



OPERATION & MANAGEMENT OF TRANSPORT SYSTEMS– FALL 2024– 250MUM012

Discussion Session #3 – LWR traffic flow theory

Mon. December 9th, 2024

Problem 1 – Diagrams between some of the traffic variables

- a) Sketch the conceptual form of the following traffic diagrams for a single freeway lane: (k-v), (q-v), (k-q fundamental diagram), (q-p), (k-p) and (s-v), where "k" is the density, "v" is the average speed, "q" is the flow, "p" is the pace (i.e. the inverse of the speed) and "s" is the average spacing.
- b) Show on your diagrams the free flow speed " v_f ", the optimal speed " v_0 ", the jam density " k_j ", and the capacity " q_{\max} ".
- c) Indicate the order of magnitude (approximate numerical value) for the parameters in part b)
- d) Show also graphically on the diagram and for any particular traffic state, the value of the third fundamental variable of traffic not directly shown represented in the axis of the diagram.

Hints:

- a) Here it is only needed that you plot a qualitative representation of each of these diagrams in a coordinate axis plane. First variable represents the horizontal axis, and the second variable represents the vertical axis. For instance (k-v) is asking for a diagram where the density (k) is represented in the horizontal axis and the average speed (v) in the vertical. Note that if you know the qualitative form of one diagram, you can derive any other by using the fundamental equation of traffic $q = k \cdot v$.
- b) Here it is asked to show graphically in your plots these parameters of the diagrams.
- c) Just give an order of magnitude of the previous parameters for a one lane of a freeway. State the units used.
- d) In any diagram, you plot 2 variables. There is one third variable not directly shown. E.g. in the (k-v) diagram, you can no directly see the flow (q). Here it is asked to use the fundamental equation of traffic $q = k \cdot v$ to show graphically in your plots how to see this third variable. It is possible that in some plots this has already been shown in part b).



Problem 2 - The Greenshields K-V model

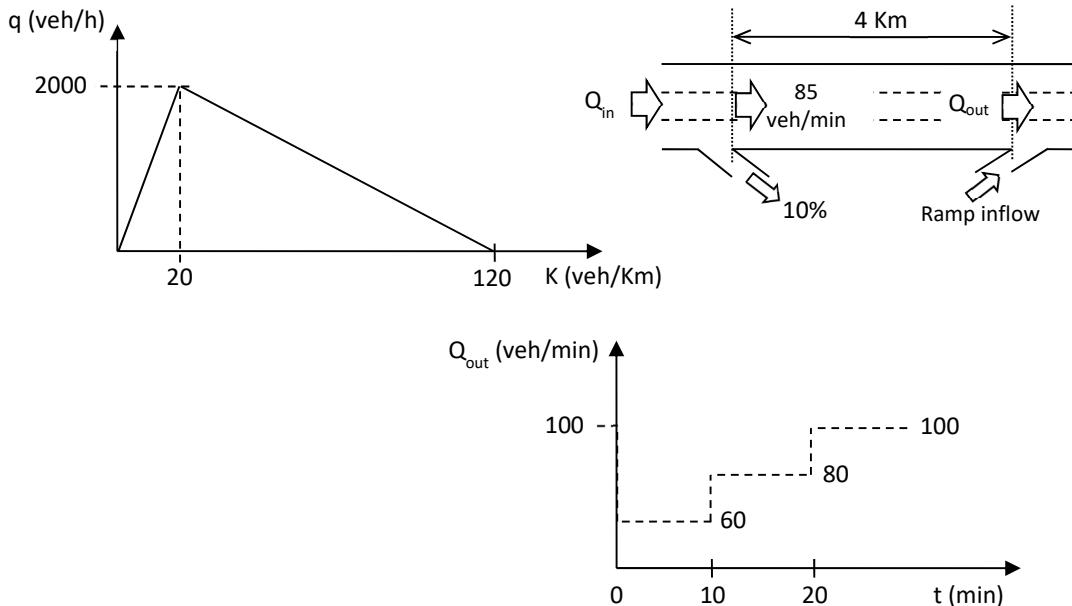
- a) Sketch the conceptual form of the Greenshields (k-v) diagram. Show on it the two basic parameters that define the traffic behavior on the infrastructure.
- b) Show qualitatively on the previous diagram the traffic state at capacity. Show graphically on the diagram the capacity value.
- c) Formulate the analytical equation for the Greenshields (k-v) model, and derive an analytical expression for the capacity " q_{\max} "
- d) Sketch the fundamental diagram (k-q) resulting from the Greenshields k-v model. Show on it the free flow speed " v_f ", the jam density " k_j ", and the capacity " q_{\max} ". Derive its analytical expression " $q(k)$ ".
- e) Is it realistic the previous fundamental diagram for freeways? Justify your answer numerically (i.e. indicating a realistic order of magnitude of the basic parameters involved).
- f) Show on the previous fundamental diagram a free-flowing traffic state and a congested traffic state. Indicate the vehicular speed in these states.
- g) Give an analytical equation for the speed of the shock wave between the previous two traffic states.

