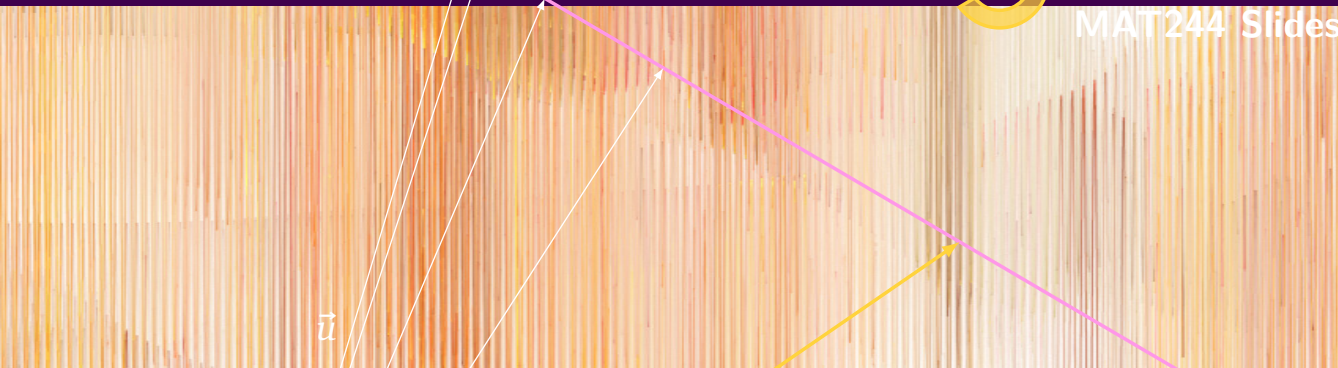
Linear Algebr

MAT244 Slides



Exercise 1

You are observing starfish that made their way to a previously uninhabited tide-pool. You'd like to predict the year-on-year population of these starfish.

You start with a simple assumption

$$\#\text{new children per year} \sim \text{size of current population}$$

1.1 Come up with a mathematical model for the number of star fish in a given year. Your model should

- Define any notation (variables and parameters) you use
- Include at least one formula/equation
- Explain how your formula/equation relates to the starting assumption

Exercise 2

Let

(Birth Rate) $K = 1.1$ children per starfish per year
(Initial Pop.) $P_0 = 10$ star fish

and define the model $\mathbf{M}_1$ to be the model for starfish population with these parameters.

2.1 Simulate the total number of starfish per year using Excel.

Exercise 3

Recall the model $\mathbf{M}_1$ (from the previous question).

Define the model $\mathbf{M}_1^*$ to be

$$P(t) = P_0 e^{0.742t}$$

3.1 Are $\mathbf{M}_1$ and $\mathbf{M}_1^*$ different models or the same?

3.2 Which of $\mathbf{M}_1$ or $\mathbf{M}_1^*$ is better?

3.3 List an advantage and a disadvantage for each of $\mathbf{M}_1$ and $\mathbf{M}_1^*$.

Exercise 4

In the model $\mathbf{M}_1$, we assumed the starfish had K children at one point during the year.

4.1 Create a model $\mathbf{M}_n$ where the starfish are assumed to have K/n children n times per year (at regular intervals).

4.2 Simulate the models $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$ in Excel. Which grows fastest?

4.3 What happens to $\mathbf{M}_n$ as $n \to \infty$?

Exercise 5

Exploring $\mathbf{M}_\infty$

We can rewrite the assumptions of $\mathbf{M}_n$ as follows:

- At time t there are $P_n(t)$ starfish.
- $P_n(0) = 10$
- During the time interval $(t, t + 1/n)$ there will be k/n new children per starfish.

5.1 Write an expression for $P_n(t + 1/n)$ in terms of $P_n(t)$.

Exercise 6

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Recall the *Magic Carpet Ride* task where the hover board could travel in the direction $\vec{h} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and the magic carpet could move in the direction $\vec{m} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

6.1 Rephrase the sentence *"Gauss can be reached using just the magic carpet and the hover board"* using formal mathematical language.

6.2 Rephrase the sentence *"There is nowhere Gauss can hide where he is inaccessible by magic carpet and hover board"* using formal mathematical language.

6.3 Rephrase the sentence "$\mathbb{R}^2$ *is the set of all linear combinations of* $\vec{h}$ *and* $\vec{m}$*"* using formal mathematical language.