mat159 1-20

Week 3

January 2026

1 Review

Rational Functions can be interpreted in finite terms.
$fintegrated \to R(t)$ by rationalization.
$R(x, y)$ where $P(x, y) = 0$.

$$y = \sqrt{ax+b}\ (y^2 = ax+b,\ \text{ie } P(x,y) = ax + b - y^2 = 0)$$

Denote by $t = \sqrt{ax+b}$, then $t^2 = ax + b$.
From there, $x = \frac{t^2-b}{a}$, and $dx = \frac{2t}{a}dt$

$$\int R(x, \sqrt{ax+b})dx = \int R(\frac{t^2-b}{a}, t)\frac{2t}{a}dt$$

1.0.1 Example

$$\int \frac{\sqrt{x+1}+2}{(x+1)^2 - \sqrt{x+1}}dx$$

Let $t = \sqrt{x+1}$, then $t^2 = x + 1$ and $2t\ dt = dx$

$$\int \frac{t+2}{t^3-1}2; dt$$

$$= 2\int \frac{t+2}{t^3-1}dt$$

$$= 2\int \frac{a}{t-1} +] \frac{bt+c}{t^2+t+1}; dt$$

We need uhhh

$$at^2 + at + a + bt^2 + (c-b)t - c = t + 2$$

$$a + b = 0$$

$$a + c - b = 1$$

$$a - c = 2$$

$$a = 1, b = -1, c = -1$$

Thus,

$$2\int \frac{1}{t-1} - \frac{t+1}{t^2+t+1} dt$$

1.1 Quadratic

$$y = \sqrt{ax^2+bx+c}$$

$$R(x,y) \text{ becomes } R(x, \sqrt{ax^2+bx+c})$$

$$\sqrt{ax^2+bx+c} = \sqrt{a(x+\frac{b}{2a})^2 + \frac{4ac-b^2}{4a}} ???????????????????$$

1. $R(u, \sqrt{u^2+1})$
   $u = \tan t,\ t \in (-\frac{\pi}{2}, \frac{\pi}{2})$
   $R(\tan t, \sec t) = \tilde{R}(\cot t, \sin t)$
   $\int R(u, \sqrt{u^2+1}du) = \int \tilde{R}(\cos tt, \sin t)\sec^2 t\ dt = \int \overline{R}\cos t, \sin t)dt$

2. $R(u, \sqrt{u^2-1})$

$$u = \sec t,\ t \in (0, \pi)$$

$$\int R(\sec t, \tan t)\sec t \tan t\ dt$$

3. $R(u, \sqrt{1-u^2})$

$$u = \sin t,\ t \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$\int R(\sin t, \cos t)\cos t\ dt$$

Something eular knew that if you could get
Function of x and y: R(x,y)
Function of x as t: x=f(t)
Function of y as t: y=g(t)
where $f, g \in R(t)$