Programming Problem 3: trapezoid.py

Directions: Download the template files I have provided on Blackboard. Then open Spyder, load the template file, and write the following program. Submit your source code via Gradescope, in `.py` format; do NOT send any other files. READ THE INSTRUCTIONS on submitting your work in the Course Documents section of Blackboard.

Be sure to read the SPECIFICATIONS carefully for all problems! And write comments!

We discussed the Trapezoidal rule for approximating definite integrals in class during Lesson 10:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left(1f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + \ldots + 2f(x_{N-2}) + 2f(x_{N-1}) + 1f(x_N)\right)$$

where

$$\Delta x = \frac{b-a}{N}$$

and

$$x_i = a + i\Delta x$$

Here, N is the number of pieces you break the interval $[a, b]$ into; the more pieces, the better. The coefficients are 1 for the first term and the last term; all the rest are 2's: 1,2,2,2,2,. . . , 2,2,1.

So, for example: if I want to approximate $\int_1^2 x^4\,dx$ with $N = 6$ steps, I have:

$$\Delta x = \frac{2-1}{6} = \frac{1}{6}$$

$$x_0 = 1,\ x_1 = 1 + \frac{1}{6} = \frac{7}{6},\ x_2 = 1 + \frac{2}{6} = \frac{8}{6},\ x_3 = 1 + \frac{3}{6} = \frac{9}{6},\ x_4 = 1 + \frac{4}{6} = \frac{10}{6},\ x_5 = 1 + \frac{5}{6} = \frac{11}{6},\ x_6 = 1 + \frac{6}{6} = 2$$

$$\int_1^2 x^4\,dx \approx \frac{1/6}{2}\left(1(1)^4 + 2\left(\frac{7}{6}\right)^4 + 2\left(\frac{8}{6}\right)^4 + 2\left(\frac{9}{6}\right)^4 + 2\left(\frac{10}{6}\right)^4 + 2\left(\frac{11}{6}\right)^4 + 1(2)^4\right) \approx 6.26478909$$

The exact answer is $\frac{31}{5} = 6.2$, so we're pretty close.

Create a program that asks the user to enter in A, B, and an integer N (this is set up for you in the provided template file) and uses the Trapezoidal rule with N intervals to approximate the value of

$$\int_A^B f(x)\,dx$$

rounded to 10 digits where f(x) is defined by

$$f(x) = \frac{1}{\ln(x)}.$$

So, when you run your program, it should look something like this:

```
Enter the lower bound: 2
Enter the upper bound: 3
Enter the number of trapezoids: 200
The definite integral is approximiately: 1.1184248145 #your answer may be slightly different.
```

Please follow the following additional guidelines:

1. Your code should contain no lists (other than a `range`, which technically isn't even a list!), and it should only have one `for` loop.
2. You should write a function called `f`, which receives one `float` named `x` as argument, and returns the value of $f(x) = \frac{1}{\ln(x)}$. Recall that this is a function with no antiderivative and so these numerical techniques (or something similar) is the best way to calculate the antiderivative.

Advice: remember that the Trapezoidal rule can be written using Σ notation (plus a couple of additional terms and times a fraction). So this does not have to be complicated!

And finally, **make sure you understand the Trapezoidal rule well enough to perform some calculations by hand**. It can be tedious, sure, but if you can't perform the Trapezoidal rule by hand, you'll never be able to code it.

Specifications: your program must

- ask the user for values of A, B, and N, where N is an integer (you may assume the user complies), and A and B should be `float`s.
- print an estimate for the value of $\int_A^B f(x)\,dx$ using Trapezoidal rule with N steps, where f(x) is as given above.
- use a Python function to represent the function f(x).
- use one `for` loop only, and **no lists** (using a `range` in your loops is okay and is probably necessary).
- the final approximation should be rounded to 10 decimal places.

This problem has two invisible and two visible test cases on Gradescope. Each test case is worth two points, and two points will be manually assigned for following all specifications. Additional points may be deducted for incorrect code or code not following specifications, even if test cases pass.