometry Factor (center crack):** $F \approx 1$, when $a/b < 0.4$
37. **Geometry Factor (double edge crack):** $F \approx 1.12$, when $a/b < 0.6$
38. **Geometry Factor (single edge crack):** $F \approx 1.12$, when $a/b < 0.13$
39. **Elliptical Crack Tip Stress:** $\sigma_{\max} = S\left(1 + 2\sqrt{\frac{c}{\rho}}\right)$, S: nominal stress, c: crack half-length, ρ: crack tip radius
40. **0.2% Offset Yield (Graphical):** Line with slope E through $\varepsilon = 0.002$, intersects stress–strain curve → σ_y

AM Manufacturing

More commonly referred too as 3D Printing is where you create new objects by adding material.

Process:

1) Design
2) File preparation
3) Machine Setup
4) Fabrication
5) Post-Processing
6) Quality check

Advantages

- Design Freedom
- Material Efficiency (sustainability)
- Customisation
- Rapid prototyping

Seven AM technologies:

- Vat Photopolymerization: Utilises a liquid photopolymer resin that is selectively cured by ultraviolet (UV) light to create solid objects
- Material Extrusion: This method entails the forcing of Thermoplastic or Composite materials through a heated nozzle, allowing for the continuous deposition of material
- Powder Bed Fusion: This category employs Thermal Energy to selectively fuse designated regions of a powder bed, resulting in the creation of solid parts.
- Material Jetting: The deposition of fine droplets of liquid photopolymer or wax materials through a printhead, allowing for the creation of high-resolution parts
- Binder Jetting: deposition of a liquid binding agent onto a powder bed, effectively binding the particles together layer-by-layer to form solid structures
- Directed Energy Deposition: Focused thermal energy is utilised to melt and deposit material onto a Substrate, enabling the creation of complex geometries and repairs of existing components
- Sheet Lamination: Bonding sheets of material together using various techniques, followed by cutting them to shape to create the final product.

Downsides:

- Material Constraints
- Scalability
- Cost

Emerging innovations:

- Bioprinting
- AI Integration
- Sustainable materials

Metal AM key points:

- Metal AM gives benefits over conventional manufacturing processes, including fabrication capabilities of internal features and complex geometries, fabrication of small-volume customised products, and lower material wastage.
- Layer-by-layer manufacturing has limitations, such as Porosity, residual Stresses due to repetitive Heating and Cooling, and surface roughness.
- PBF and DED are the most common melting-based Metal AM processes.
- In BJT, binding agents are used to form the bonds among the powder particles. The Density of the binder jet component is improved using high-Temperature sintering.
- MEX is easier to operate and 60–80 % more economical than PBF.
- SHL combines Additive and Subtractive processes, where sheets are cut and stacked using Ultrasonic consolidation, Diffusion Bonds, or Resistance Welding.
- In the melting-based AM process, input Energy density is one of the crucial parameters for controlling defects.
- The quality of binder jetting components is governed by the powder properties, selection of binders, and process parameters, including print orientation and curing time. The size distribution and surface morphology of powder particles are the primary reasons for defect formation.
- The immense research in Metal AM applications in the biomedical and aerospace industries, etc.

Polymer AM materials used:

- Polylactic Acid
- ABS
- HiPS
- PLA+
- HDPE
- Polyamide 12
- Photopolymer Resins

Creep Failure

Creep Failure is the slow Deformation of a material due to constant Load being applied.

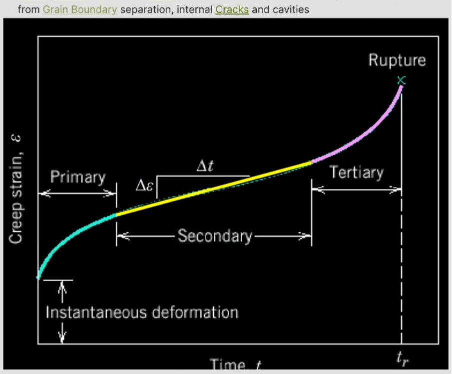
- Stress is constant
- Deformation (or Strain) increases gradually over time

It is often seen in Metals that are exposed to constant Load over a extended period of time.
Temperature can have a significant impact on Creep Failure, often being the cause of such failure.
If the Creep is very small, and the total Deformation is exclusively in the Elastic Region then the material will be able to fully recover from the Deformation.
If driven by a Tensile load, then Necking may occur.

Temperature:
Temperature plays an important factor in Creep Failure, often being the cause of such Failure. For most Metals it only becomes an important consideration at $0.4Tm$. With Tm being the Absolute Melting Point of the Metal.
Amorphous polymers are especially vulnerable to Creep.

