Computtional Methods and C++ Assigment

Vincent Richard

02/11/17

Abstract

Sample text

Contents

List of Figures

List of Tables

1 Introduction

In this assignment we are asked to examine the application of numerical schemes for the solution of partial diferential equations. In order to do so we will consider the following probem.

A wall 1 ft. thick and infinite in other directions has an initial uniform temperature Tin of 100°F. The surface temperatures Tsur at the two sides are suddenly increased and maintained at 300°F. The wall is composed of nickel steel (40% Ni) with a diffusivity of D = 0:1 ft2=hr. Please compute the temperature distribution within the wall as a function of time. The governing equation to be solved is the unsteady one-space dimensional heat conduction equation, which in Cartesian coordinates is:

$$\frac{\partial T}{\partial t} = D\frac{\partial T}{\partial x^2} \tag{1}$$

1.1 Presentation of the different methods used

1.1.1 DuFort-Frankel

The DuFort-Frankel scheme is an explicit scheme unconditionnaly stable the parabolic PDE is:

$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = D\frac{T_{i+1}^n - (T_i^{n+1} + T_i^{n-1}) + T_{i-1}^n)}{\Delta x^2} \tag{2}$$

This equation leads to an explicit form which is:

$$T_i^{n+1}(1+2r) = T_i^{n-1} + 2r(T_{i+1}^n - T_i^{n-1} + T_{i-1}^n), r = \frac{D\Delta t}{\Delta x^2} \tag{3}$$

1.1.2 Richardson

The Richardson scheme is an explixit scheme, unconditionnaly unstable:

$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = D\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} \tag{4}$$

This equation leads to an explicit form which is:

$$T_i^{n+1} = 2r(T_{i+1}^n - 2T_i^n + T_{i-1}^n) + T_i^{n-1}, r = \frac{D\Delta t}{\Delta x^2} \tag{5}$$

1.1.3 Laasonen

The Laasonen scheme is an implicit scheme, that as for equation:

$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = D\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} \tag{6}$$

This equation leads to a form that result in asystem of linear equation:

$$-rT_{i+1}^{n+1} + (1+2r)T_i^{n+1} - rT_{i-1}^{n+1} = T_i^n, r = \frac{D\Delta t}{\Delta x^2} \tag{7}$$

1.1.4 Cranck-Nicholson

The Crank-Nicholson scheme is an implicit scheme, that as for equation:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{D}{2}(\frac{T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}}{\Delta x^2} + \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}) \tag{8}$$

This equation leads to a form that result in asystem of linear equation:

$$-\frac{r}{2}T_{i+1}^{n+1} + (1+r)T_i^{n+1} - \frac{r}{2}T_{i-1}^{n+1} = \frac{r}{2}T_{i+1}^n + (1-r)T_i^n + \frac{r}{2}T_{i-1}^n \tag{9}$$

2 Methods and Procedures