Now imagine that the previous diagrams were measured on a 3 lane freeway. At some time during the peak hour, an incident blocks one of the three lanes, so that a queue arises.

- h) Sketch qualitatively how the fundamental diagram would be modified at the incident location.
- i) Show on the 3-lane diagram, the resulting traffic state downstream of the incident location in two time instants i) during the incident blockage ii) just after the clearance of the incident.
- j) Assuming that the freeway suffers recurrent congestion, and that the incident happens during the peak hour, show conceptually on a N-t cumulative diagram the following:
 - i. The total delay
 - ii. The part of the total delay that is due to the incident.

Problem 3 – Freeway traffic management

Consider the 3 lane freeway section sketched below. The upstream traffic demand is “ Q_{in} ”. 10% of this traffic exits at the off-ramp, so that the through traffic is 85 veh/min. An on-ramp is located 4 km downstream of this off-ramp. The input flows at the on-ramp during the peak period (duration of 20 minutes) severely limit the through traffic “ Q_{out} ” at this location (as shown in the figure below). This creates a bottleneck. During off-peak periods, the on-ramp input traffic is negligible. Clarification: You are to analyze a 4 km section (in between ramps) of a three lane freeway where the upstream demand is 85 veh/min and the downstream capacity is 100 veh/min except for a 20 minute period, when it is reduced to 60 veh/min (10 minutes) and 80 veh/min (10 minutes).



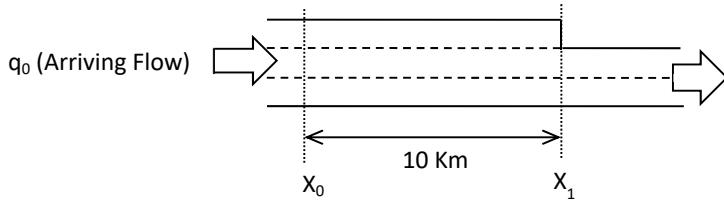
Assuming the above flow (q) – density (k) diagram for one lane and that the diagram for a “ x ”-lane section exhibits the same speed for “ x ” times the density, answer the following questions:

- Will the queue generated at the on-ramp bottleneck block the off-ramp? Justify your answer by using a x - t diagram, showing the back of the queue. It is enough to show traffic states only for the duration that allows you to answer the question.
- Assume that a ramp meter is implemented at the on-ramp, so that the available capacity for through traffic is never below 76 veh/min. Does it solve the problem? Justify your answer by showing on a “ x - t ” diagram the complete traffic state evolution during the rush period.
- Find the minimum through flow that must allow the ramp metering strategy in order not to block the upstream off-ramp.

Problem 4 – Rush hour on the freeway

The figure below shows a 10Km section of freeway of interest. Traffic on the 3 lane section behaves according to a triangular flow(q) – density(k) diagram, defined by a free flow speed “ $V_f=100\text{km/h}$ ”, a capacity “ $q_{\max}=7000\text{veh/h}$ ” and a jam density “ $k_j=420\text{veh/km}$ ”. At the downstream end of the section (location “ x_1 ”) there is a lane drop, causing a bottleneck. The capacity of this bottleneck is “ $q_{bn}=5000\text{veh/h}$ ”. During most of the day the demand “ q_0 ” (measured at location “ x_0 ”) of this freeway section is 3500 veh/h, except during the 30 minute rush period, when the demand doubles. Given this context:

- a) Show on a “x-t” diagram the traffic state evolution during the rush period.
- b) On a “N-t” diagram, draw the cumulative “N(t)” measurements at locations “ x_0 ”, “ x_1 ”. Afterwards, construct a queuing diagram by constructing the virtual arrivals curve “V(t)” (i.e. a translation of the arrivals curve, “N(t)”) at “ x_0 ”.



- c) Use the most suitable of the previous plots to determine:
 - The maximum number of vehicles in the physical queue.
 - The maximum excess accumulation.
 - The maximum time in the physical queue of any vehicle.
 - The maximum delay of any vehicle.
- d) If the jam density was larger (i.e. “ $k_j>420\text{veh/km}$ ”) and all the other inputs to the problem were maintained, the resulting queue would be larger or shorter? Justify your answer.

Extra credit:

- e) Draw on the “N-t” diagram (i.e. Figure 4) the “N(t)” measurement at the point exactly in the middle of locations “ x_0 ” and “ x_1 ”.