The three Phases
Creep Failure is composed of three distinct Phases, listed below:

- Primary Phase: Characterised by a slowly decreasing rate of Creep.
- Secondary Phase: Characterised by a constant level of Creep, the graph becomes Linear . This balance is achieved by the balancing of the competing processes of Strain Hardening and Recovery. This rate is called the Steady State Strain Rate
- Tertiary Phase: The rate of Creep increases until Rupture, driven by internal fissures from Grain Boundary separation, internal Cracks and cavities



Design Considerations:

- The most important factor in Creep is the slope of the Secondary Phase. This is important for long-term applications.
- For relatively short-life creep situations, such as Turbines we may accept a short part lifespan, and look at the time to Rupture quality of a material. (Rupture Lifetime, t_r)
- With increasing Stress or Temperature, we will see the following effects:
  1) The instantaneous Strain at the time of Stress application increases
  2) The steady-state (Secondary Phase) Creep rate increases
  3) Rupture Lifetime decreases

AM techniques

BJT is a powder-bed AM process that uses a binding agent to join powder particles

Key Features:

- Rapid production of complex structures
- Isotropic properties in 3D-printed samples
- Room Temperature printing process
- Suitable for various powder Metals
- Lower distortions due to less residual stress

Process Steps:

1) Printing
2) Curing
3) Sintering (to improve density and mechanical properties)

Critical Parameters:

- Powder properties (size, morphology, distribution)
- Binder characteristics (wettability, penetration, binding strength)
- Print orientation
- Curing time

Advantages of BJT:

- Low capital cost and high scalability
- No support structures required
- Higher production rate
- Better surface Finish compared to PBF
- Low residual stress

Disadvantages of BJT:

- Porosity issues
- Difficult to predict and control Shrinkage
- Complex binder-particle relationship

DED simultaneously supplies Energy input and feedstock material to generate 3D objects

Key Features:

- Feedstock (powder or wire) is injected directly onto the Substrate
- Heat sources: Laser beam, Electron beam, or Electric arc
- Faster production rate and larger build volume compared to PBF
- Suitable for repairing functional parts

Advantages of DED:

- Can manufacture large components
- Ability to add material to existing surfaces (repair)
- High deposition and build rates
- Can produce fully dense parts
- Flexibility to change materials during build

Disadvantages of DED:

- High residual Stress and distortion
- High surface roughness (for blown powder machines)
- Requires post-processing
- Challenging process control

MEX creates 3D parts by extruding material through a nozzle

Key Features:

- No high-Energy Lasers required
- More economical than PBF (60-80% cheaper)
- Originally developed as Fused Deposition Modelling (FDM) for polymers

Process Steps:

1) Printing
2) Debinding (thermal decomposition of binder)
3) Sintering (thermal densification of metal particles)

Polymer MEX:

Polymer FDM Process:

1) Filament (PLA, ABS) is fed into a heated nozzle (180–260°C)
2) Molten Polymer is extruded onto a build platform
3) Layers solidify as the nozzle moves in X-Y axes; platform lowers for subsequent layers.

Key Components:

- Nozzle: Diameter (0.2–1.0 mm) controls extrusion width and resolution.
- Build Platform: Heated (50–110°C) to prevent warping.

Critical Parameters:

- Nozzle Temperature: Affects layer adhesion and flow.
- Layer Thickness: 0.1–0.4 mm; thinner layers improve resolution but increase print time.
- Raster Angle: Orientation of deposited strands (e.g., ±45° for isotropic strength).
- Challenges: o Warping: Caused by uneven cooling; mitigated by bed adhesion (glue, rafts).
- Buckling: Filament buckling due to insufficient Stiffness; Euler's formula:

$$\sigma_c = \frac{\pi^2 E d^2}{(16L^2)}$$

Advantages of MEX:

- Safer feedstock handling
- Economical and easy to use
- High geometrical accuracy

Disadvantages of MEX:

- High Porosity
- Poor surface Finish due to stair-stepping effect
- Requires debinding stage
- Anisotropic properties

PBF is a process where thermal energy selectively melts or sinters regions of a powder bed. It is further classified based on heat sources:

Variants

Laser Powder Bed Fusion

- Uses a Laser beam to melt or sinter Metal powders
- Key parameters: Laser power, spot diameter, scanning speed, powder layer thickness, shielding gas, and gas Flow rate
- Suitable for small components with complex geometries
- Limitations: small build size, longer production time for larger parts

Electron Beam Powder Bed Fusion

- Uses an Electron beam as the Energy source
- Employs electromagnetic coils to focus the beam
- Features semi-sintering of powder particles, unlike L-PBF

Polymer PBF

Selective Laser sintering (SLS), a variant of PBF and widely used AM technique, is a process used to produce objects from powdered materials using one or more lasers to selectively fuse the particles at the surface, layer upon layer, in an enclosed chamber.

