Econ 373: Labor Economics
Prof. Sullivan
Assignment 4: Labor Demand

Goals for the assignment:

- Learn how to compute labor demand under the three types of production function that we have studied: perfect substitutes, perfect complements, and Cobb-Douglas.
- Start thinking about how the assumption of price taking (perfectly competitive markets) affects labor demand.
- Gain more experience with key concepts such as 1) profit maximization, 2) finding demand functions, 3) computing elasticities, 4) learning how the labor demand elasticity relates to an important policy question: the minimum wage.

1. In the near future, Uber will produce car trips using the following technology, $f(H, R) = H + 2R$, where H is hours of human driven cars, and R is hours of robot-driven cars. Note that robot-driven cars are superior to human driven cars (they are more productive). The cost of human driven hours is the wage, w_H. Self-driven cars are rented at rate w_R from Google. Suppose Uber is a price taker in both input markets. Furthermore, in the future antitrust laws are no-longer enforced, so Uber and Lyft have agreed to split the market for car trips, so each will sell $N = 1,000$ trips. They have also agreed on a flat per-trip fee of \$20.
   (a) Suppose that initially, $w_H = \$10$ and $w_R = \$25$. Find Uber's optimal choice of inputs (H^*, R^*).
   (b) The wage of humans falls to \$8 because of an increase in the supply of labor, and the rental rate of robot cars falls to \$15 because of technological breakthroughs. Find Uber's new optimal choice of inputs (H^*, R^*).
   (c) What do your answers to parts (a) and (b) suggest about employment in jobs where technology may in the future provide inexpensive, and close, substitutes to human labor? It might be informative to look up online what happened to travel agents after internet shopping became widespread.
   (d) Is the assumption that Uber is a price taker in the labor market reasonable? Answer this carefully, using specific economic concepts. You will need to think about the labor market that Uber is operating in.
   (e) If Uber is not a price taker in the labor market, how would the firm's problem change? Specifically, what should Uber take into account now that they did not have to consider when a price taker in the labor market?

2. Suppose that a firm produces output using a perfect complements technology, $f(L, K) = \min\{L, 4K\}$. The wage rate is w and the rental rate of capital is r.

   (a) Find the firms optimal choice of L and K to produce q units of output. Draw a diagram that illustrates this.

   (b) What is the cost of producing q units of output, in terms of w and r? Assume that the firm is producing output efficiently. This is the cost function.

   (c) A production function exhibits constant returns to scale (CRS) if doubling both inputs leads to exactly twice as much output. Show that this is this a constant returns to scale production function. (more precisely, a production function is CRS if scaling up each input by some amount $c > 0$ leads to exactly c times as much output).

   (d) Find the firm's profit at two different levels of output $q^* = 4$ and $q^* = 8$. Assume that the price of output is $p = \$100$, the wage is $w = \$10$, and the rental rate of capital is $r = \$2$. What do you notice about what happens to profit as output increases? This is a general property of any constant returns to scale production function, it is not just a specific property of perfect complements.

3. A firm produces output using only labor according to the following production function, $F(L) = 2L^{1/2}$. The firm operates in competitive output and input markets, receiving price p per unit of output and paying w per hour of labor.

   (a) What is the marginal product of labor?

   (b) Are there diminishing returns to labor? You could show this several ways, but an easy way is to just plug in different values for L. A figure may help also.

   (c) Find the firms labor demand curve, $L^*(w, p)$.

   (d) Find the elasticity of labor demand with respect to the wage. Explain what the elasticity means.

   (e) Using your answer from (d), what happens to labor demand at this firm if the government imposes a minimum wage that is 1 percent above the current market wage? Note: this should illustrate why the elasticity of labor demand is such an important number from the perspective of policies such as the minimum wage.

   (f) Find the elasticity of labor demand with respect to the price of output, p. Explain what the elasticity means.

   (g) What are the firms profits at $L^*(w, p)$? Hint: you need to take your answer from (c), use it to figure out how much output the firm will produce, and then figure out what profits are. The answer is called the profit function $\pi^*(w, p)$, and it will depend on w and p. Are profits increasing, decreasing, or constant in w and p? In other words, what direction do profits move in when w increases (p increases)? Does this make sense intuitively?