



OPERATION & MANAGEMENT OF TRANSPORT SYSTEMS– FALL 2024– 250MUM012

Discussion Session #2 – Queuing processes

Mon. November 12^h, 2024

Problem 1 – Queuing to enter work

Every morning workers arrive at an employment center at a constant rate of 2000 people/hr between 7:00 and 7:30 am. The entrance gate to the employment center is a (FIFO) bottleneck with a fixed capacity of 1000 people/hr.

- a) Sketch the queuing diagram for this bottleneck showing the “V(t)” and “D(t)” curves.
- b) At what time does the queue dissipate?
- c) What is the maximum excess accumulation? At what time does this happen?
- d) What is the maximum delay?
- e) Compute the total delay.
- f) Which of the following three improvements is the best one? Compute the new total delay.
 - I. The service rate is increased by 1000 people/hr to serve only the last 500 workers.
 - II. The service rate is increased by 1000 people/hr to serve only the first 500 workers.
 - III. The service rate is increased by 500 people/hr to serve all the 1000 workers.

Problem 2 – Tandem queues at the airport

At an origin airport, passengers arrive at the ticket counters to check in their luggage and go through the security before they arrive to the departure gate. The time-dependent arrival rates of passengers to the ticket counters are 130 pax/hr for the first hour and 10 pax/hr afterwards. The capacities at the ticket counters and security checkpoints are 70 pax/hr and 40 pax/hr, respectively.

- a. Calculate the total passenger delay at the ticket counters, between the ticket counters and security checkpoints, and for the entire system.
- b. Calculate the change in total delay for the entire system if the capacity at the ticket counters is increased to 130 pax/hr.
- c. Calculate the change in total delay for the entire system if the capacity at the security checkpoints is increased to 70 pax/hr or to 130 pax/hr.



Problem 3 – Waiting in a restaurant

The demand in a restaurant takes place during a duration “T”, 2 times a day: lunch and dinner. The demand rate for lunch is “ 3μ ”, where “ μ ” is the maximum rate at which the restaurant can serve customers. The demand rate for dinner is “ 2μ ”. Determine:

- The average customer delay for lunch
- The average customer delay for dinner
- The average delay per customer to eat in the restaurant

Problem 4 – Freeway ramp metering

A ramp meter is being considered at an entrance to a freeway. Currently, rush hour traffic arrives at the on-ramp at a rate “ q_1 ”, from time “ $t=0$ ” to time “ $t=t^*$ ”. After “ $t=t^*$ ”, vehicles arrive at a (lesser) rate “ q_2 ”.

- Assuming that drivers will not change their trips, draw and label a queuing diagram showing a metering (i.e. departure) rate of “ μ ($q_2 < \mu < q_1$)”. Label the maximum delay “ W_{max} ”.
- If an alternate route is available to drivers, and it is known that they will take this route if their expected delay at the ramp meter is greater than “ $W_{max}/2$ ”, add this new scenario to your diagram. Show graphically the following:
 - The number of vehicles which will divert.
 - How much earlier the queue will dissipate (compared to part (a)).

Problem 5 – The traffic signal

Consider a unidirectional street with a traffic light. The traffic light has two phases: Green for duration “G” and red for the remaining of the cycle, “R”. Vehicles arrive at a rate “ λ ”, and the maximum service rate is “ μ ” during the green phase. The traffic light is under-saturated (i.e. the queue created dissipates every cycle). Considering that the units for “ λ ” and “ μ ” are [veh/min] and neglecting the stochastic fluctuations and lost times, determine:

- Show a queuing diagram (cumulative N versus time) for a complete cycle of the traffic light.
- Determine the total delay per cycle (vehicles·minutes) and the average delay per vehicle (minutes).
- Determine the condition to be fulfilled for the traffic signal to remain under-saturated (in the considered direction and in terms of the variables given in the problem statement).
- What would happen if you add one lane at the intersection?
- What would happen if you extend the green time?

Now, if the traffic light is just the first of a series of N consecutive traffic lights in the same street, prove using the most suitable tools if these assertions are true or false:

- If the N traffic lights are coordinated, the average delay per vehicle in crossing the whole street (the N traffic lights) is the same as in the case of the isolated signal.
- If the N traffic lights are NOT coordinated, the average delay per vehicle in crossing the whole street (the N traffic lights) is N times the delay of crossing an isolated signal.