Polymer SLS Process:

1) Powder (PA12) is spread in a thin layer (~0.1 mm) by a roller
2) CO_2 Laser (10–100W) selectively sinters powder particles
3) Platform lowers; the process repeats until part completion
4) Un-sintered powder is recycled (50–70% reuse)

Key Parameters:

- Laser Power: Higher Power increases Fusion but risks overheating
- Hatch Spacing: Distance between Laser scans (0.1–0.2 mm); affects Density
- Preheat Temperature: Reduces Thermal gradient (e.g., 170°C for PA12)

Advantages:

- No support structures (powder acts as support)
- High-Density parts with good mechanical properties

Limitations:

- Rough surface Finish (grainy texture)
- Limited material options (primarily Thermoplastics)

Advantages of PBF

- High accuracy and material utilisation
- Near net-shaped production
- Ability to recycle powder
- High geometrical complexity

Disadvantages of PBF

- Poor surface Finish
- High Porosity and residual Stress
- Requires post-processing and support structures
- Health and safety issues with powder handling
- Low productivity and limited build envelope

Sheet Lamination involves stacking and bonding sheets of material (often using adhesive, Ultrasonic Welding, or Laser cutting) to build a 3D part. It combines Additive (layer stacking) and Subtractive (material removal by cutting/Milling) approaches.

Its variants include:

- Laminated Object Manufacturing (LOM): Sheets are cut to shape and then bonded; may require sintering to improve Density.
- Ultrasonic Additive Manufacturing (UAM): Sheets are joined via Ultrasonic Welding; subsequent machining can create complex internal geometries.
- Selective Deposition Lamination (SDL): Uses paper or similar materials as feedstock. Considerations of using this method: Porosity and distortion due to shrinkage during sintering or bonding are key concerns. LOM is less suitable for highly complex shapes because excess material removal can be challenging.

SLA, a vat-based and the early adopted AM technique, works on the process of 3D printing by using Photopolymerization in which the Photocurable Resin is solidified through Photopolymerization initiated by absorbing Light.

Polymer SLA Process:

1. UV Laser (355-405 nm) scans a vat of liquid Resin.
2. Resin solidifies layer by layer on a build platform.
3. Post-curing under UV Light enhances mechanical Properties.

Key Parameters:

- Cure Depth: Determined by Laser Energy ($E = P \times t$).
- Layer Thickness: 25-100 µm; thinner layers improve resolution.

Resin Types:

- Standard Resins: High detail for prototypes.
- Engineering Resins: Heat-resistant, flexible (e.g., ABS-like, Silicone-like).
- Biocompatible Resins: For Dental/Medical applications.

Challenges:

- Support Structures: Required for overhangs; removal causes surface defects.
- Resin Handling: Toxicity requires gloves/ventilation.

Fatigue

Types of Fatigue

Applied Stress can come in three forms:

1) Axial (Tension-Compression)
2) Flexural (Bending)
3) Torsional (Twisting)

And there are three ways to apply this Stress:

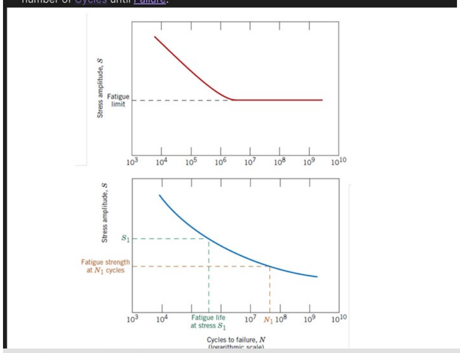
1) Reversed Stress Cycle: Where the Stress follows a regular and symmetrical Sinusoidal curve (a)
2) Repeated Stress Cycle: Same as the Reversed Stress Cycle, except it is not symmetrical (b)
3) The last one is Irregular (c)

Load parameters

- Mean Stress: The Mean Stress experienced by the component. For Reversed Stress Cycle this is 0
- Stress Range: The Range of Stress experienced by the component. Stress Amplitude is half of Stress Range
- Stress Ratio (R) is the Ratio of minimum and maximum Stress

Plotting a graph of S against $\log N$, we see two distinct types of S-N behaviour.

- We have a Fatigue Limit, where the graph levels off, typically seen in some Ferrous and Titanium Alloys. In general, the material will not fail below the Fatigue Limit. Typically, Fatigue Limits range between 35% and 60% of their Tensile Strength.
- For most Alloys, we have a gentle curve, meaning Failure will occur eventually, regardless of Load. Sometimes a quantity called Fatigue Life will be used, which is the number of Cycles until Failure.



Improve Fatigue Behaviours of Materials

- Decrease Load
- Surface treatment
- Shot Peening
- Case Hardening (For Steel and other Alloys)
- Engineering design

Linear Elastic Fracture Mechanics

There is a limit to the overarching assumption for Linear Elastic Fracture Mechanics: "*brittle fractures occur at the stress levels at the yielding stress of materials*"

We can calculate the limit for when this is true:

$$a_t = \frac{1}{\pi}\left(\frac{K_c}{\sigma_o}\right)^2$$

Where K_c is the Fracture Toughness and σ_o is the Yield Stress.
For Cracks shorter than a_t, we cannot use Linear Elastic Fracture Mechanics in order to describe when the Crack will propagate, as the material will Yield under the Stress.