$$\int \frac{8+t+6t^2-12t^3}{(3t^2+4)(t^2+7)}dt$$

$$\frac{8+t+6t^2-12t^3}{(3t^2+4)(t^2+7)} = \frac{At+B}{3t^2+4} + \frac{Ct+D}{t^2+7}$$

$$\begin{aligned}\implies 8+t+6t^2-12t^3 &= (At+B)(t^2+7)+(Ct+D)(3t^2+4)\\ &= (A+3C)t^3+(B+3D)t^2+(7A+4C)t+(7B+4D)\end{aligned}$$

$$\begin{aligned}A+3C &= -12\\ B+3D &= 6\\ 7A+4C &= 1\\ 7B+4D &= 8\end{aligned}$$

$$\begin{pmatrix}1 & 3\\ 7 & 4\end{pmatrix}^{-1} = \frac{-1}{17}\begin{pmatrix}4 & -3\\ -7 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 3\\ 7 & 4\end{pmatrix}\begin{pmatrix}A\\ C\end{pmatrix} = \begin{pmatrix}-12\\ 1\end{pmatrix} \implies \begin{pmatrix}A\\ C\end{pmatrix} = \begin{pmatrix}5\\ 5\end{pmatrix}$$

$$\begin{pmatrix}1 & 3\\ 7 & 4\end{pmatrix}\begin{pmatrix}B\\ D\end{pmatrix} = \begin{pmatrix}6\\ 8\end{pmatrix} \implies \begin{pmatrix}B\\ D\end{pmatrix} = \begin{pmatrix}0\\ 2\end{pmatrix}$$

$$= \frac{3t}{3t^2+4}dt + \int \frac{5t}{t^2+7}dt + \int \frac{2}{t^2+7}dt$$

$$\int \frac{8+t+6t^2-12t^3}{(3t^2+4)(t^2+7)}dt$$

$$\frac{8+t+6t^2-12t^3}{(3t^2+4)(t^2+7)} = \frac{At+B}{3t^2+4} + \frac{Ct+D}{t^2+7}$$

$$\implies 8+t+6t^2-12t^3 = (At+B)(t^2+7)+(Ct+D)(3t^2+4)$$

Let $t^2 = -7$

$$8+t+6t^2-12t^3 = (At+B)(t^2+7)+(Ct+D)(3t^2+4)$$
$$\implies 8+t-42+84t = (Ct+D)(-17)$$

$$\implies -17C = 85,\ -17D = -34$$
$$\implies C = -5,\ D = 2$$

Let $t^2 = -\frac{4}{3}$

$$8+t-8+16t = (At+B)(-\frac{4}{3}+7)$$
$$\implies 17t = (At+B)(\frac{17}{3})$$

$$\implies \frac{17}{3}A = 17,\ \frac{17}{3}B = 0$$
$$\implies A = 3,\ B = 0$$

$$= \frac{3t}{3t^2+4}dt + \int \frac{5t}{t^2+7}dt + \int \frac{2}{t^2+7}dt$$